Quasi Resonance State in 3N System

-- He calls me Mr. Singularity --

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LENPIC 2022
Conference in Bochum
24-26 Aug. 2022


He is
Walter Glöckle.

Outline

- Our Bible
- Complex Energy Method
- Coulmb transformation
- Faddeev Equation for Excited States
- Low-momentum NN interaction
- Ay puzzle

Our Bible

ELSEVIER

Physics Reports 274 (1996) 107–285

PHYSICS REPORTS

The three-nucleon continuum: achievements, challenges and applications

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Received November 1995; editor: G.E. Brown

we end up with

$$T\phi = tP\phi + tP\ G_0 T\phi \tag{21}$$

$$\tilde{t}_{\bar{\alpha}\bar{\alpha}'}\left(p, p', E - \frac{3}{4m}q^2\right) \equiv \begin{cases} \dfrac{\hat{t}_{\bar{\alpha}\bar{\alpha}'}(p, p', E - (3/4m)q^2)}{E + i\epsilon - (3/4m)q^2 - \epsilon_d} & \text{for } \alpha = \alpha_d \\ \tilde{t} = \hat{t} & \text{for } \alpha \neq \alpha_d \end{cases} \tag{188}$$

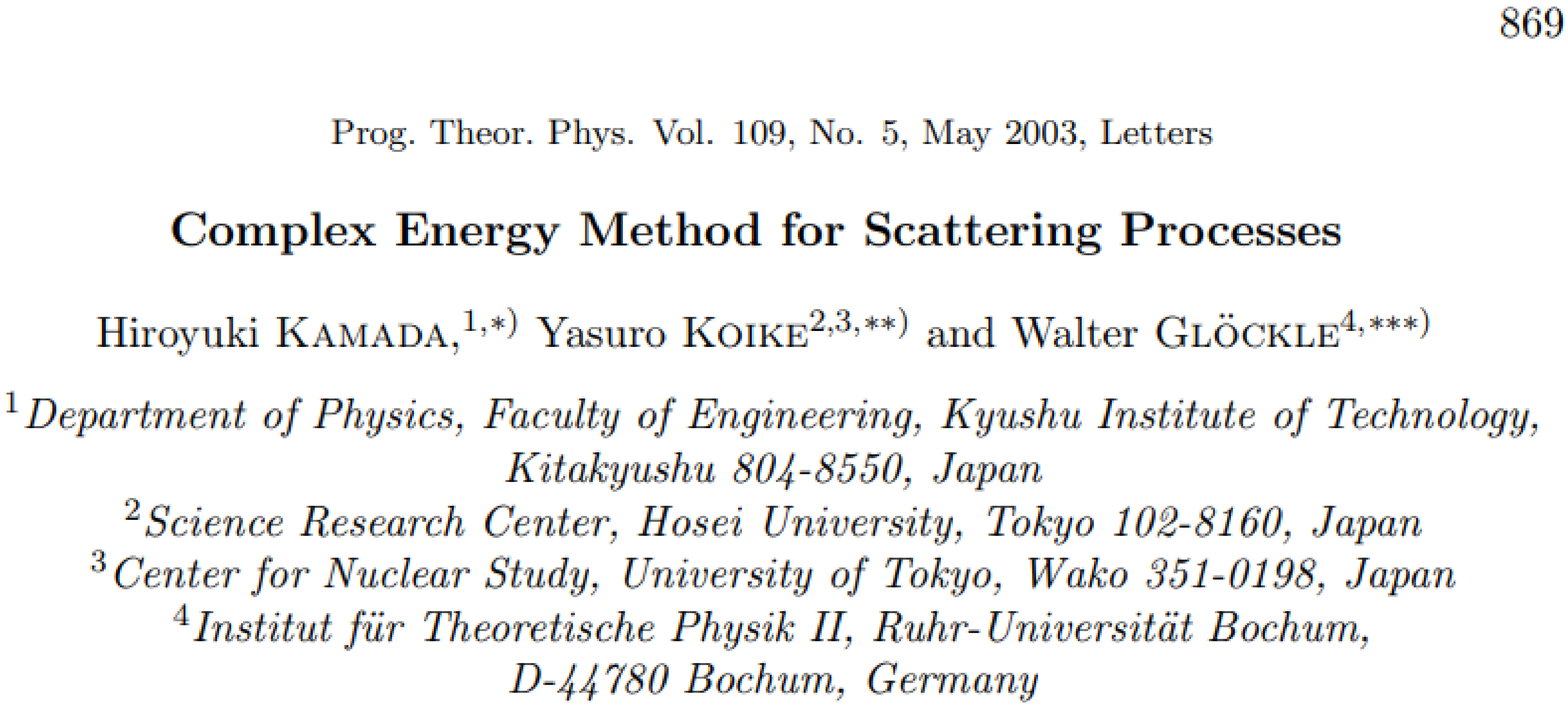
It results

$$\begin{aligned} \langle pq\alpha|\hat{T}|\phi\rangle &= \langle pq\alpha|\hat{t}P|\phi\rangle + \sum_{\alpha'}\sum_{\alpha''}\int_0^\infty dq'\ q'^2 \int_{-1}^{+1} dx\ \frac{\hat{t}_{\bar{\alpha}\bar{\alpha}'}(p, \pi_1, E - (3/4m)q^2)}{\pi_1^{l'}} \\ &\times G_{\alpha'\alpha''}(qq'x)\frac{1}{E + i\epsilon - q^2/m - q'^2/m - qq'x/m} \\ &\times\left(\delta_{\alpha''\alpha_d}\frac{\langle\pi_2 q'\alpha''|\hat{T}|\phi\rangle}{\pi_2^{l''}}\frac{1}{E + i\epsilon - (3/4m)q^2 - \epsilon_d} + \bar{\delta}_{\alpha''\alpha_d}\frac{\langle\pi_2 q'\alpha''|\hat{T}|\phi\rangle}{\pi_2^{l''}}\right) \end{aligned} \tag{189}$$

$$\frac{1}{E+i\epsilon-p''^{2}/m-(3/4m)q''^{2}}\,\frac{1}{E+i\epsilon-(3/4m)q''^{2}-\epsilon_d}$$
$$=\left\{\frac{1}{E+i\epsilon-p''^{2}/m-(3/4m)q''^{2}}-\frac{1}{E+i\epsilon-(3/4m)q''^{2}-\epsilon_d}\right\}\frac{1}{p''^{2}/m-\epsilon_d} \tag{200}$$

Since $\epsilon_d < 0$ the last factor is nonsingular and well behaved. The first part on the right hand side can be treated as above using Eq. (197), which just leads to a simple pole. In the second part one can use the first type of permutation operator given in Eq. (162), which does not affect q'' and again one just encounters a simple pole. This form has been applied in [237] to realistic calculations. Though the

Complex Energy Method



Prog. Theor. Phys. Vol. 109, No. 5, May 2003, Letters

Complex Energy Method for Scattering Processes

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[redacted]

PHYSICAL REVIEW C **68**, 061001(R) (2003)

Complex energy method in four-body Faddeev-Yakubovsky equations

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(Received 1 October 2003; published 4 December 2003)

The Complex Energy Method [Prog. Theor. Phys. **109**, 869L (2003)] is applied to the four-body Faddeev-Yakubovsky equations in the four-nucleon system. We obtain a well converged solution in all energy regions below and above the four-nucleon breakup threshold.

$$E^c = E + i\epsilon, \qquad \epsilon \text{ is a positive finite number.}$$

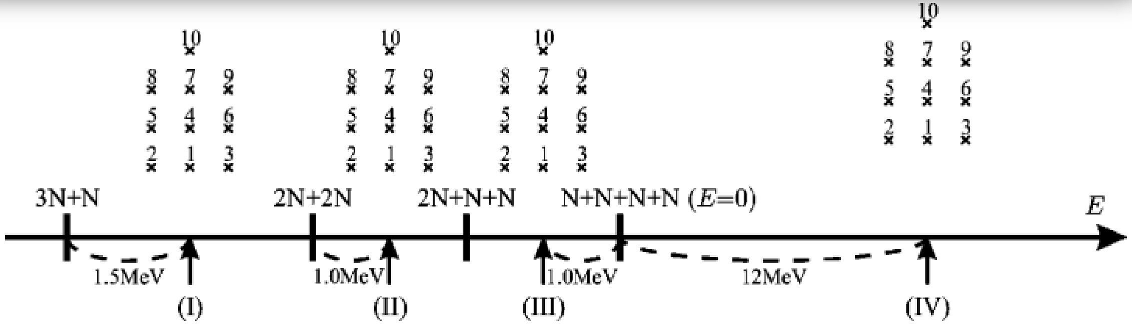


FIG. 4. Illustration of threshold energies for the 4N system. We choose E=0 at the four-body threshold. The various energies for the calculations are measured relative to the thresholds. The crosses indicate the complex energies where we solve the FY equations in the CEM. They are numbered by n for each choice of the energy region (i–iv).

Coulomb Transformation

Few-Body Syst (2013) 54:1629–1632
DOI 10.1007/s00601-012-0554-4

M. Yamaguchi · H. Kamada · W. Glöckle

Coulombic Transformation in Momentum Space

$$H = H_0 + V^C + V^S \xrightarrow[CFtrans.]{} \mathcal{H} = \langle \psi^C_{\mathbf{k}'} | H | \psi^C_{\mathbf{k}} \rangle = \mathcal{T} + \mathcal{V} \tag{1}$$

$$\text{with } \mathcal{T} = \langle \psi^C_{\mathbf{k}'} | (H_0 + V^C) | \psi^C_{\mathbf{k}} \rangle = \frac{\mathbf{k}^2}{2\mu} \delta^3(\mathbf{k} - \mathbf{k}') \quad \text{and} \quad \mathcal{V}(\mathbf{k}', \mathbf{k}) = \langle \psi^C_{\mathbf{k}'} | V^S | \psi^C_{\mathbf{k}} \rangle, \tag{2}$$

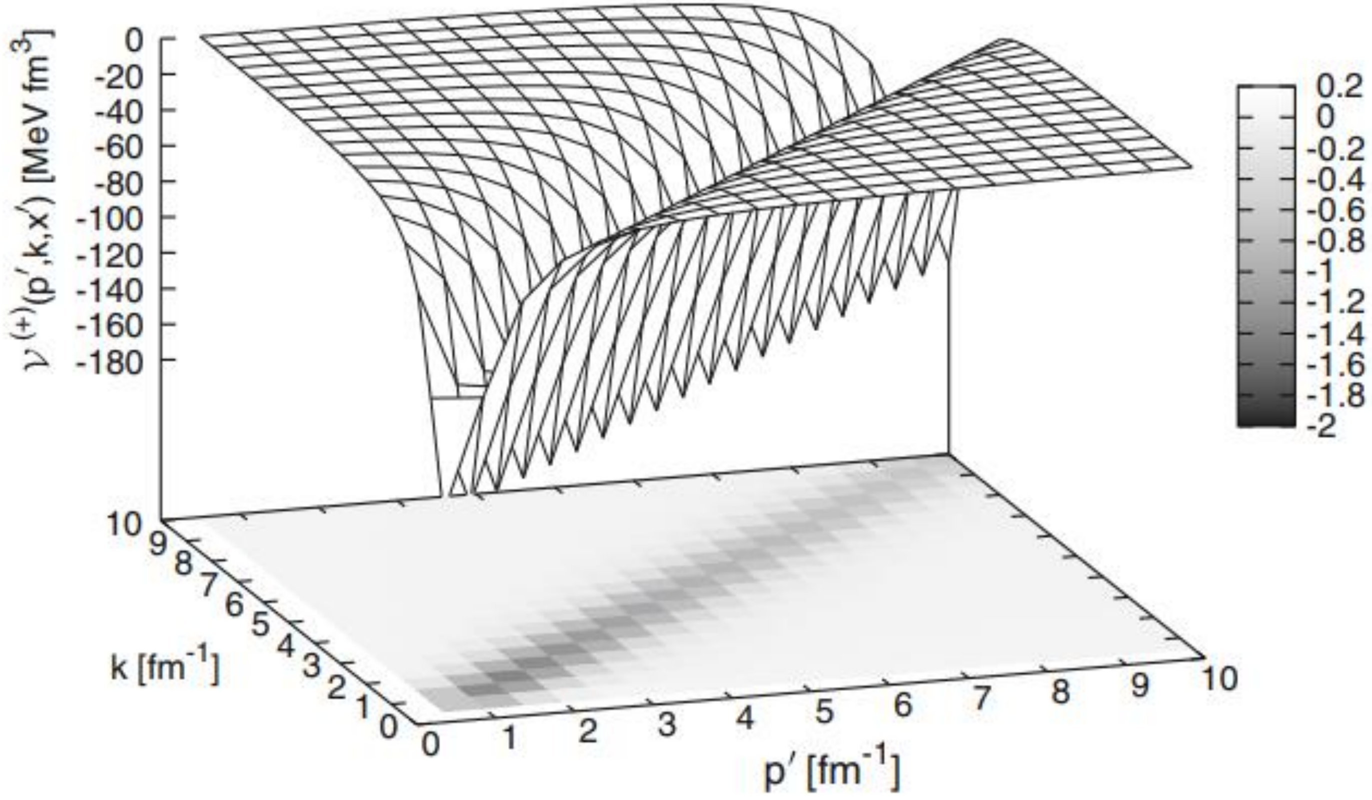


Fig. 1 The transformed Malfliet-Tjon potential (real part) at $x' = 1$

Faddeev Equations for Excited States

Faddeev Eq.

Faddeev Equation

$\mathcal{H} = H_0 + V_1 + V_2 + V_3$

$\mathcal{H}\Psi = E\Psi$

$(E - H_0)\Psi = (V_1 + V_2 + V_3)\Psi$

Faddeev Component: ψ_i

$$\psi_i \equiv \frac{1}{E - H_0} V_i \Psi = G_0 V_i \Psi,$$

$$\psi_1 + \psi_2 + \psi_3 = \frac{1}{E - H_0}(V_1 + V_2 + V_3)\Psi = \Psi$$

$\psi_i = G_0 V_i \Psi = G_0 V_i(\psi_1 + \psi_2 + \psi_3) = G_0 V_i (1 + P)\psi_i$

$\psi = G_0 V(1 + P)\psi,$

$(1 - G_0 V)\psi = G_0 V \psi$

$\psi = G_0 t P \psi$

$(1 - G_0 V)^{-1} G_0 V \equiv G_0 t,$

$(1 - G_0 V) G_0 t = G_0 V$

$t = V + V G_0 t$

t :t-matrix

$$\boxed{\psi = G_0 t P \psi}$$

Unphysical pole

$$\eta\psi = G_0 t P \psi$$

$$\eta(E_b) = 1$$

$$\text{Iteration} \; ; \; E = E_b + \delta E$$

$$\psi^{(i+1)} = G_0 t P \psi^{(i)}$$

$$\eta \equiv \frac{\psi^{(i+1)}(p_0, q_0; \alpha_0)}{\psi^{(i)}(p_0, q_0; \alpha_0)},$$

We often get a negative eigen value.

$$\eta_{neg} < 0$$

$$\eta \to -1.7, \quad i \to \infty$$

$$\eta \equiv \frac{\psi^{(i+1)} - \eta_{neg}\psi^{(i)}}{(1 - \eta_{neg})\psi^{(i)}},$$

$$\psi^{(i+1)} \leftarrow G_0 t P \frac{\psi^{(i+1)} - \eta_{neg}\psi^{(i)}}{(1 - \eta_{neg})}$$

$$\psi = G_0 t P \psi$$
$$(1 - G_0 t P)\psi = 0, D(E) = det[1 - G_0 t P] = 0.$$
$$D(E_g) = 0, \quad E_g = E_b \quad \text{Ground bound state}$$
$$D(E_x) = 0. \quad E_x \quad \text{Excited state}$$

Faddeev Equations for Excited States

There is a question:
$\langle \psi_g | \psi_x \rangle = 0?$
$\mathcal{H}\Psi_g = E_g\Psi_g, \mathcal{H}\Psi_x = E_x\Psi_x$
$0 = \langle\Psi_g|\mathcal{H}|\Psi_x\rangle - \langle\Psi_g|\mathcal{H}|\Psi_x\rangle = (E_g - E_x)\langle\Psi_g|\Psi_x\rangle \; E_g \neq E_x \rightarrow \langle\Psi_g|\Psi_x\rangle = 0.$
$0 = \langle\psi_x|G_0tP|\psi_g\rangle - \langle\psi_x|G_0tP|\psi_g\rangle = \langle\psi_x|\psi_g\rangle - \langle\psi_x|\psi_g\rangle = 0???$
Even G_0tP is not Hermite.

Orthogonality

A Solution;

$$\psi_1 + \psi_2 + \psi_3 = \Psi = (1+P)\psi$$

$$\langle \Psi_g | \Psi_x \rangle = \langle \Psi_g | (1+P) | \psi_x \rangle = 3 \langle \Psi_g | \psi_x \rangle = 0$$

In general, $\langle \psi_g | \psi_{ex} \rangle \neq 0$.

But $\langle \Psi_g | \psi_x \rangle = 0$

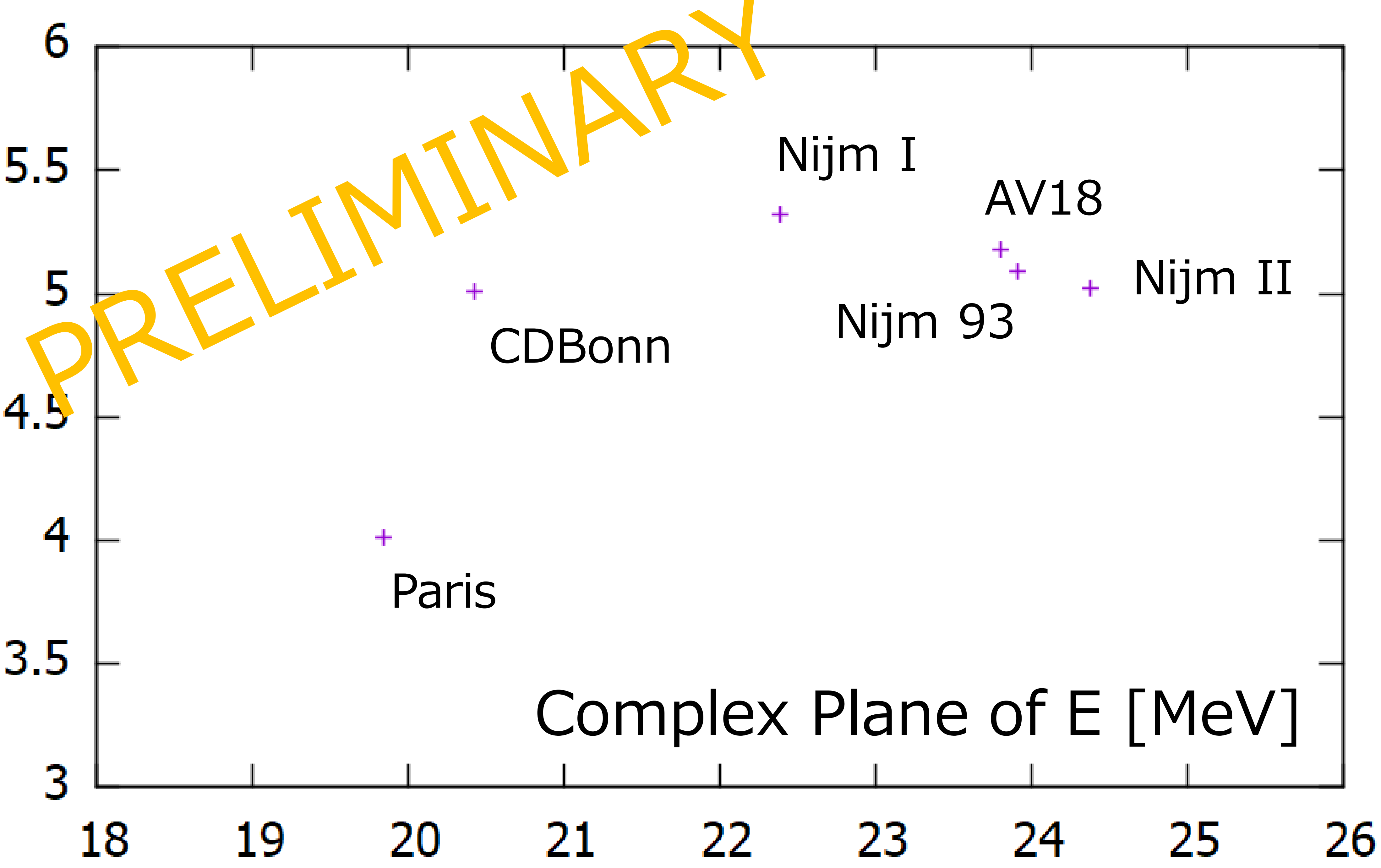
$\psi' = \psi - \langle \Psi_g | \psi \rangle \Psi_g$,(Projection)

$$\langle \Psi_g | \psi' \rangle = \langle \Psi_g | \{ \psi - \langle \Psi_g | \psi \rangle \Psi_g \} \rangle = \langle \Psi_g | \psi \rangle - \langle \Psi_g | \psi \rangle \langle \Psi_g | \Psi_g \rangle = 0$$

$$\psi^{(i+1)} \leftarrow \psi^{(i+1)} - \langle \Psi_g | \psi^{(i+1)} \rangle \Psi_g$$

Potential	Binding Energy [MeV]	Quasi Energy [MeV]
Paris	-7.46	19.84 + i 4.01
AV18	-7.83	23.91 + i 5.09
Nijmegen 93	-7.86	23.80 + i 5.18
Nijmegen I	-8.01	22.39 + i 5.32
Nijmegen II	-7.90	24.40 + i 5.04
CDBonn	-8.25	20.43 + i 5.01

PRELIMINARY


PRELIMINARY
6
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3.5
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19
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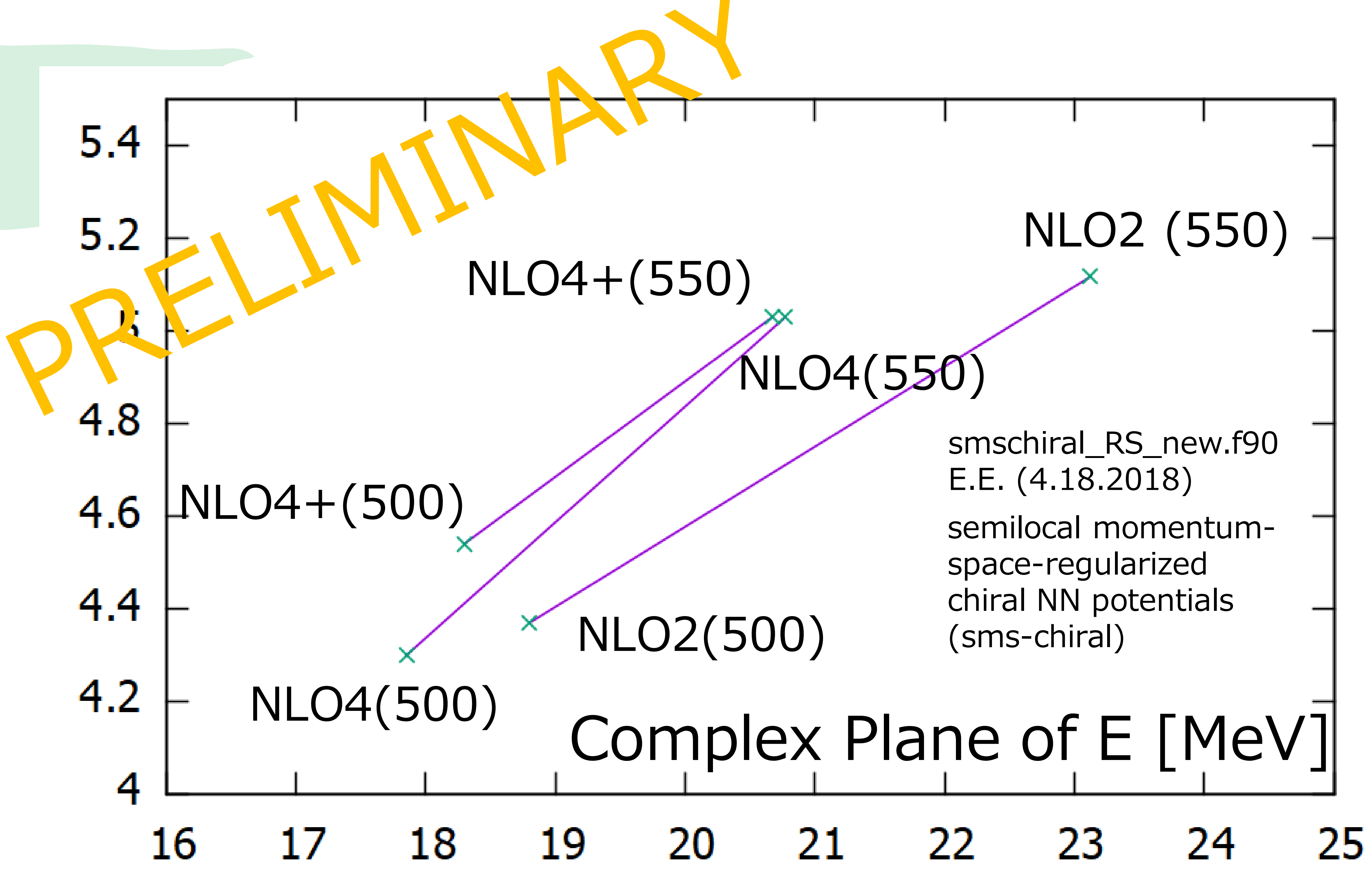
26
Nijm I
AV18
Nijm II
Nijm 93
CDBonn
Paris
Complex Plane of E [MeV]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4.6
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17
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22
23
24
25
NLO2 (550)
NLO4+(550)
NLO4(550)
NLO4+(500)
NLO2(500)
NLO4(500)
smschiral_RS_new.f90
E.E. (4.18.2018)
semilocal momentum-
space-regularized
chiral NN potentials
(sms-chiral)
Complex Plane of E [MeV]

Low-momentum NN interaction

PHYSICAL REVIEW C **70**, 024003 (2004)

Low-momentum nucleon-nucleon interaction and its application to few-nucleon systems

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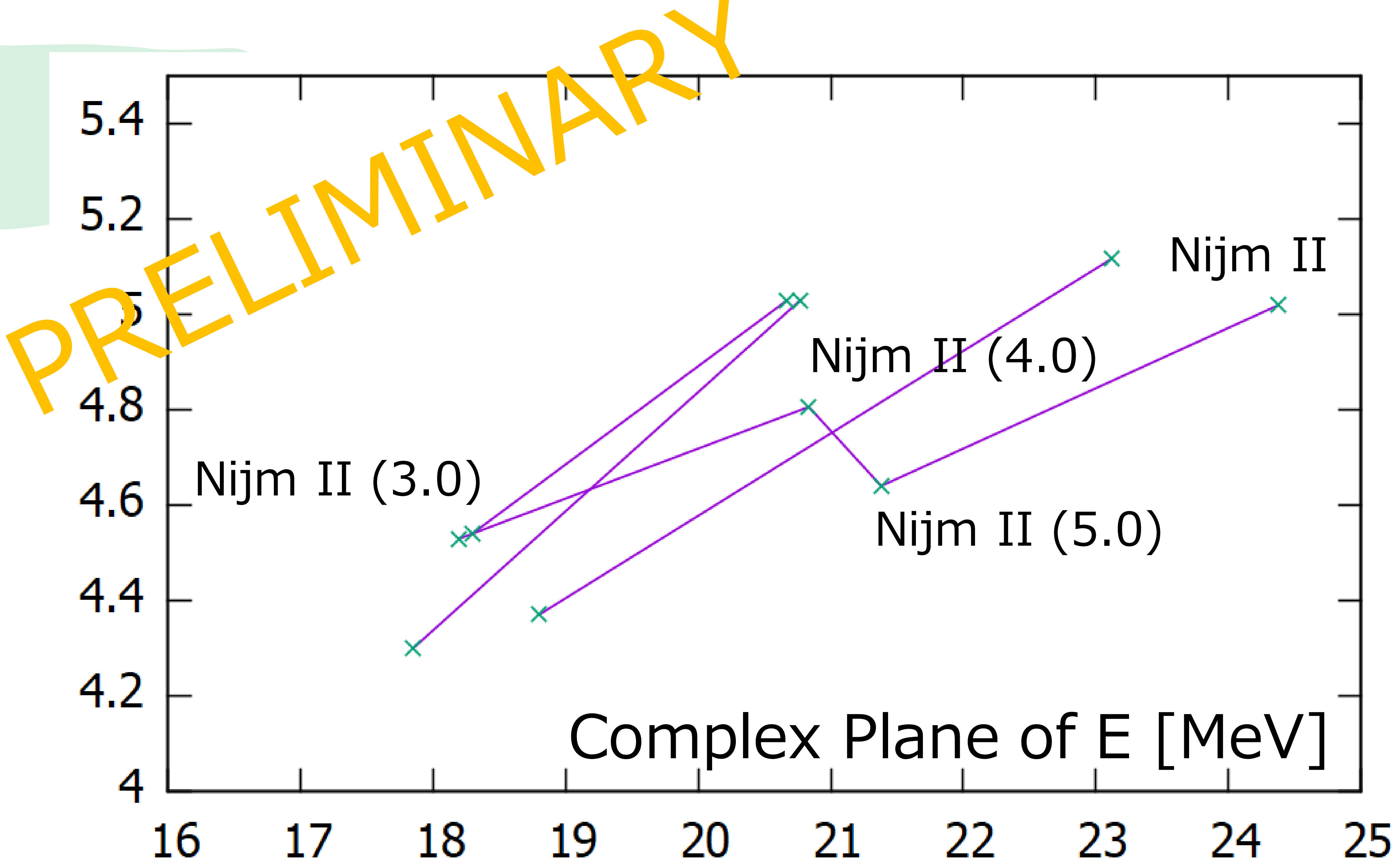
(Received 28 April 2004; published 26 August 2004)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17
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21
22
23
24
25
Nijm II
Nijm II (4.0)
Nijm II (3.0)
Nijm II (5.0)
Complex Plane of E [MeV]

Three- and Four-Nucleon Systems from Chiral Effective Field Theory

E. Epelbaum,[1] H. Kamada,[1,2] A. Nogga,[2] H. Witała,[3] W. Glöckle,[2] and Ulf-G. Meißner[1]

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(Received 26 July 2000)

Recently developed chiral nucleon-nucleon (*NN*) forces at next-to-leading order (NLO), that descr
NN phase shifts up to about 100 MeV fairly well, have been applied to 3*N* and 4*N* systems. Fadde
Yakubovsky equations have been solved rigorously. The resulting 3*N* and 4*N* binding energies are in
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are very well reproduced as for standard *NN* forces. The long-standing A_y puzzle is absent at NLO. T
cutoff dependence of the scattering observables is rather weak.

I went back to Japan in the year.

DOI: 10.1103/PhysRevLett.86.4787 PACS numbers: 21.45.+v, 21.30.−x, 25.10.+s, 27.10

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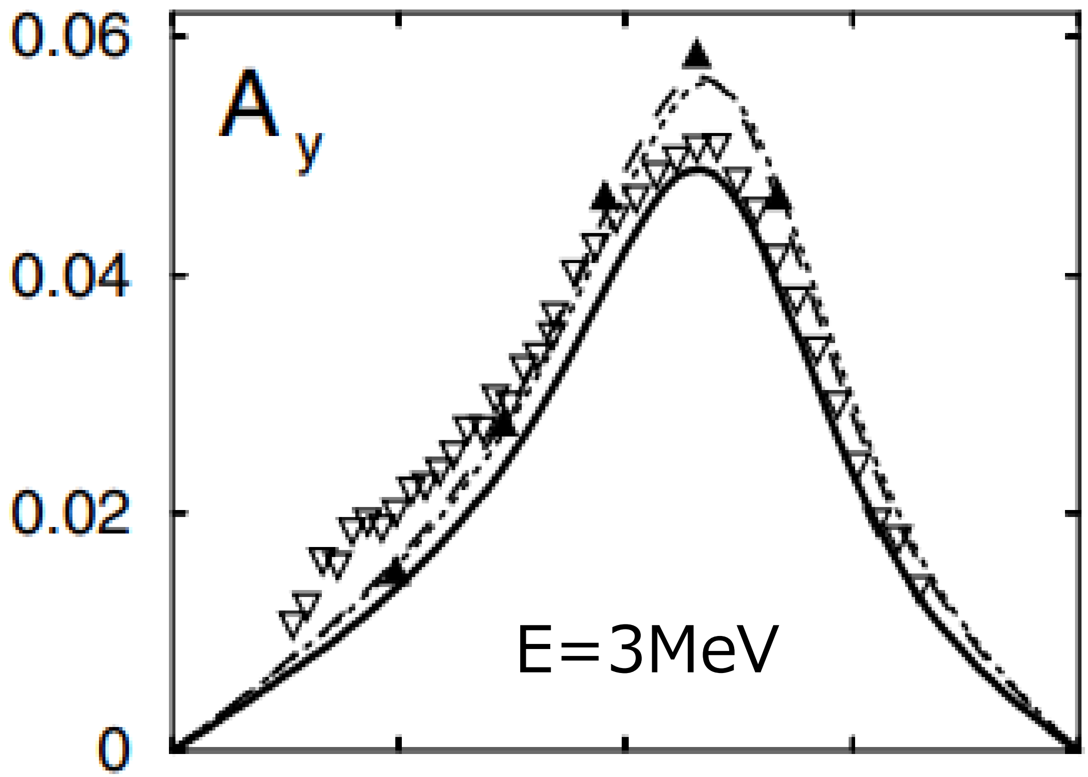
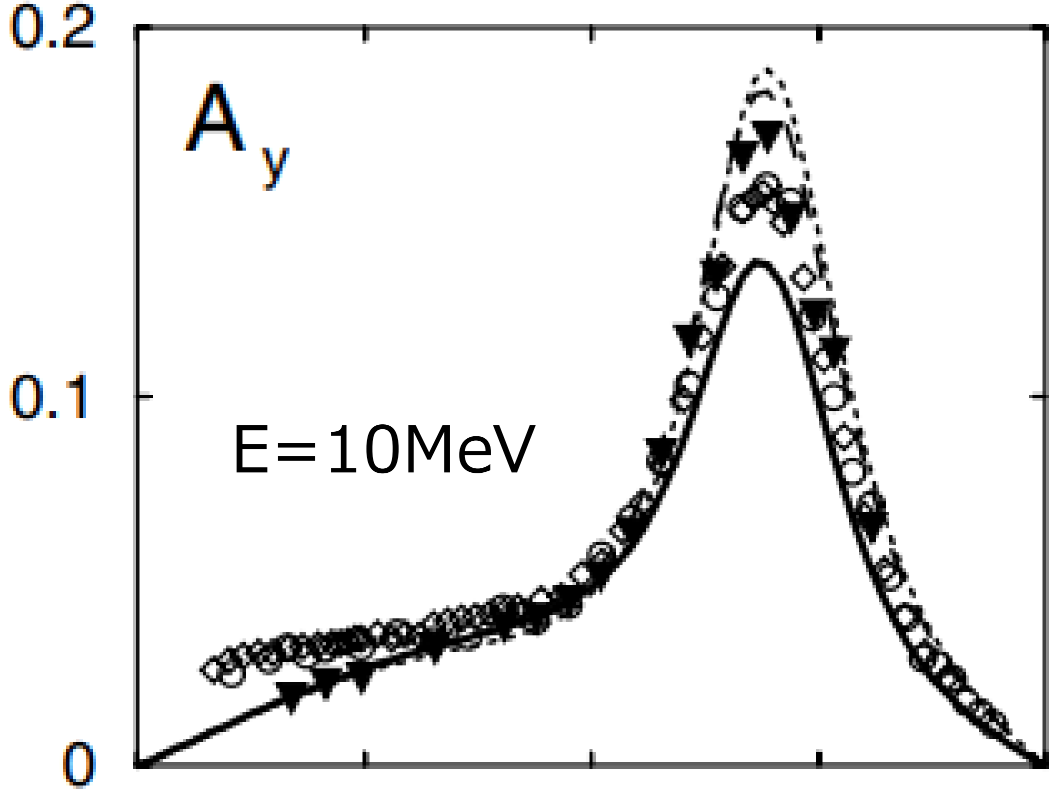
servables rather quantitatively. On top of that, the chiral force predictions are now significantly higher in the maxima of A_y than for CD-Bonn and break the long-standing situation that all standard realistic *NN* forces up to now underpredict the maxima by about 30%. This is called the A_y puzzle [21]. We are now in fact rather close to the experimental *nd* values. Since we restrict ourselves to NLO we cannot expect a final answer from the point of view of chiral dynamics, but this result for A_y is very interesting.

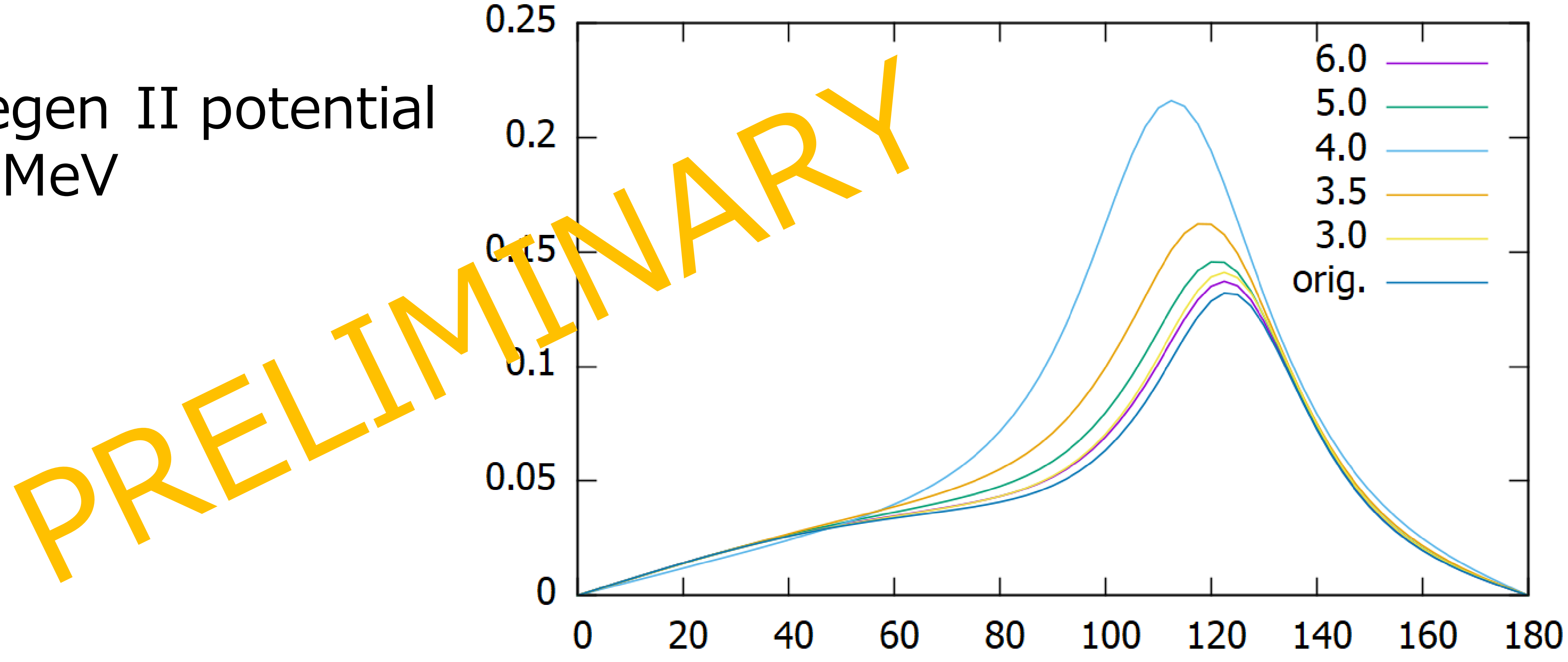
Λ=540 MeV, dotted curve,
Λ= 600 MeV, dashed curve,
CD-Bonn thick solid curve





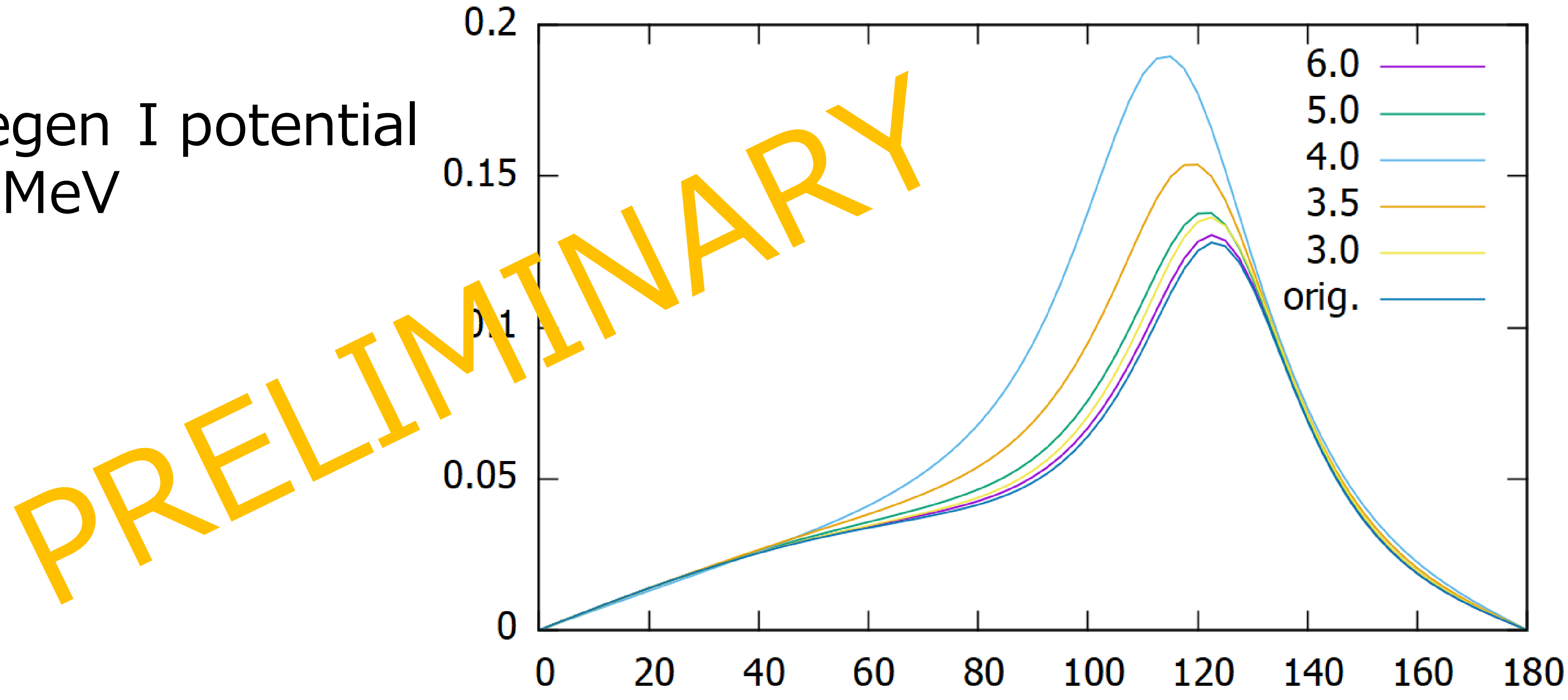
Ay puzzle

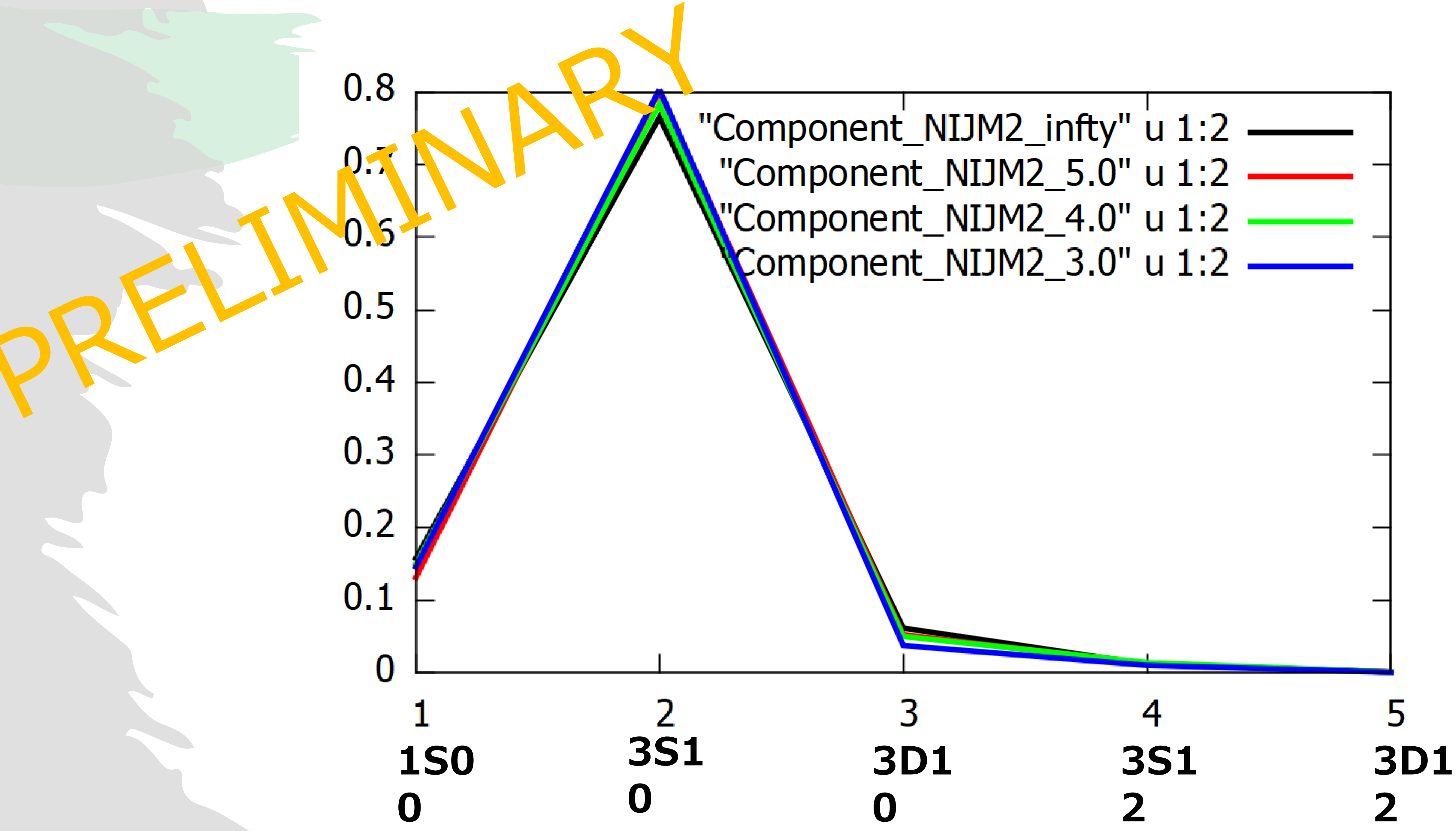
Nijmegen II potential
E=10MeV



Ay puzzle

Nijmegen I potential
E=10MeV




PRELIMINARY
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0.4
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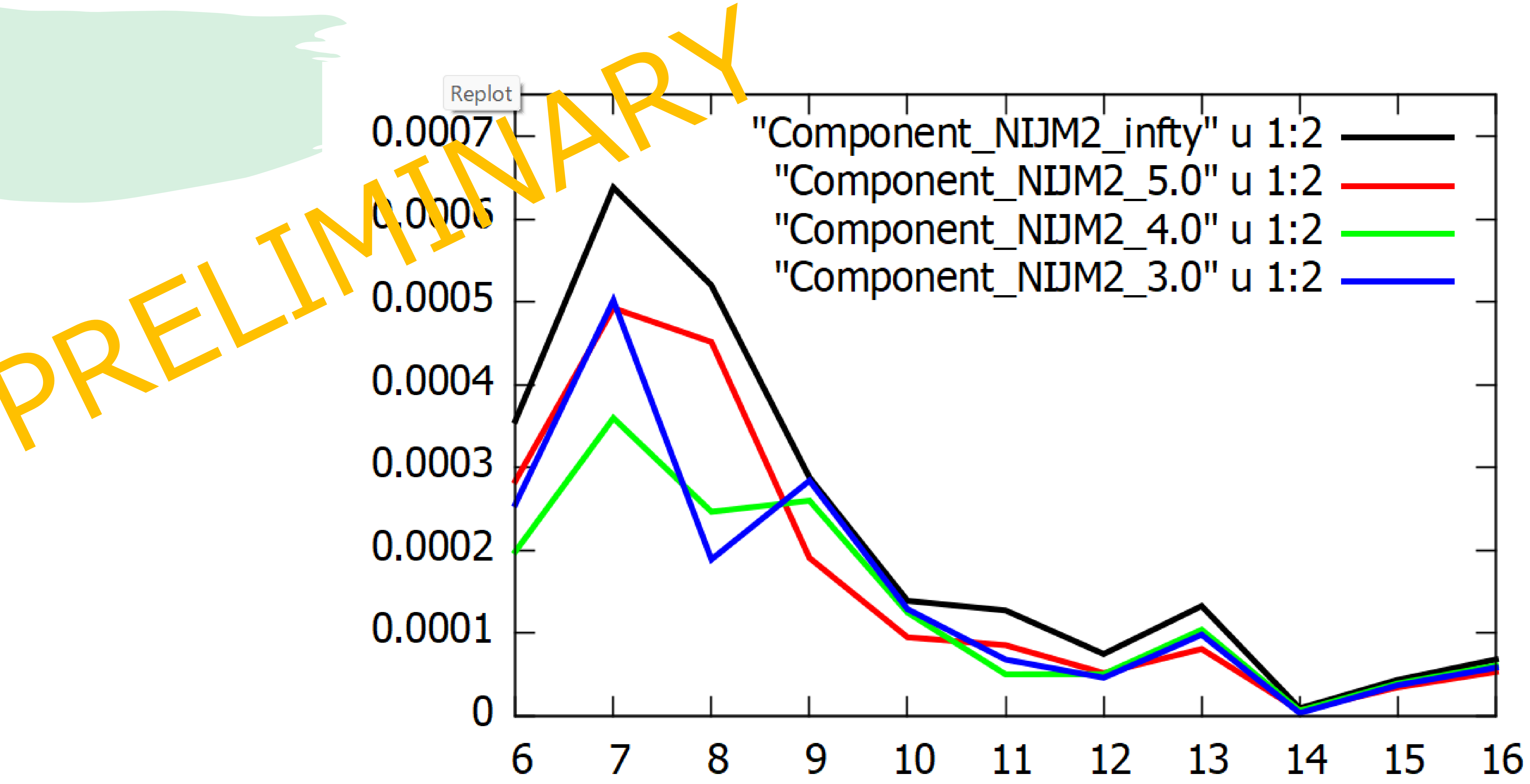
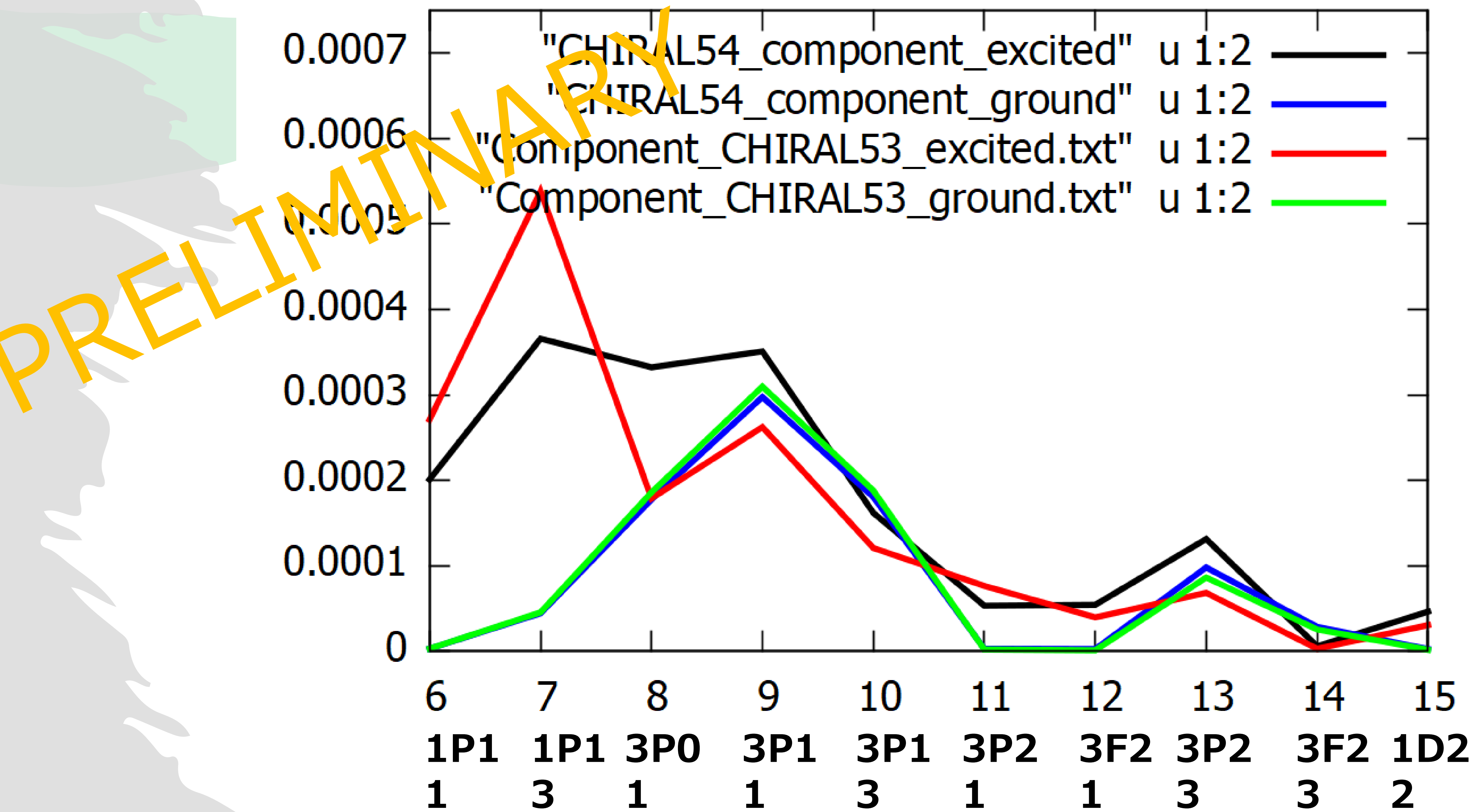
3S1
2
3D1
2



1P1	1P1	3P0	3P1	3P1	3P2	3F2	3P2	3F2	1D2	1D2
1	3	1	1	3	1	1	3	3	2	2,5/2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"Component_CHIRAL53_ground.txt" u 1:2
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0.0003
0.0002
0.0001
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6
7
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9
10
11
12
13
14
15
1P1 1
1P1 3
3P0 1
3P1 1
3P1 3
3P2 1
3F2 1
3P2 3
3F2 3
1D2 2

Summary

- Ay puzzle might be linked to the Quasi resonance pole.
- It may be necessary to adjust the cutoff function of the chiral potential.
- The Coulomb problem needs to work out.