Magnetic moments of light nuclei

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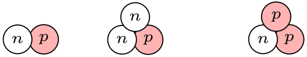
Outline

Introduction

Current status of few-nucleon magnetic form factors

Experiment



- Magnetic moments of deuteron, triton, and helion known extremely precisely e.g. $\mu_d = 0.8574382346(53)\mu_N$ Puchalski_2015
- Magnetic radii known poorly (muonic spectroscopy measurements discussed Antognini:2015vxo)
- Some data of deuteron magnetic form factor, less for 3N ones

Chiral EFT

- Other recent studies Marcucci:2015rca; Schiavilla:2018udt; Seutin:2021hls are to $\mathrm{N^3LO}$ and lack proper error analysis or put different focus
- Can use the same setup as for *charge* form factors Arseniy's talk, in particular:
  – $\mathrm{N^4LO^+}$ chiral 2N forces
  – $\mathrm{N^2LO}$ chiral 3N forces (plus selected $\mathrm{N^4LO}$ 3N forces) Patrick's talk
  – uncertainty estimation (most importantly truncation errors)
- Spatial components of isoscalar vector current operators also known up through $\mathrm{N^4LO}$

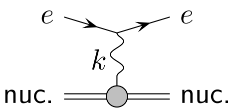
Motivation

- All ingredients to perform precision calculations of magnetic observables
- Accurate experimental magnetic moments provide benchmarks for ChEFT
  $\rightarrow$ for light as well as heavier nuclei
- Improve predictions of magnetic form factors and radii
- Fix unknown LECs for reuse in other calculations

Magnetic form factors

Scattering cross section: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{NS}} F(Q^2,\theta)^2$

$Q^2 := -k_\mu k^\mu$



Spin-1 particle (deuteron)

$F \rightarrow G_C,\ G_Q,\ G_M$

Spin-$\frac{1}{2}$ particle (triton, helion)

$F \rightarrow F_C,\ F_M$

$$F_M^S = \frac{1}{2}\left(F_M^{3\mathrm{He}} + F_M^{3\mathrm{H}}\right)$$
$$F_M^V = \frac{1}{2}\left(F_M^{3\mathrm{He}} - F_M^{3\mathrm{H}}\right)$$

Magnetic moments: $\mu \propto X_M(0)$ Magnetic radii: $r_M^2 \propto X'_M(0)$

Yennie_1957; Drell:1963ej; Garcon:2001sz; Gilman:2001yh

Calculation

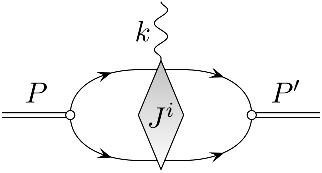
Calculation of magnetic form factors

Convolutions $\langle P', \lambda' | J^i | P, \lambda \rangle$ of

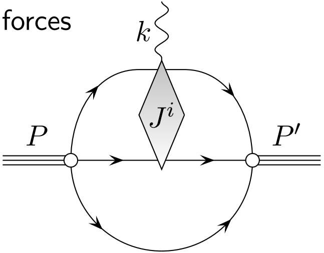
a) spatial components of the vector current operator J^i

b) with wave functions $|P, \lambda\rangle$ obtained from SMS chiral forces

G_M:



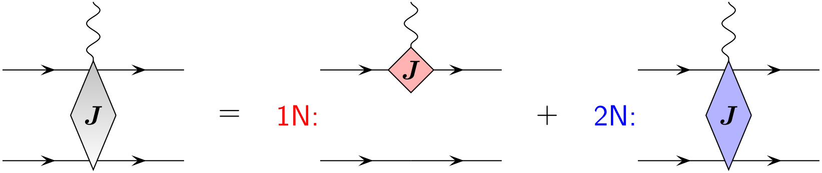
F_M:



1. Solution of Schrödinger and Faddeev equations in PW basis
2. Decomposition of current operators in PW basis
3. Analytic integration over angles (either directly or by Fourier transform to coordinate space)
4. Cheap numeric integration over absolute values of momenta or coordinates
5. Fitting of unknown LECs
6. Error analysis

Current operators

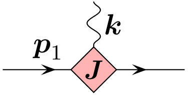
- Single-nucleon and two-nucleon operators:



  3N operators appear at higher orders
- No leading-order (LO, q^{-3}) three-current operator in our power counting, first contribution at NLO (q^{-1})
- Operators at N^{3}LO only known in dim-reg Kolling:2011mt; Krebs:2019aka
- Current operators require consistent regularization with potential
  $\rightarrow$ for isovector currents beyond N^{2}LO still work in progress Hermann's talk
  $\Rightarrow$ Focus on isoscalar currents and observables

Single-nucleon contributions

From NLO: $\boldsymbol{J}_{\mathrm{1N}} = -\frac{\mathrm{i}e}{2m_N}\boldsymbol{k}\times\boldsymbol{\sigma}\mathcal{G}_M + \frac{e}{2m_N}\left(2\boldsymbol{p}_1 + \boldsymbol{k}\right)\mathcal{G}_E$



Single-nucleon Sachs FFs $\mathcal{G}_E$ and $\mathcal{G}_M$

- Keep unexpanded and replace by phenomenological parametrization: Ye:2017gyb, check consistency with Belushkin:2006qa; Lin:2021umz
- $\mathcal{G}_{E(M)} = \frac{1+\tau^3}{2}\mathcal{G}^p_{E(M)} + \frac{1-\tau^3}{2}\mathcal{G}^n_{E(M)}$
- $\mathcal{G}^S_{E(M)} := \mathcal{G}^p_{E(M)} + \mathcal{G}^n_{E(M)}$
  $\mathcal{G}^V_{E(M)} := \mathcal{G}^p_{E(M)} - \mathcal{G}^n_{E(M)}$

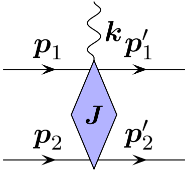
No further 1N contributions up to and including N^{4}LO
(also no relativistic boost corrections to rest-frame wave functions up to this order)

Two-nucleon contributions

From NLO:

$$\boldsymbol{J}_{1\pi}^{\mathrm{NLO}} = \mathrm{i}e\mathcal{G}_E^V \frac{g_A^2}{4F_\pi^2}[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 \frac{\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_2}{\boldsymbol{q}_2^2 + M_\pi^2}\left(\boldsymbol{q}_1 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{q}_1^2 + M_\pi^2} - \boldsymbol{\sigma}_1\right)$$

- parameter-free
- fully isovector



$\boldsymbol{q}_i := \boldsymbol{p}_i' - \boldsymbol{p}$

From N^{3}LO: $\boldsymbol{J}_{1\pi}^{S} = \mathrm{i}e2\mathcal{G}_E^S \frac{-3g_A}{F_\pi^2} d_9 \frac{\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_2}{\boldsymbol{q}_2^2 + M_\pi^2}\boldsymbol{k} \times \boldsymbol{q}_2 \quad \to \quad$ isoscalar 1π exch.

$\boldsymbol{J}_{\mathrm{cont}}^{S} = \mathrm{i}e\mathcal{G}_E^S L_2(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \boldsymbol{k} \quad \to \quad$ isoscalar contact

$\boldsymbol{J}_{\mathrm{cont}}^{V} = \mathrm{i}e\mathcal{G}_E^V L_1 \tau_1^3(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times \boldsymbol{k} \quad \to \quad$ single isovector contact

- d_9 is determined in πN ChPT
- L_2, L_1 are determined in this study

No further isoscalar contributions at N^{4}LO

Regularization of two-nucleon contributions

Regularization 1π exchange contributions:

Replace propagators the same way as in the SMS potential

$$\frac{1}{\boldsymbol{q}^2+M_\pi^2} \to \frac{e^{-\frac{\boldsymbol{q}^2+M_\pi^2}{\Lambda^2}}}{\boldsymbol{q}^2+M_\pi^2}, \qquad \frac{1}{(\boldsymbol{q}^2+M_\pi^2)^2} \to \frac{e^{-\frac{\boldsymbol{q}^2+M_\pi^2}{\Lambda^2}}}{(\boldsymbol{q}^2+M_\pi^2)^2} + \frac{1}{\Lambda^2}\frac{e^{-\frac{\boldsymbol{q}^2+M_\pi^2}{\Lambda^2}}}{\boldsymbol{q}^2+M_\pi^2}$$

Regularization of contact contributions:

Transversal $\boldsymbol{k}\cdot\boldsymbol{J} = 0 \Rightarrow$ current conservation does not help

Comparing with similar terms at higher order or other isovector terms at the same order, the most consistent approach appears to be

$$\boldsymbol{J}_{\rm cont} \to \frac{1}{4}\Bigg(\exp\left[-\frac{(\boldsymbol{p}+\boldsymbol{k}/2)^2+\boldsymbol{p}'^2}{\Lambda^2}\right] + \exp\left[-\frac{(\boldsymbol{p}-\boldsymbol{k}/2)^2+\boldsymbol{p}'^2}{\Lambda^2}\right] + \exp\left[-\frac{\boldsymbol{p}^2+(\boldsymbol{p}'+\boldsymbol{k}/2)^2}{\Lambda^2}\right] + \exp\left[-\frac{\boldsymbol{p}^2+(\boldsymbol{p}'-\boldsymbol{k}/2)^2}{\Lambda^2}\right]\Bigg)\boldsymbol{J}_{\rm cont},$$

with $\boldsymbol{p}^{(\prime)} = (\boldsymbol{p}_1^{(\prime)} - \boldsymbol{p}_2^{(\prime)})/2$

$\to$ to be verified once results from higher-derivative regularization are available

Fits and uncertainties

Fitting of unknown LECs from N^{3}LO operators:

- Isoscalar 1π-exch. LEC $d_9 \xrightarrow{\text{set to}} 0$
  compatible with determination in πN sector Rijneveen:2021bfw
- Isoscalar contact LEC $L_2 \xrightarrow{\text{fit to}} \mu_d^{\text{exp}}$
- Isovector contact LEC $L_1 \xrightarrow{\text{fit to}} \mu_3^{V,\text{exp}}$

Error analysis:

- Bayesian analysis considering chiral expansion order by order
  Furnstahl:2015rha; Melendez:2017phj; Wesolowski:2018lzj; Epelbaum:2019zqc
- No LO for magnetic observables → modified $\bar{C}^{650}_{0.5-10}$ model starting at NLO:
  $$X_M = X_M^{(2)} + \Delta X_M^{(3)} + \Delta X_M^{(4)} + \ldots =: X_M^{\text{ref}} \left(c_2 q^2 + c_3 q^3 + c_4 q^4 + \ldots\right)$$
  with $X_M^{\text{ref}} = \max\left(\frac{|X_M^{(2)}|}{q^2}, \frac{|\Delta X_M^{(3)}|}{q^3}, \frac{|\Delta X_M^{(4)}|}{q^4}\right)$
- Inclusion of uncertainties beyond truncation errors is work in progress (assumed to be less dominant)

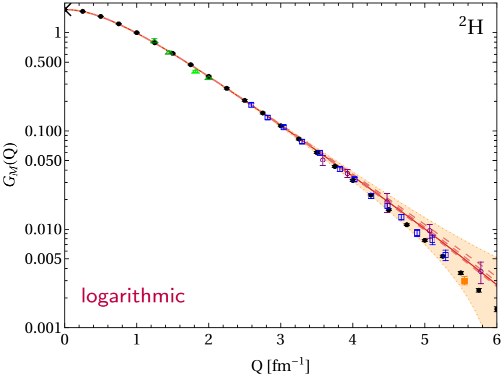
Results (all preliminary)

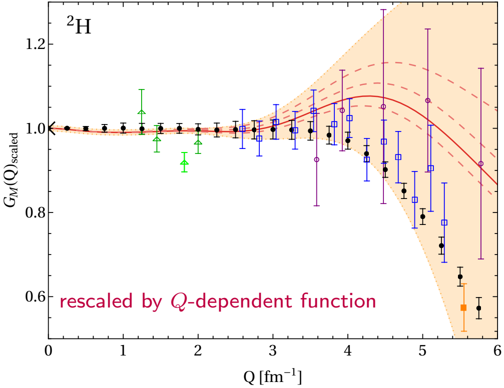
Deuteron magnetic form factor

Prediction of FF shape at N^{4}LO with $\Lambda = 450$ MeV

band is 68% DoB truncation error, dashed are other cutoffs

Experimental data (colored) and parametrization by Sick (black) from Marcucci:2015rca



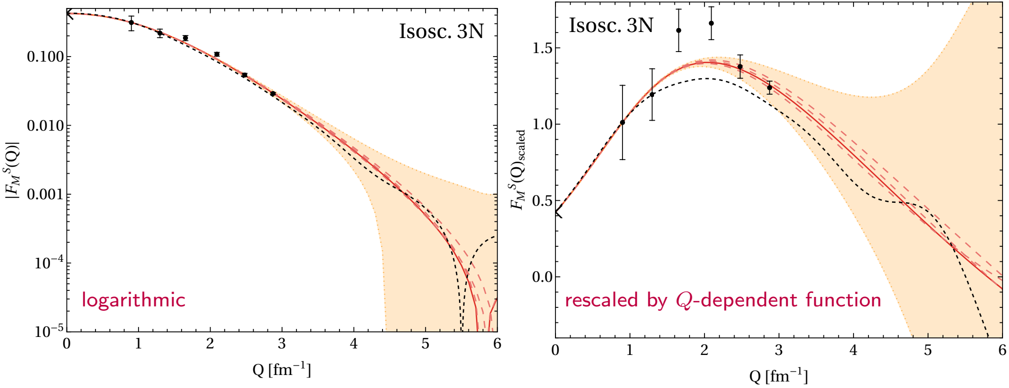


→ Excellent description of data within truncation error

3N isoscalar magnetic form factor $nnp + ppn$

Prediction of FF shape and magnetic moment (N^{4}LO, $\Lambda = 450\,\mathrm{MeV}$)

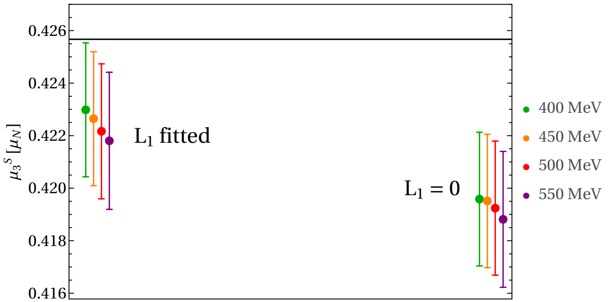
Data from Beck:1987zz and parametrization by Sick (black dashed) from Marcucci:2015rca



$\rightarrow$ Prediction overall consistent with (poor) data

Isoscalar magnetic moment

Prediction of isoscalar 3N magnetic moment at N^{4}LO, 68% DoB truncation errors



- Successful precision test of ChEFT (regarding order and DoB)
- Essential to include isovector current at N$^{\geq 3}$LO, fully covered by L_1 here

Isoscalar magnetic moment and magnetic radii

Results for magnetic moment and radius predictions and corresponding LECs

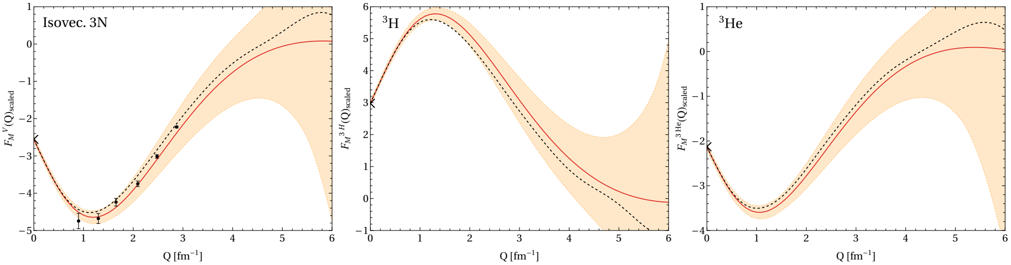
(N^4LO, 68% DoB truncation errors)

Λ in MeV	L_2 in $F_\pi^{-2}\Lambda_b^{-2}$	L_1 in $F_\pi^{-2}\Lambda_b^{-2}$	μ_3^S in μ_N	r_{Md}^2 in fm^2	$(r_{M3}^S)^2$ in fm^2
400	0.023	1.285	0.4230(26)	4.481(27)	2.283(14)
450	0.052	1.201	0.4226(26)	4.481(27)	2.279(14)
500	0.079	1.187	0.4222(26)	4.481(27)	2.273(14)
550	0.109	1.225	0.4218(26)	4.480(27)	2.261(15)
experiment	—	—	0.425 668 622(6)	4.29(7)	2.1(24)

- (Isovector) L_1 large compared to (isoscalar) L_2 but of natural size
- Predicted radii much more precise than current experimental knowledge

Isovector and individual 3N FFs (n n p) & (p n p)

Prediction of FF shapes and ^{3}H and ^{3}He magnetic moments at (incomplete) N^{3}LO



Λ in MeV	μ_{3H} in μ_N	μ_{3He} in μ_N	$(r^V_{M3})^2$ in fm^2	r^2_{M3H} in fm^2	r^2_{M3He} in fm^2
450	2.98(7)	−2.13(6)	3.53(8)	3.35(8)	3.78(9)
500	2.98(7)	−2.13(6)	3.54(8)	3.36(8)	3.79(9)
experiment	2.978 962 471(10)	−2.127 625 227(8)	3.6(4)	3.4(7)	3.90(19)

→ Good agreement with data, including just a single N^{3}LO isovector contribution

Summary

Summary and outlook

- Calculation of *isoscalar* magnetic few-nucleon observables pushed to N^{4}LO
- Calculation of *isovector* magnetic few-nucleon observables pushed beyond N^{2}LO
- Good agreement with experimental data, in particular
  – successfully benchmarked ChEFT with isoscalar 3N magnetic moment
  – unprecedented precision for magnetic radii

Outlook

- Room for improvements, in particular for isovector and individual ^{3}H, ^{3}He form factors via consistently regularized (isovector) N$^{\geq 3}$LO current operators
- Calculation should be easily extensible to magnetic properties of larger nuclei, magnetic moments can provide ChEFT benchmark in addition to charge radii
  $\rightarrow$ We can provide "magnetic moment operators" to be directly plugged into existing frameworks.

Supplementary slides

Fitting d_9 to μ_3^S at N^{4}LO

- Having three precise experimental values μ_d, μ_3^S, μ_3^V, one can principally fit all LECs L_2, L_1, and d_9 simultaneously as done in Schiavilla:2018udt.
- Similar as there, we obtain a range of

  $$d_9 = 0.3 \ldots 0.5\,\text{GeV}^{-2}$$

  by varying cutoffs.
  (d_9 actually cutoff-independent, since πN quantity)
- Quite sizable contribution but practically completely cancelled by L_2 contribution
  $\rightarrow$ sum of both is very small and form factors as well as radii change only insignificantly compared to $d_9 = 0$
- Change to μ_3^S is only within truncation error

$\Rightarrow$ Unable to reliably extract non-zero d_9 from few-nucleon data

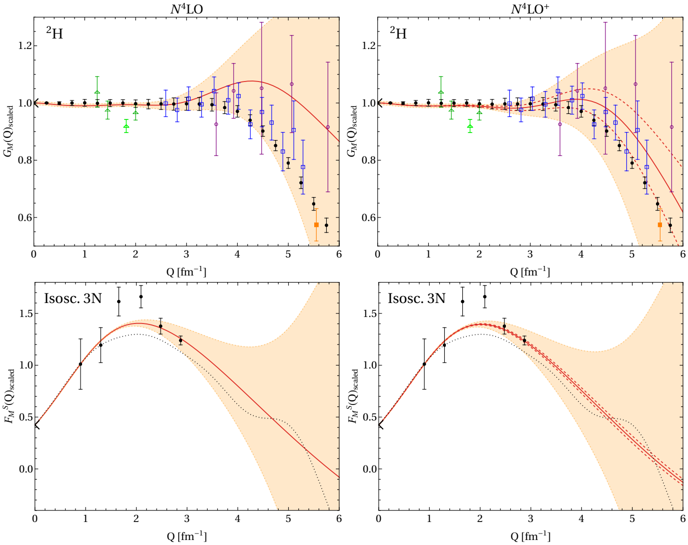
$\mathrm{N^5LO}$ contact current

- Derivable analogously to higher-order *charge* corrections in Filin:2020tcs via unitary transformation $U = e^{AT_1+BT_2+CT_3}$ of the leading current operator:

$$\hat{\boldsymbol{J}}^{\mathrm{N^5LO}}_{\mathrm{cont}} = \hat{U}^\dagger \hat{\boldsymbol{J}}_{\mathrm{1N}} \hat{U} - \hat{\boldsymbol{J}}_{\mathrm{1N}} \simeq \left[\hat{\boldsymbol{J}}_{\mathrm{1N}}, \hat{\mathcal{T}}\right]$$

- Regularization of the generators T_i consistent with the chiral potential from Reinert:2017usi as $T_i \to T_i \exp\left(-\frac{\boldsymbol{p}^2 + \boldsymbol{p}'^2}{\Lambda^2}\right)$
- Gives three more LECs, only two contribute to deuteron
- Fitting to G_M data yields high correlation of these LECs (not enough data from isoscalar 3N form factor)
- Fit only one of them to G_M shape (no contribution to magnetic moments)
- χ^2-fit including experimental and theoretical errors (1N form factor parametrization and truncation [iteratively])

Results beyond N^{4}LO

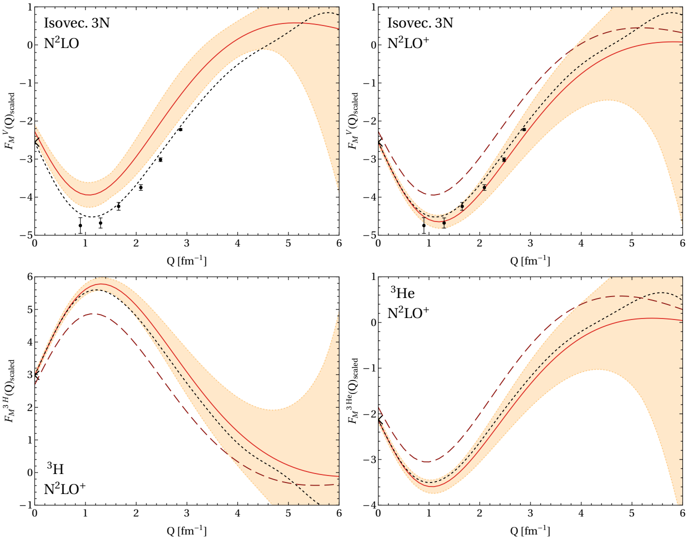


- Slightly improved description of G_M data after fitting single LEC
- Statistical uncertainty (red dashed) within truncation error
- Effect on F_M^S negligible
- Radii only get a few per mille larger

⇒ N^{4}LO prediction already does a good job

Details on isovector and individual 3N FFs

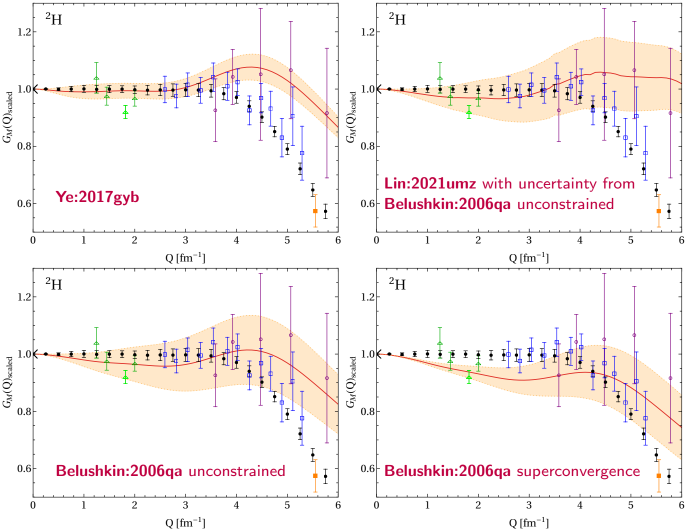
- Up through N^{2}LO all operators included
- At N^{3}LO only single isovector operator included in addition
- Fitting L_1 seems to do a good job compared to $L_1 = 0$ (dashed)
- Magnetic moment and radius predictions well in line with data but large (few-percent) truncation errors



Effects of changing 1N FF parametrizations

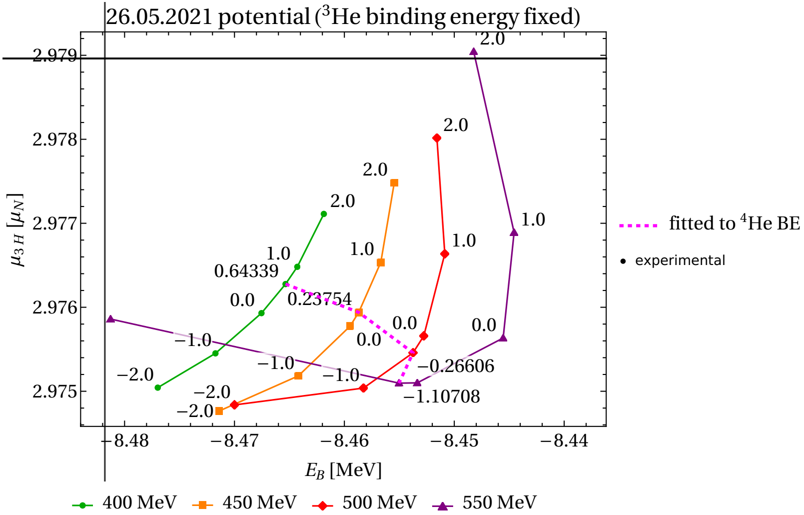
Scaled G_M for different parametrizations of the 1N Sachs form factors $\mathcal{G}_E, \mathcal{G}_M$ (bands are propagated uncertainties of the parametrizations)

$\Rightarrow$ Within errors not relevant if Ye:2017gyb or Lin:2021umz are used though deviation is larger than for deuteron charge form factors (due to $\mathcal{G}_M$ being more important here)



Correlation of ^{3}H binding energy and magnetic moment

Varying c_{E_1} 3N force contribution yields similar correlation as for charge radii:



Chiral convergence of magnetic moments

μ_d 0.857 438 234 6(53)

Λ	NLO	N^{2}LO	N^{3}LO fit	N^{4}LO fit
400	0.86(31)	0.86(7)	0.857(18)	0.857(5)
450	0.86(31)	0.85(7)	0.857(18)	0.857(5)
500	0.86(31)	0.85(7)	0.857(18)	0.857(5)
550	0.85(31)	0.84(7)	0.857(18)	0.857(5)

μ_3^S 0.425 668 622(6)

Λ	NLO	N^{2}LO	N^{3}LO	N^{4}LO
400	0.42(15)	0.416(34)	0.423(9)	0.4230(26)
450	0.42(15)	0.412(33)	0.423(9)	0.4226(26)
500	0.42(15)	0.407(33)	0.424(9)	0.4222(26)
550	0.42(15)	0.403(33)	0.424(9)	0.4218(26)

μ_3^V −2.553 293 849(6)

Λ	NLO	N^{2}LO	N^{3}LO fit	N^{4}LO fit
400	−2.3(8)	−2.26(18)	−2.55(7)	−2.553(20)
450	−2.3(8)	−2.28(18)	−2.55(7)	−2.553(19)
500	−2.3(8)	−2.30(18)	−2.55(6)	−2.553(18)
550	−2.3(9)	−2.32(19)	−2.55(6)	−2.553(17)

$\mu_{3\mathrm{H}}$ 2.978 962 471(10) beginning from N^{3}LO incomplete

Λ	NLO	N^{2}LO	N^{3}LO	N^{4}LO
400	2.7(10)	2.67(21)	2.98(8)	2.976(21)
450	2.7(10)	2.69(22)	2.98(7)	2.976(20)
500	2.7(10)	2.70(22)	2.98(7)	2.975(19)
550	2.8(10)	2.72(22)	2.98(7)	2.975(19)

$\mu_{3\mathrm{He}}$ −2.127 625 227(8) beginning from N^{3}LO incomplete

Λ	NLO	N^{2}LO	N^{3}LO	N^{4}LO
400	−1.8(7)	−1.84(15)	−2.13(7)	−2.130(19)
450	−1.9(7)	−1.86(15)	−2.13(6)	−2.131(18)
500	−1.9(7)	−1.89(15)	−2.13(6)	−2.131(16)
550	−1.9(7)	−1.91(15)	−2.13(5)	−2.131(15)

Individual contributions to magnetic moments

Percentages of individual contributions to deuteron and isoscalar 3N magnetic moments relative to the experimental value at N^{4}LO

μ_d in %:

Λ	1N contrib.	L_2-term	truncation error
400	99.72	0.28	0.60
450	99.36	0.64	0.60
500	99.03	0.97	0.60
550	98.74	1.26	0.60

μ_3^S in %:

Λ	1N contrib.	NLO OPE	L_2-term	L_1-term	missing	truncation error
400	97.91	-0.07	0.72	0.80	0.63	0.60
450	96.90	0.02	1.63	0.74	0.71	0.60
500	95.96	0.10	2.44	0.69	0.82	0.60
550	95.10	0.15	3.14	0.70	0.91	0.61

$\rightarrow$ Similar as for charge radii, 2N contributions drastically reduce cutoff dependence and their importance grows with number of nuclei

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