

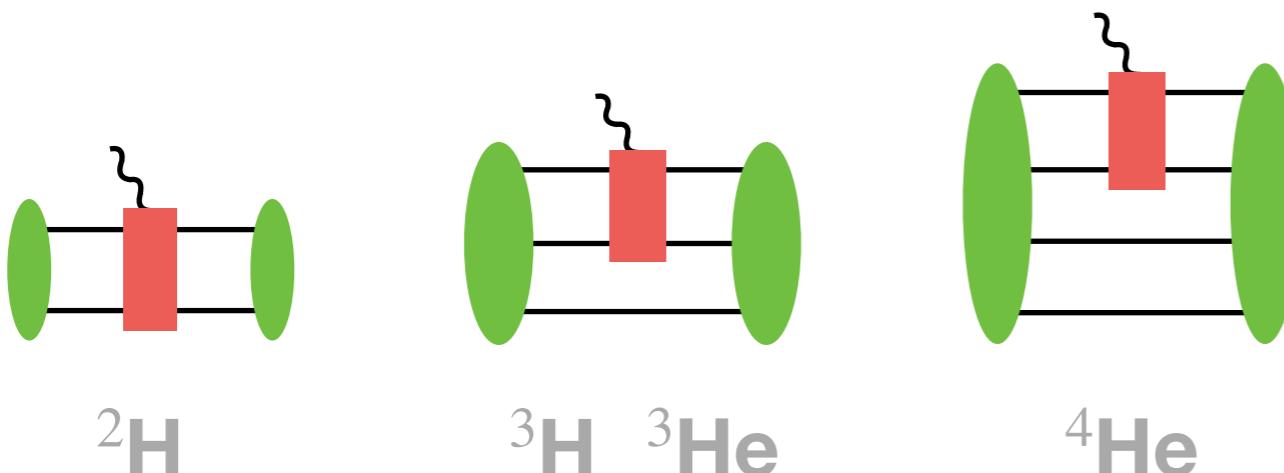
Precision calculations of charge radii of light nuclei

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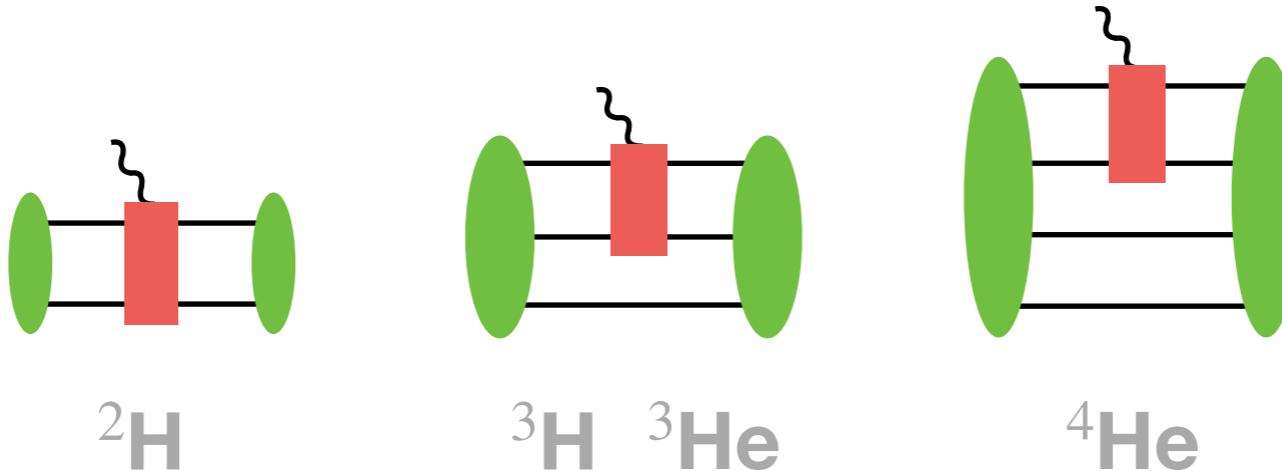
in collaboration with

V. Baru, E. Epelbaum, C. Körber, H. Krebs, D. Möller, A. Nogga, and P. Reinert



PRL 124 082501 (2020)
Phys.Rev.C 103 024313 (2021)

Precision calculations of charge radii of light nuclei



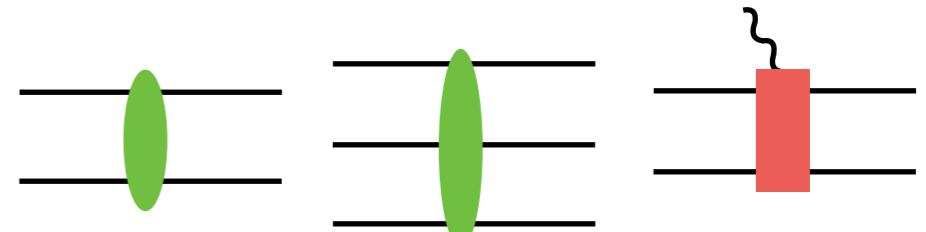
Motivation:

- Precision **tests of nuclear chiral effective field theory (EFT)**
- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**
- More applications:
 - A new way to **extract the neutron and the proton charge radii** from few-nucleon data
 - Search for **Beyond-Standard-Model** physics

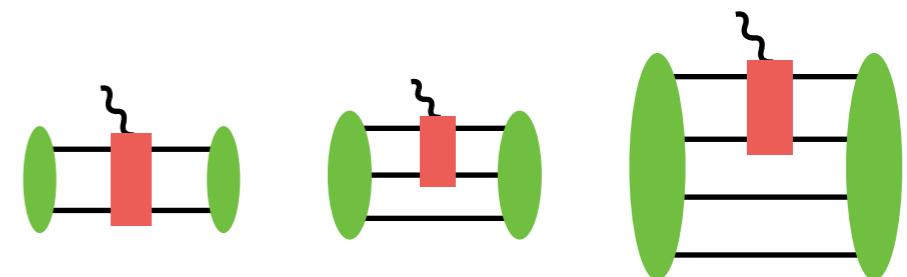
Precision calculations of charge radii of light nuclei

Introduction

- precision measurements of charge radii $A \leq 4$
- precision and accuracy of chiral EFT



Results for $A=2,3,4$ charge form factors and radii



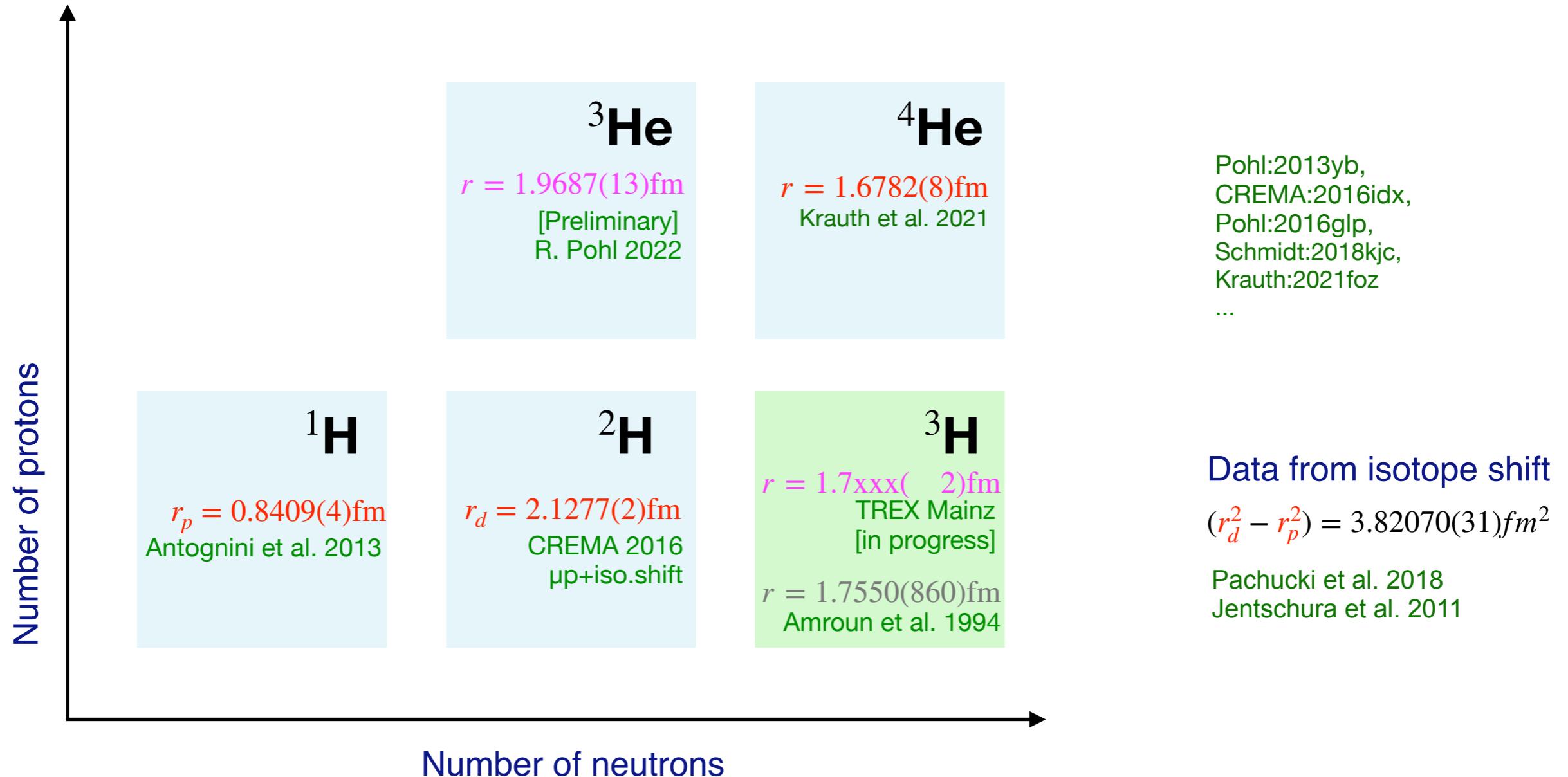
Applications and tests

- extraction of the neutron and proton charge radii
- predictions for $3N$ structure radii

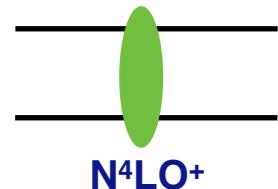


Takeaway from $A=2,3,4$ calculations

Precision measurements of charge radii for $A = 1, 2, 3, 4$ nuclei

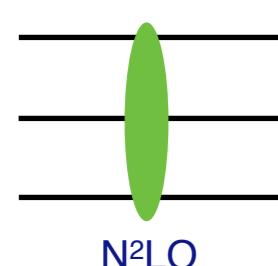


Chiral effective field theory - precise, accurate and consistent



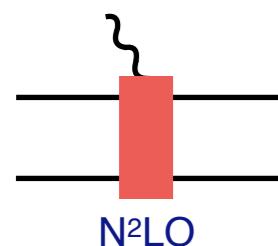
New high-precision chiral NN forces (N⁴LO⁺) Reinert et al. PRL 126, 092501 (2021) talk by Patrick

- Nearly perfect description of pp and pn scattering data up to pion production threshold



Chiral 3N forces (general N²LO; selected terms at N⁴LO) Epelbaum:2019kcf

- LECs cD and cE (N²LO) are fitted to RIKEN Nd DCS data and ³He binding energy
- **Consistent** regularisation of N³LO is also in progress, **talk by Hermann**



Isoscalar: N⁴LO⁻

2N Chiral electromagnetic currents (general N²LO; isoscalar N⁴LO⁻)

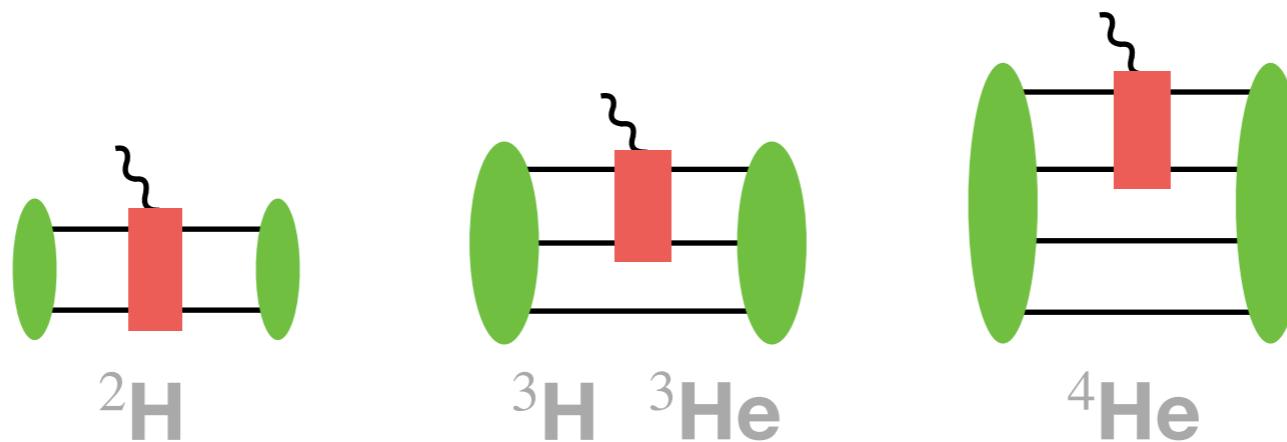
- N²LO (**isoscalar N⁴LO⁻**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N³LO (isovector) is in progress

Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)

Reliable methods to quantify truncation uncertainty of the EFT expansion

Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017),
Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

Chiral EFT calculation of charge radii



Goals:

- **consistent** calculation of **isoscalar** charge radii of $A = 2, 3, 4$ nuclei
- **aim at N⁴LO level of accuracy** even when not all forces are available at N⁴LO
- careful estimation of uncertainties (truncation, statistical, incompleteness of 3NFs, ...)

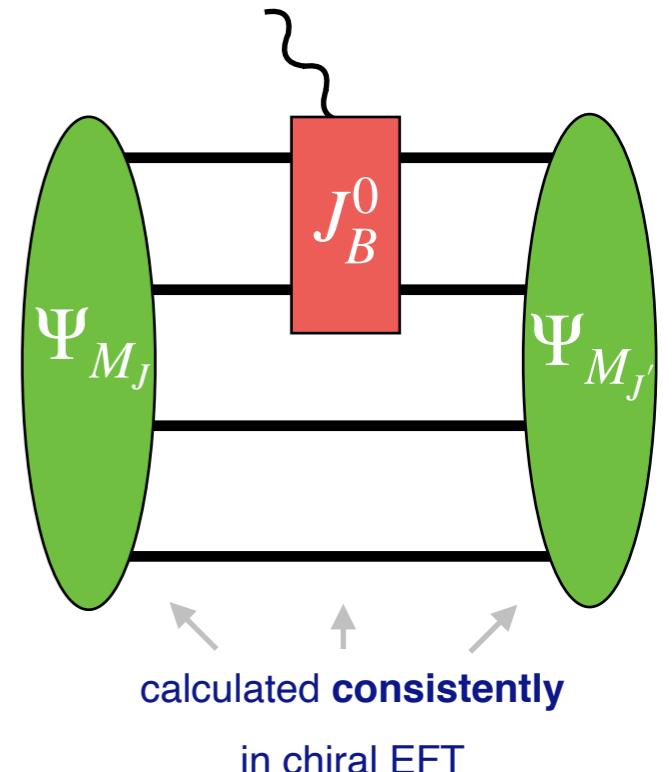
Chiral EFT calculation of the nuclear charge radius

Charge radius r_C is related to the charge form factor $F_C(Q)$

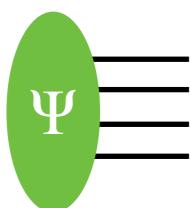
$$r_C^2 = (-6) \frac{\partial}{\partial Q^2} F_C(Q^2) \Big|_{Q=0}$$

Charge form factor F_C can be computed (in the Breit frame) as

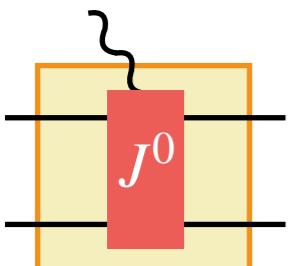
$$F_C(Q^2) = \frac{1}{2J+1} \sum_{M_J} \langle P', M_J | J_B^0 | P, M_J \rangle$$



The matrix element is a convolution of nuclear wave function and charge density operator



Nuclear wave function - based on high-precision chiral EFT interactions



Charge density operator - consistent with chiral nuclear forces

Wave functions of A=2,3,4 nuclei



A=2,3,4 wave functions - solutions of Schrödinger / FY equations in the partial wave basis

- 2N forces at N⁴LO+
- 3N forces at N²LO LECs cD and cE (N²LO) are fitted to RIKEN Nd DCS data and ^3He binding energy

Extra 3N forces at N⁴LO:

- added selected 3NF at N⁴LO (cE1 or cE3) (tree-level & regularised consistently)
- fitted LEC cE1 **or** cE3 to exactly reproduce ^4He physical BE

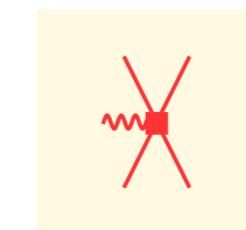
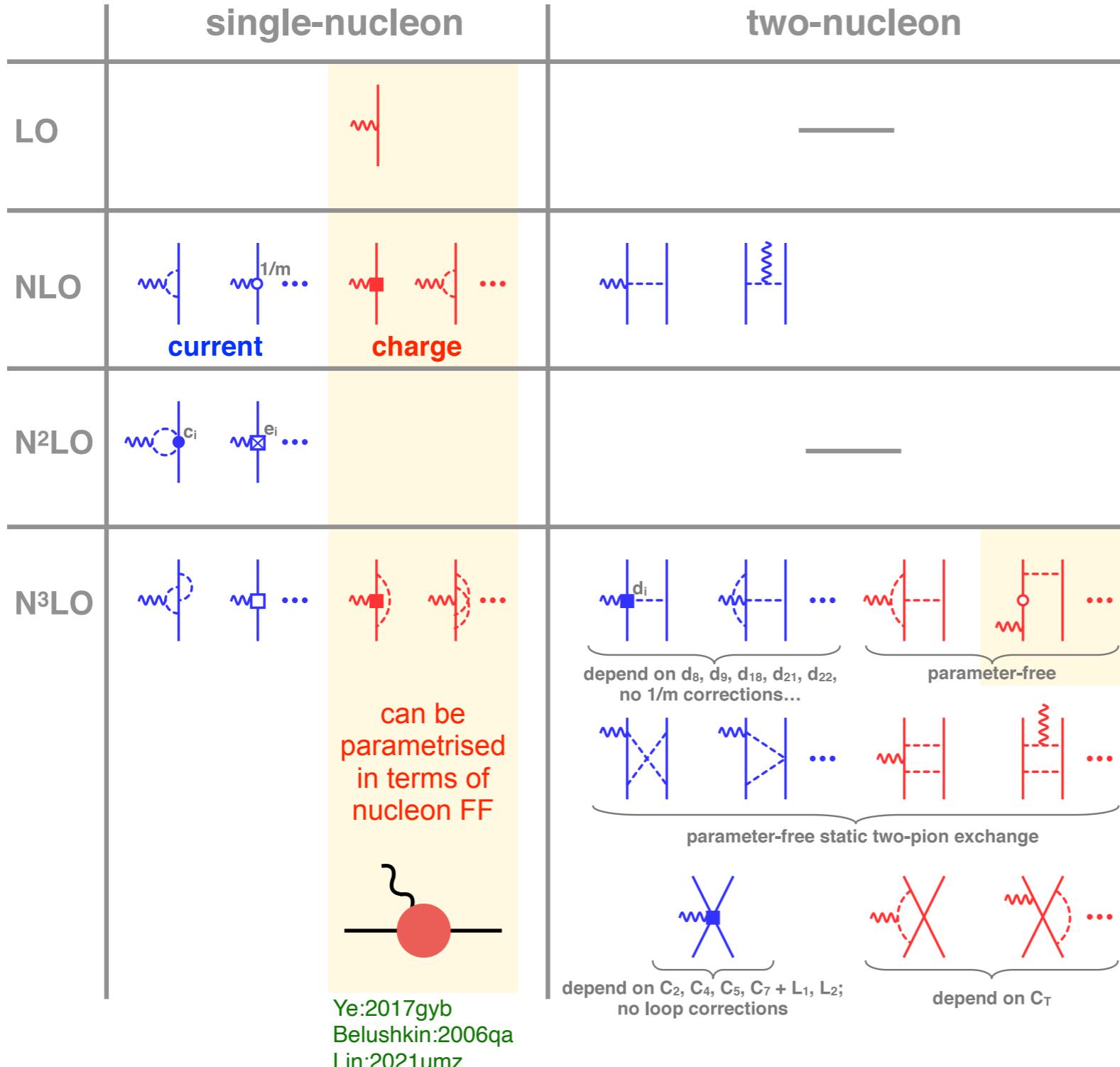
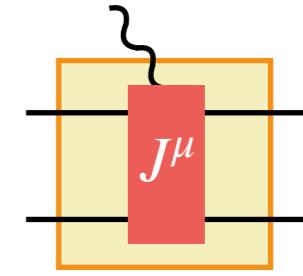
In progress

- relativistic treatment of 3N and 4N equations (talk by Anderas)

Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka

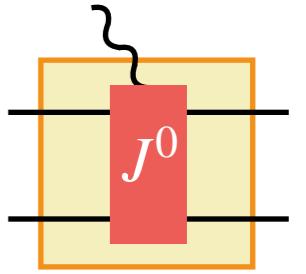
Review: H. Krebs, EPJA 56 (2020) 240



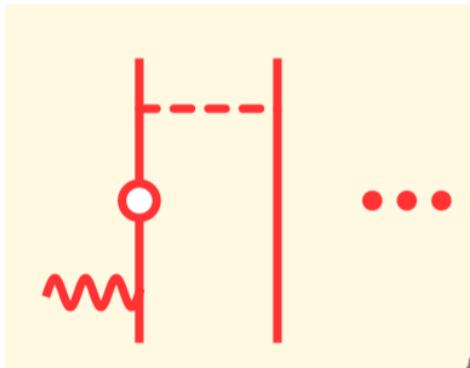
depend on **3 LECs**
 3S_1 - 3S_1 - can be fitted to deuteron FF data
 3S_1 - 3D_1 - this one too
 1S_0 - 1S_0 - can be fitted to ${}^4\text{He}$ FF data
Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

three-nucleon isoscalar charge operators are beyond N⁴LO

2N Charge density operators (N³LO)



Isoscalar 2N one-pion exchange charge density

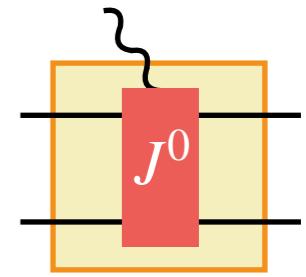


- parameter free
- regularised consistently with NN forces
- local operator (simple PWD)
- depends on the off-shell parameters β_8, β_9

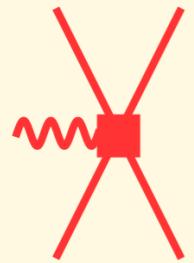
β_8, β_9 are chosen consistently with NN potential and checked

$$\begin{aligned} \rho_{2N}^{1\pi, \text{reg}} &= (1 - 2\bar{\beta}_9) G_E^S(Q^2) \frac{eg_A^2}{16F_\pi^2 m_N} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{\mathbf{q}_2^2 + M_\pi^2} \exp\left(-\frac{\mathbf{q}_2^2 + M_\pi^2}{\Lambda^2}\right) \\ &+ (2\bar{\beta}_8 - 1) G_E^S(Q^2) \frac{eg_A^2}{16F_\pi^2 m_N} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2)(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2) (\mathbf{q}_2 \cdot \mathbf{k}) \left(\frac{1}{(\mathbf{q}_2^2 + M_\pi^2)^2} + \frac{1}{\Lambda^2(\mathbf{q}_2^2 + M_\pi^2)} \right) \exp\left(-\frac{\mathbf{q}_2^2 + M_\pi^2}{\Lambda^2}\right) \end{aligned}$$

2N Charge density operators (N³LO)



Isoscalar 2N contact charge density



- **3 LECs** which we fit to reproduce the deuteron and 4He form factors
- **regularised consistently** with NN forces
- **separable** operator (also simple PWD)

$$\rho_{\text{Cont}} = 2eG_E^S(\mathbf{k}^2) \left[\left(A + B + \frac{C}{3} \right) \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + 3}{4} \frac{1 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{4} \mathbf{k}^2 \right.$$

^{3S₁-3S₁} LEC

$$+ C \frac{1 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{4} \left((\mathbf{k} \cdot \boldsymbol{\sigma}_1)(\mathbf{k} \cdot \boldsymbol{\sigma}_2) - \frac{1}{3} \mathbf{k}^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \right)$$

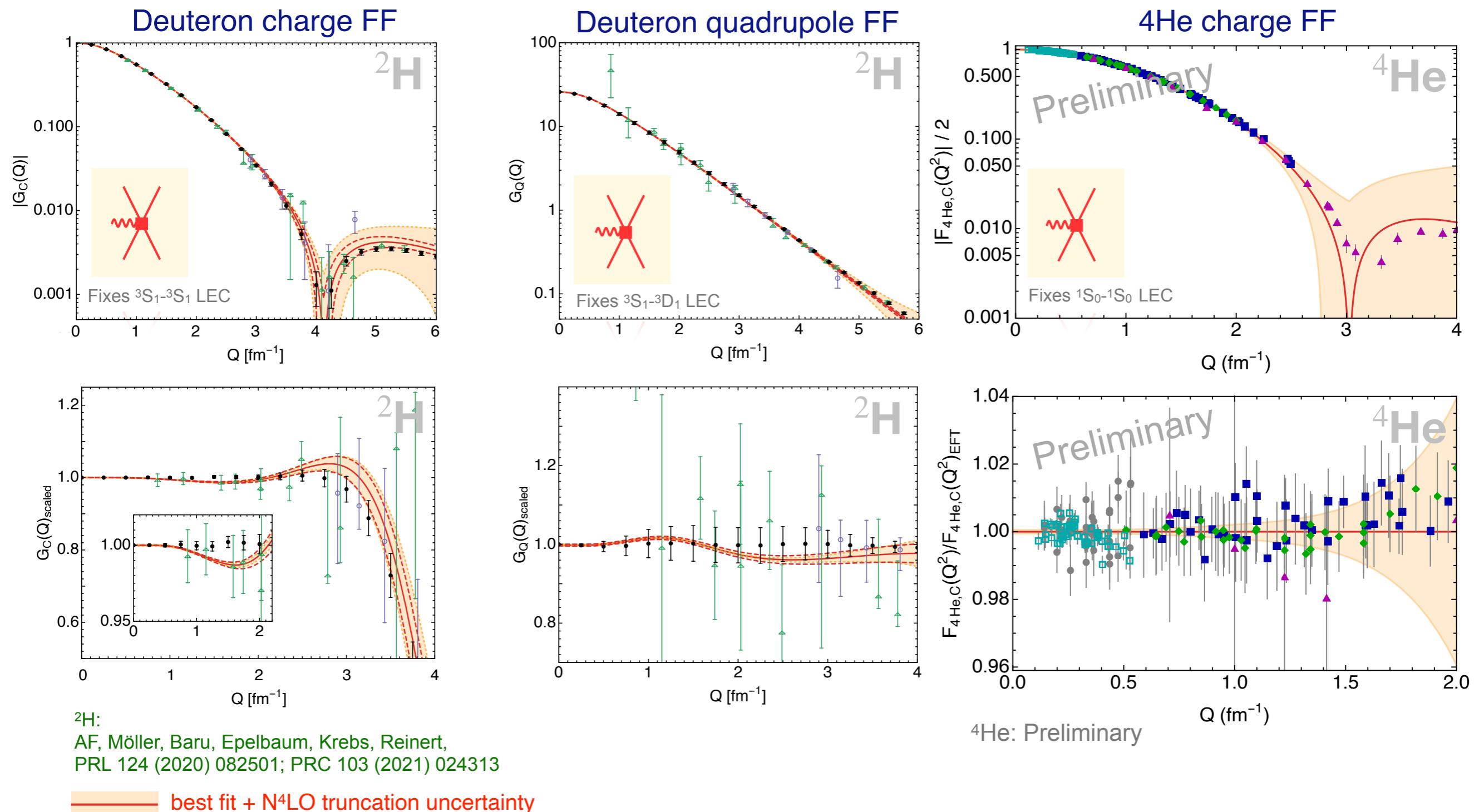
^{3S₁-3D₁} LEC

$$\left. + (A - 3B - C) \frac{1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{4} \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + 3}{4} \mathbf{k}^2 \right]$$

^{1S₀-1S₀} LEC

(regulator not shown)

Low-energy constants from a fit to charge and quadrupole form factors



3 LECs in J^0 are fixed from the form factor data of deuteron and ⁴He

Parameter-free prediction of structure radii

After all three LECs in charge density operators are fixed we get predictions for the structure radii

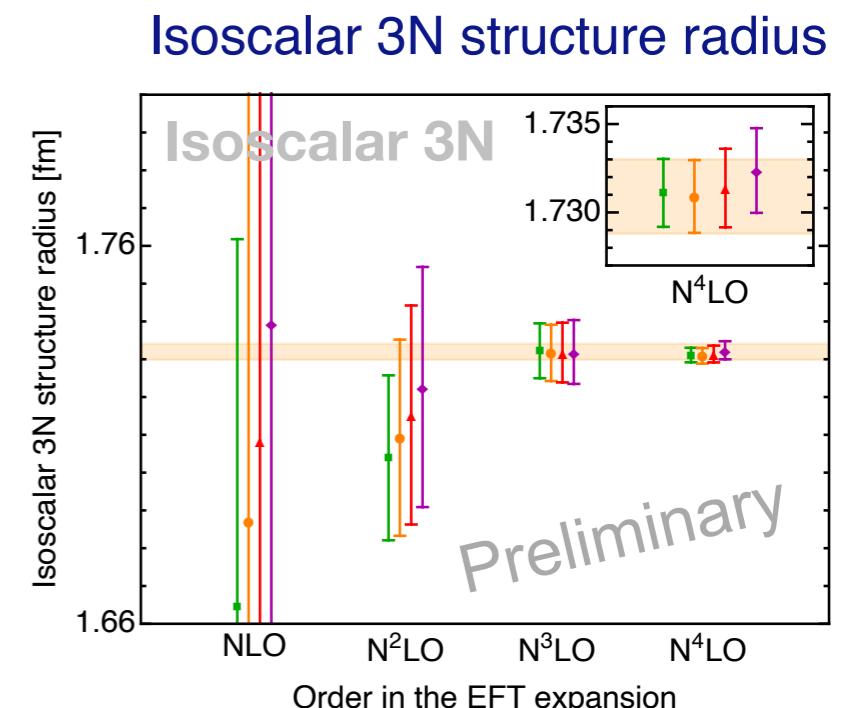
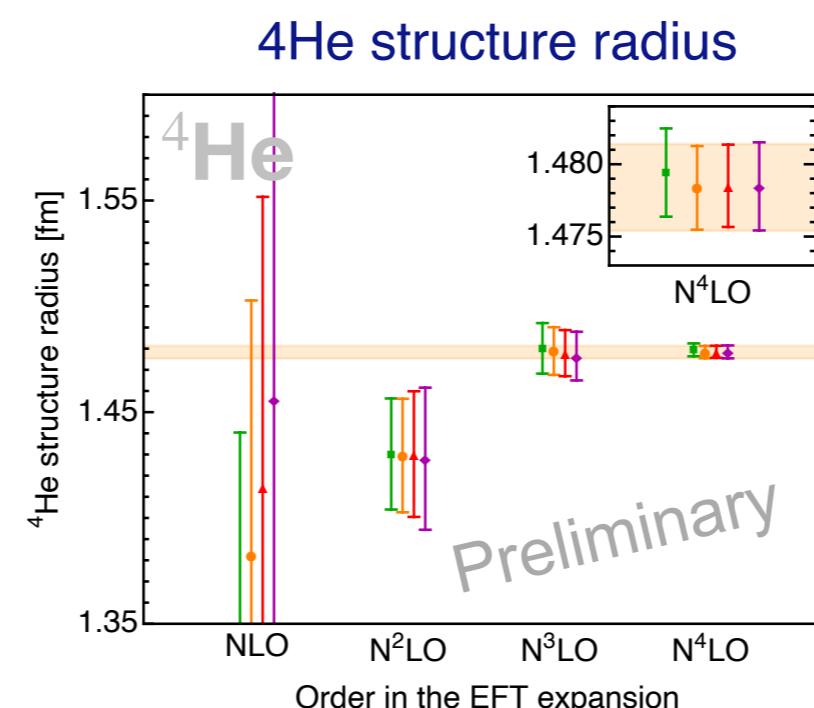
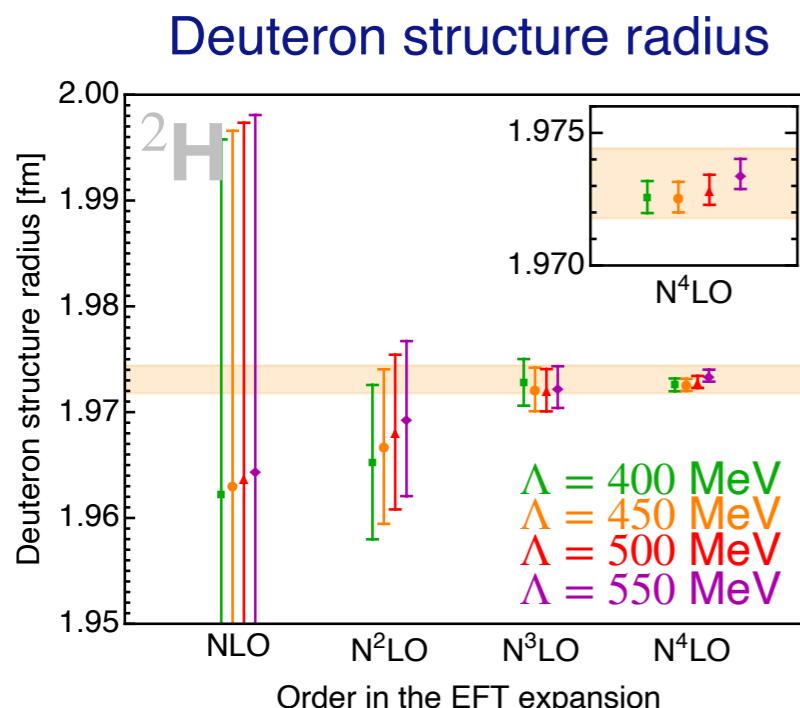
$$r_{str}(^2\text{H}) = 1.9729 \pm 0.0006_{\text{trunc}}^{+0.0012}_{-0.0008} \text{ stat} \text{ fm}$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

$$r_{str}(^4\text{He}) = 1.4784 \pm 0.0030_{\text{trunc}} \pm 0.0013_{\text{stat}} \pm 0.0007_{\text{num}} \text{ fm} \text{ (Preliminary)}$$

$$r_{str}(\text{Isoscalar 3N}) = 1.7309 \pm 0.0020_{\text{trunc}} \pm 0.0006_{\text{stat}} \pm 0.0002_{\text{iso-v}} \pm 0.0003_{\text{num}} \text{ (Preliminary)}$$

Using Bayesian model to estimate truncation uncertainty at each order Epelbaum et al. EPJA 56, 92 (2020)



error bands = χ EFT truncation uncertainty

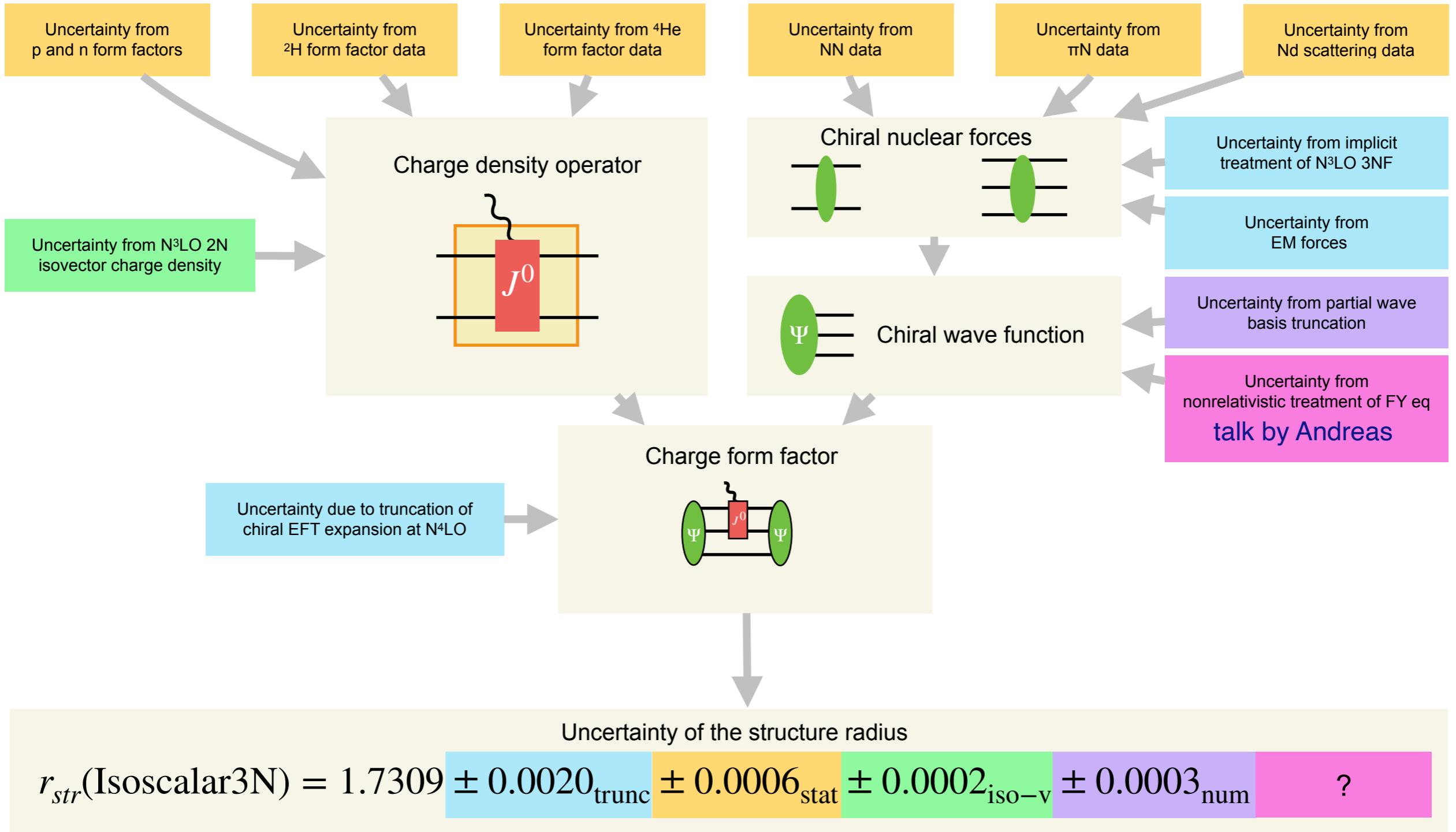
orange band = our prediction \pm total uncertainty

Chiral EFT expansion converges well

Regulator dependence is smaller than the truncation uncertainty

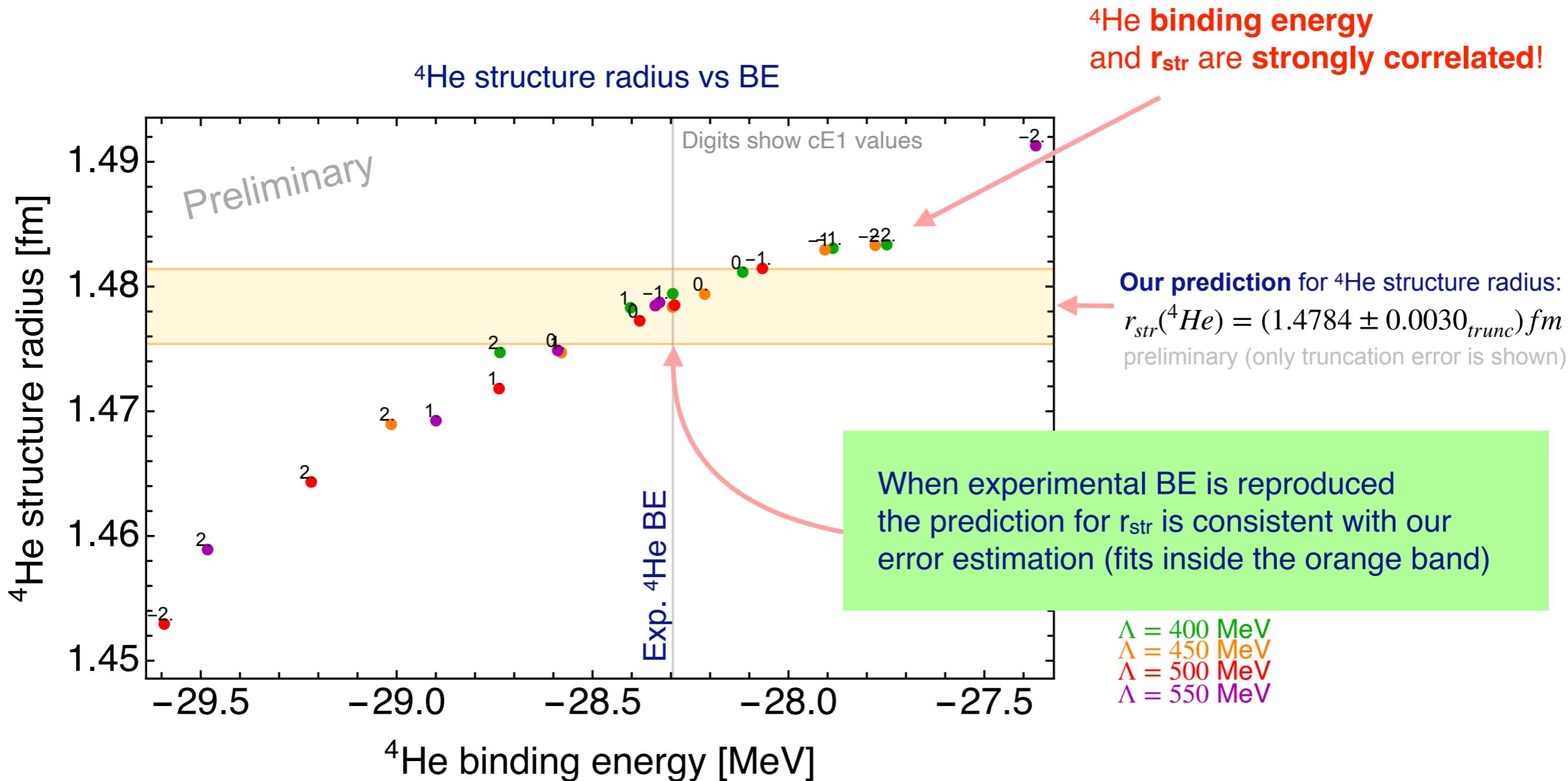
Extensive uncertainty analysis

Propagation of uncertainties from data and theory



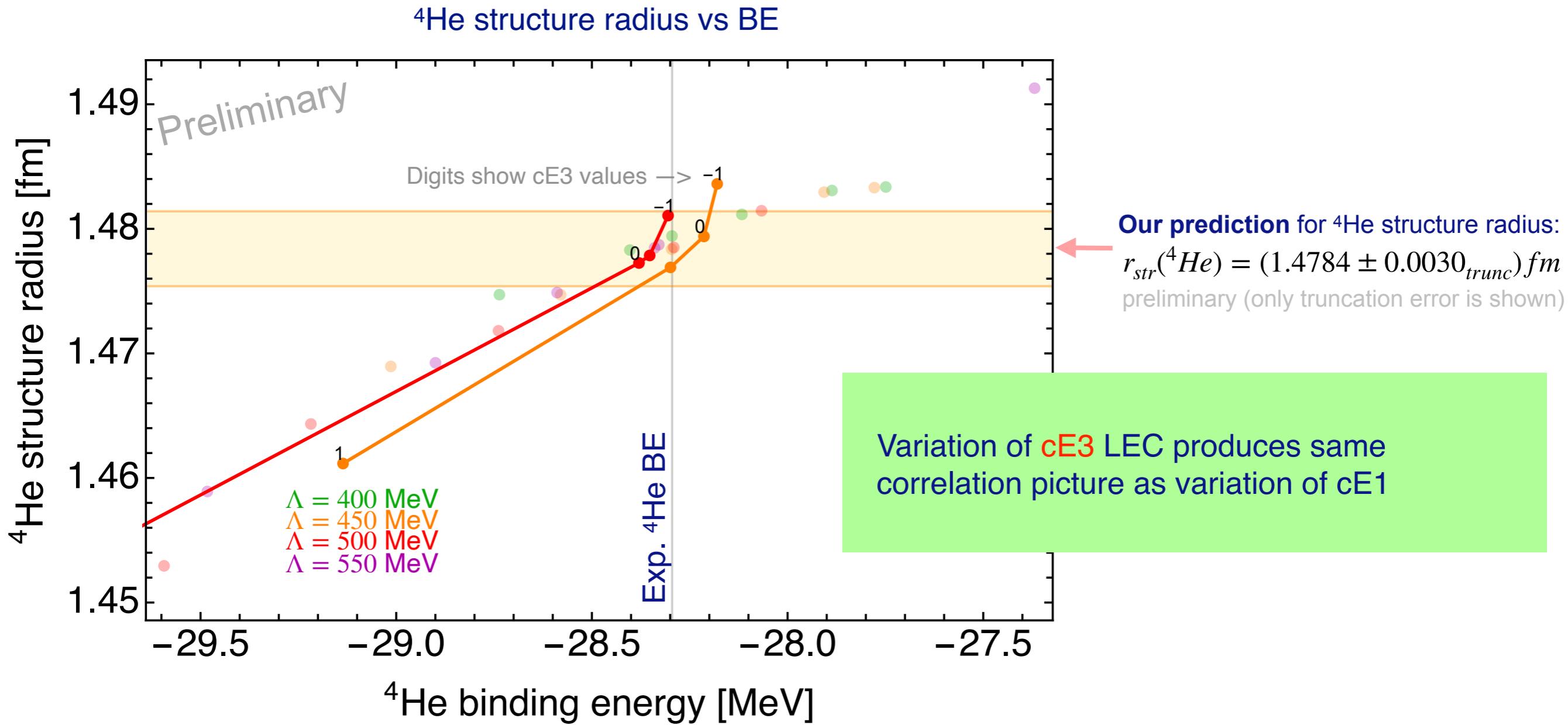
Correlation between ${}^4\text{He}$ structure radius and binding energy

using variation of **cE1** (N⁴LO 3NF LEC)



Correlation between ${}^4\text{He}$ structure radius and binding energy

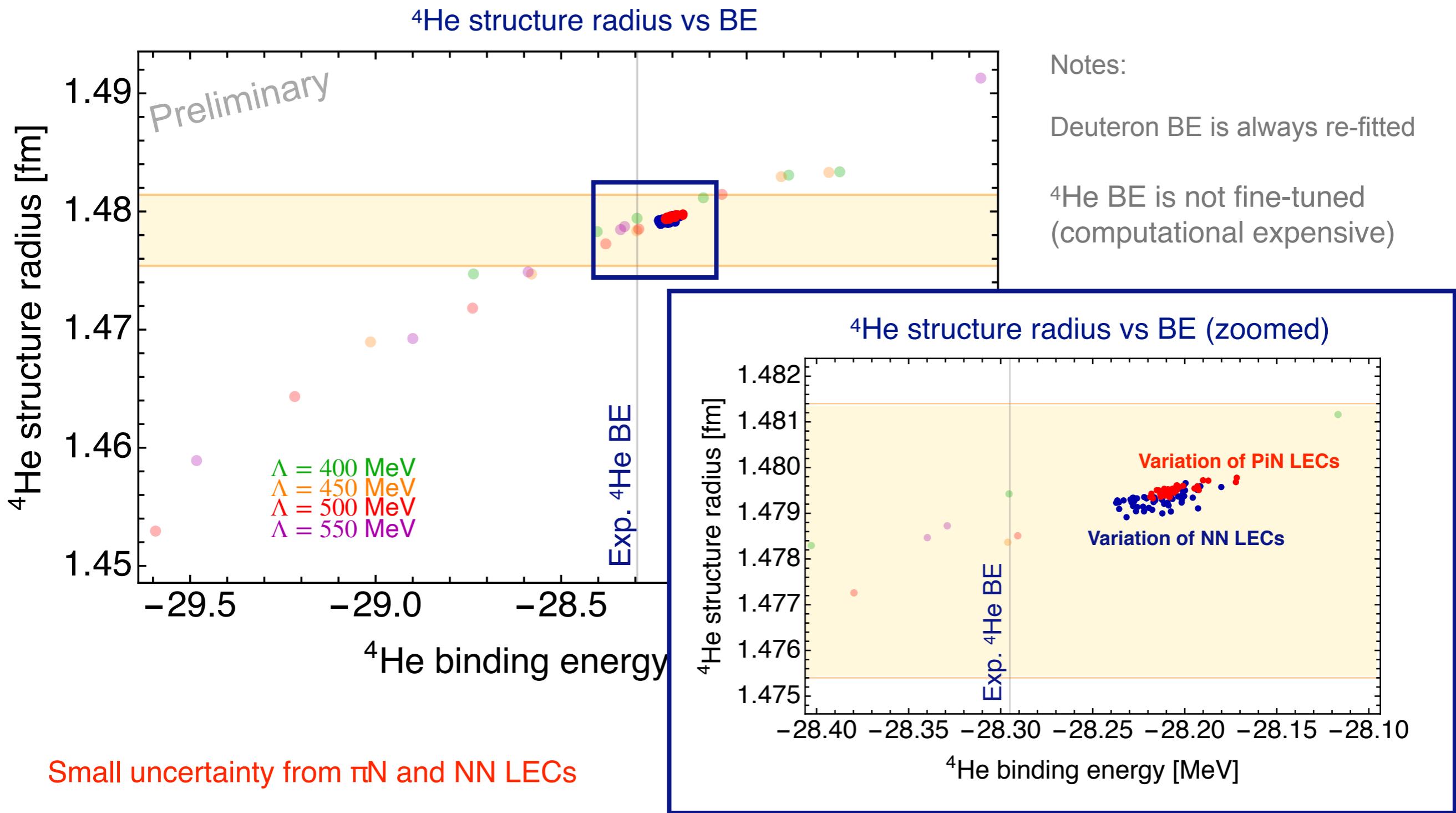
using variation of **cE3** (N⁴LO 3NF LEC)



r-BE correlations help to precisely extract r_{str} even with incomplete 3NF!

Propagation of uncertainty from πN and NN LECs

Prepared **50 sets of NN LECs** and **50 sets of πN LECs** correlation information from NN and RS analysis
Full calculation is repeated from scratch for each set



Applications

Relation between charge and structure radii

Nuclear **charge radius** can be decomposed into **structure**, **proton** and **neutron** radii

General $r_C^2 = r_{str}^2 + \left(\frac{r_p^2}{4m_p^2} + \frac{3}{4m_p^2} \right) + \frac{A-Z}{Z} r_n^2$

We focus on isoscalar A=2,3,4 radii

Deuteron $r_d^2 = r_{str}^2(^2\text{H}) + \left(\frac{r_p^2}{4m_p^2} + \frac{3}{4m_p^2} \right) + r_n^2$

${}^4\text{He}$ $r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \left(\frac{r_p^2}{4m_p^2} + \frac{3}{4m_p^2} \right) + r_n^2$

Isoscalar 3N $\frac{r_C^2(^3\text{H}) + 2r_C^2(^3\text{He})}{3} = \frac{r_{str}^2(^3\text{H}) + 2r_{str}^2(^3\text{He})}{3} + \left(\frac{r_p^2}{4m_p^2} + \frac{3}{4m_p^2} \right) + r_n^2$

Some applications of the accurate **xEFT calculation** of the **nuclear structure** radii:

- extract **proton** and **neutron** charge radii from precisely measured **nuclear charge radii**
- **predict** other nuclear charge radii

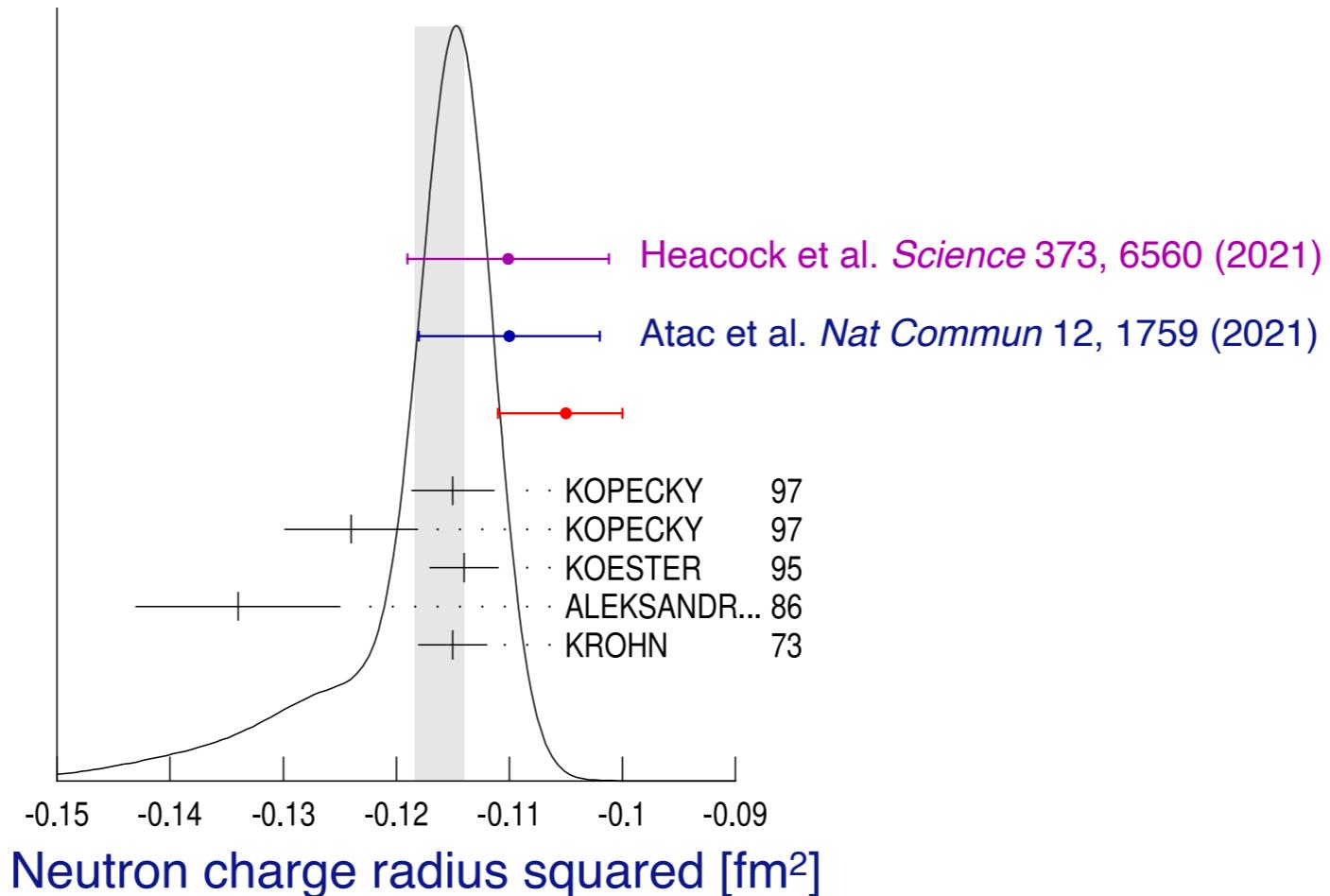
Extraction of the neutron charge radius

$$r_d^2 = r_{str}^2(^2\text{H}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2 \quad \rightarrow \quad r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$$

Extraction of the **neutron radius** from $(r_d^2 - r_p^2) = 3.82070(31)\text{fm}^2$ (atomic spectroscopy + QED corrections)

$$r_n^2 = -0.105^{+0.005}_{-0.006}\text{fm}^2$$

$\sim 2\sigma$ deviation from the **PDG (2020)** weighted average $r_n^2 = -0.1161(22)\text{fm}^2$



Neutron charge radius in PDG 2022

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm ²)	DOCUMENT ID	COMMENT
-0.1155±0.0017 OUR AVERAGE		
-0.115 ±0.002 ±0.003	KOPECKY 97	ne scattering (Pb)
-0.124 ±0.003 ±0.005	KOPECKY 97	ne scattering (Bi)
-0.114 ±0.003	KOESTER 95	ne scattering (Pb, Bi)
-0.115 ±0.003	¹ KROHN 73	ne scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •		
-0.1101±0.0089	² HEACOCK 21	n interferometry
-0.106 ^{+0.007} _{-0.005}	³ FILIN 20	chiral EFT analysis
-0.117 ^{+0.007} _{-0.011}	BELUSHKIN 07	Dispersion analysis
-0.113 ±0.003 ±0.004	KOPECKY 95	ne scattering (Pb)
-0.134 ±0.009	ALEKSANDR... 86	ne scattering (Bi)
-0.114 ±0.003	KOESTER 86	ne scattering (Pb, Bi)
-0.118 ±0.002	KOESTER 76	ne scattering (Pb)
-0.120 ±0.002	KOESTER 76	ne scattering (Bi)
-0.116 ±0.003	KROHN 66	ne scattering (Ne, Ar, Kr, Xe)

¹ KROHN 73 measured -0.112 ± 0.003 fm². This value is as corrected by KOESTER 76.

² HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

³ FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

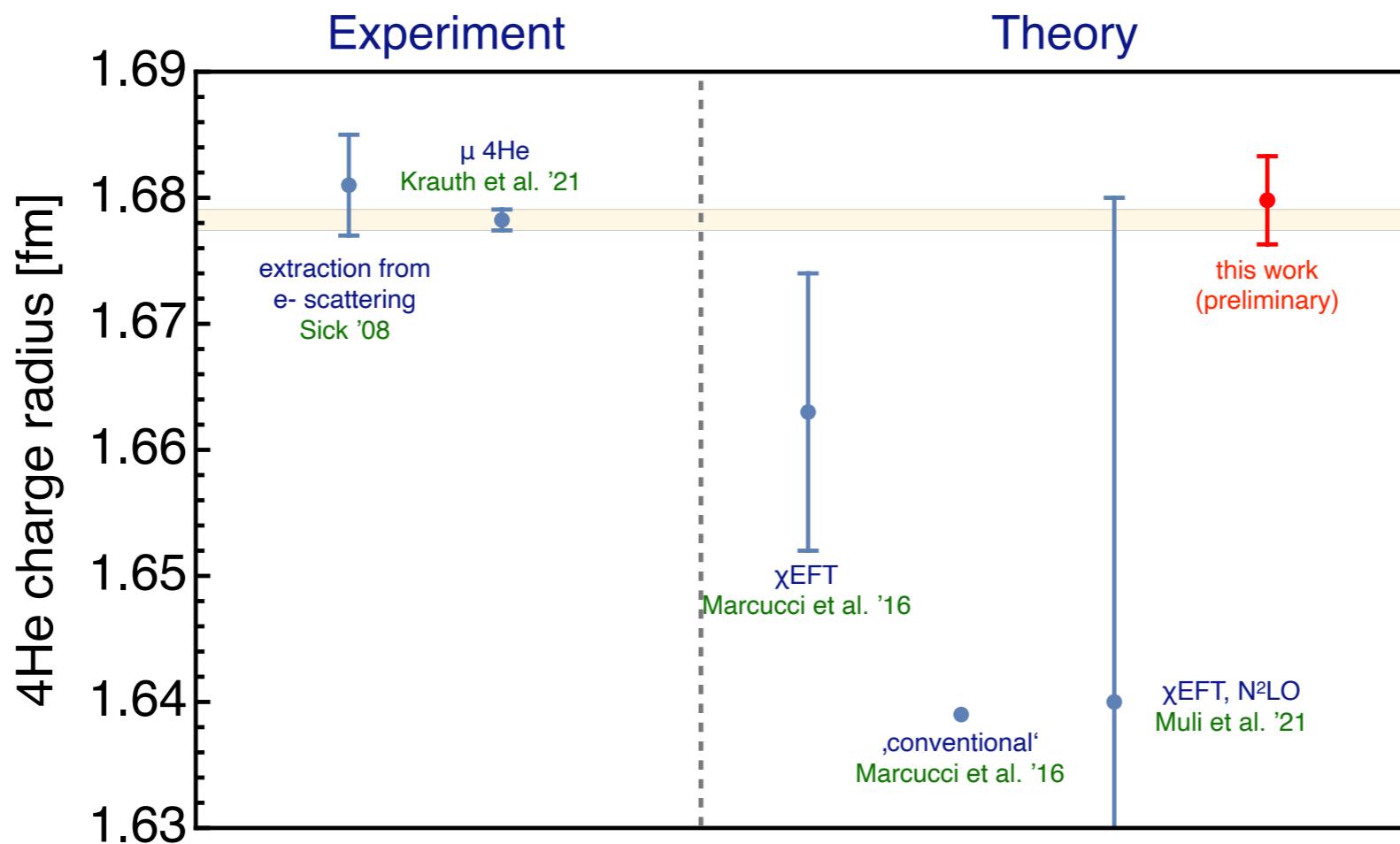
^4He charge radius: effective field theory and experiment

$$r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \left(\frac{3}{4m_p^2} + r_p^2 \right) + r_n^2$$

Our prediction for ^4He charge radius

$$r_C(^4\text{He}) = (1.6798 \pm 0.0035) \text{ fm}$$

preliminary, using CODATA 2018 r_p and own determination of r_n



Our prediction for ^4He charge radius is fully consistent with the muonic-atom spectroscopy

Indications of BSM physics in ${}^4\text{He}$?

All data used to constrain chiral EFT LECs are from strong interaction / electron-based experiments:

πN Roy-Steiner analysis [Hoferichter:2015tha](#), [Hoferichter:2015hva](#)

NN pn and pp scattering data, deuteron BE [Reinert:2020mcu](#)

Deuteron charge and quadrupole FF data [JLABt20:2000qyq](#), [Nikolenko:2003zq](#)

Deuteron-proton radii difference from atomic spectroscopy [Pachucki:2018yxg](#), Jentschura et al. PRA 83 (2011)

Proton charge radius CODATA2018

${}^4\text{He}$ form factor data [Erich:1971rhg](#), [Mccarthy:1977vd](#), [VonGunten:1982yna](#), [Ottermann:1985km](#), [Frosch:1967pz](#),
[Arnold:1978qs](#), [Camsonne:2013df](#)

Binding energies of ${}^3\text{He}$ and ${}^4\text{He}$

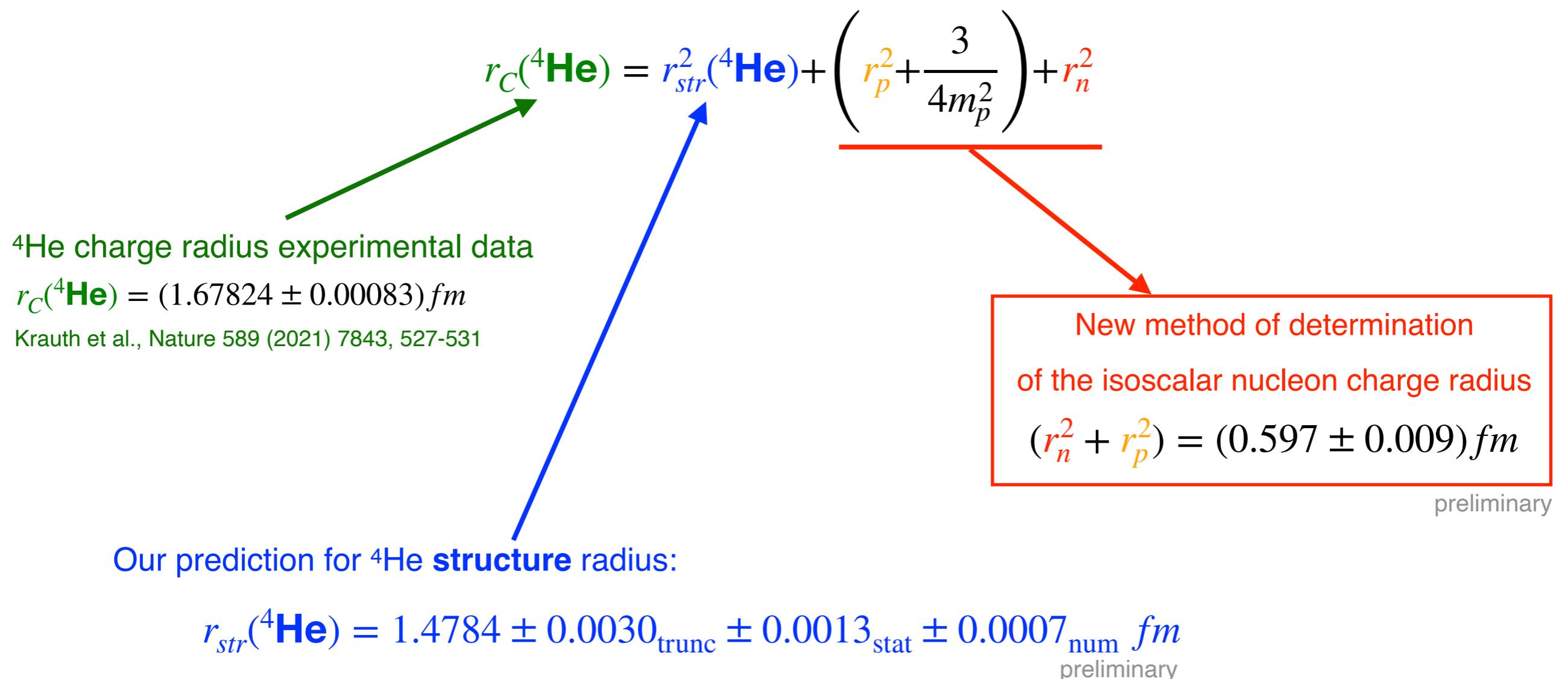
Nd DCS minimum @ 70 MeV [RIKEN](#) data

No muonic data is used in our chiral EFT predictions

Our prediction for ${}^4\text{He}$ charge radius is consistent with the muonic experiment

No indication of BSM physics in ${}^4\text{He}$ at this accuracy level

Isoscalar nucleon charge radius from experimental ${}^4\text{He}$ charge radius



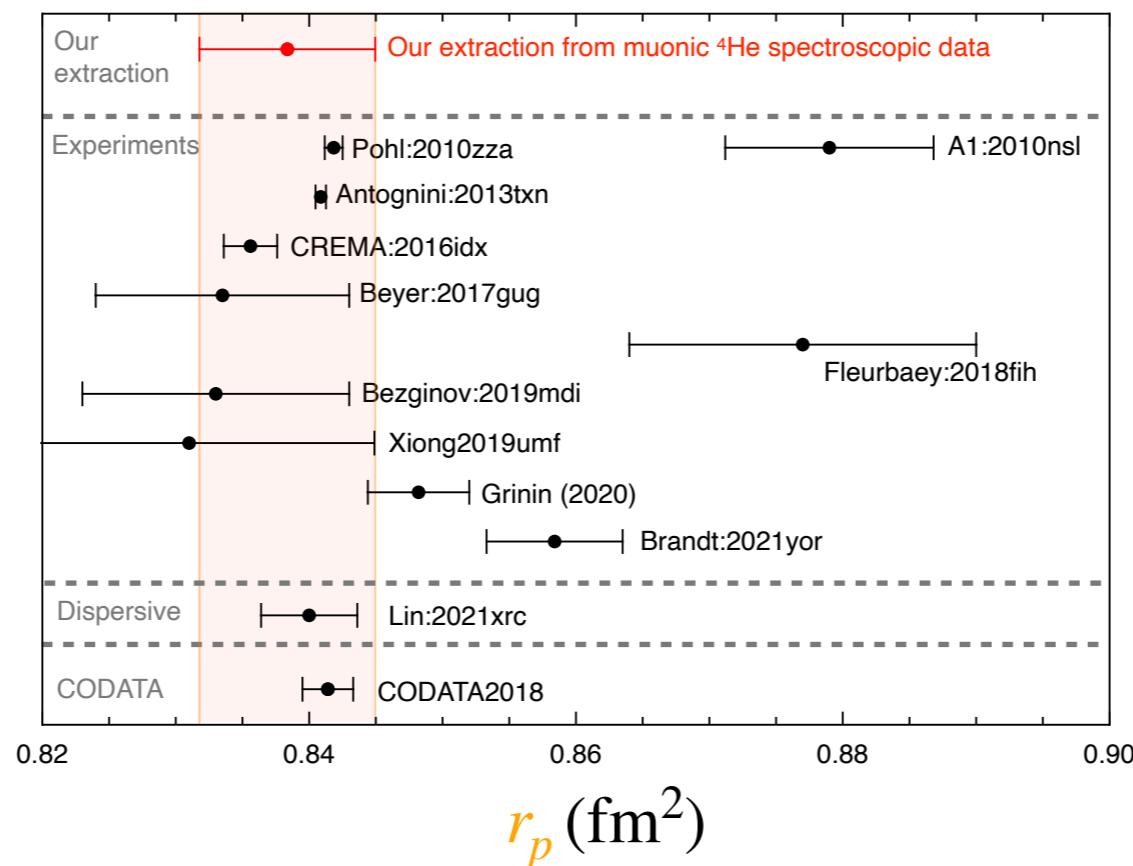
Proton charge radius from isoscalar nucleon radius

Our determination of the
isoscalar nucleon charge radius from ${}^4\text{He}$
 $(r_n^2 + r_p^2) = (0.597 \pm 0.009)\text{fm}^2$ preliminary

Our determination of the
neutron charge radius from ${}^2\text{H}$
 $r_n^2 = -0.105^{+0.005}_{-0.006}\text{fm}^2$
AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius: $r_p = (0.838 \pm 0.007)\text{fm}$

preliminary



Our extraction supports the „small“ proton radius

Our uncertainty is comparable with the experimental one

Prediction for isoscalar 3N charge radius

With all LECs being fixed, we can predict the isoscalar 3N charge radius:

$$\frac{r_C^2(^3\text{H}) + 2r_C^2(^3\text{He})}{3} = \frac{r_{str}^2(^3\text{H}) + 2r_{str}^2(^3\text{He})}{3} + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

$$r_C^{isoscalar3N} = \sqrt{\frac{r_C^2(^3\text{H}) + 2r_C^2(^3\text{He})}{3}} = (1.9058 \pm 0.0026) \text{ fm}$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Our result is **10x more precise** than current experimental data:

$$r_{C, exp.}^{isoscalar3N} = (1.9030 \pm 0.0290) \text{ fm}$$

using muonic ${}^3\text{He}$ and old ${}^3\text{H}$:

$$r_C^{^3\text{He}} = (1.9687 \pm 0.0013) \text{ fm} \quad \text{Pohl '20 (preliminary)}$$

$$r_C^{^3\text{H}} = (1.7550 \pm 0.0860) \text{ fm} \quad \text{Amroun et al. '94 (world average)}$$

T-REX experiment in Mainz [Pohl et al.] aims at measuring $r_C^{^3\text{H}}$ within $\pm 0.0002 \text{ fm}$ (400x more precise)

The isoscalar 3N radius will be then known within $\pm 0.0009 \text{ fm}$

⇒ **precision tests of nuclear chiral EFT!**

Estimation of ${}^3\text{H}$ charge radius

Our **isoscalar 3N charge** radius calculation:

$$r_C^{isoscalar3N} = \sqrt{\frac{r_C^2({}^3\text{H}) + 2r_C^2({}^3\text{He})}{3}} = (1.9058 \pm 0.0026) \text{ fm}$$

preliminary, using CODATA 2018 r_p and own determination of r_n

${}^3\text{H}$ charge radius:

$$(r_C^{3H})^2 = 3(r_C^{isoscalar3N})^2 - 2(r_C^{3He})^2$$

Preliminary ${}^3\text{He}$ charge radius [Pohl et al.]

$$r_C^{3He} = (1.9687 \pm 0.0013) \text{ fm}$$

Coefficients 2 and 3 amplify both theoretical and experimental uncertainties

Our ${}^3\text{H}$ radius estimation:

$$r_C^{(3H)} = (1.7734 \pm 0.0088) \text{ fm}$$

preliminary

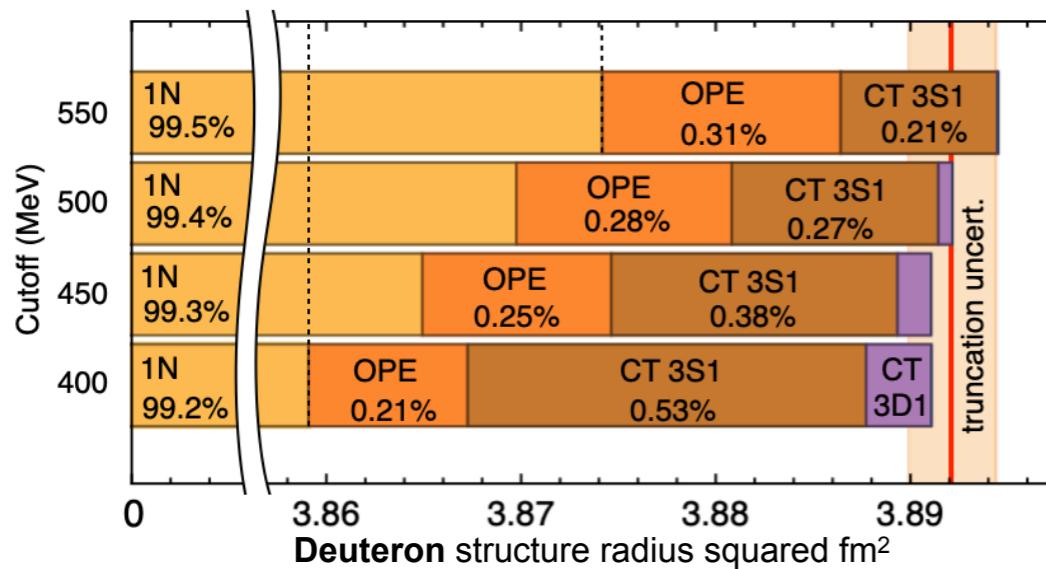
This estimation is 10x more precise than e- data $r_C^{3H} = (1.7550 \pm 0.0860) \text{ fm}$ Amroun et al. '94 (world average)

But it suffers from parametric amplification of uncertainties (both from theory and from ${}^3\text{He}$ data)

=> isoscalar 3N charge radius should be used for precision tests

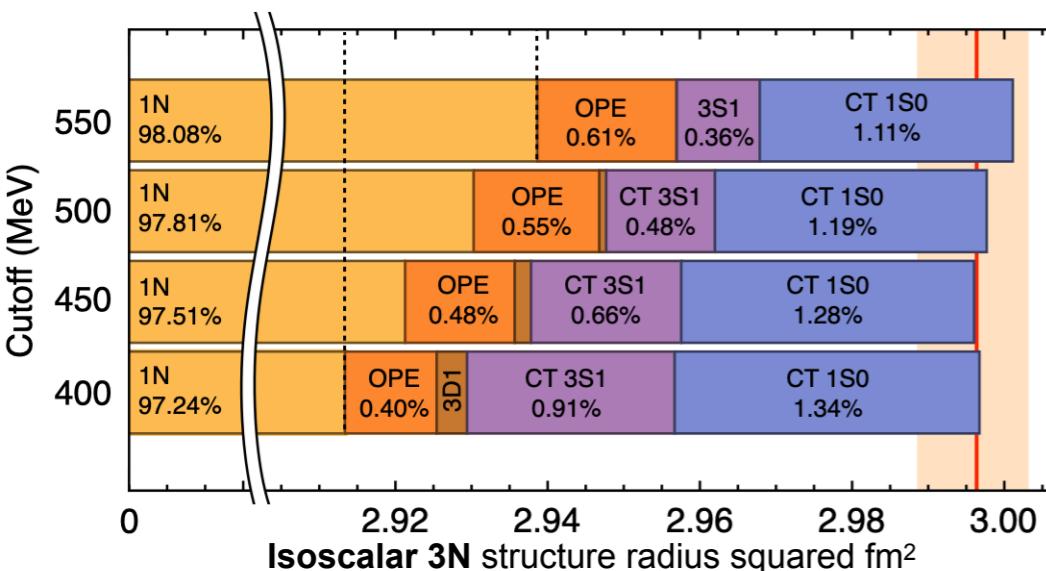
Takeaway from $A=2,3,4$ calculations

Importance of 2N charge density



Individual contributions to $A=2,3,4$ stucture radii from

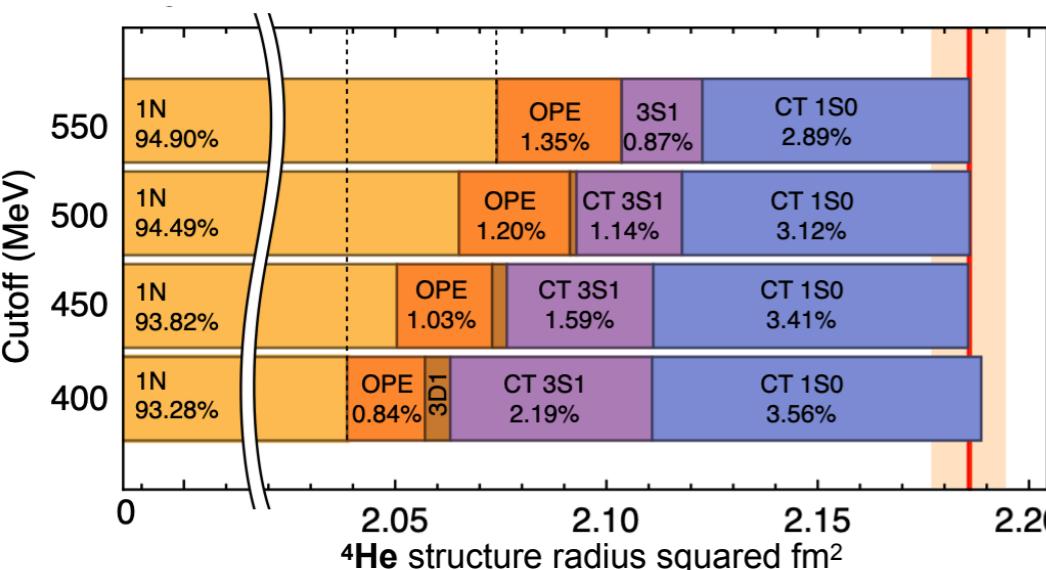
- single-nucleon charge density (1N)
- 2N one-pion exchange density (OPE)
- 2N contact densities (CT 3S1, 3D1, 1S0)



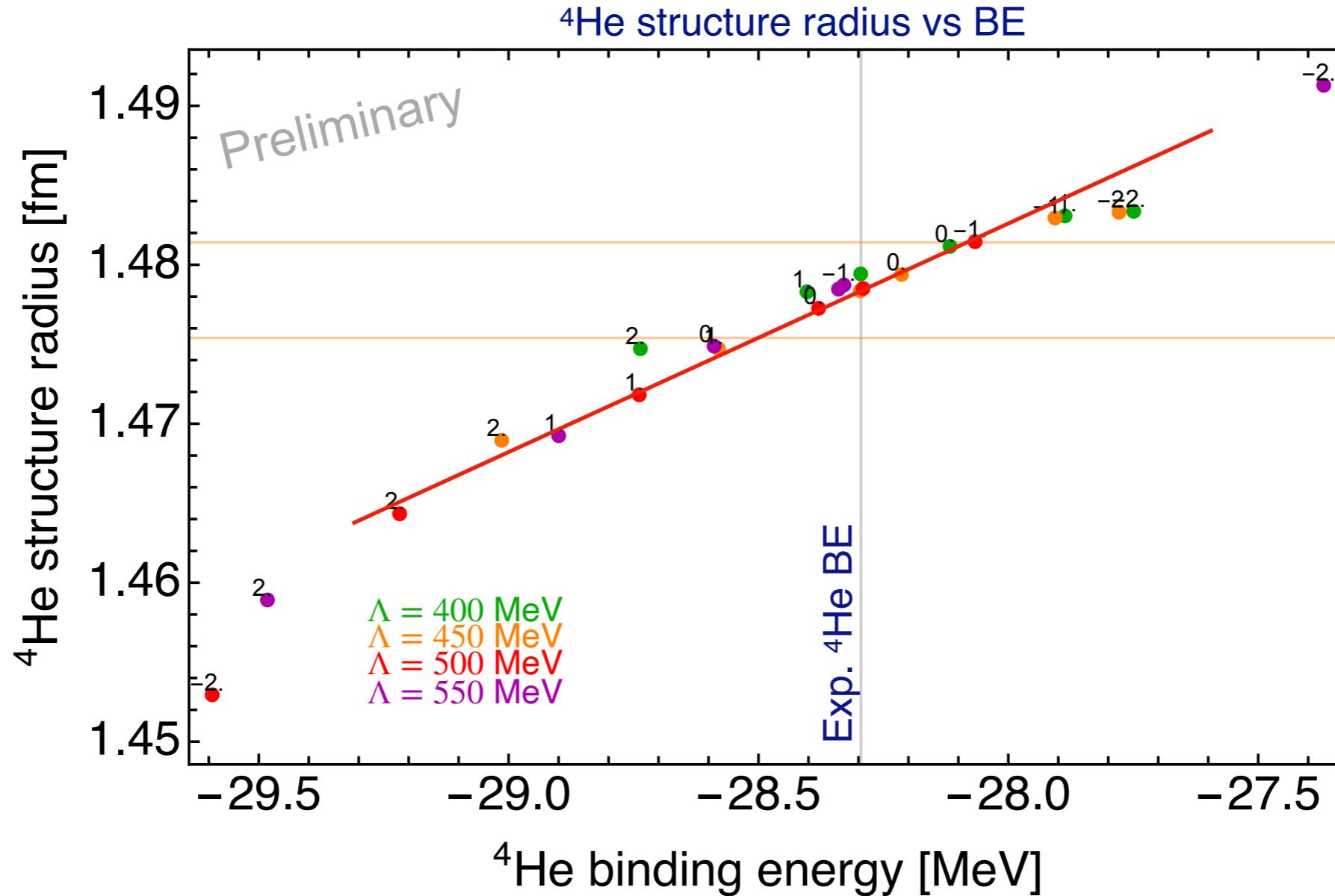
2N charge density contribution to structure radii squared:

deuteron	$\sim 0.7\%$
isoscalar 3N	$\sim 2.5\%$
^4He	$\sim 6\%$

For $A=2,3,4$ importance of 2N charge grows with A



Importance of correct binding energy



Charge radius is correlated with BE

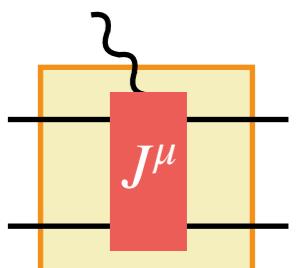
- correlation helps to calculate radii with incomplete 3NFs
- effects of over/underbinding cannot be compensated by short range charge density

Summary

Precise calculation of $A = 2, 3, 4$ charge radii in chiral effective field theory & uncertainty analysis

Isoscalar 2N charge density operators

- derived, regularized, PW-decomposed and **LECs are fixed**
- produce important contributions to $A=2,3,4$ structure radii (**importance grows with A**)
- can be used to calculate corrections to charge radii of $A>4$ nuclei



Applications and tests: preliminary

${}^4\text{He}$:

- calculated ${}^4\text{He}$ charge radius (0.2% accuracy) agrees with the new $\mu{}^4\text{He}$ measurement

${}^3\text{H}-{}^3\text{He}$:

- predicted the isoscalar 3N charge radius r_c (0.1% accuracy)
- our r_c is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX (${}^3\text{H}$) exp. in Mainz will allow for a precision test of nuclear chiral EFT

p and n charge radii from light nuclei:

- ${}^2\text{H}$ r_{str} combined with isotope-shift data => extracted the neutron charge radius (2 σ tension with PDG)
- ${}^4\text{He}$ r_{str} combined with spectroscopic data => extracted isoscalar nucleon and proton charge radii

Outlook

- estimation of the effects from non-relativistic treatment of the SE (talk by Anderas)
- Analysis of **magnetic form factors** of ^2H , ^3H and ^3He (talk by Daniel)
- **Consistent inclusion of N³LO, N⁴LO three-nucleon forces**
- **Consistent inclusion of isovector currents** (individual predictions for ^3H and ^3He)
- Application to processes with two photons (**polarizabilities**, ...)