Electron and neutrino scattering off the deuteron

Yet another relativistic approach

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Outline

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Inclusive neutrino scattering off the deuteron from threshold to GeV energies

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Background: Neutrino-nucleus quasi-elastic scattering is crucial to interpret the neutrino oscillation results in long baseline neutrino experiments. There are rather large uncertainties in the cross section, due to insufficient knowledge on the role of two-body weak currents.

Purpose: Determine the role of two-body weak currents in neutrino-deuteron quasi-elastic scattering up to GeV energies.

Methods: Calculate cross sections for inclusive neutrino scattering off deuteron induced by neutral and charge-changing weak currents, from threshold up to GeV energies, using the Argonne v_{18} potential and consistent nuclear electroweak currents with one- and two-body terms.

Results: Two-body contributions are found to be small, and increase the cross sections obtained with one-body currents by less than 10% over the whole range of energies. Total cross sections obtained by describing the final two-nucleon states with plane waves differ negligibly, for neutrino energies $\gtrsim$500 MeV, from those in which interaction effects in these states are fully accounted for. The sensitivity of the calculated cross sections to different models for the two-nucleon potential and/or two-body terms in the weak current is found to be weak. Comparing cross sections to those obtained in a naive model in which the deuteron is taken to consist of a free proton and neutron at rest, nuclear structure effects are illustrated to be non-negligible.

Conclusion: Contributions of two-body currents in neutrino-deuteron quasi-elastic scattering up to GeV are found to be smaller than 10%. Finally, it should be stressed that the results reported in this work do not include pion production channels.

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Relativistic effects in neutron-deuteron elastic scattering

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We solved the three-nucleon ($3N$) Faddeev equation including relativistic features at incoming neutron lab energies $E_n^{\mathrm{lab}} = 28$, 65, 135, and 250 MeV. Those features are relativistic kinematics, boost effects and Wigner spin rotations. As dynamical input a relativistic nucleon-nucleon (NN) interaction exactly on-shell equivalent to the AV18 NN potential has been used. The effects of Wigner rotations for elastic scattering observables were found to be small. The boost effects are significant at higher energies. They diminish the transition matrix elements at higher energies and lead in spite of the increased relativistic phase-space factor as compared to the nonrelativistic one to rather small effects in the cross section, which are mostly restricted to the backward angles.

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Realistic two-nucleon potentials for the relativistic two-nucleon Schrödinger equation

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Abstract

The potentials V (v) in the nonrelativistic (relativistic) nucleon–nucleon (NN) Schrödinger equation are related by a quadratic equation. That equation is numerically solved, thus providing phase equivalent v-potentials related for instance to the high precision NN potentials, which are adjusted to NN phase shift and mixing parameters in a nonrelativistic Schrödinger equation. The relativistic NN potentials embedded in a three-nucleon (3N) system for total NN momenta different from zero are also constructed in a numerically precise manner. They enter into the relativistic interacting 3N mass operator, which is needed for relativistic 3N calculations for bound and scattering states.

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Relativity and the low-energy *nd* A_y puzzle

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We solve the Faddeev equation in an exactly Poincaré invariant formulation of the three-nucleon problem. The dynamical input is a relativistic nucleon-nucleon (NN) interaction that is exactly on-shell equivalent to the high-precision CD Bonn NN interaction. S-matrix cluster properties dictate how the two-body dynamics is embedded in the three-nucleon mass operator (rest Hamiltonian). We find that for neutron laboratory energies above ≈20 MeV relativistic effects on A_y are negligible. For energies below ≈20 MeV dynamical effects lower the nucleon analyzing power maximum slightly by ≈2% and Wigner rotations lower it further up to ≈10%, thereby increasing disagreement between data and theory. This indicates that three-nucleon forces (3NF) must provide an even larger increase of the A_y maximum than expected up to now.

Formalism

In our approach a relativistically invariant quantum mechanical model is defined by a unitary representation, *U(Λ, a),* of the Poincaré group acting on the Hilbert space of the theory.

(1) For a particle of mass *m* and spin *j* a single-particle unitary representation of the Poincaré group is given on the single-particle basis states by:

$$U(B(\mathbf{p}/m))|(m,j)\mathbf{0},\mu\rangle = \sqrt{\frac{E(\mathbf{p})}{m}}\,|(m,j)\mathbf{p},\mu\rangle$$

$$U(\Lambda,a)|(m,j)\mathbf{p},\mu\rangle =$$

$$e^{-ip'\cdot a}\sum_{\nu=-j}^{j}|(m,j)\mathbf{p}',\nu\rangle\sqrt{\frac{E(\mathbf{p}')}{E(\mathbf{p})}}D^{j}_{\mu\nu}[R_w(\Lambda,p)]$$

$$B(\mathbf{p}/m)^0{}_0 = E(\mathbf{p})/m, \qquad B(\mathbf{p}/m)^i{}_0 = B(\mathbf{p}/m)^0{}_i = p^i/m,$$

$$B(\mathbf{p}/m)^i{}_j = \delta^{ij} + p^i p^j/(m(m+E(\mathbf{p})).$$

$$R_w(\Lambda,p) := B^{-1}(\mathbf{p}'/m)\Lambda B(\mathbf{p}/m)$$

rotationless Lorentz transformation that maps *(m, 0, 0, 0)* to *(E(**p**), **p**)*

$E(\mathbf{p}) = \sqrt{m^2+\mathbf{p}^2}$

$p'^{\mu} := \Lambda^{\mu}{}_{\nu}p^{\nu}$

Wigner **rotation !**

In this representation the spin observable in an arbitrary frame is defined as the spin that would be measured in the particle's rest frame if it was boosted to the rest frame using $B^{-1}(\mathbf{p}/m)$.

Dirac spinors as defined in the Bjorken and Drell's textbook provide for j= ½ a four dimensional representation of the Poincare group with the so-called canonical spin !

(2) The free dynamics on a multi-particle Hilbert space is given by the tensor product of the single particle unitary representations, $U_i(\Lambda, a)$, of the Poincaré group:

$$U_0(\Lambda, a) = \otimes_i U_i(\Lambda, a)$$

The components of the free four momentum and the free Lorentz generators are the infinitesimal generators of $U_0(\Lambda, a)$. The free mass Casimir operator, M_0, and the components of the canonical spin operator, $\mathbf{j}_0^i$, are functions of the infinitesimal generators.

(3) Dynamical representation of the Poincare group is achieved through:

1. Building simultaneous eigenstates of the commuting observables for the noninteracting system (*M_0*, momentum, $\mathbf{j}_0^2$, $\mathbf{j}_{0,z}$)
2. Adding an interaction *V* to *M_0 : M = M_0 + V,* where *V* fulfills

$$\langle (m', j')\mathbf{p}', \mu', d' | V | (m, j)\mathbf{p}, \mu, d \rangle = \delta(\mathbf{p}' - \mathbf{p}) \delta_{j'j} \delta_{\mu\mu'} \langle m' \, d' \| V^j \| m \, d \rangle$$

d (d') – degeneracy parameters; typically invariant masses and squares of angular momenta of subsystems

3. Diagonalizing *M* in the basis from point 1 yields simultaneous eigenstates of (*M,* momentum, $\mathbf{j}_0^2$, $\mathbf{j}_{0,z}$). These eigenstates are complete on the N-body Hilbert space.
4. The dynamical representation of the Poincaré group is defined by requiring that the eigenstates from point 3 transform like the single-particle states with the mass *m* replaced by the eigenvalue of *M*, m_I :

$$U(\Lambda, a) | (m_I, j)\mathbf{p}, \mu, \tilde{d} \rangle =$$

$$e^{-ip' \cdot a} \sum_{\nu=-j}^{j} | (m_I, j)\mathbf{p}', \nu, \tilde{d} \rangle \sqrt{\frac{E(\mathbf{p}')}{E(\mathbf{p})}} D^j_{\mu\nu}[R_w(\Lambda, p)]$$

$$E(\mathbf{p}) = \sqrt{m_I^2 + \mathbf{p}^2}$$

This construction is due to Bakamjian and Thomas and corresponds to what Dirac called „instant-form dynamics”. It is formally applicable to calculations at energies below the threshold for pion production.

In our work the construction discussed above will be used to model the strong interactions; the electromagnetic and weak interactions are treated using the **one-boson exchange approximation**.

In the one-boson exchange approximation *U(Λ, a)* factors into a tensor product of unitary representations of the Poincaré group for the strongly interacting baryons (B) and the leptons (L):

$$U(\Lambda, a) \approx U_B(\Lambda, a) \otimes U_L(\Lambda, a)$$

A current couples to the exchanged boson. The current is a sum of a lepton and strong (here nuclear) current plus an interaction current, which does not contribute in the one-boson exchange approximation:

$$J^\mu(x) = J^\mu_B(x) + J^\mu_L(x) + J^\mu_I(x) \to J^\mu_B(x) + J^\mu_L(x)$$

The strong and lepton currents transform covariantly (*A=B,L*)

$$U_A(\Lambda, a) J^\mu_A(x) U_A(\Lambda, a) = (\Lambda^{-1})^\mu{}_\nu J^\nu_A(\Lambda x + a)$$

The strong and lepton currents are sums of one-body, two-body, …, operators

$$J_A^\mu(x) = \sum_i J_{Ai}^\mu(x) + \frac{1}{2}\sum_{i\neq j} J_{Aij}^\mu(x) + \cdots$$

The lepton currents can be approximately treated at tree level.

The many-body parts of the baryon currents must be non-zero in order to satisfy the covariance and current conservation !
This follows from the commutation relations of the current operator with the dynamical generators of the Poincaré group. The cluster expansions for the current and rotationless boost generators, ***K***, have the form

$$J_B^\mu(0) = J_{B0}^\mu(0) + J_{BI}^\mu(0)\,, \qquad J_{B0}^\mu(0) = \sum_i J_{Bi}^\mu(0)$$

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I\,, \qquad \mathbf{K}_0 = \sum_i \mathbf{K}_i$$

$$[K_0^i + K_I^i, J^i(0)] = iJ^0(0)\,, \qquad [K_0^i + K_I^i, J^0(0)] = iJ^i(0)$$

By cancelling the one-body terms, one arrives at the following condition:

$$[K_0^i, J_I^i(0)] + [K_I^i, J_I^i(0)] - iJ_I^0(0) = [J_0^i(0), K_I^i],$$

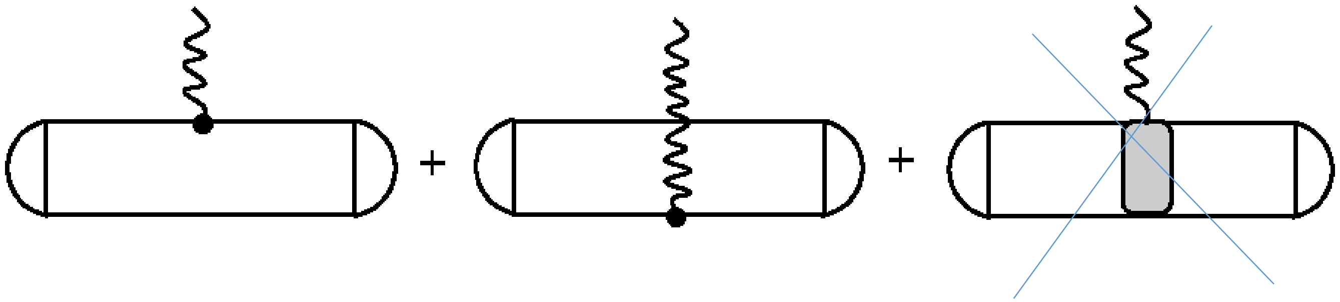
$$[K_0^i, J_I^0(0)] + [K_I^i, J_I^0(0)] - iJ_I^0(0) = [J_0^0(0), K_I^i].$$

If the right side of either equation is non zero then the current must have many-body parts in order to satisfy current covariance. Similar conditions follow if the current is conserved.

These many-body contributions to the current appear in addition to the many-body currents that arise from physical processes such as exchange of charged mesons. They are **not** uniquely determined from current covariance.

One way to ensure covariance is to use current matrix elements. Since current matrix elements transform covariantly, all current matrix elements can be generated from any independent set of matrix elements using covariance.

Elastic electron scattering off the deuteron



$$\langle \mathbf{p}'_D, \mu'_D, D | J^{\mu}_{nuc}(0) | \mathbf{p}_D, \mu_D, D \rangle$$

$$d\sigma = \frac{(2\pi)^4 E_D E_e}{\sqrt{(p_D \cdot p_e)^2 - m_D^2 m_e^2}} |\langle \mathbf{p}'_D, \mu'_D, D, \mathbf{p}'_e, \mu'_e \| T_{eD} \| \mathbf{p}_D, \mu_D, D, \mathbf{p}_e, \mu_e \rangle|^2$$

$$\times E'_D E'_e \delta^4(p_D + p_e - p'_D - p'_e) \frac{d\mathbf{p}'_D}{E'_D} \frac{d\mathbf{p}'_e}{E'_e}$$

$$\langle \mathbf{p}'_D, \mu'_D, D, \mathbf{p}'_e, \mu'_e \| T_{eD} \| \mathbf{p}_D, \mu_D, D, \mathbf{p}_e, \mu_e \rangle =$$

$$-e^2 (2\pi)^3 \langle \mathbf{p}'_D, \mu'_D, D | J^{\mu}_{nuc,EM}(0) | \mathbf{p}_D, \mu_D, D \rangle \frac{g_{\mu\nu}}{(p'_e - p_e)^2 + i\epsilon} \langle \mathbf{p}'_e, \mu'_e | J^{\nu}_e(0) | \mathbf{p}_e, \mu_e \rangle ,$$

$$\langle \mathbf{p}_e', \mu_e' | J_e^\nu(0) | \mathbf{p}_e, \mu_e \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_e^2}{E_e E_e'}} \bar{u}_e(\mathbf{p}_e', \mu_e') \gamma^\nu u_e(\mathbf{p}_e, \mu_e)$$

electron current matrix element

$$\equiv \frac{1}{(2\pi)^3} \sqrt{\frac{1}{4E_e E_e'}} L_e^\nu (\mathbf{p}_e', \mu_e', \mathbf{p}_e, \mu_e) \ ,$$

(this form can be also used for neutrino scattering)

$$\langle \mathbf{p}_D', \mu_D', D | J^\mu_{nuc,EM}(0) | \mathbf{p}_D, \mu_D, D \rangle$$

$$= \int d\mathbf{p}' \sum_{l'=0,2} \int dk' k'^2 \int d\mathbf{p}_1' \sum_{\mu_1', \tau_1'} \int d\mathbf{p}_2' \sum_{\mu_2', \tau_2'}$$

$$\int d\mathbf{p}_1 \sum_{\mu_1, \tau_1} \int d\mathbf{p}_2 \sum_{\mu_2, \tau_2} \int d\mathbf{p} \sum_{l=0,2} \int dk k^2$$

$$\boxed{\langle \mathbf{p}_D', \mu_D', D | (1, k') \mathbf{p}', \mu'; l', 1; 0, 0 \rangle}$$

$$\boxed{\langle (1, k') \mathbf{p}', \mu'; l', 1; 0, 0 | \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' \rangle}$$

$$\langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | J^\mu_{nuc,EM}(0) | \mathbf{p}_1, \mu_1, \tau_1, \mathbf{p}_2, \mu_2, \tau_2 \rangle$$

$$\boxed{\langle \mathbf{p}_1, \mu_1, \tau_1, \mathbf{p}_2, \mu_2, \tau_2 | (1, k) \mathbf{p}, \mu; l, 1; 0, 0 \rangle}$$

$$\boxed{\langle (1, k) \mathbf{p}, \mu; l, 1; 0, 0 | \mathbf{p}_D, \mu_D, D \rangle} \ ,$$

deuteron current matrix element

???

At present the nuclear matrix elements comprise contributions only from the single-nucleon current:

$$
\begin{aligned}
&\langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | J^{\mu}_{nuc,EM}(0) | \mathbf{p}_1, \mu_1, \tau_1, \mathbf{p}_2, \mu_2, \tau_2 \rangle \\
=\;& \delta(\mathbf{p}_1' - \mathbf{p}_1)\, \delta_{\mu_1'\mu_1}\, \delta_{\tau_1'\tau_1}\, \langle \mathbf{p}_2', \mu_2', \tau_2' | J^{\mu}_{2,EM}(0) | \mathbf{p}_2, \mu_2, \tau_2 \rangle \\
+\;& \delta(\mathbf{p}_2' - \mathbf{p}_2)\, \delta_{\mu_2'\mu_2}\, \delta_{\tau_2'\tau_2}\, \langle \mathbf{p}_1', \mu_1', \tau_1' | J^{\mu}_{1,EM}(0) | \mathbf{p}_1, \mu_1, \tau_1 \rangle \\
&\cancel{+\; \langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | J^{\mu}_{\{1,2\},EM}(0) | \mathbf{p}_1, \mu_1, \tau_1, \mathbf{p}_2, \mu_2, \tau_2 \rangle}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathbf{p}', \mu', \tau' | J^{\mu}_{k,EM}(0) | \mathbf{p}, \mu, \tau \rangle &= \delta_{\tau'\tau} \frac{1}{(2\pi)^3} \sqrt{\frac{m^2}{E(\mathbf{p}')E(\mathbf{p})}} \\
\times\, \bar{u}(\mathbf{p}', \mu') \left(\gamma^{\mu} F_{1,\tau}(Q^2) + i \frac{(p_\alpha' - p_\alpha)\sigma^{\mu\alpha}}{2m} F_{2,\tau}(Q^2) \right) u(\mathbf{p}, \mu)\,, &
\end{aligned}
$$

single-nucleon current matrix element

$$
u(\mathbf{p}, \mu) = \sqrt{\frac{\sqrt{m^2 + |\mathbf{p}|^2} + m}{2m}} \begin{pmatrix} \chi_\mu \\ \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{\sqrt{m^2+|\mathbf{p}|^2}+m} \chi_\mu \end{pmatrix}
$$

Explicit formula for the Dirac spinor (Bjorken-Drell normalization)

We need to express 2N states with the relative momentum **k** and total momentum **P**

$$|(M_0, \frac{1}{2}, \frac{1}{2}), \mathbf{k}, \mathbf{P}, \mu_1, \mu_2\rangle$$

through product states

$$|(m, \frac{1}{2})\mathbf{p}_1, \mu_1'\rangle \, |(m, \frac{1}{2})\mathbf{p}_2, \mu_2'\rangle$$

Group theory helps to avoid combersome algebra !! We use the general formula

$$U(B(\mathbf{p}/m))|(m,j)\mathbf{0},\mu\rangle = \sqrt{\frac{E(\mathbf{p})}{m}}\,|(m,j)\mathbf{p},\mu\rangle$$

and evaluate

$$U(B(\mathbf{P}/M_0))\left(|(m,\frac{1}{2})\mathbf{k},\mu_1\rangle\,|(m,\frac{1}{2}){-\mathbf{k}},\mu_2\rangle\right)$$

$$M_0 = 2\sqrt{m^2 + \boldsymbol{k}^2}$$

in two ways:

$$U(B(\mathbf{P}/M_0))\left(|(m,\frac{1}{2})\mathbf{k},\mu_1\rangle\,|(m,\frac{1}{2}){-\mathbf{k}},\mu_2\rangle\right)$$

$$= \left(U(B(\mathbf{P}/M_0))|(m,\frac{1}{2})\mathbf{k},\mu_1\rangle\right)\,\left(U(B(\mathbf{P}/M_0))|(m,\frac{1}{2}){-\mathbf{k}},\mu_2\rangle\right)$$

1. individually

$$= \sqrt{\frac{\sqrt{M_0^2+\mathbf{P}^2}}{M_0}}\,|(M_0,\frac{1}{2},\frac{1}{2}),\mathbf{k},\mathbf{P},\mu_1,\mu_2\rangle$$

2. treating noninteracting 2N system with total momentum zero as one object with mass M_0

Relativistic partial wave states

$$|(j,k)\mathbf{P},\mu;l,s\rangle$$

are constructed in the following way:

First we build

$$|(j,k)\mathbf{0},\mu;l,s\rangle = \int \mathrm{d}\hat{\mathbf{k}} \sum_{\mu_l \mu_s \mu_1 \mu_2} Y_{l\mu_l}(\hat{\mathbf{k}})\,(l,\mu_l,s,\mu_s|j,\mu)$$
$$\times\, (1/2,\mu_1,1/2,\mu_2|s,\mu_s)\,\left(|(m,\frac{1}{2})\mathbf{k},\mu_1\rangle\, |(m,\frac{1}{2}){-\mathbf{k}},\mu_2\rangle \right)$$

and then calculate (again in two ways)

$$U(B(\mathbf{P}/M_0))\,|(j,k)\mathbf{0},\mu;l,s\rangle.$$

In one way we get simply

$$|(j,k)\mathbf{P},\mu;l,s\rangle = \sqrt{\frac{\sqrt{M_0^2+\mathbf{P}^2}}{M_0}}\,|(j,k)\mathbf{P},\mu;l,s\rangle$$

treating noninteracting 2N system as one object with mass M_0

In this way we arrive at the Poincaré Clebsch-Gordan coefficients

$$
\begin{aligned}
&\langle \mathbf{p}_1, \mu_1, \mathbf{p}_2, \mu_2 | (j,k)\mathbf{p}, \mu; l, s\rangle \\
&= \sum_{\mu_l \mu_s \mu_1' \mu_2'} \delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2) \frac{\delta(k - k(\mathbf{p}_1, \mathbf{p}_2))}{k^2} \mathcal{N}^{-1}(\mathbf{p}_1, \mathbf{p}_2)\, Y_{l\mu_l}(\hat{\mathbf{k}}(\mathbf{p}_1, \mathbf{p}_2)) \\
&\quad \times D^{1/2}_{\mu_1 \mu_1'}[R_w(B(\mathbf{p}/m_{120}), \mathbf{k}_1)]\, D^{1/2}_{\mu_2 \mu_2'}[R_w(B(\mathbf{p}/m_{120}), \mathbf{k}_2)] \\
&\quad \times (l, \mu_l, s, \mu_s | j, \mu)(1/2, \mu_1', 1/2, \mu_2' | s, \mu_s) \\
&= \int \mathrm{d}\hat{\mathbf{k}} \sum_{\mu_l \mu_s \mu_1' \mu_2'} \delta(\mathbf{p}_1 - \mathbf{p}_1(\mathbf{p}, \mathbf{k}))\delta(\mathbf{p}_2 - \mathbf{p}_2(\mathbf{p}, \mathbf{k}))\, \mathcal{N}(\mathbf{p}_1, \mathbf{p}_2)\, Y_{l\mu_l}(\hat{\mathbf{k}}) \\
&\quad \times D^{1/2}_{\mu_1 \mu_1'}[R_w(B(\mathbf{p}/m_{120}), \mathbf{k}_1)]\, D^{1/2}_{\mu_2 \mu_2'}[R_w(B(\mathbf{p}/m_{120}), \mathbf{k}_2)] \\
&\quad \times (l, \mu_l, s, \mu_s | j, \mu)(1/2, \mu_1', 1/2, \mu_2' | s, \mu_s)\,.
\end{aligned}
$$

$$\mathbf{k} = \mathbf{k}_1 \text{ and } -\mathbf{k} = \mathbf{k}_2$$

$$\mathcal{N}^{-2}(\mathbf{p}_1, \mathbf{p}_2) = \frac{E(\mathbf{k})E(\mathbf{k})(E(\mathbf{p}_1) + E(\mathbf{p}_2))}{E(\mathbf{p}_1)E(\mathbf{p}_2)(E(\mathbf{k}) + E(\mathbf{k}))}$$

$$\boldsymbol{k} = \frac{1}{2}\left(\mathrm{p}_1 - \mathrm{p}_2 - \frac{(E(\mathrm{p}_1) - E(\mathrm{p}_2))\,(\mathrm{p}_1 + \mathrm{p}_2)}{E(\mathrm{p}_1) + E(\mathrm{p}_2) + \sqrt{(E(\mathrm{p}_1) + E(\mathrm{p}_2))^2 - (\mathrm{p}_1 + \mathrm{p}_2)^2}}\right)$$

relativistic relative 2N momentum

$$R_w(B(\mathbf{p}/m_{120}), \mathbf{k}_i) = B^{-1}(\mathbf{p}_i/m)B(\mathbf{p}/m_{120})B(\mathbf{k}_i/m)$$

Wigner rotations we need

brings the particle with the momentum $\boldsymbol{p}_i$ to its rest frame

takes a particle of mass m at rest to momentum $\boldsymbol{k}_i$

takes a system of two particles with the same mass, m, and momenta $\mathbf{k} = \boldsymbol{k}_1$ and $-\mathbf{k} = \boldsymbol{k}_2$, respectively, to the total two-particle momentum $\mathbf{p}$, by which the momentum $\boldsymbol{k}_i$ is changed to $\boldsymbol{p}_i$

$$\mathrm{p}_1 \equiv \mathrm{p}_1(\mathrm{p}, \mathrm{k}) = \mathrm{k} + \frac{1}{2}\mathrm{p} + \frac{(\mathrm{p}\cdot\mathrm{k})\,\mathrm{p}}{2E(\mathrm{k})(E_{12}(\mathrm{p},\mathrm{k}) + 2E(\mathrm{k}))},$$

$$\mathrm{p}_2 \equiv \mathrm{p}_2(\mathrm{p}, \mathrm{k}) = -\mathrm{k} + \frac{1}{2}\mathrm{p} - \frac{(\mathrm{p}\cdot\mathrm{k})\,\mathrm{p}}{2E(\mathrm{k})(E_{12}(\mathrm{p},\mathrm{k}) + 2E(\mathrm{k}))}$$

$$m_{120} = M_0$$

$$E_{12}(\mathbf{p},\mathbf{k}) = \sqrt{4\boldsymbol{k}^2 + 4m^2 + \boldsymbol{p}^2} = \sqrt{{M_0}^2 + \boldsymbol{p}^2}$$

The relativistic deuteron at rest

$$\left(2\sqrt{\hat{k}^2 + m^2} + V\right)|D\rangle \;=\; m_D\,|D\rangle$$

V – relativistic NN potential in the total 2N momentum zero frame

The relativistic deuteron with the total momentum ***q***

$$\left(\sqrt{4\hat{k}^2 + 4m^2 + \boldsymbol{q}^2 + Vq}\right)|D\rangle \;=\; \sqrt{m_D^2 + \boldsymbol{q}^2}\,|D\rangle$$

V_q – „boosted" NN potential

The quantum numbers *l* and s are kinematically invariant quantities that distinguish representations with the same mass (k) and spin (j). For a two-nucleon system they have the same spectrum as the orbital and spin angular momentum operators in a partial wave representation of the nonrelativistic basis.

deuteron component

$$\langle (j,k)\mathbf{P},\mu;l,s;t,\tau|\mathbf{p}_D,\mu_D,D\rangle \;=\; \delta\left(\mathbf{P}-\mathbf{p}_D\right)\delta_{j1}\,\delta_{\mu\mu_D}\,\delta_{s1}\,\delta_{t0}\,\delta_{\tau 0}\,\phi_{D,l}(k)$$

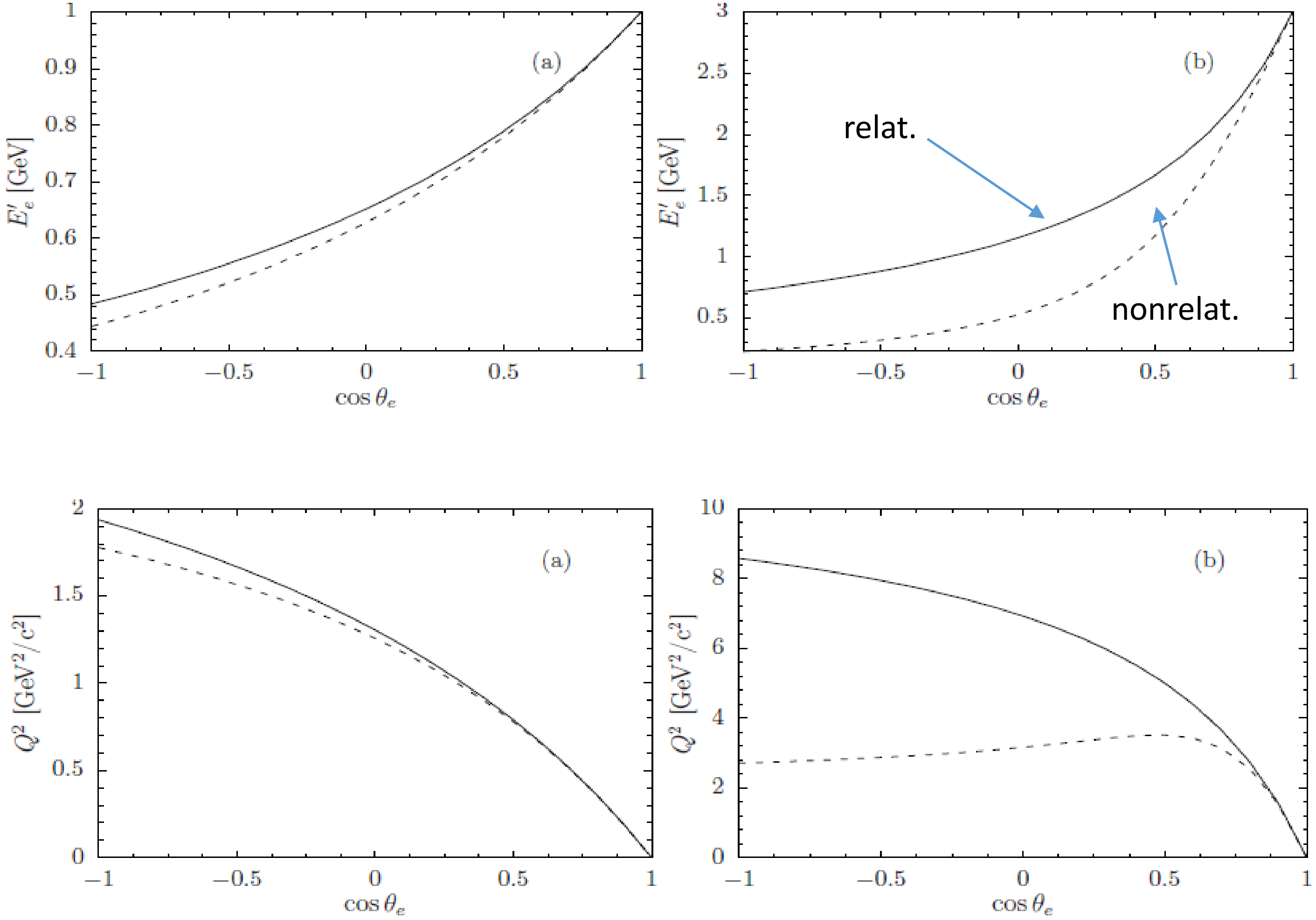
For the construction of *V* and V_q from the nonrelativistic NN potential see H. Kamada and W. Glöckle, Phys. Lett. B **655**, 119 (2007). We use the Argonne V18 NN force.

Note that the relativistic interaction *V* reproduces the same phase shifts as calculated nonrelativistically - the phase shifts extracted from experiment are relativistically invariant !

Elastic electron scattering off the deuteron

First things first:
Kinematics

E_e = 1 GeV　　E_e = 3 GeV

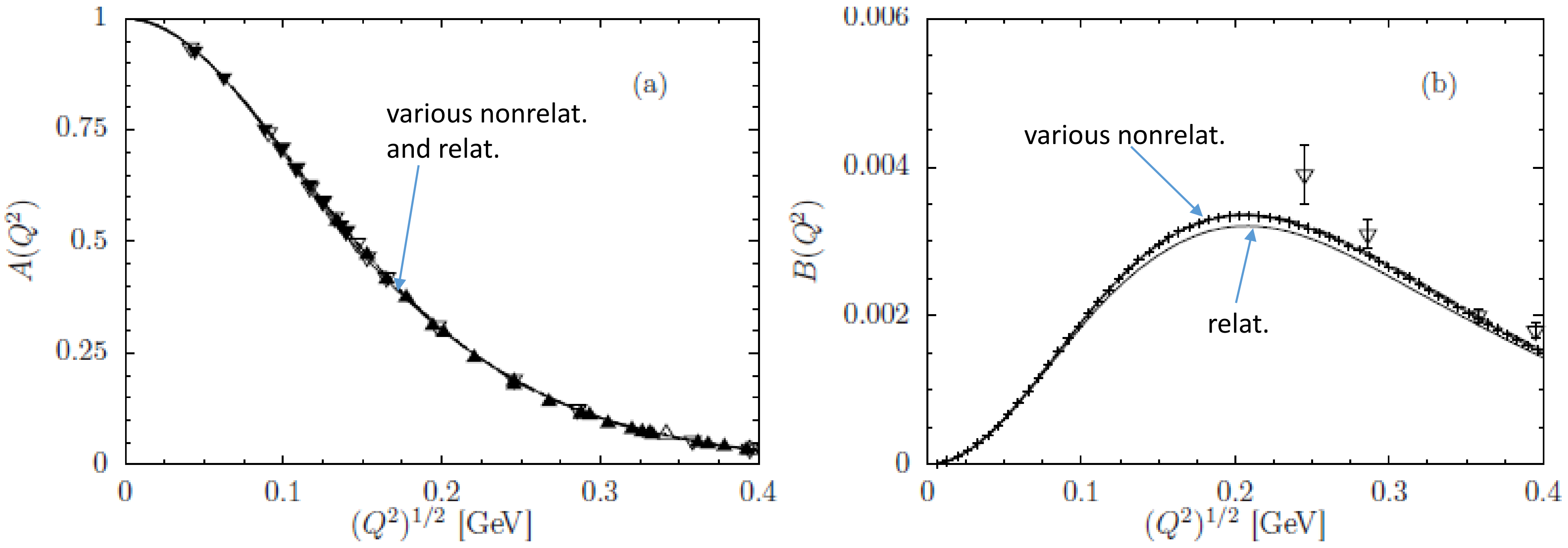


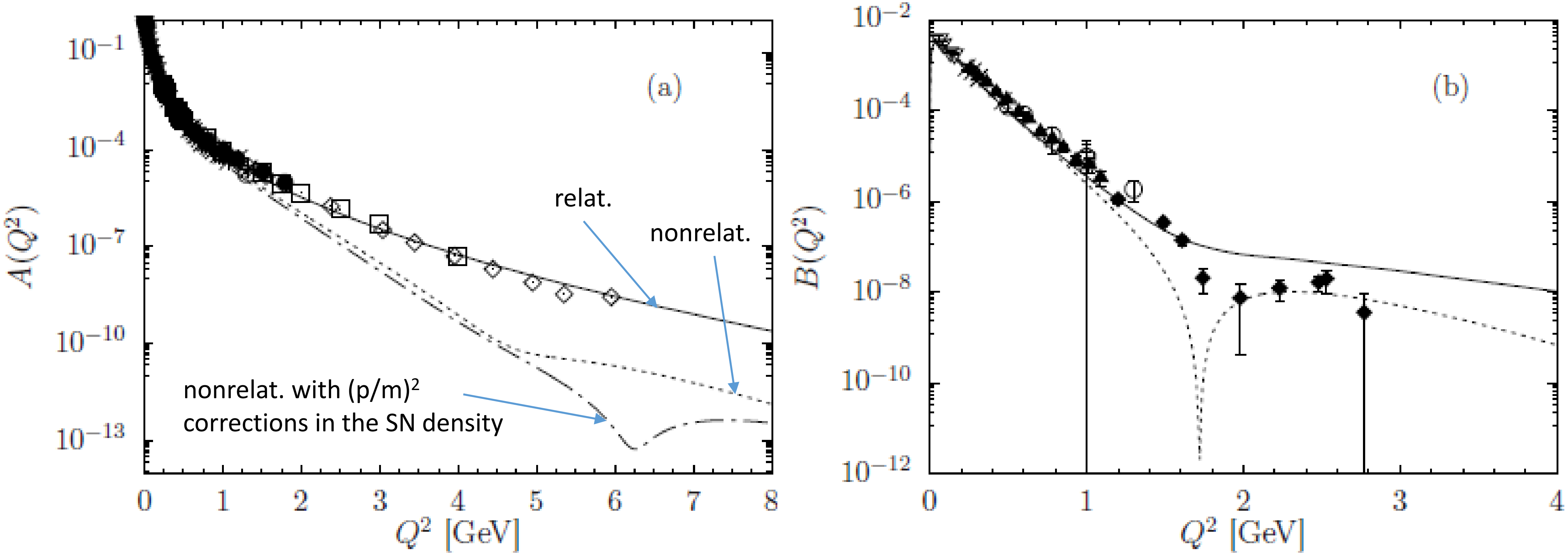
final electron energy

four momentum transfer squared

Unpolarized cross section

$$\frac{d\sigma}{d\hat{p}'_e}(Q^2,\theta_e) = \sigma_{Mott}\left(A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right)\frac{|p'_e|}{|p_e|}$$



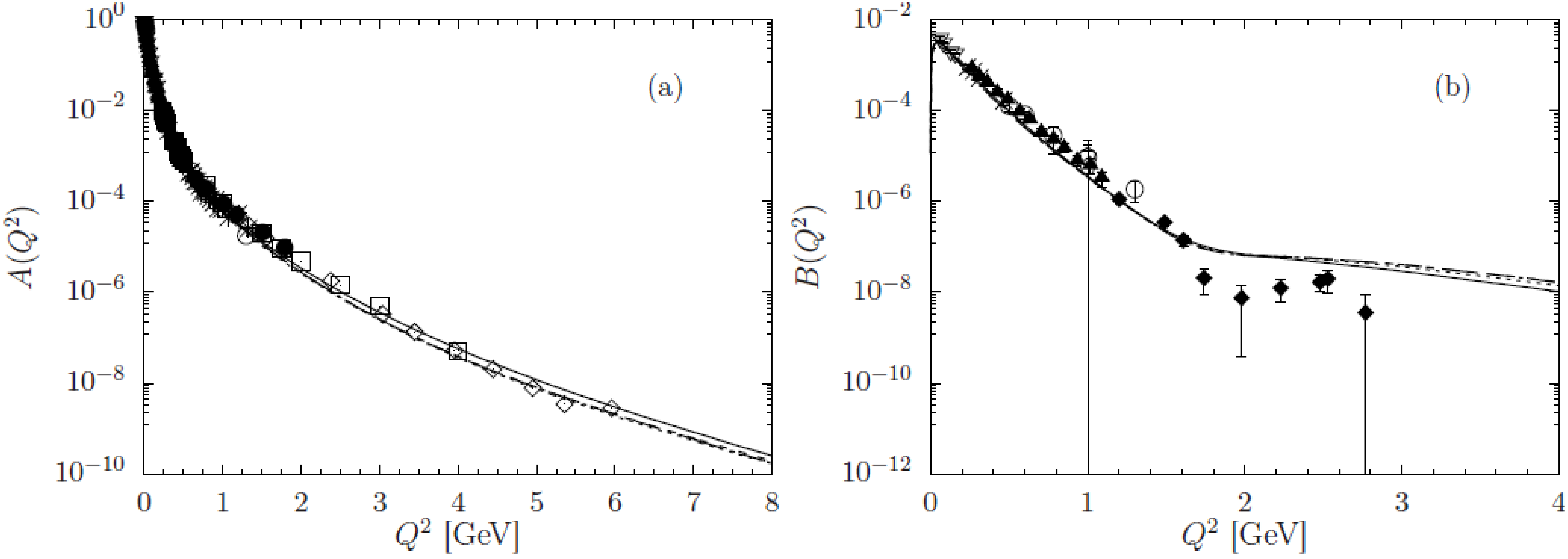


See also

Y. Huang and W. N. Polyzou, Phys. Rev. C **80**, 025503 (2009).

A. A. Filin et al., Phys. Rev. C **103**, 024313 (2021).

Dependence on the model of the nucleon EM form factors



dash-dotted: H. Budd, A. Bodek, and J. Arrington, arXiv:hep-ex/0308005v2 (2003)
dashed: J. J. Kelly, Phys. Rev. C **70**, 068202 (2004)
dotted: E. L. Lomon, Phys. Rev. C **66**, 045501 (2002)

solid: G. Shen, L. E. Marcucci, J. Carlson, S. Gandolfi, and R. Schiavilla, Phys. Rev. C **86**, 035503 (2012)

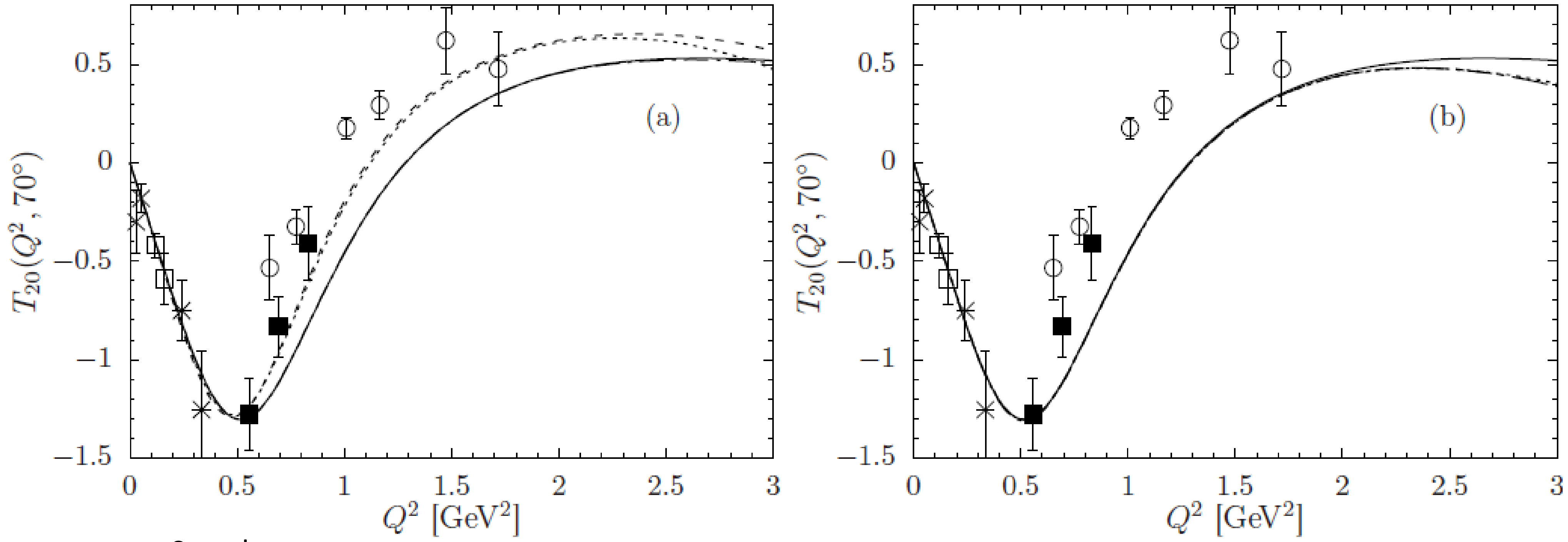
dipole parametrization of FF sticks out

Deuteron tensor analyzing power

$$T_{20}(Q^2,\theta_e) = \frac{\sqrt{2}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\hat{\mathbf{p}}'_e}(Q^2,\theta_e;\mu_D=1) - \frac{\mathrm{d}\sigma}{\mathrm{d}\hat{\mathbf{p}}'_e}(Q^2,\theta_e;\mu_D=0)\right)}{\frac{\mathrm{d}\sigma}{\mathrm{d}\hat{\mathbf{p}}'_e}(Q^2,\theta_e)}$$

dashed: nonrelat. calculations
dotted: nonrelat. calculations with relat. kinematics
solid: relat. calculations

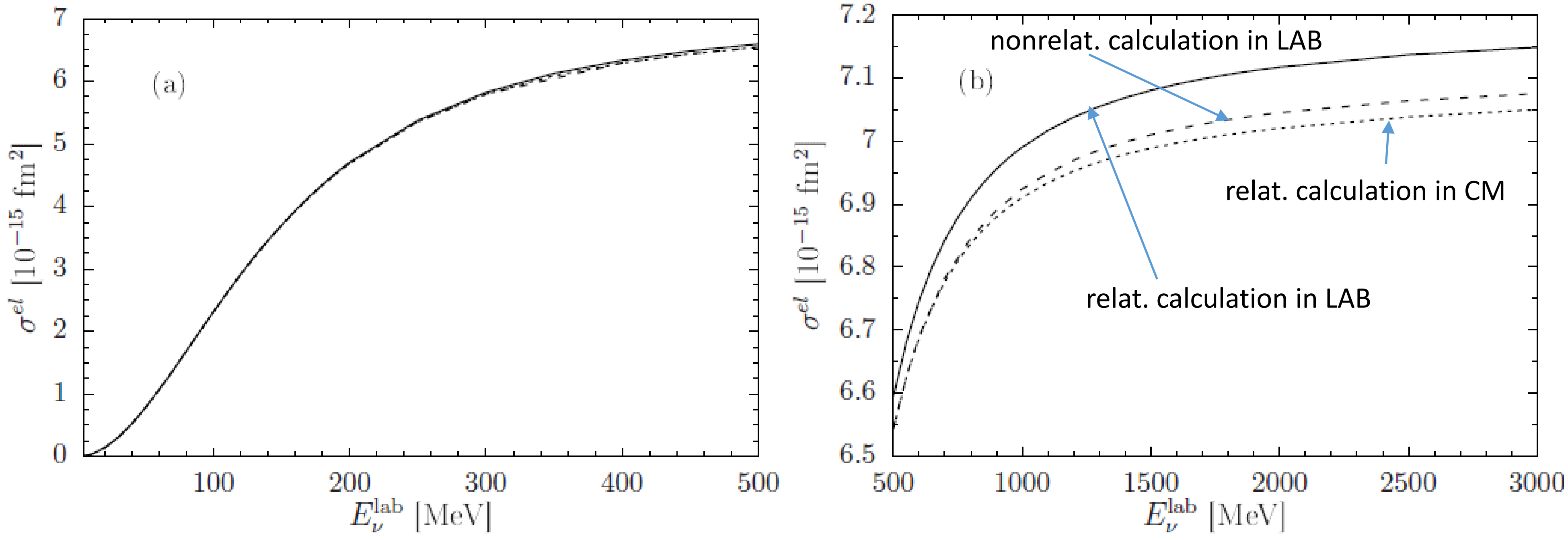
various models of the nucleon EM form factors



See also
A. F. Krutov, V. E. Troitsky, Phys. Rev. C **75**, 014001 (2007)

Elastic neutrino (NC) scattering off the deuteron

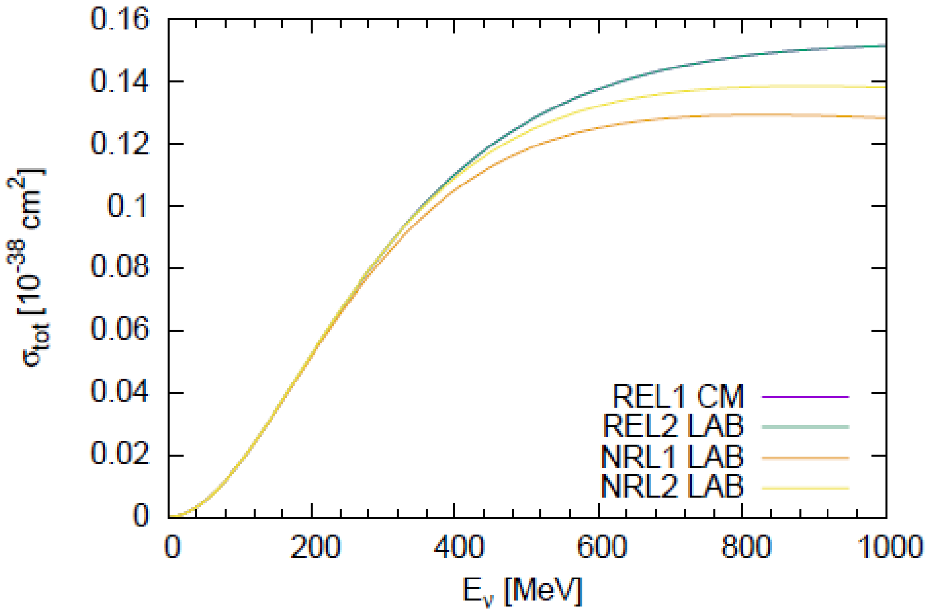
Total elastic cross section



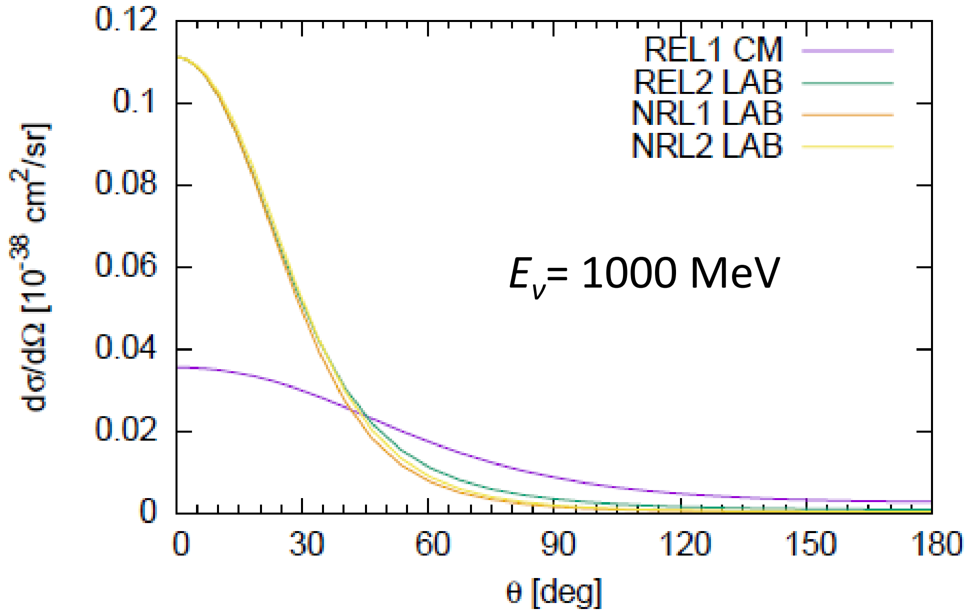
Lack of covariance in the nuclear current matrix element !
The effect amounts to approx. 1.5 % at 3 GeV

… in contrast to elastic neutrino (NC) scattering off the proton

total elastic cross section



angular distribution of the cross section



REL1 – relat. with the FeynCalc package of *Mathematica*® in CM

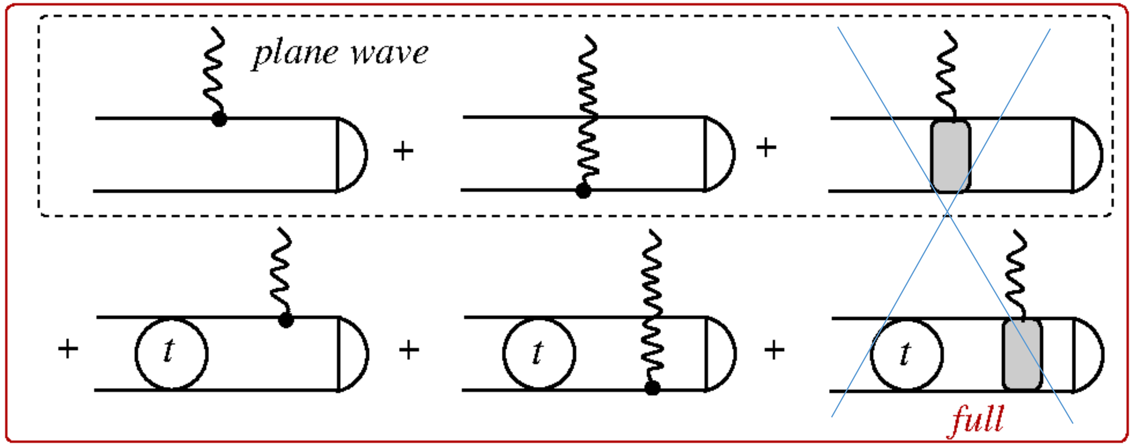
REL2 – relat. with the „nuclear" formula: $d\sigma/d\Omega \sim (v_L R_L + v_T R_T + \cdots)\,\rho$ in LAB

NRL1 – nonrelat. with $(p/m)^2$ corrections in LAB

NRL2 – nonrelat. without $(p/m)^2$ corrections in LAB

relativistically we get ONE result for total elastic cross section !

Inelastic electron scattering off the deuteron: *e + d → e' + p + n*



$$\langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | J^{\mu}_{nuc}(0) | \mathbf{p}_D, \mu_D, D \rangle$$

$$\langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | \, t(E+i\epsilon) G_0(E+i\epsilon) \, J^{\mu}_{nuc}(0) \, | \mathbf{p}_D, \mu_D, D \rangle$$

The rescattering contribution to the nuclear current matrix element is calculated in two steps. We calculate first

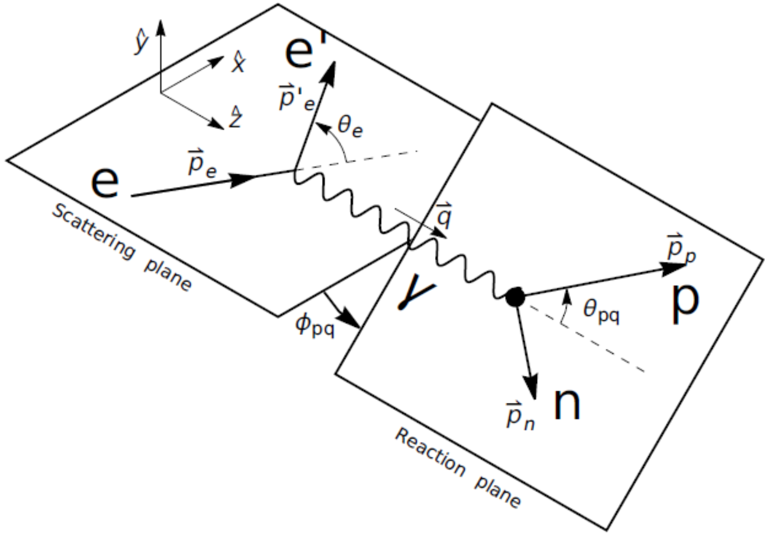
$$\langle (j', k') \mathbf{p}', \mu'; l' s' t' \tau' | \, t(E+i\epsilon) G_0(E+i\epsilon) \, J^{\mu}_{nuc}(0) \, | \mathbf{p}_D \mu_D D \rangle$$

and then

$$\langle \mathbf{p}_1', \mu_1', \tau_1', \mathbf{p}_2', \mu_2', \tau_2' | \, t(E+i\epsilon) G_0(E+i\epsilon) \, J^{\mu}_{nuc}(0) \, | \mathbf{p}_D, \mu_D, D \rangle$$

Here *t* is the boosted t-matrx obtained with the boosted potential by solving the Lippmann-Schwinger equation for the 2N system with total momentum ***q***. G_0 is the relativistic free 2N propagator.

Exclusive cross section for the $^2H(e,e'p)n$ reaction



Fixed electron parameters (chosen arbitrarily)

(a) E_e= 800 MeV, Θ_e= 38.7◦, E'_e = 635.3 MeV, $E_{c.m.}$=100 MeV, |q|= 500 MeV/c

(b) E_e= 800 MeV, Θ_e= 106.5◦, E'_e= 414.4 MeV, $E_{c.m.}$= 150 MeV, |q|= 1000 MeV/c

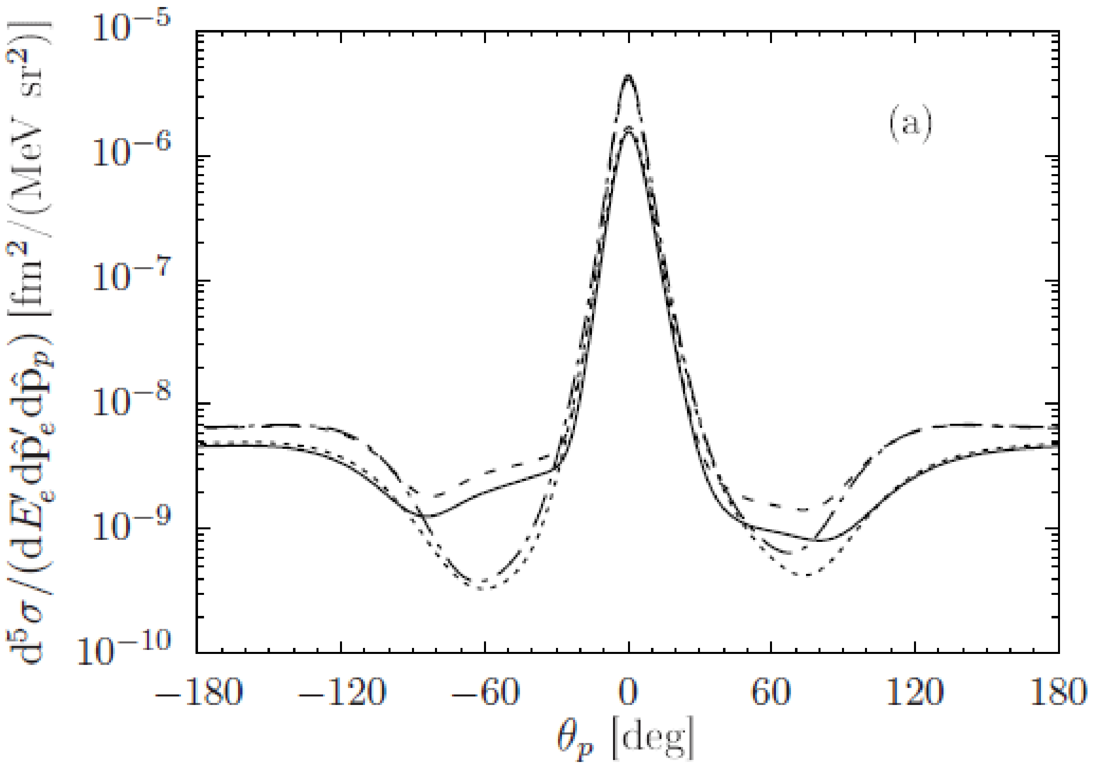
Negative values of Θ_p or p_{miss} correspond to ϕ_{pq}= 0 and their positive values to ϕ_{pq}= 180◦.

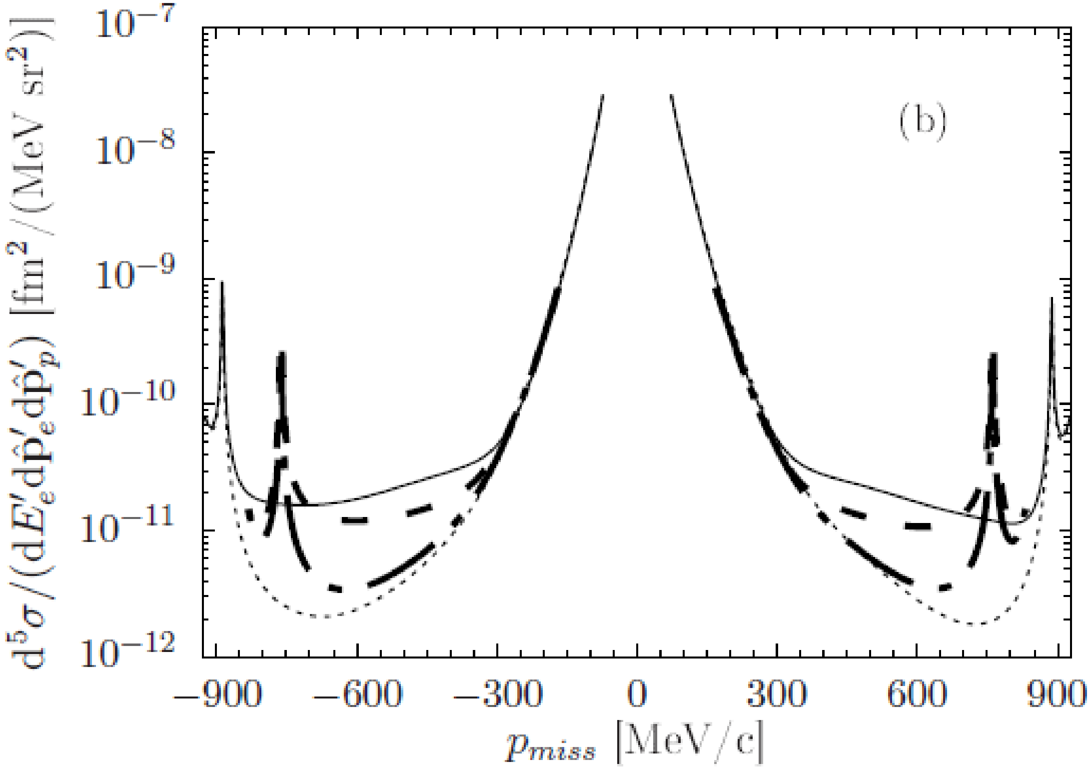
dotted: relativ. plane wave results

solid: relativvistic full results

dash-dotted: nonrelat. plane wave results

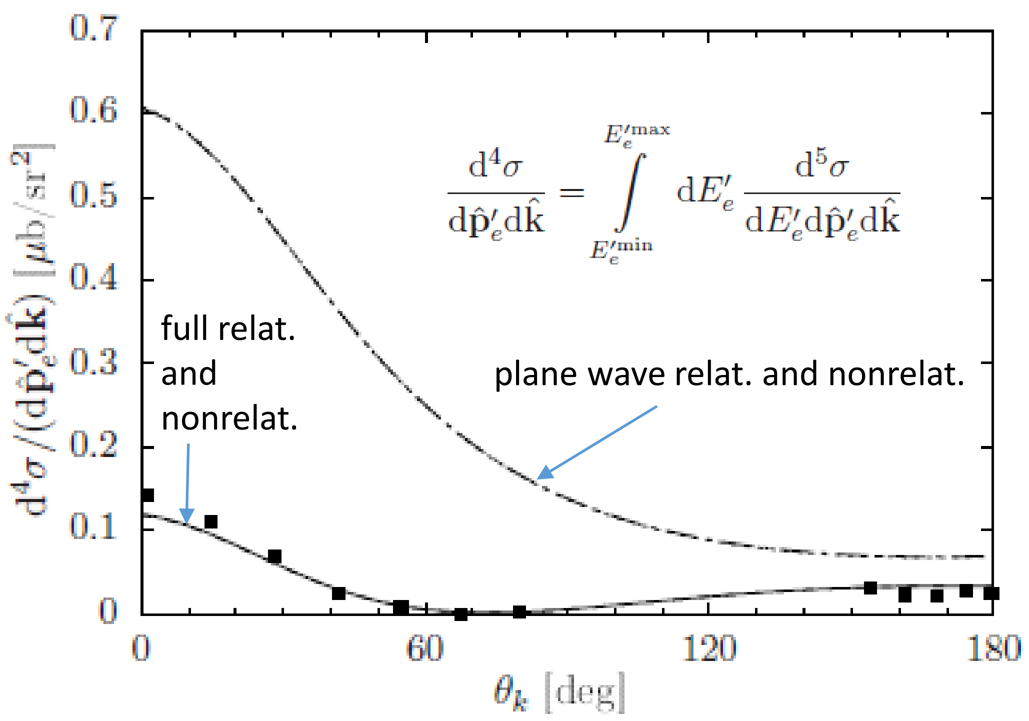
dashed: nonrelat. full results



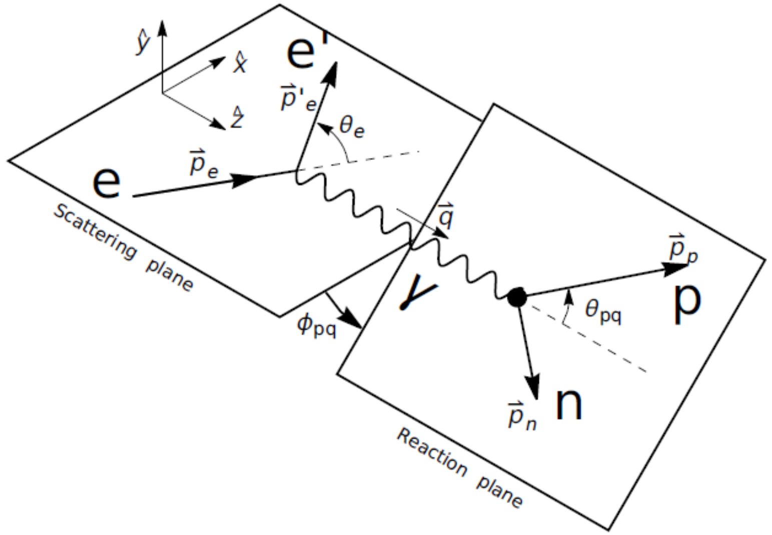


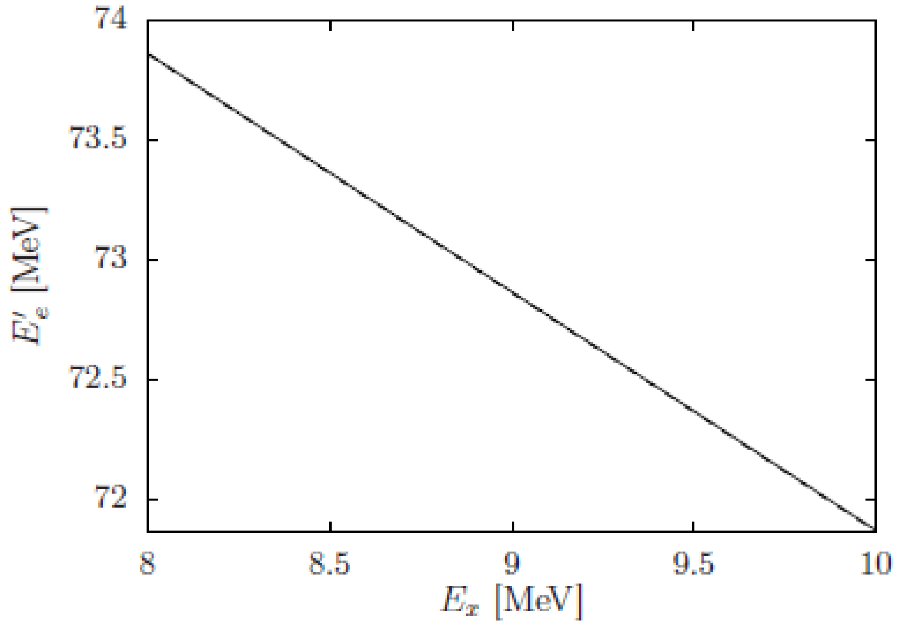
Example of „mixed kinematics” for the $^2H(e,e'p)n$ reaction

Electron arm parameters: E_e= 85 MeV, Θ_e= 40◦, 8 MeV < E_x < 10 MeV, ϕ_{pq}= 45◦



P. von Neumann-Cosel *et al.*, Phys. Rev. Lett. **88**, 202304 (2002)

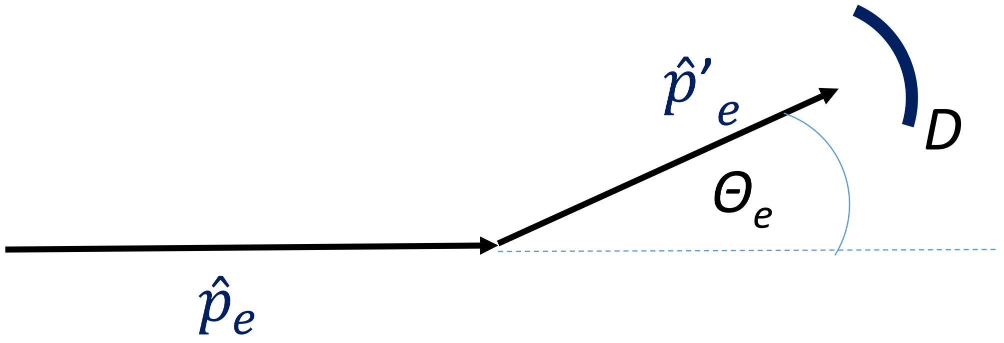




rescattering contribution is decisive at this kinematics !

Semi-inclusive cross section for the $^2H(e,e')$ reaction

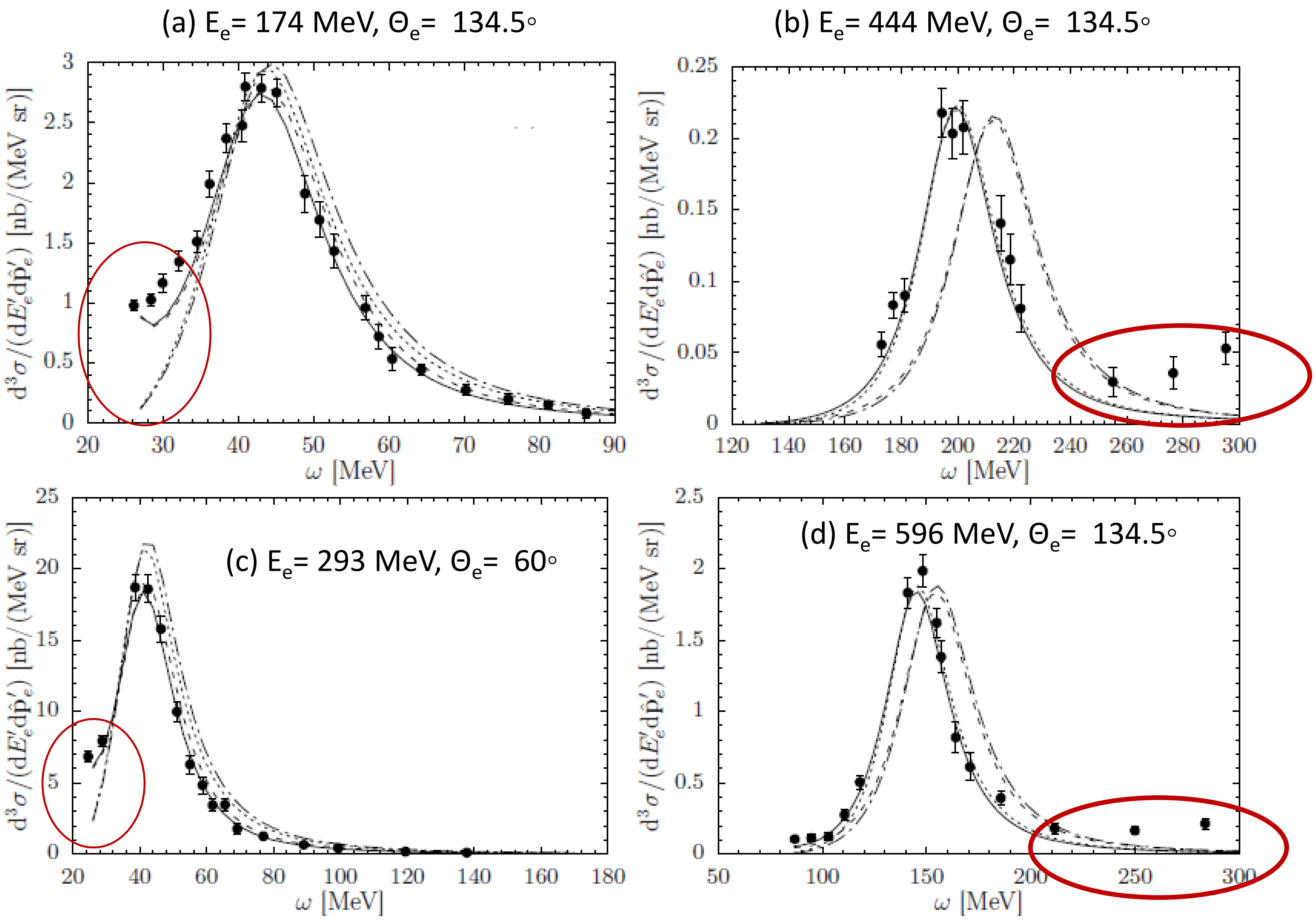
(only electron in the final state is detected)



In B. P. Quinn et al., Phys. Rev. C **37**, 1609 (1988) a rich data set for the $d^3\sigma/(dE'_e\, \mathrm{d}\hat{p}'_e)$ differential cross sections is compared with Arenhövel and Leidemann's predictions as well as with the results obtained by Laget.

Data taken in the vicinity of the quasi elastic peak, where one expects electron scattering on a nucleon moving inside a nucleus.

B. P. Quinn et al., Phys. Rev. C **37**, 1609 (1988), Fig. 8

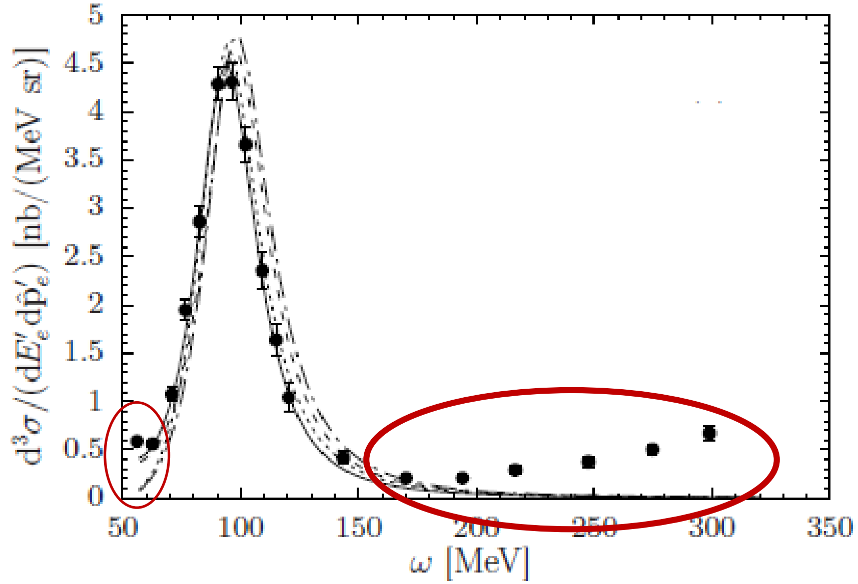


Enhancement of the cross sections shown by full calculations on left slopes in the region where the internal two-nucleon energy is very small

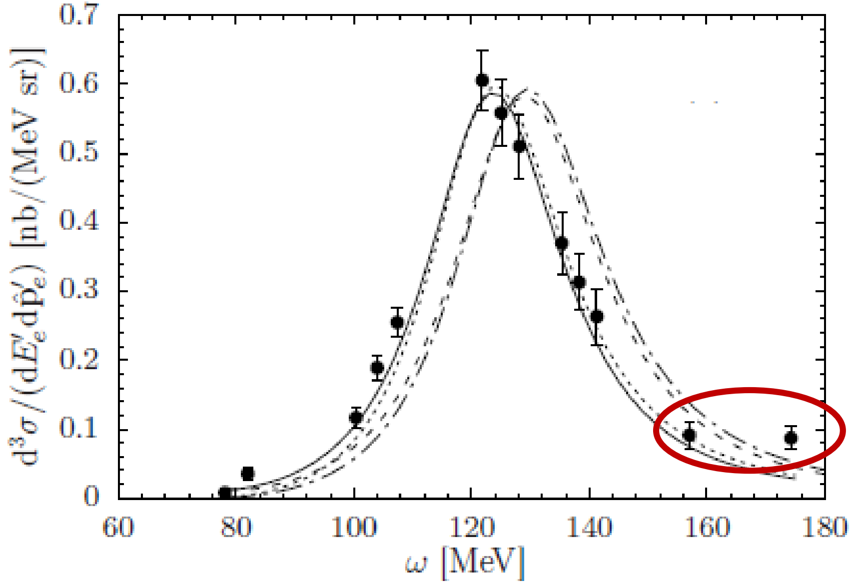
On right slopes and for large energy transfers, where the internal two-nucleon energy exceeds the pion mass, new channels (pion production, isobar excitation) are open, which cannot be described by our theory.

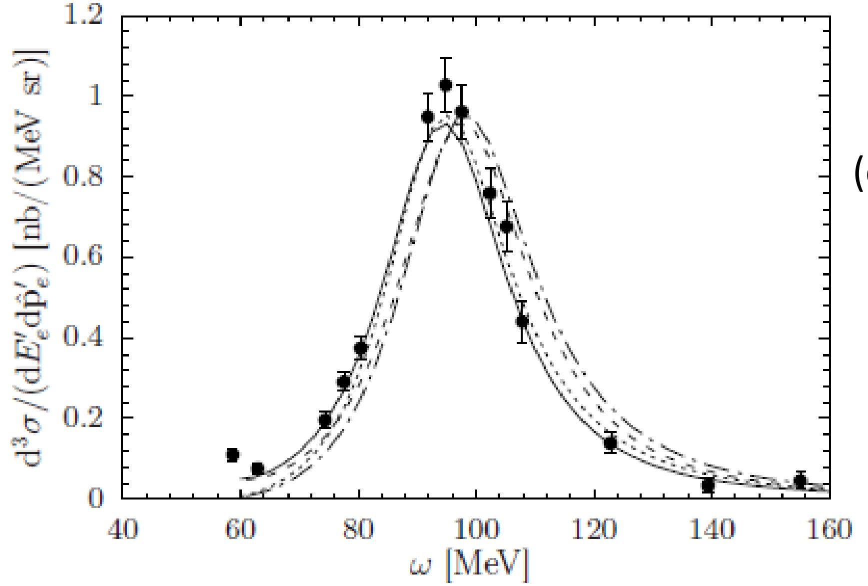
B. P. Quinn et al., Phys. Rev. C 37, 1609 (1988), Fig. 9

(a) E_e= 465 MeV, Θ_e= 60◦



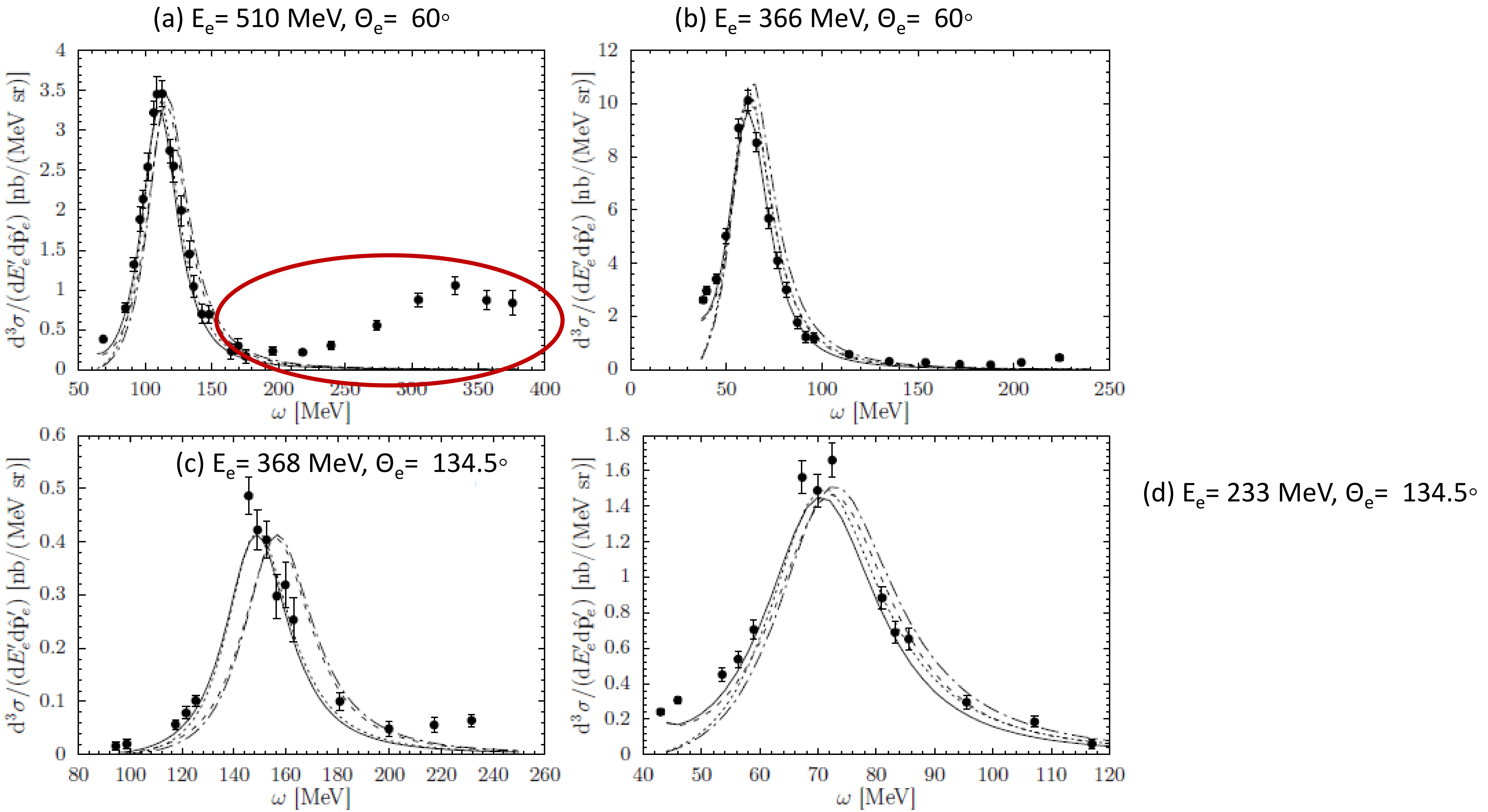
(b) E_e= 328 MeV, Θ_e= 134.5◦





(c) E_e= 278 MeV, Θ_e= 134.5◦

B. P. Quinn et al., Phys. Rev. C 37, 1609 (1988), Fig. 10



Dealing with „Q^2-p_{miss}" kinematics in the $e + d \rightarrow e' + p + n$ reaction

(From $\mathrm{d}^5\sigma / \left(\mathrm{d}\hat{\mathrm{p}}'_e \mathrm{d}E'_e \mathrm{d}\hat{\mathrm{k}}\right)$ to $\mathrm{d}^2\sigma / \left(\mathrm{d}Q^2\, \mathrm{d}p_{\mathrm{miss}}\right)$)

Following the experimental approach, we take finite bins in Q^2 and p_{miss}, scan the whole four dimensional parameter space to see which combinations of (θ_e,E'_e, θ_k, ϕ_k) lead to required Q^2 and p_{miss} intervals.

We set *Ee*= 500 MeV and chose four Q^2 intervals:

1. (0.0875, 0.1125) GeV^2,
2. (0.175, 0.225) GeV^2,
3. (0.35, 0.45) GeV^2,
4. (0.525, 0.675) GeV^2.

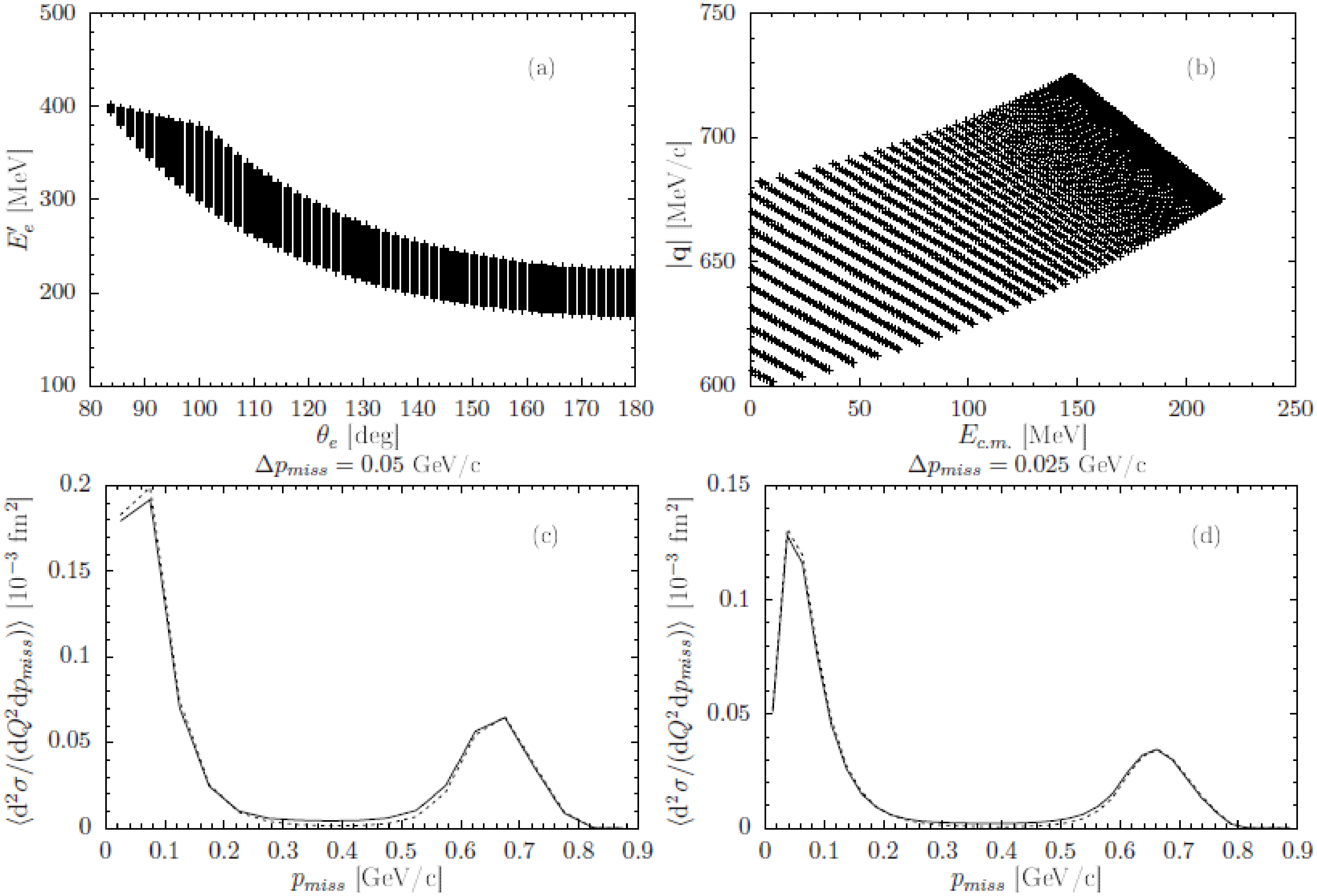
Values of the $\mathrm{d}^5\sigma / \left(\mathrm{d}\hat{\mathrm{p}}'_e \mathrm{d}E'_e \mathrm{d}\hat{\mathrm{k}}\right)$ cross section for each point on a four dimensional grid were written to a file together with the complete integral weight

$$2\pi\ \Delta\theta_e(i)\ \sin\left(\theta_e(i)\right)\ \Delta E'_e(j)\ \Delta\theta_k(l)\ \sin\left(\theta_k(l)\right)\ \Delta\phi_k(n)$$

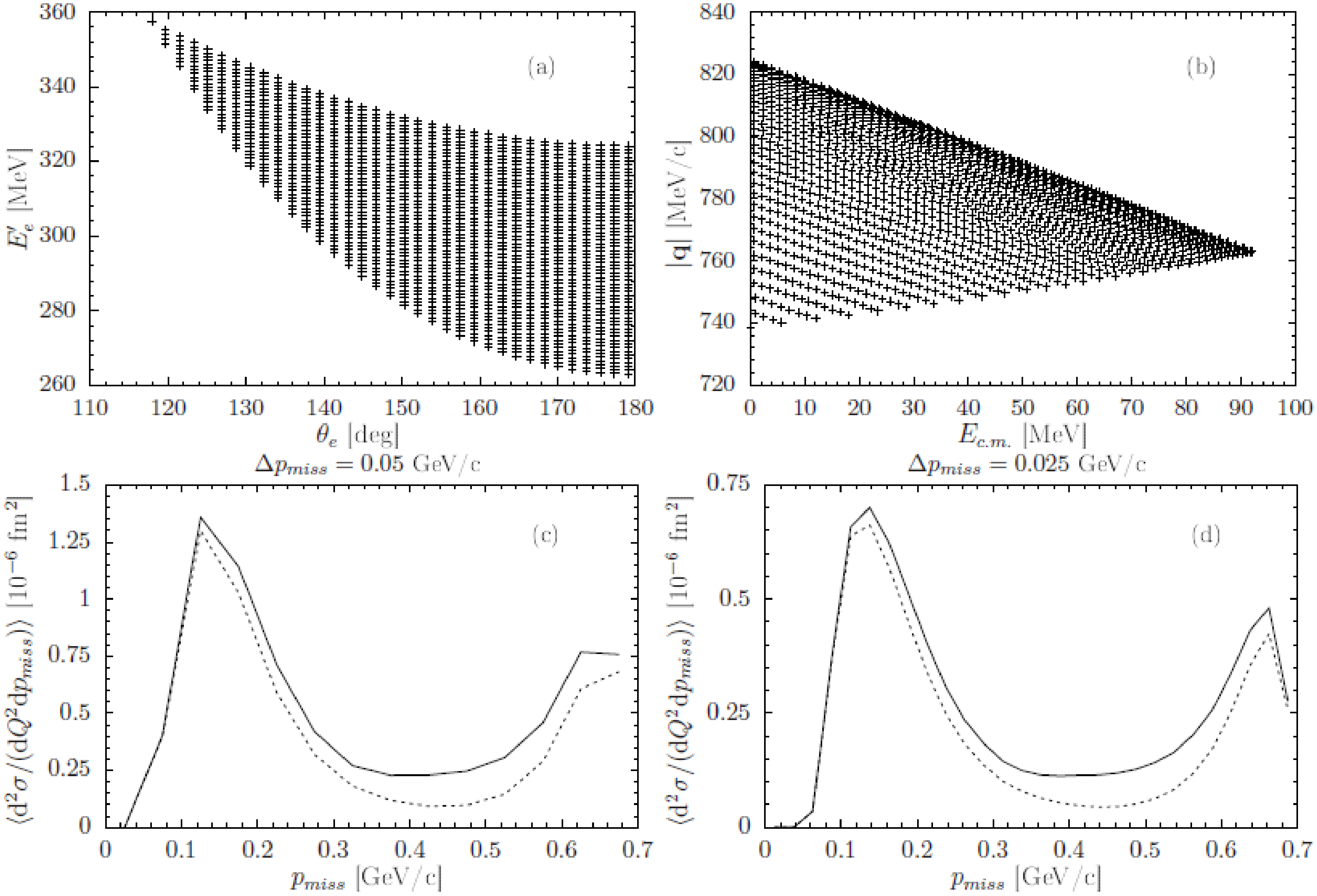
and with the value of the missing momentum. For each Q^2 interval a file containing several millions of „events" can be later sorted to arrive at

$$\left\langle \frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}p_{\mathrm{miss}}} \right\rangle \equiv \int\limits_{Q^2_{min}}^{Q^2_{max}} \mathrm{d}Q^2 \int\limits_{\bar{p}_{\mathrm{miss}}-\frac{1}{2}\Delta p_{\mathrm{miss}}}^{\bar{p}_{\mathrm{miss}}+\frac{1}{2}\Delta p_{\mathrm{miss}}} \mathrm{d}p_{\mathrm{miss}}\, \frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}p_{\mathrm{miss}}}\ ;$$

$0.35\,\mathrm{GeV}^2 < Q^2 < 0.45\,\mathrm{GeV}^2$

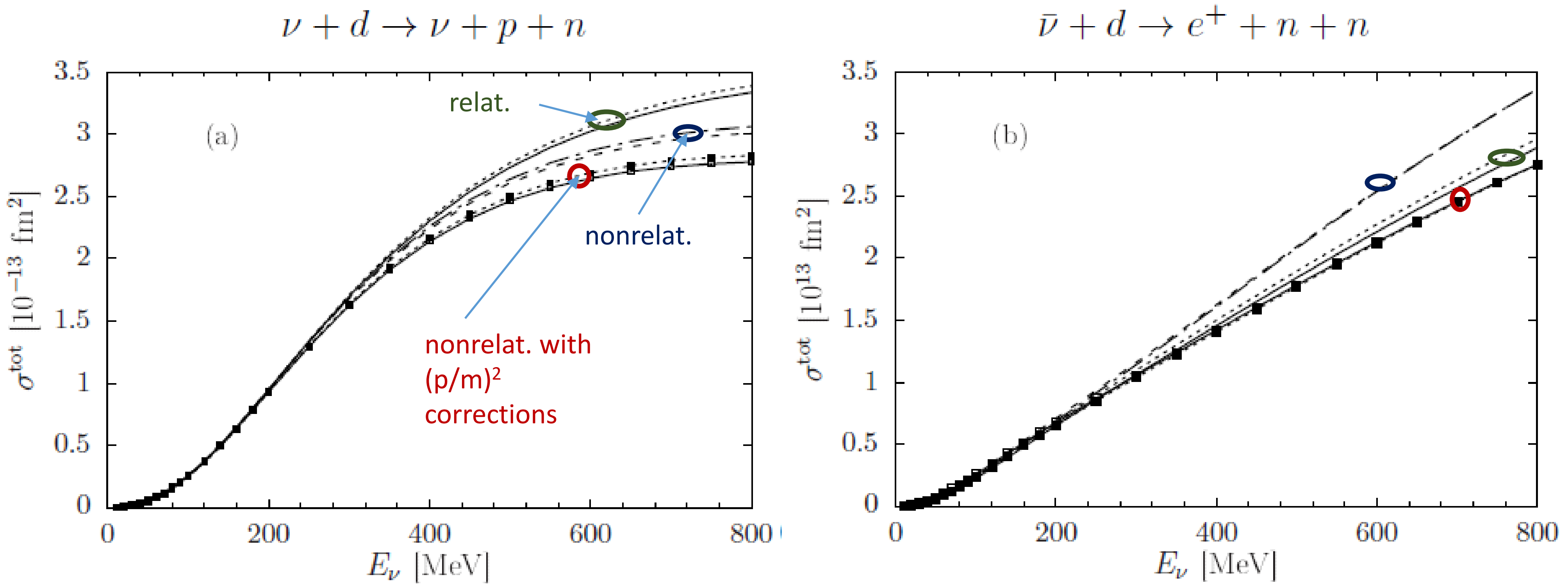


$$0.525\,\mathrm{GeV}^2 < Q^2 < 0.675\,\mathrm{GeV}^2.$$



Inelastic neutrino NC and CC scattering off the deuteron

Total breakup cross section



Conclusions and outlook

1. Based on our experience from nucleon-deuteron scattering as well as from electron and neutrino scattering on ^{2}H, ^{3}H and ^{3}He we are building a relativistic framework to describe elastic and inelastic electron and neutrino scattering off the deuteron under the one-boson-exchange approximation.
2. The treatment of the nuclear sector corresponds to Dirac's „instant-form dynamics" and dates back to Bakamjian and Thomas; Wayne Polyzou's expertise proved to be crucial.
3. Our framework is formally applicable to calculations at energies below the pion production threshold and at arbitrary magnitudes of the three momentum transfer.
4. At low energy and momentum transfers our results coincide (nearly by construction !) with the nonrelativistic predictions.
5. We can deal with various kinematics (exclusive, semi-exclusive, inclusive) and make predictions for unpolarized cross sections and various polarization observables.
6. The framework presently lacks 2N parts in the nuclear current matrix elements, which restricts its applicability to specific electron kinematics; should better work for neutrinos.
7. We (mainly Wayne) see possible extensions of the framework

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Thank you !