Relativistic NN potential in chiral EFT

– – – based on time-ordered perturbation theory – – –

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OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspective

Nuclear forces — Weinberg's seminal work

- Apply chiral perturbation theory (ChPT), as the low-energy EFT of QCD, to derive nuclear forces

S. Weinberg, PLB251(1990)288-292; NPB363(1991)3-18

  - **Self-consistently** include many-body forces

$$V = V_{2N} + V_{3N} + V_{4N} + \cdots$$

  - **Systematically improve** order by order (in heavy baryon ChPT)

$$V_{iN} = V_{iN}^{\mathrm{LO}} + V_{iN}^{\mathrm{NLO}} + V_{iN}^{\mathrm{NNLO}} + \cdots$$

  - Scattering amplitude obtained by solving the **Schrödinger equation** (or the integral equations in momentum space)

$$\left[\left(\sum_{i=1}^{A} -\frac{\nabla_i^2}{2m_N}\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right] |\Psi\rangle = E|\Psi\rangle$$

Provide a systematic and solid theoretical approach to study the few-nucleon scattering

Renormalization issue of NN scattering

Iteration of the chiral truncated NN potential within LSE

$$T(p',p) = V(p',p) + \boxed{\int_0^\infty \frac{k^2 dk}{(2\pi)^3} V(p',k) \frac{m_N}{p^2 - k^2 + i\epsilon} T(k,p)}\,.$$

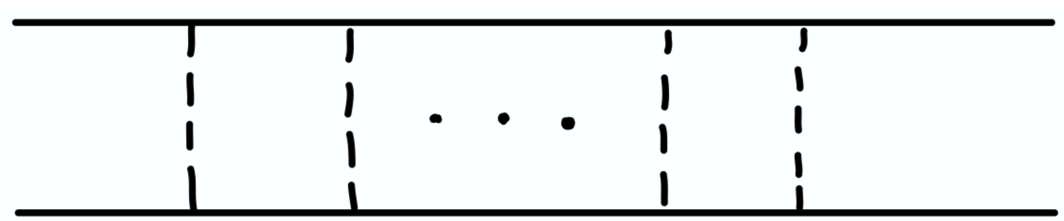
➡ **Generated UV divergencies cannot be absorbed by contact terms!**

A. Gasparyan talk

- Leading order NN potential

$$V_{\rm LO} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4f_\pi^2} \tau_1 \cdot \tau_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

- Iterated one-pion exchange potential (ladder diagrams) for



M. Savage_arXiv:nucl-th/9804034

$$\xrightarrow{k \to \infty}$$

Logarithmic Divergence

$$\sim (Q m_N)^n$$

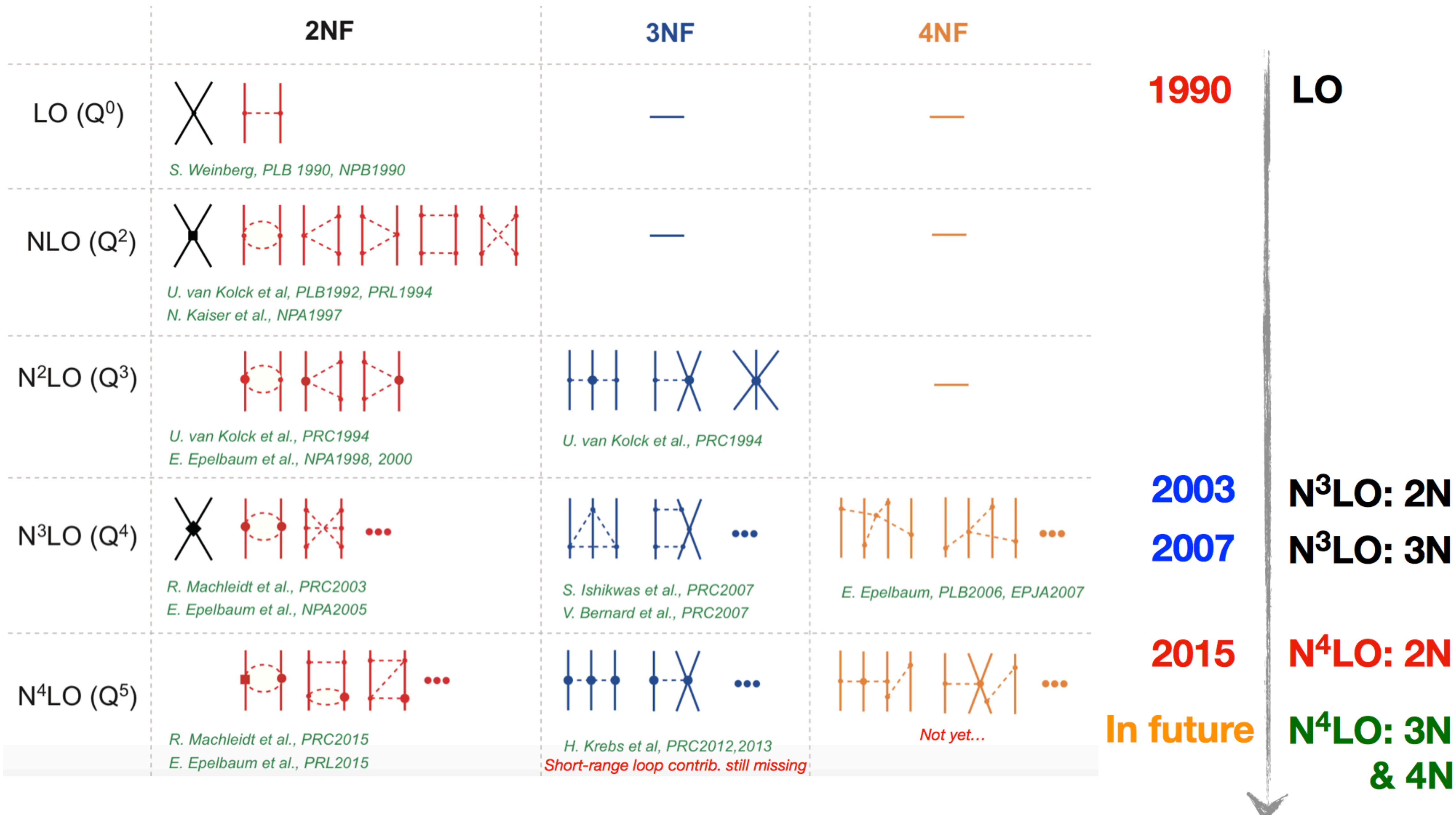
cannot be absorbed by C_S, C_T

WPC is inconsistent with renormalization, even at LO!

Deal with the renormalization issue

Possible solutions (still controversial...)

- Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
  - ✓ WPC is consistent *G.P. Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*
    A.M. Gasparyan, E. Epelbaum (2021), 2110.15302
  - ✓ Achieve great successes ***E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773***
    R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1



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- Kaplan, Savage, and Wise (KSW) power counting
  - ✓ Treat the exchange of pions perturbatively *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*
  - ✓ Fail to converge in certain spin-triplet channels *S. Fleming, et al., Nucl.Phys. A677 (2000) 313*
  - ✓ Recently, some improvements of KSW proposed by Kaplan *D.B. Kaplan, arXiv: 1905.07485*

- Modified WPC with renormalization group invariance (RGI)
  - ✓ Rearrange the higher order contact terms to the lower chiral order

    A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002.
    B. Long and C.-J. Yang, PRC84(2011)057001 …

Deal with the renormalization issue

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- Lorentz invariant framework to reformulate chiral force
  - ✓ The fundamental symmetry of our nature

Chiral forces in Lorentz invariant framework

Modified Weinberg approach

E. Epelbaum and J. Gegelia, PLB716(2012)338-344

- Based on the Lorentz invariant chiral Lagrangians
- Adopt Weinberg power counting to expand the NN potential and the relativistic corrections are perturbatively included

$$V(p',p) = \bar{u}_1 \bar{u}_2 \mathscr{A}\, u_1 u_2 \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

✓ Leading order (same as the non-rel. case):

$$V_{\rm LO} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4F_\pi^2} \tau_1 \cdot \tau_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

- Use the Kadyshevsky equation to calculate the scattering T-matrix

$$T(p',p) = V(p',p) + \int \frac{k^2 dk}{(2\pi)^3} V(p',k) \frac{m_N^2}{2(k^2+m_N^2)} \frac{1}{\sqrt{p^2+m_N^2} - \sqrt{k^2+m_N^2} + i\epsilon} T(k,p) \,.$$

✓ Milder ultraviolet behavior than in Lippmann-Schwinger equation

- Result in **a renormalizable framework!**

✓ LO potential: non-perturbatively and renormalizable (except 3P0 channel)

✓ Higher order: perturbatively

Chiral forces in Lorentz invariant framework

- Modified Weinberg approach E. Epelbaum and J. Gegelia, PLB716(2012)338-344
  - Based on the Lorentz invariant chiral Lagrangians
  - Adopt Weinberg power counting to expand the NN potential and the relativistic corrections are perturbatively included

$$V(p',p) = \bar{u}_1 \bar{u}_2 \mathscr{A} u_1 u_2 \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

This idea has not been systematically explored,

especially beyond leading order!

- ✓ Milder ultraviolet behavior than in Lippmann-Schwinger equation
- Result in **a renormalizable framework!**
  - ✓ LO potential: non-perturbatively and renormalizable (except 3P0 channel)
  - ✓ Higher order: perturbatively

In this work

- We proposed a systematic framework within the time-ordered perturbation theory using the Lorentz invariant chiral Lagrangians
  - Derive the rules of time-ordered diagrams, especially for the rules with spin-1/2 fermion *(as far as we know, there was no such rules in the literature)*
  - Formulate the nucleon-nucleon interaction up to **next-to-next-to-leading order**
    - ✓ Calculate the two-pion-exchange contributions at one-loop level
    - ✓ Describe the partial wave phase shifts

V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798 (2019) 134987

XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101 (2020) 034001

XLR, E. Epelbaum, J. Gegelia, arXiv: 2202.04018, PRC in press

XLR, PoS(CD2021)007

OUTLINE

- Introduction
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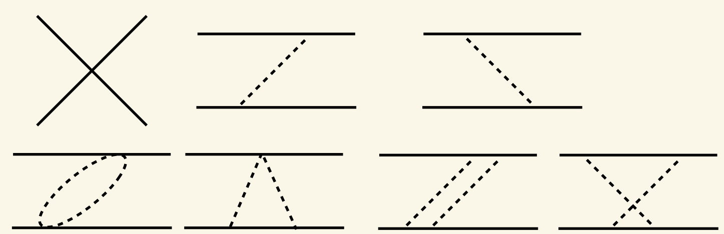
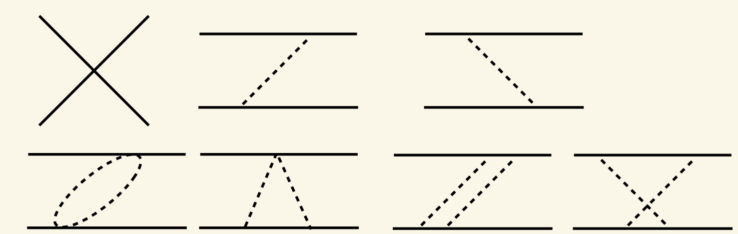
Theoretical framework

- Follow the standard procedure of formulating chiral forces in time-ordered perturbation theory

S. Weinberg, PLB1990, NPB1991;
C. Ordóñez, U. van Kolck PLB(1992) …

	Non-relativistic (Heavy-baryon)	**Manifestly Lorentz invariant**
Chiral Lagrangians	$N^\dagger\left[i(v\cdot D)+g_A(S\cdot u)\right]N$ $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$	$\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$
Potential TOPT diagrams		
Scattering equations (T = V + VGT)	Lippmann-Schwinger eq.	Kadyshevsky eq.
Power counting	Weinberg p.c.	Weinberg p.c.

$1/m_N$

Chiral Lagrangian up to NNLO

Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

- Purely pionic sector

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

J.Gasser, H. Leutwyler, Ann.Phys.(1984)

$f_\pi = 92.4$ MeV, $g_A = 1.267$

$c_{1,2,3,4}$ are determined by πN scattering

- One-nucleon sector

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N\left\{i\not{D} - m_N + \frac{1}{2}g_A \not{u}\gamma^5\right\}\Psi_N$$

J. Gasser, M. E. Sainio, and A. Svarc, NPB(1988)

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N\left\{c_1\langle\chi+\rangle - \frac{c_2}{4m_N^2}\langle u^\mu u^\nu\rangle\left(D_\mu D_\nu + \text{ h.c. }\right) + \frac{c_3}{2}\langle u^\mu u_\mu\rangle - \frac{c_4}{4}\gamma^\mu\gamma^\nu\left[u_\mu, u_\nu\right]\right\}\Psi_N$$

N.Fettes, U.-G. Meißner, S. Steininger, NPA(1998)

- Two-nucleon sector

$$\begin{aligned}\mathcal{L}_{NN}^{(0)} =& \frac{1}{2}\left[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N) + C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right) + C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)\right.\\ &\left.+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right) + C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\right]\end{aligned}$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1}\bar{\Psi}_N\bar{\Psi}_N\mathcal{O}_i\Psi_N\Psi_N$$

L.Girlanda, S. Pastore, R. Schiavilla, M. Viviani, PRC(2010)

Yang Xiao, Li-Sheng Geng, XLR, PRC(2019)

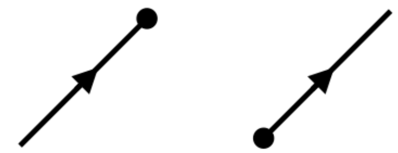
Diagrammatic rules in TOPT

- **External lines**

Spin 0 boson (in, out)

1

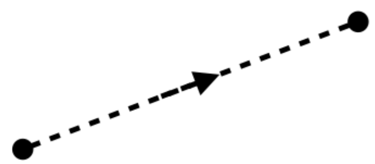
Spin 1/2 fermion (in, out)



$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

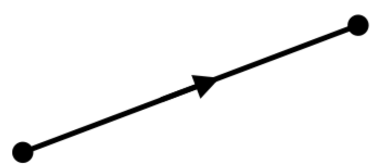
- **Internal lines**

Spin 0 (anti-)boson



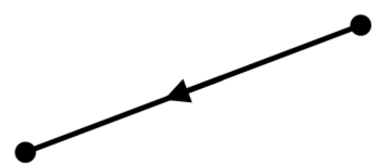
$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

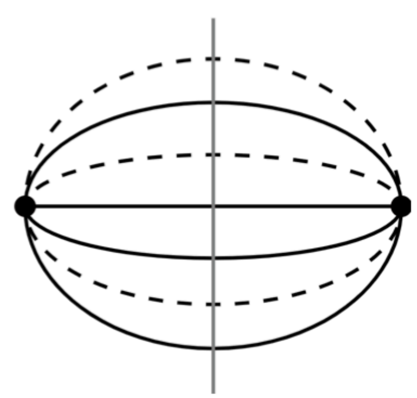
anti-fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

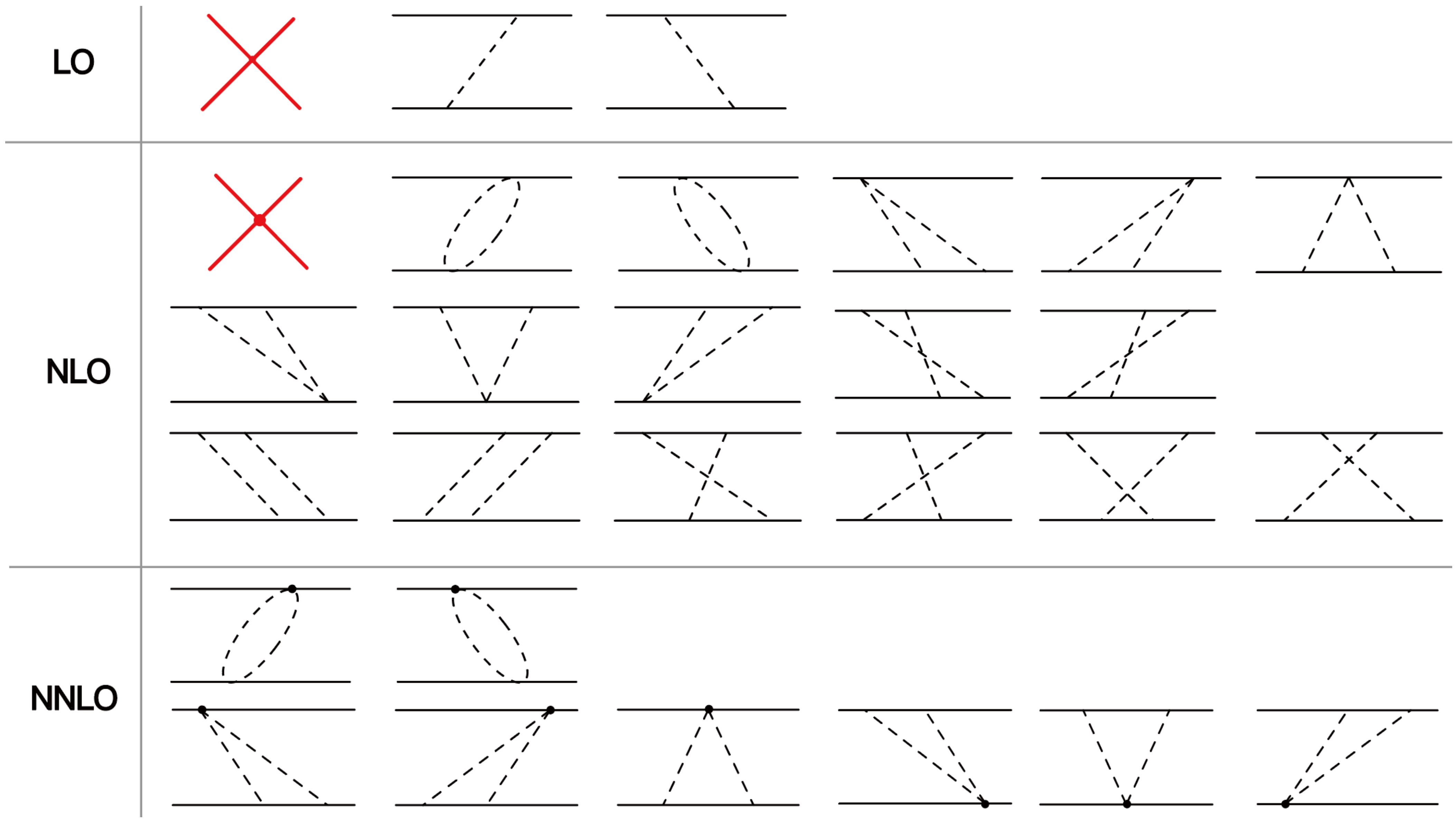
- **Interaction vertices:** the standard Feynman rules
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Nucleon-nucleon potential in TOPT

Two-nucleon **irreducible time-ordered diagrams**

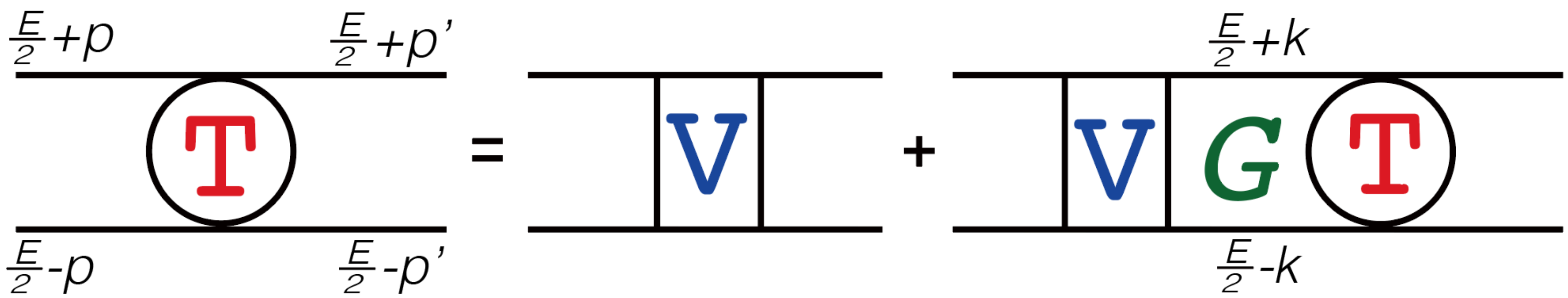


- Use the obtained TOPT rules to evaluate these diagrams
- Apply Weinberg power counting to organize
  - ✓ Expand the nucleon energy appearing in the numerator

$$\nu = 2l + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

Scattering equations in TOPT

- Following the above diagrammatic rules of TOPT, we obtain the scattering equations



- Two-nucleon Green function

$$G(E) = \frac{m_N^2}{\omega^2(k, m_N)} \frac{1}{E - 2\omega(k, m_N) + i\epsilon}$$

✓ This is the Kadyshevsky propagator of NN scattering *V. Kadyshevsky, NPB (1968)*

✓ **SELF-CONSISTENTLY obtained** in TOPT

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{2(\boldsymbol{k}^2 + m_N^2)} \frac{1}{\sqrt{\boldsymbol{p}^2 + m_N^2} - \sqrt{\boldsymbol{k}^2 + m_N^2} + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

✓ Milder UV behaviour than the Lippmann-Schwinger equation

Green function $G \xrightarrow{k\longrightarrow\infty}$ Kady. $\frac{1}{k^3}$ vs. LS $\frac{1}{k^2}$

OUTLINE

- Introduction

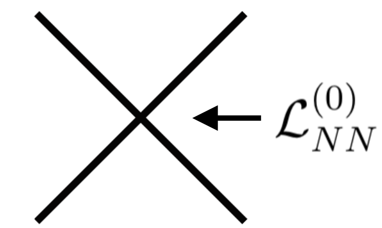
- Theoretical framework

- Results and discussion
  - Leading order study   Phys. Lett. B 798 (2019) 134987
  - Next-to-next-to-leading order calculation
    XLR, E. Epelbaum, J. Gegelia, arXiv:2022.04018, PRC in press

- Summary and perspective

Leading order potentials

- Contact nucleon-nucleon interaction



  - According to our TOPT rules

$$V_{0,C} = C_S(\bar{u}_3\, u_1)(\bar{u}_4\, u_2) + C_A(\bar{u}_3\, \gamma_5\, u_1)(\bar{u}_4\gamma_5 u_2) + C_V(\bar{u}_3\, \gamma_\mu\, u_1)(\bar{u}_4\, \gamma^\mu\, u_2)$$
$$+ C_{AV}(\bar{u}_3\, \gamma_\mu\gamma_5\, u_1)(\bar{u}_4\, \gamma^\mu\gamma_5\, u_2) + C_T(\bar{u}_3\, \sigma_{\mu\nu}\, u_1)(\bar{u}_4\, \sigma^{\mu\nu}\, u_3)$$

  - Contain higher order contributions according to Weinberg P.C.
  - Perform the expansion for the nucleon energies

$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

$$\boxed{V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}$$

✓ Two independent parameters to be fixed

✓ Consistent with the non-relativistic contact terms

S. Weinberg, PLB251(1990)288-292

Leading order potentials

One-pion-exchange (OPE) potential

- According to our TOPT rules

$$V_{0,\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{2\omega(q,M_\pi)}\left[\frac{\left(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1\right)\left(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2\right)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right.$$

$$\left.+\frac{\left(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1\right)\left(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2\right)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right]$$

- Contains higher order contributions according to Weinberg P.C.
- Perform the expansion for the nucleon energies in numerator

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

✓ Keep the nucleon energies in denominator (consistent with Kadyshevsky eq.)

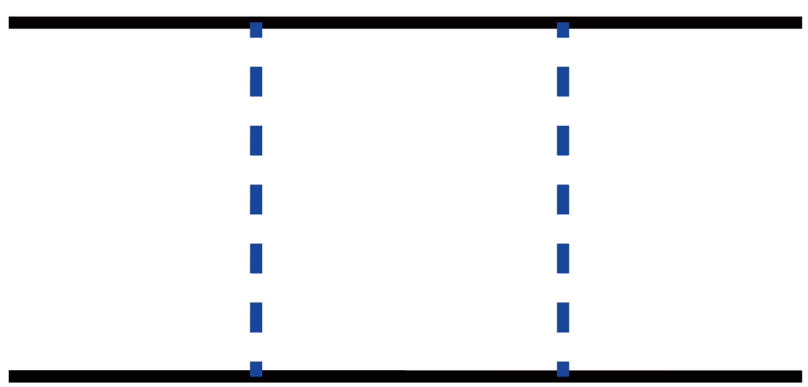
$$V_{\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{4m_N^2}{\omega(q,M_\pi)\left(m_N+\omega(p,m_N)\right)\left(m_N+\omega(p',m_N)\right)}$$

$$\times\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}$$

It has a milder UV behaviour than the non-relativisitc OPEP

UV Behavior of the long-range potential

- One-loop integral $V\,G\,V$:



$$I_{VGV} = \int \frac{d^3k}{(2\pi)^3}\, V_{\rm OPE}\, G(E)\, V_{\rm OPE}$$

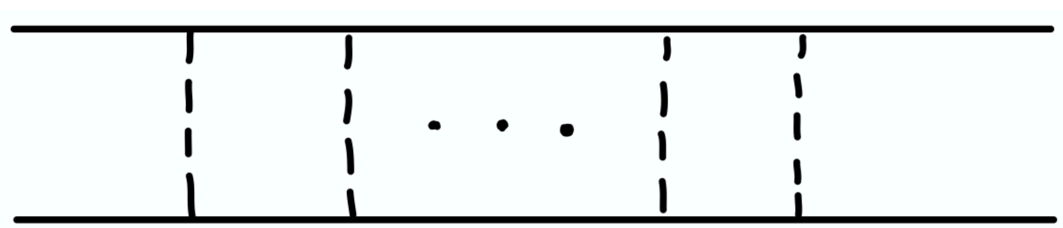
$$\xrightarrow{k\to\infty}$$

$$\begin{cases} \textbf{Our:}\ I_{VGV}^{\rm Our} \to \int dk^3\, \frac{1}{k}\,\frac{1}{k^3}\,\frac{1}{k} = \int dk^3\, \frac{1}{k^5} \\ \textbf{NR:}\ I_{VGV}^{\rm NR} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3\, \frac{1}{k^2} \end{cases}$$

Ultraviolet convergent!

Ultraviolet divergent!

- Iteration of our OPEP



$$\xrightarrow{k\to\infty}$$

Finite diagram!

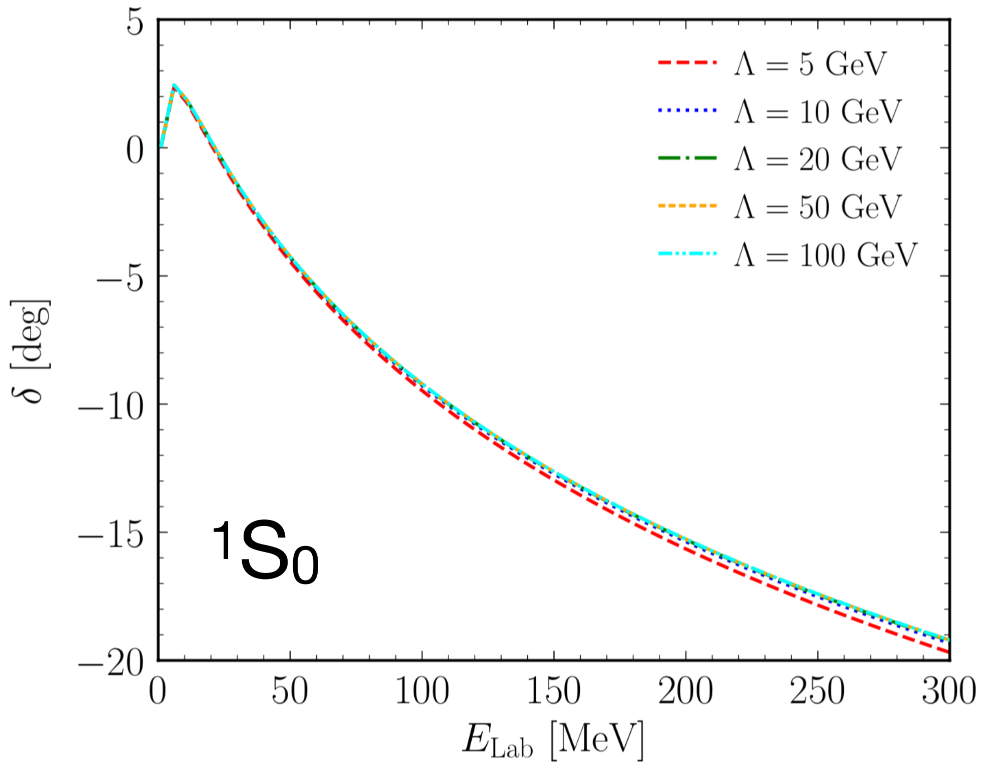
- Scattering amplitude from OPEP is cutoff independent

$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}\, G\, T_{\rm OPE}$$

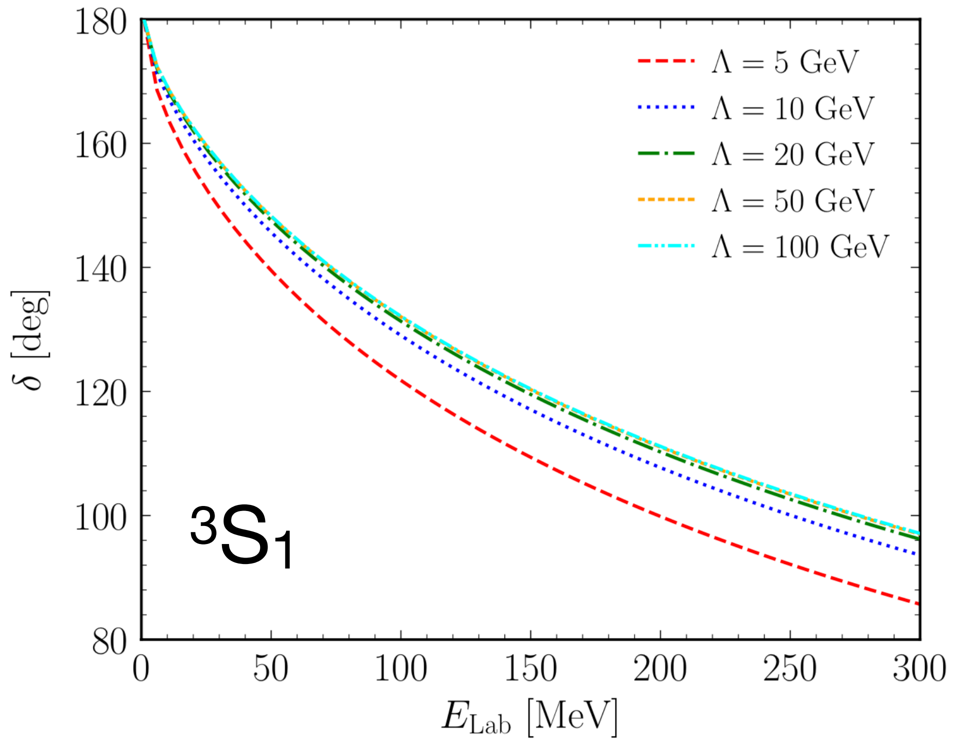
Renormalizable!

Phase shifts: cutoff-independent

- NN single channel: e.g. 1S0



- NN couple channels: e.g. 3S1



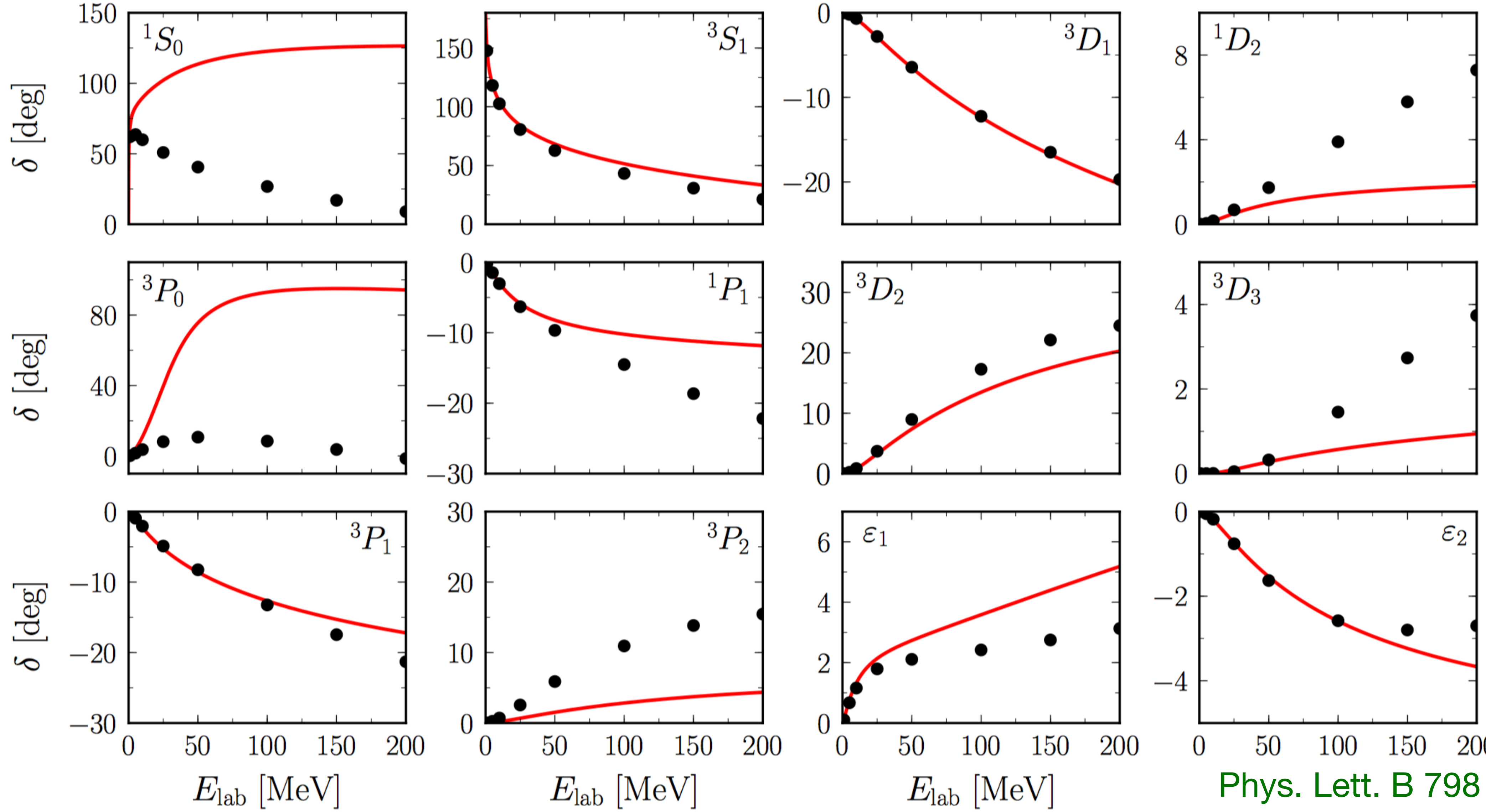
✤ **Our LO potential is perturbatively renormalizable!**

- All divergences appearing from its iterations can be absorbed in the coupling constant of the contact interaction
- Scattering equation has unique solutions for all partial waves
- Avoid finite-cutoff artefacts inherent to the conventional non-relativistic framework

Phase shifts of NN scattering

Phase shifts at LO with cutoff → ∞

- Two LECs are fixed by the scattering lengths of 1S0 and 3S1



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✓ **Provides a reasonable description of the empirical phase shifts**

✓ $^1S_0$ and $^3P_0$: Large deviation

➡ Part of the subleading corrections must be treated non-perturbatively

Beyond LO

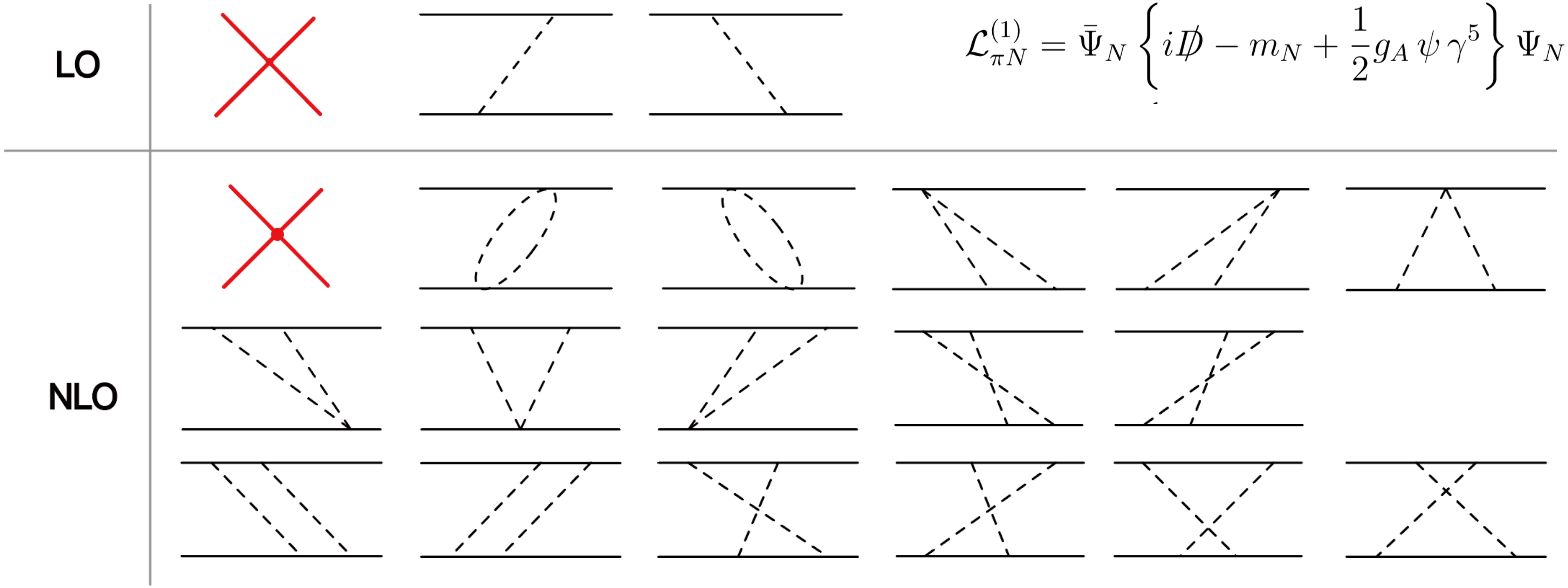
Beyond Leading order studies

- Two strategies to include higher orders
  - Restricting the non-perturbative treatment to the (non-singular) LO potential and higher-order interactions are treated perturbatively
    - ✓ Systematically remove all divergences from the amplitude
  - Full effective potential (LO + higher orders) are treated non-perturbatively
    - ✓ Milder UV behavior offers a larger flexibility regarding admissible cutoff
    - ✓ Direct input for few-/many-body problems
- Here, we focus on the second strategy (as a first step)
  - Since the derivation of higher order contributions is computationally more demanding
  - **Formulate the chiral nuclear potential up to NLO and NNLO**
  - Calculate the two-pion exchange contribution at one-loop level

XLR, E. Epelbaum, J. Gegelia, 2022.04018, PRC in press

Study of NLO potential in TOPT

- Time ordered diagrams up to NLO



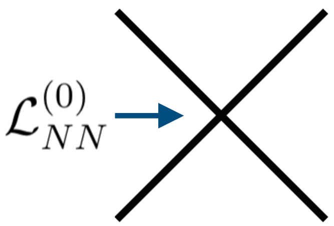
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2}\left[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N) + C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right) + C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)\right.$$
$$\left. + C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right) + C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\right]$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N\bar{\Psi}_N\mathcal{O}_i\Psi_N\Psi_N$$

Contact terms up to NLO

LO contact term (5 LECs)



$$V_{\rm LO} = C_S(\bar{u}_3 u_1)(\bar{u}_4 u_2) + C_A(\bar{u}_3 \gamma_5 u_1)(\bar{u}_4 \gamma_5 u_2) + C_V(\bar{u}_3 \gamma_\mu u_1)(\bar{u}_4 \gamma^\mu u_2)$$
$$+ C_{AV}(\bar{u}_3 \gamma_\mu \gamma_5 u_1)(\bar{u}_4 \gamma^\mu \gamma_5 u_2) + C_T(\bar{u}_3 \sigma_{\mu\nu} u_1)(\bar{u}_4 \sigma^{\mu\nu} u_3)$$

- Expand the nucleon energy up to $\mathcal{O}(p^2)$ / NLO

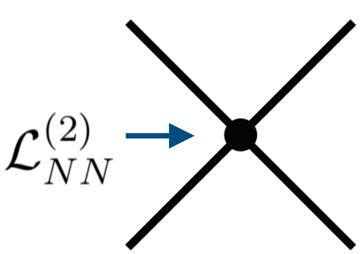
$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}\, m_N^{3/2}} + \mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for LO contact terms

➡ Keep the full form of Dirac spinors

NLO contact term

- Expand the nucleon energy $\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$
- Same form as the non-relativistic case with 7 LECs



$$V_{\rm NLO} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{P}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{P}^2\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{i}{2} C_5 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}$$
$$+ C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7 (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{P} \cdot \boldsymbol{\sigma}_2)$$

Partial wave decomposition for contact terms

- J=0: 1S0 and 3P0 partial waves

$$V(^1S_0) = \xi_N C^{LO}_{^1S_0}\left(1 + R_p^2 R_{p'}^2\right) + \xi_N\left[\hat{C}^{LO}_{^1S_0} + 4m_N^2 C^{NLO}_{^1S_0}\right]\left(R_p^2 + R_{p'}^2\right)$$
$$= \xi_N C^{LO}_{^1S_0}\left(1 + R_p^2 R_{p'}^2\right) + \xi_N \tilde{C}_{^1S_0}\left(R_p^2 + R_{p'}^2\right)$$

$$V(^3P_0) = -2\xi_N\left[C^{LO}_{^3P_0} - 2m_N^2 C^{NLO}_{^3P_0}\right] R_p R_{p'}$$
$$= -2\xi_N \tilde{C}_{^3P_0} R_p R_{p'} = -2\tilde{C}_{^3P_0}\frac{p\,p'}{4m_N^2}$$

$$\xi_N = \frac{(\omega_p + m_N)(\omega_{p'} + m_N)}{4m_N^2}, \quad R_p = \frac{p}{\omega_p + m_N}, \text{ and } R_{p'} = \frac{p'}{\omega_{p'} + m_N}$$

- J=1: 1P1, 3P1, 3S1-3D1 partial waves

$$V(^1P_1) = \frac{2}{3}\xi_N\left[-C^{LO}_{^1P_1} - 6m_N^2\, C^{NLO}_{^1P_1}\right] R_p R_{p'}$$
$$= \frac{2}{3}\xi_N \tilde{C}_{^1P_1} R_p R_{p'} = \frac{2}{3}\tilde{C}_{^1P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3P_1) = -\frac{4}{3}\xi_N\left[C^{LO}_{^3P_1} - 3m_N^2 C^{NLO}_{^3P_1}\right] R_p R_{p'}$$
$$= -\frac{4}{3}\xi_N \tilde{C}_{^3P_1} R_p R_{p'} = -\frac{4}{3}\tilde{C}_{^3P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3S_1) = \frac{\xi_N}{9} C^{LO}_{^3S_1}\left(9 + R_p^2 R_{p'}^2\right) + \frac{\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 36m_N^2 C^{NLO}_{^3S_1}\right]\left(R_p^2 + R_{p'}^2\right)$$
$$= \frac{\xi_N}{9} C^{LO}_{^3S_1}\left(9 + R_p^2 R_{p'}^2\right) + \xi_N \tilde{C}_{^3S_1}\left(R_p^2 + R_{p'}^2\right)$$

$$V(^3D_1) = \frac{8\xi_N}{9} C^{LO}_{^3S_1} R_p^2 R_{p'}^2$$

$$V(^3S_1 - {}^3D_1) = \frac{2\sqrt{2}\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 9\sqrt{2}m_N^2 C^{NLO}_{^3D_1 - {}^3S_1}\right] R_p^2 + \frac{2\sqrt{2}\xi_N}{9} C^{LO}_{^3S_1} R_p^2 R_{p'}^2$$
$$= \xi_N \tilde{C}_{^3D_1 - {}^3S_1} R_p^2 + \frac{2\sqrt{2}\xi_N}{9} C^{LO}_{^3S_1} R_p^2 R_{p'}^2$$

$$V(^3D_1 - {}^3S_1) = \xi_N \tilde{C}_{^3D_1 - {}^3S_1} R_{p'}^2 + \frac{2\sqrt{2}\xi_N}{9} C^{LO}_{^3S_1} R_p^2 R_{p'}^2$$

- J=2: 3P2 partial wave $V(^3P_2) = C^{NLO}_{^3P_2}\, p\,p' = \tilde{C}_{^3P_2}\frac{p\,p'}{4m_N^2}$

- Finally, we have **9 LECs to be fixed:** $C^{LO}_{^1S_0}, C^{LO}_{^3S_1}, \tilde{C}_{^1S_0}, \tilde{C}_{^3P_0}, \tilde{C}_{^1P_1}, \tilde{C}_{^3P_1}, \tilde{C}_{^3S_1}, \tilde{C}_{^3D_1 - {}^3S_1}, \tilde{C}_{^3P_2}$

Same number of contact terms as the non-relativistic NLO case

One-Pion exchange potential up to NLO

OPE potential

$$V_{\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{\omega_q}\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p+\omega_{p'}+\omega_q-E-i\epsilon}$$

- Expand the nucleon energy expansion for OPEP at NLO

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}\,m_N^{3/2}} + \mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for OPE potential

➡ Keep the full form of Dirac spinors

- Eliminate the energy dependence of OPEP (avoid the pole contribution)

✓ Expand E at $\omega_p + \omega_p'$, then, we obtain contribution of OPEP at NLO

$$V_{\mathrm{OPE}}^{\not E} = -\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\frac{1}{\omega_q^2}(\bar{u}_3\gamma_\mu\gamma_5 q^\mu u_1)(\bar{u}_4\gamma_\nu\gamma_5 q^\nu u_2) \longrightarrow$$

LO correction $V_{\mathrm{OPE},\not E}^{(0)}$

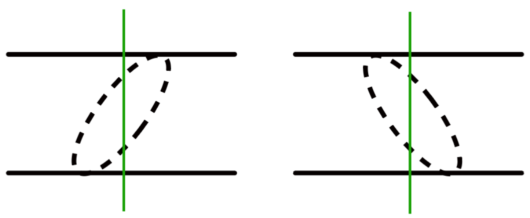
NLO correction $V_{2\pi,\not E}^{(2)}$

$$\left\{\begin{aligned} &+\frac{1}{2}\left(\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\right)^2\int\frac{d^3k}{(2\pi)^3}\frac{m_N^2}{k^2+m_N^2}\frac{\omega_{p'-k}+\omega_{p-k}}{\omega_{p'-k}^3\omega_{p-k}^3} \\ &\times\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}-\boldsymbol{p})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}-\boldsymbol{p})\right].\end{aligned}\right.$$

Two-pion exchange potential at NLO

- Follow our TOPT rules:
  - Football diagram

$$V_F = \frac{1}{16 f_\pi^4}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \int \frac{d^3k}{(2\pi)^3}\,\frac{(\omega_k+\omega_{k+q})(\omega_p+\omega_{p'})+4\omega_k\omega_{k+q}-E(\omega_k+\omega_{k+q})}{2\omega_k\omega_{k+q}\,\underline{(\omega_k+\omega_{k+q}+\omega_p+\omega_{p'}-E)}}.$$

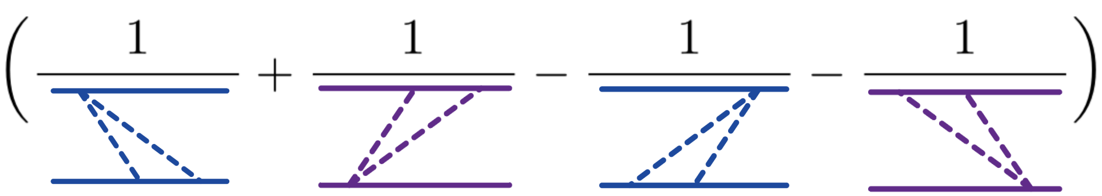


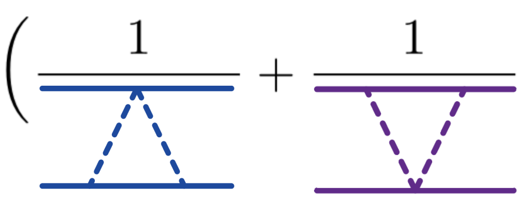
Energy denominator of football diagram

  - Triangle diagrams

$$V_{T+\tilde T}^{NN} = \frac{4m_N g_A^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{128 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[\left(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k}\right)+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}(a+b)\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

$$\times\left[(\omega_{k+q}-\omega_k)\left(\ \cdots\ \right)+(\omega_k+\omega_{k+q})\left(\ \cdots\ \right)\right]$$



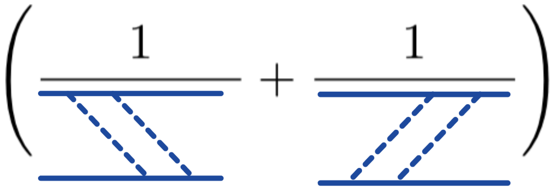


Energy denominator

  - Planar and crossed box diagrams

$$V_B = \frac{m_N^2 g_A^4(3-2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{64 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_1+X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+X_3\frac{i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}+X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}^2}\left(\ \cdots\ \right)$$



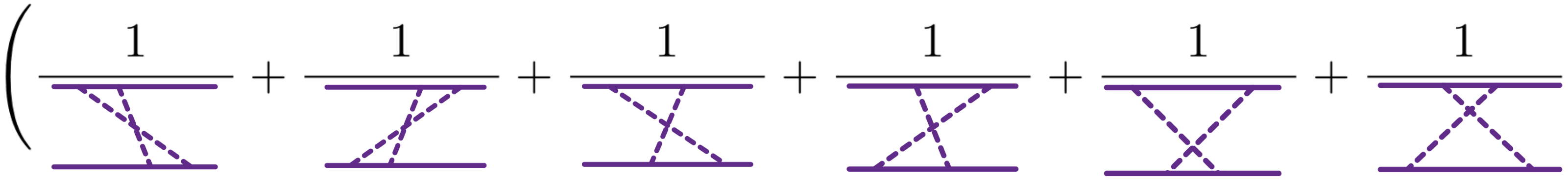
$$\boldsymbol{k} = a\,\boldsymbol{p}+b\,\boldsymbol{p}'+c\,(\boldsymbol{p}'\times\boldsymbol{p})$$

$X_1=\left[\boldsymbol{k}^2+\boldsymbol{q}\cdot\boldsymbol{k}\right]^2,\quad X_2=-c^2\boldsymbol{q}^2\left[\boldsymbol{P}^2\boldsymbol{q}^2-(\boldsymbol{q}\cdot\boldsymbol{P})^2\right],\quad X_3=-2(a+b)\left(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k}\right),$

$X_4=-(a+b)^2+c^2\boldsymbol{q}^2,\quad X_5c^2\left[\boldsymbol{P}^2\boldsymbol{q}^2-(\boldsymbol{q}\cdot\boldsymbol{P})^2\right]$

$$V_{\tilde B} = \frac{m_N^2 g_A^4(3+2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{64 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_1+X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}\omega_{p'+k}}\left(\ \cdots\ \right)$$

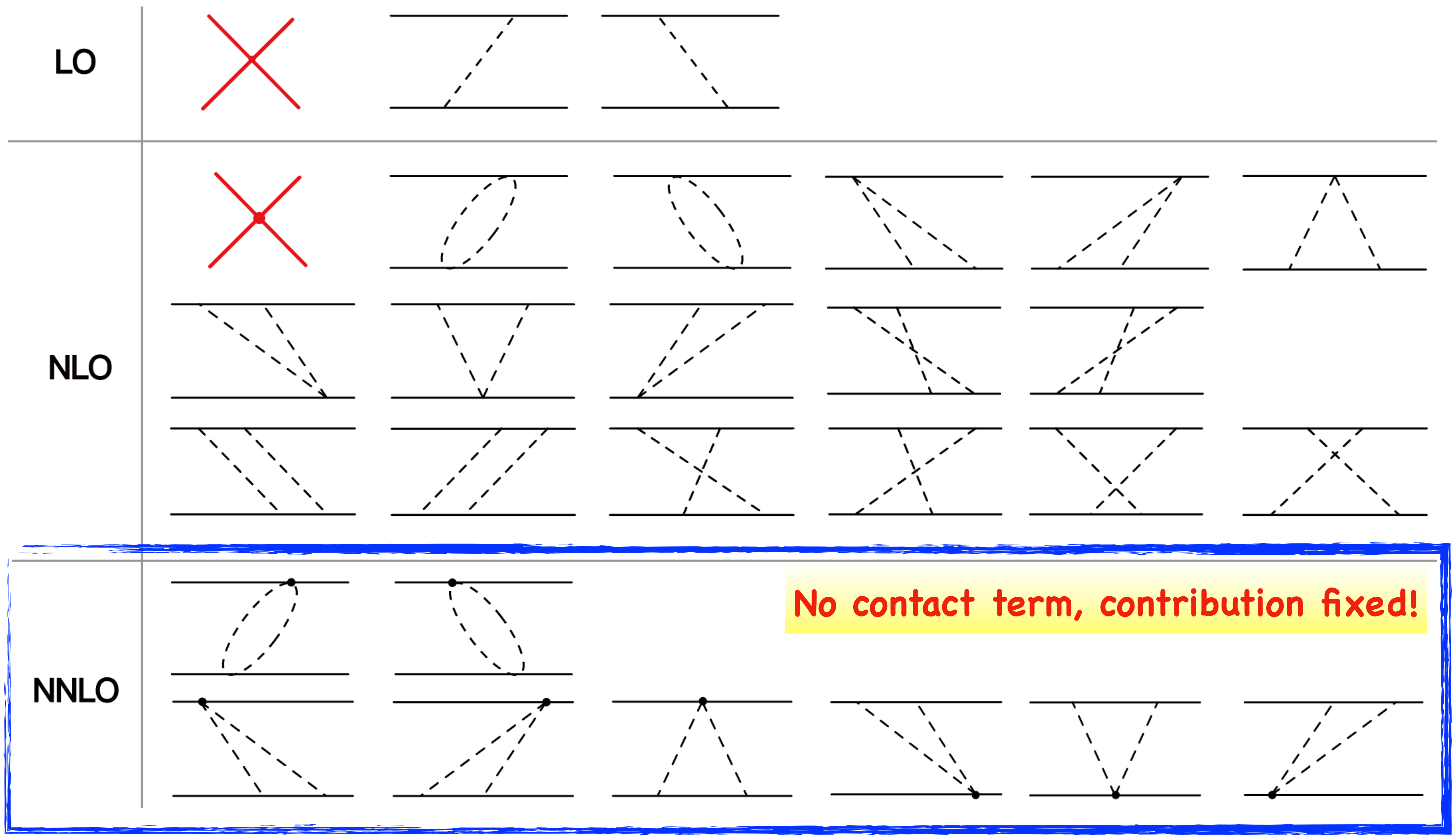


2022.04018

UV Divergent terms and power counting breaking terms are removed by using **the subtractive renormalization**

Study of NNLO potential in TOPT

- Time ordered diagrams up to NNLO



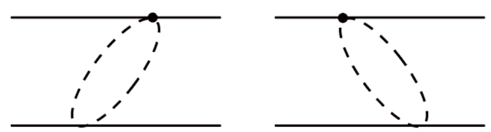
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle (D_\mu D_\nu + \text{ h.c. }) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] \right\} \Psi_N$$

$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \mathrm{GeV}^{-1}$

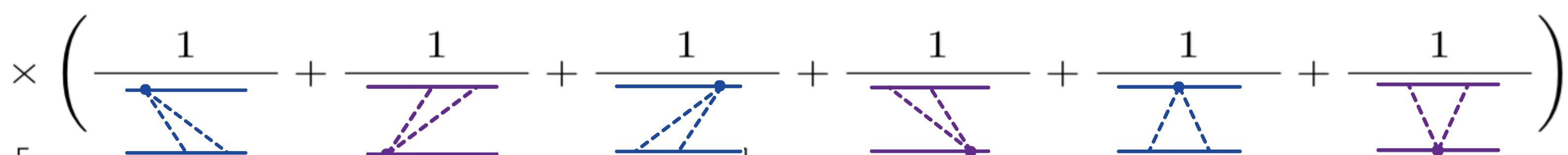
D. Siemens, et al., 1610.08978

Two-pion exchange potential at NNLO

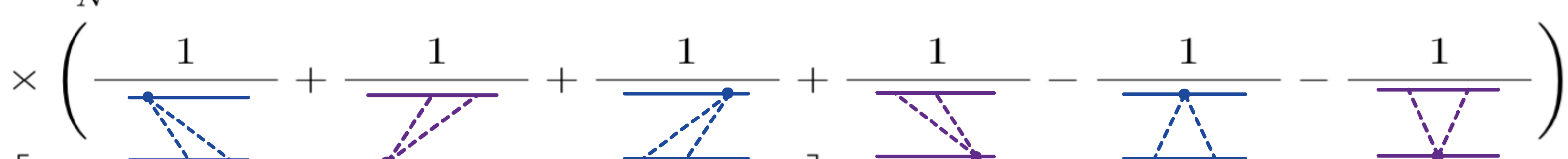
- Follow our TOPT rules:
  - Football diagrams  No contribution!
  - Triangle diagrams

$$V_{T+\tilde{T}} = \frac{3 m_N g_A^2}{16 f_\pi^4}\int \frac{d^3k}{(2\pi)^3}\left[(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k})-(a+b)\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

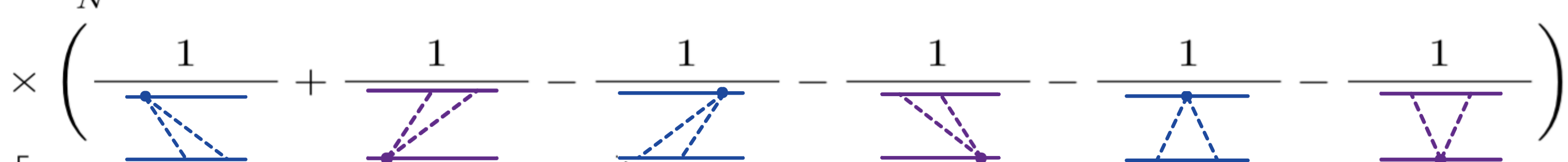
$$\times\left\{\left[4c_1M_\pi^2-\frac{c_2}{m_N^2}\Big(\boldsymbol{p}\cdot\boldsymbol{k}\,\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\boldsymbol{p}'\cdot\boldsymbol{k}\,\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\Big)+2c_3\boldsymbol{k}\cdot(\boldsymbol{k}+\boldsymbol{q})\right]\right.$$



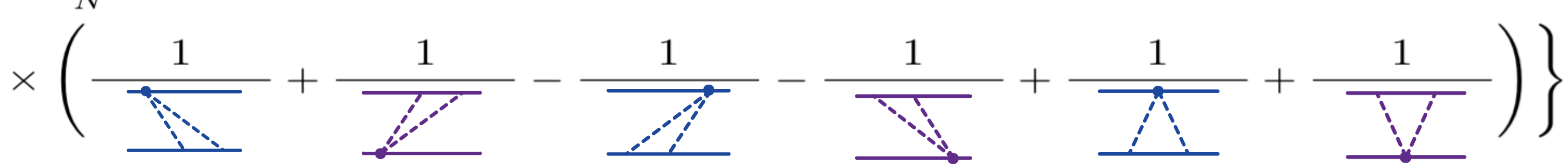
$$+\left[\frac{c_2}{m_N^2}\omega_k\omega_{k+q}(\omega_p+\omega_{p'})+2c_3\omega_k\omega_{k+q}\right]$$



$$+\left[\frac{c_2}{m_N^2}\omega_k\left(\omega_p\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\omega_{p'}\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\right)\right]$$



$$-\left[\frac{c_2}{m_N^2}\omega_{k+q}\left(\omega_p\boldsymbol{p}\cdot\boldsymbol{k}+\omega_{p'}\boldsymbol{p}'\cdot\boldsymbol{k}\right)\right]$$



$$\left.\right\}$$

- UV Divergent terms
- Power-counting breaking terms
- are removed by using **the subtractive renormalization**

$$+\frac{c_4 m_N g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{8f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{X_3}{2}\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}+X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

2022.04018

Prediction for peripheral phases

(D, F, … waves)

XLR, E. Epelbaum, J. Gegelia, arXiv:2022.04018, PRC in press

T-matrix and phase shifts

- **On-shell T-matrix:** Born series truncated at one-loop order

$$T(p',p) = V^{(0)}_{\text{OPE},\not{E}}(p',p) + V^{(2)}_{2\pi,\not{E}}(p',p) + V^{(2)}_{2\pi,irr}(p',p) + V^{(3)}_{2\pi,irr}(p',p) + V_{2\pi,it}(p',p)$$

$$= V^{(0)}_{\text{OPE},\not{E}}(p',p) + V_{2\pi,irr}(p',p) + V_{2\pi,it}(p',p), \qquad V_{2\pi,it}(p',p) = V^{(0)}_{\text{OPE},\not{E}}\, G\, V^{(0)}_{\text{OPE},\not{E}}$$

  - D, F and higher partial waves: no contact-interaction contributions

- **Phase shifts and mixing angles of the partial waves**

$$\delta_l^{sj} = -\frac{p\, m_N^2}{16\pi^2\sqrt{p^2+m_N^2}}\,\text{Re}\langle lsj|T|lsj\rangle,$$

$$\epsilon_j = \frac{p\, m_N^2}{16\pi^2\sqrt{p^2+m_N^2}}\,\text{Re}\langle j-1,1,j|T|j+1,1,j\rangle,$$

J. Gasser and U.-G. Meißner, PLB 258, 219 (1991).

  - Phase shifts: $\delta = \delta^{(\nu=0)} + \delta^{(\nu=2)} + \delta^{(\nu=3)} + \delta^{\mathcal{O}(\nu=3)}$
  - Higher order: $\delta^{\mathcal{O}(\nu=3)}$ depends on the method of unitarizing, but negligible!

Consistency check of the TPEP

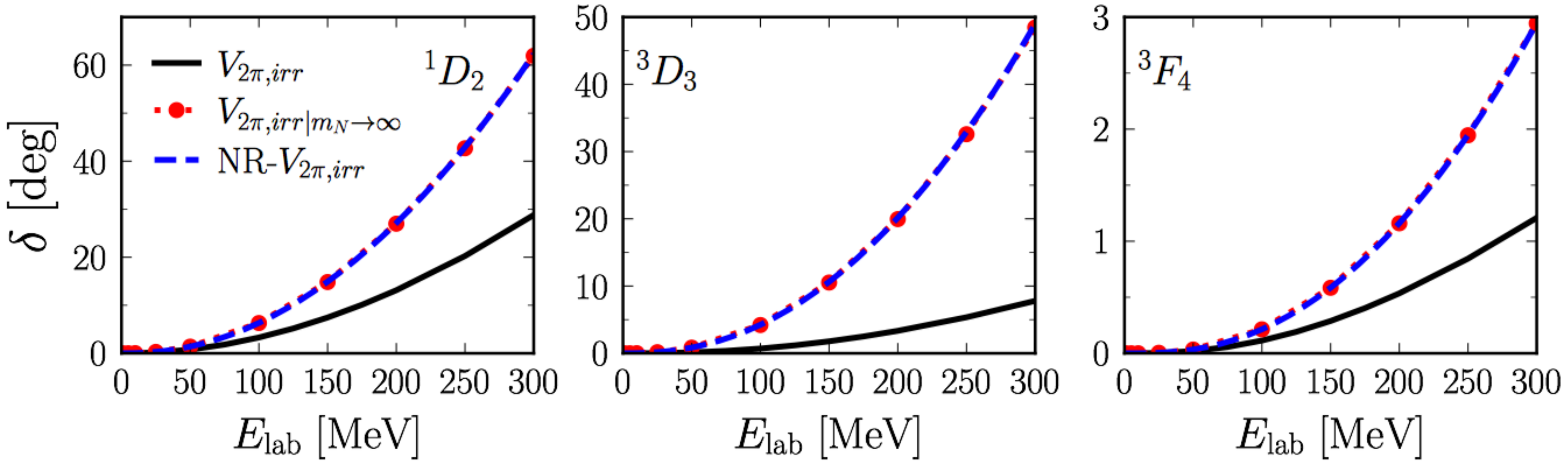
- For D, F waves, since no contact terms contribution, it provides a good chance to check our TPEP
  - Our TPEP obtained using the subtractive renormalization
  - Non-relativistic TPEP calculated using the DimReg + MS
  - Use the same parameters:

$$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \mathrm{GeV}^{-1}$$

D. Siemens, et al., 1610.08978

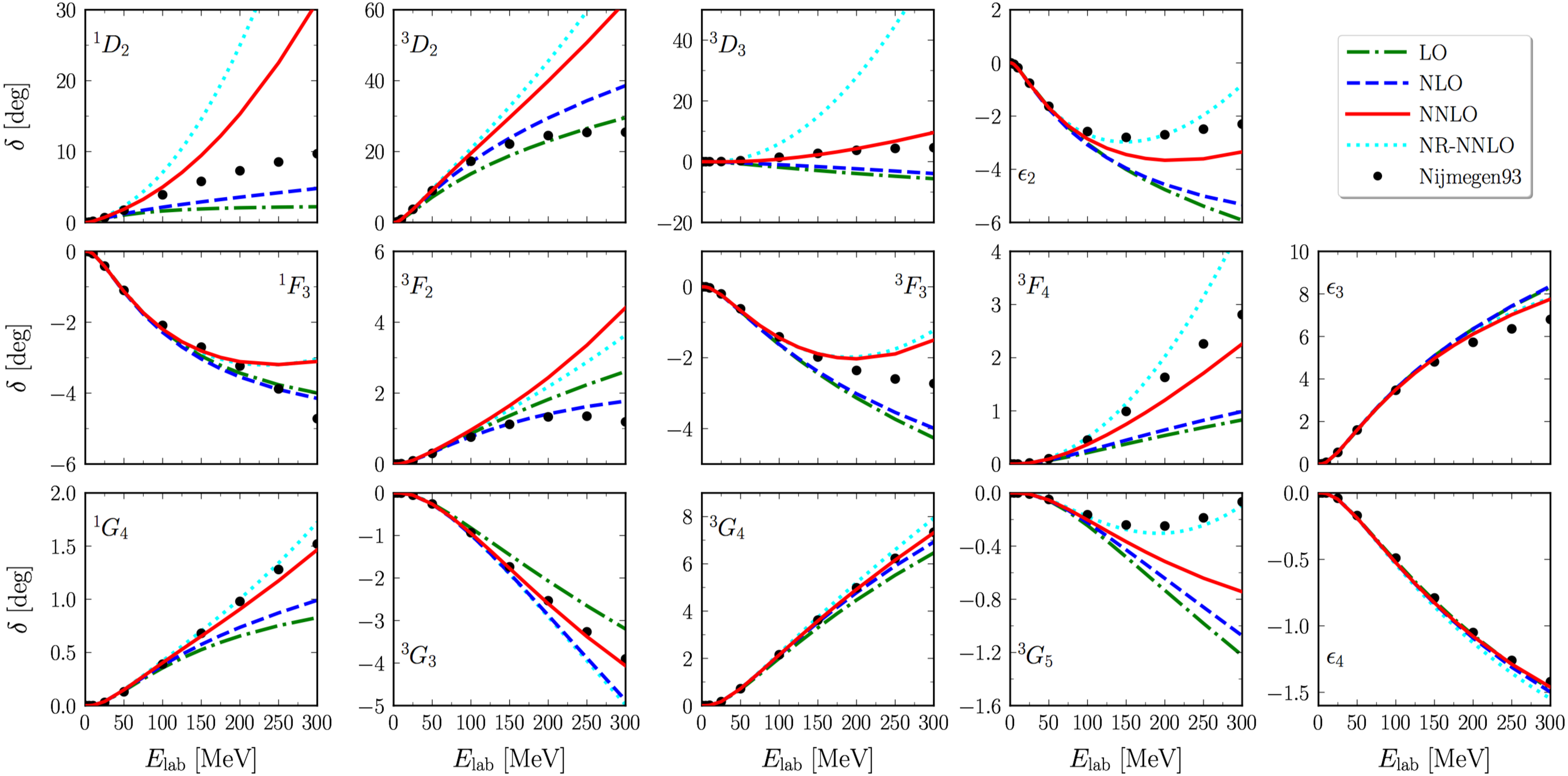
$$f_\pi = 92.4\ \mathrm{MeV}, \quad g_A = 1.29$$

  - **Achieve the same results when m_N is taken to infinity**



Peripheral phase shifts

- Prediction for the D, F, G partial waves



- Improve the description of D waves, especially for $^3D_3$
- Give the globally similar results for F, G waves
  - ✓ $^3G_5$: non-rel. result is accidental, c_i/m_N effect (N^4LO) is large.

D. Entem, et al., 1411.5335

Low phases and the deuteron

XLR, E. Epelbaum, J. Gegelia, in progress

Summary

- We proposed a systematic framework to formulate the NN interactions based on the time-ordered perturbation theory using the manifestly Lorentz invariant effective Lagrangian
  - Obtained the rules of time-ordered diagrams with spin-1/2 fermions
  - Derived the Kadyshevsky equation self-consistently
    - ✓ Effective potential and the scattering equation are obtained within the same framework
  - Obtained **non-singular LO potential**, which is perturbatively renormalizable
    - ✓ Avoid finite-cutoff artefacts and take cutoff $\Lambda \to \infty$
  - **Formulated the chiral potential up to NNLO**
    - ✓ Calculated the complicated two-pion-exchange potential at one-loop level
    - ✓ Achieved a rather reasonable description of peripheral shifts

Future perspectives

- Investigate the energy-independent potential at NNLO
  - Follow the idea of energy-independent OPEP

$$V(E;k,q) = V\left(\omega_k+\omega_q;p',p\right) + (E-\omega_k-\omega_q)\,\frac{\partial V(E;k,p)}{\partial E}|_{E=\omega_k+\omega_q} + \frac{(E-\omega_k-\omega_q)^2}{2!}\frac{\partial^2 V(E,k,p)}{\partial E^2}|_{E=\omega_k+\omega_q} + \cdots$$

  - Can be applied to the whole two-pion-exchange potentials
  - This will be more convenient for many-body calculations
- Perturbatively include NLO/NNLO contributions
  - Based on our non-singular LO potential, all divergences of the amplitude can be systematically removed $(\Lambda \sim \infty)$
- In the long run, apply **symmetry preserving regularization** to investigate the chiral potential

H. Krebs talk

  - e.g. preserve chiral symmetry

J. Behrendt, E. Epelbaum, J. Gegelia, U.-G. Meißner and A. Nogga, Eur. Phys. J. A 52,296 (2016).

Thank you for your attention!