Renormalization of nuclear chiral EFT

A. M. Gasparyan, Ruhr-Universität Bochum

in collaboration with E. Epelbaum

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Outline

- Explicit renormalization: motivation
- NN chiral EFT. Finite cutoff. NLO: perturbative renormalization
- Cutoff dependence
- Non-perturbative renormalization
- Infinite cutoff scheme
- Summary

EFT: systematic expansion.
Power counting. Theoretical error estimation.

Expansion parameter: (soft scale)/(hard scale) $Q = \dfrac{q}{\Lambda_b}$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$$

“Perturbative” calculation of the S-matrix, spectrum, etc.

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$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Contains bare parameters → Renormalization: power counting for renormalized quantities

Explicit renormalization of nuclear chiral EFT is a complicated matter.
Non-perturbative effects.

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Recent progress: NN EFT at NLO

AG, Epelbaum, **PRC105**, 024001 (2022),
PoS **PANIC2021**, 371 (2022), *In preparation* (2022)

Power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)

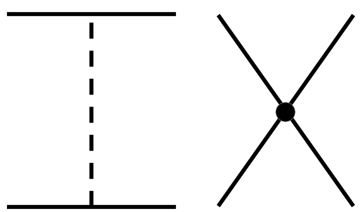
For potential (2N-irreducible) contributions:

$$D = 2L + \sum_{i=\text{vertices}} \left(d_i + \frac{n_i}{2} - 2 \right)$$

d_i − number of derivatives and quark masses

n_i − number of nucleon fields, L − number of loops

$\mathcal{O}(Q^0)$



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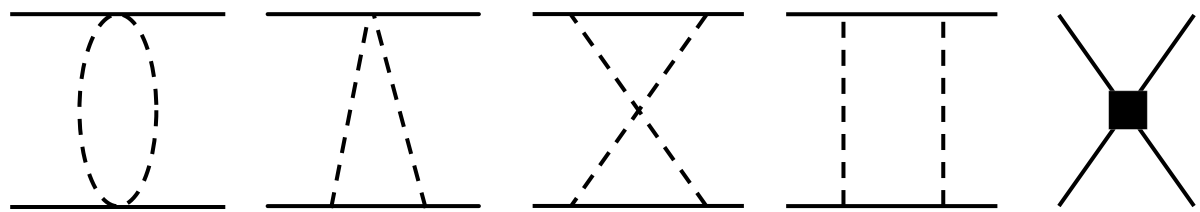

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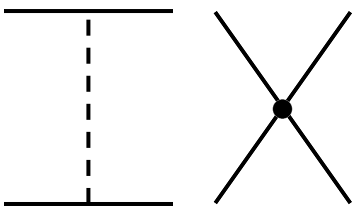

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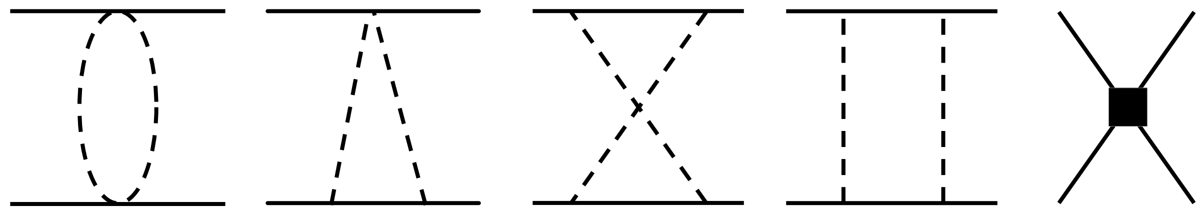

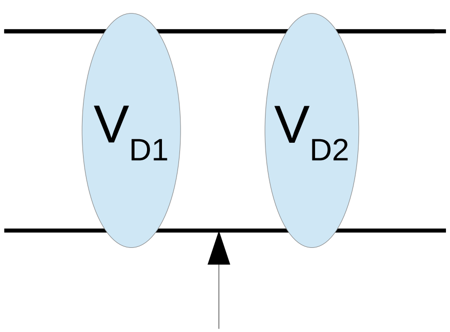

$$\sim \frac{m_N q}{\Lambda_b^2} \sim 1$$

Enhancement due to the infrared singularity: V_0 must be iterated

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m \, V_2 \, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

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→ Regulator: cutoff Λ

Infinite number of counter terms to absorb positive powers of Λ

Two intuitive approaches:
Infinite cutoff ($\Lambda >> \Lambda_b$) scheme, "RG invariant"

All positive powers of Λ cancel

$$T \approx 1 + \Lambda + \Lambda^2 + \cdots = \frac{1}{1-\Lambda}$$

A. Nogga, R. Timmermans,
U. van Kolck, **PRC72**, 054006 (2005)
B. Long, C. Yang, **PRC85**, 034002 (2012)
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Motivation: singular potentials

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Rev. Mod. Phys. 43, 36 (1971)

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Criticism

E. Epelbaum, J. Gegelia, **EPJA41**, 341 (2009)
E. Epelbaum, AG, J. Gegelia,U.-G. Meißner, **EPJA54**, 186 (2018)

Two intuitive approaches:
Finite cutoff

$\Lambda \approx \Lambda_b$

Cutoff dependence gets weaker when chiral order increases

Phenomenological success (NN): $\geq N^4LO$

P. Reinert, H. Krebs, and E. Epelbaum, **EPJA54**, 86 (2018)
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Explicit renormalization: power counting?

Power counting. Leading order. Perturbative case.

Perturbative: the series in V_0 is convergent, but the number of terms is arbitrary

$$T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0)$$

$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1} dp}{(\Lambda_{\rm V})^n} \sim \left(\frac{\Lambda}{\Lambda_V} \right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b} \right)^n \sim \mathcal{O}(Q^0)$$

G. P. Lepage, nucl-th/9706029
J. Gegelia, **JPG25**, 1681 (1999)

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Rigorously proved
under rather general conditions on V_0
if T_0 is perturbative
(P-waves and higher except for $^3P_0$):

$$\mathrm{T}_0 = \sum_{n=0}^{\infty} T_0^{[n]}$$

$$T_0^{[n]} \leq \mathcal{M}_1 \left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^n \qquad \mathcal{M}_1\,, \mathcal{M}_2 \sim 1$$

AG, E.Epelbaum,
PRC 105, 024001 (2022)

Renormalization at NLO. Perturbarive case

Renormalization: power counting in terms of renormalized quantities

$$T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from momenta: $p \sim \Lambda \, , p' \sim \Lambda$ in $V_2(p', p)$

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Can be absorbed by LO contact interactions? $\mathbb{R}\left(T_2^{[m,n]}\right) \sim \frac{q^2}{\Lambda_b^2} \left(\frac{\Lambda}{\Lambda_V}\right)^{m+n}$?

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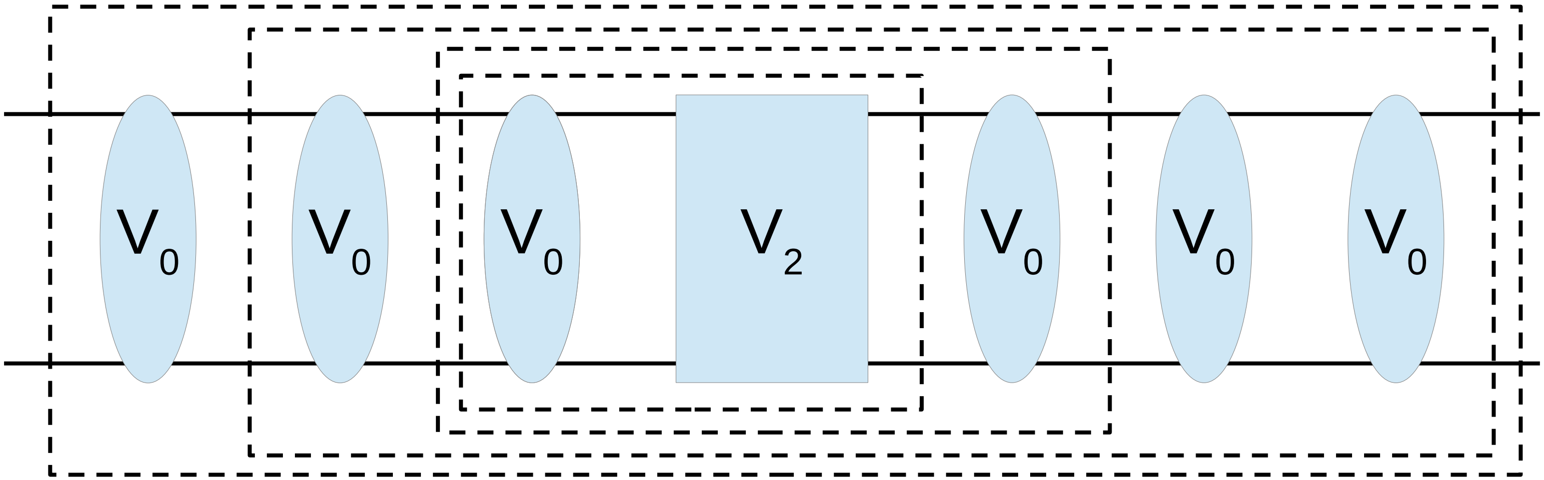
Non-perturbative effects?

Renormalization of NLO amplitude to arbitrary order in V_0. BPHZ subtraction scheme

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



Subtraction operation:

$$\mathbb{T}(X)(p', p, p_{\rm on}) = X(p'=0, p=0, p_{\rm on}=0)$$

Renormallized amplitude (forest formula):

$$\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}$$

$$U_k = ((m_{k,1}, n_{k,1}), (m_{k,2}, n_{k,2}), \ldots), \quad m \geq m_{k,i+1} \geq m_{k,i} \geq 0, \; n \geq n_{k,i+1} \geq n_{k,i} \geq 0 .$$

Power counting in the perturbative case, NLO

AG, E.Epelbaum, PRC 105, 024001 (2022)

Convergent series in V_0 :

$$\mathbb{R}\left(T_2\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$$

$$\left|\mathbb{R}(T_2^{[m,n]})(p)\right| \leq \mathcal{M}_1 \left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^{m+n} \frac{p^2}{\Lambda_b^2} \log \Lambda/M_\pi$$

$$\mathcal{M}_1\,, \mathcal{M}_2 \sim 1$$

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☺

$$\mathcal{M}_1\,, \mathcal{M}_2 \sim 1$$

Cutoff dependence. Systematic study.

Regulated potential: $V_0 \equiv V_\Lambda = V_{\Lambda=\infty} + \delta V_\Lambda$

Perturbative inclusion of δV_Λ: $\delta T_2^\Lambda = (1 + T_0 G)\delta V_0^\Lambda (1 + G T_0) \sim \mathcal{O}(Q^0)$

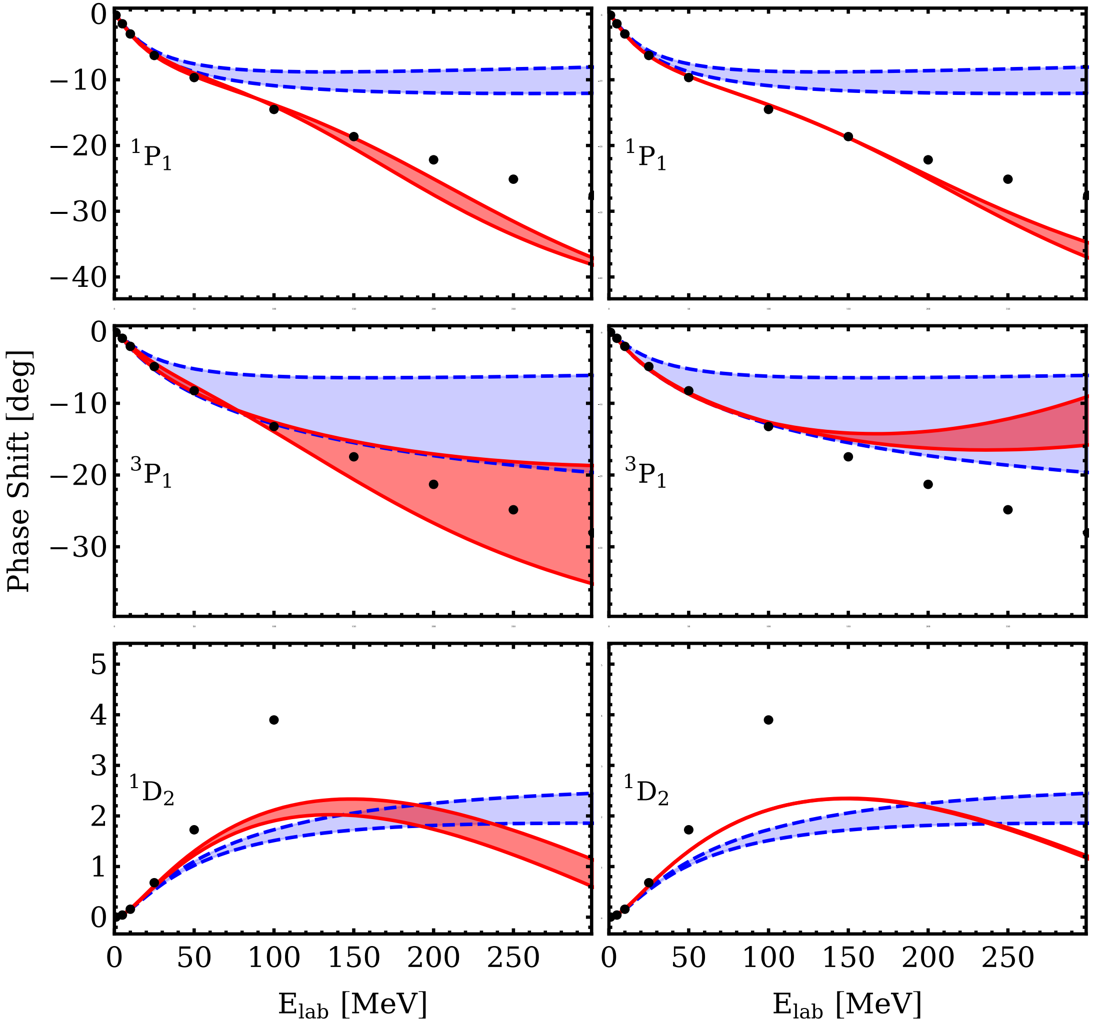
After renormalization: $\mathbb{R}\left(\delta T_2^\Lambda\right) \sim \mathcal{O}(Q^2)$

Removing Λ-dependence perturbatively

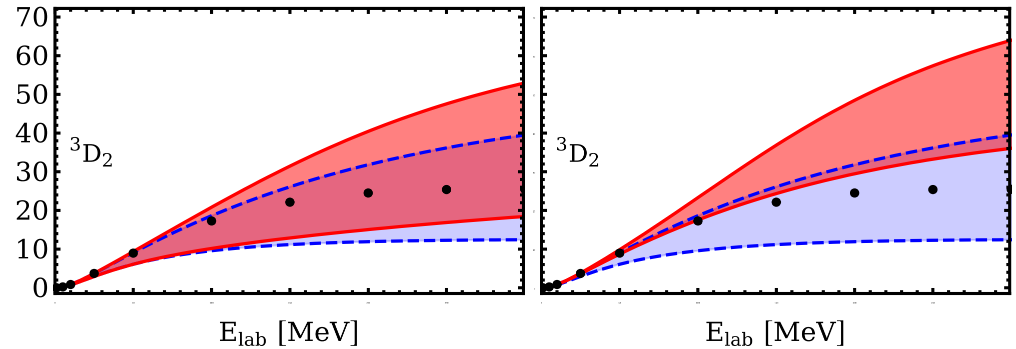
Cutoff dependence: P and D-waves. Uncoupled perturbative channels

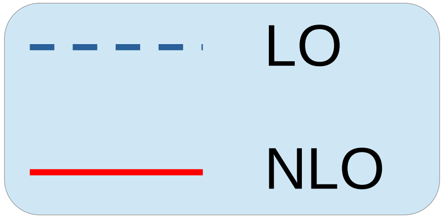
AG, E.Epelbaum, PRC 105, 024001 (2022)

with δV_Λ



with δV_Λ





$$F_\Lambda = \frac{\Lambda_\pi^2 - M_\pi^2}{\Lambda_\pi^2 + \vec{q}^{\,2}}$$

$$\Lambda_\pi \in (300, 800)\mathrm{MeV}$$

Cutoff dependence with δV_Λ *is weaker*

S-waves. Non-perturbative LO. Fredholm formula

$$T_0 = V_0 R = \bar{R} V_0 \qquad R = \frac{1}{\mathbb{1} - G V_0} = \frac{N}{D}\,, \quad \bar{R} = \frac{1}{\mathbb{1} - V_0 G} = \frac{\bar{N}}{D}$$

Convergent series in V_0:

$$N = \sum_{i=0}^{\infty} N^{[i]}\,, \quad D = \sum_{i=0}^{\infty} D^{[i]}$$

(Quasi-) bound state:

$$D(p) \sim \frac{p}{M_\pi}$$

Enhancement at threshold:

$$T_0(p) = \frac{N_0(p)}{D(p)} \sim \mathcal{O}(Q^{-1})$$

NLO. Using Fredholm formula.

$$T_2(p) = (1 + T_0 G) V_2 (1 + G T_0) = \frac{N_2(p)}{D(p)^2}$$

Convergent series in V_0 :

$$N_2 = \sum_{i=0}^{\infty} N_2^{[i]} \, , \;\; D = \sum_{i=0}^{\infty} D^{[i]}$$

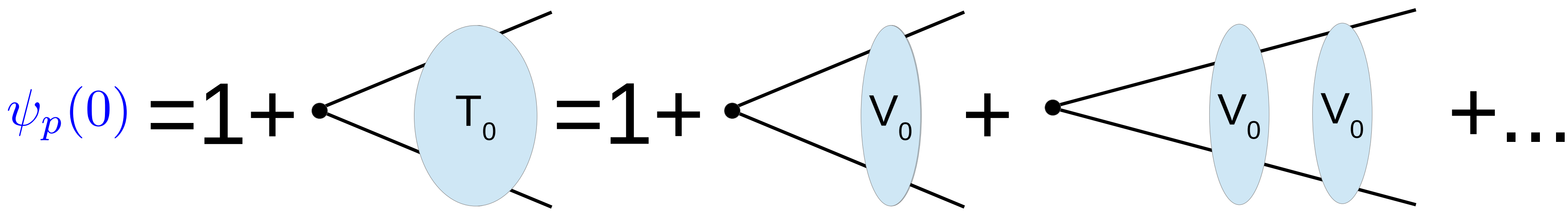
The same for the counter terms:

$$\delta T_2 = (1 + T_0 G) \delta V_0^{ct} (1 + G T_0)$$

S-waves. NLO.
Subtractions in the non-perturbative case

The series for $R(T_2^{[m,n]})$ can be summed explicitly

$$\mathbb{R}\left(T_2\right)(p) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p) = T_2(p) - T_2(p=0)\left[\frac{\psi_p(0)}{\psi_{p=0}(0)}\right]^2$$



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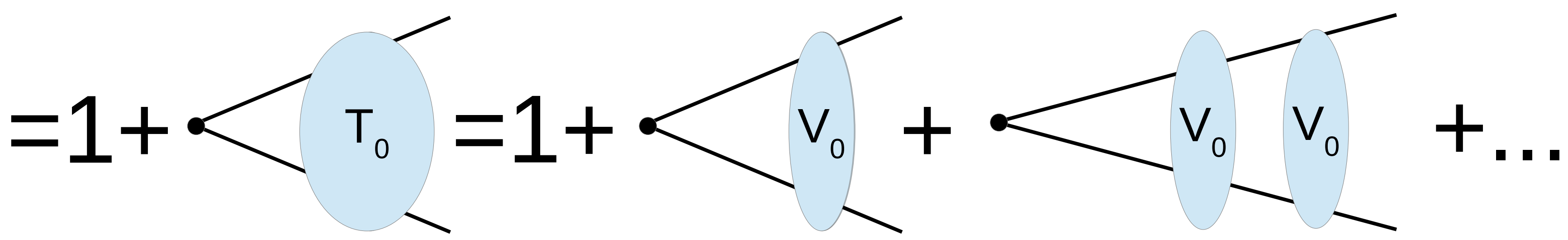
$\psi_p(0)$



$$\mathbb{R}\left(T_2\right)(p=0) = 0$$

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Renormalizability constraints on the the short-range part of the LO potential:

$$\psi_{p=0}(0) \not\approx 0$$

AG, E.Epelbaum, PoS **PANIC2021**, 371 (2022), *In preparation* (2022)

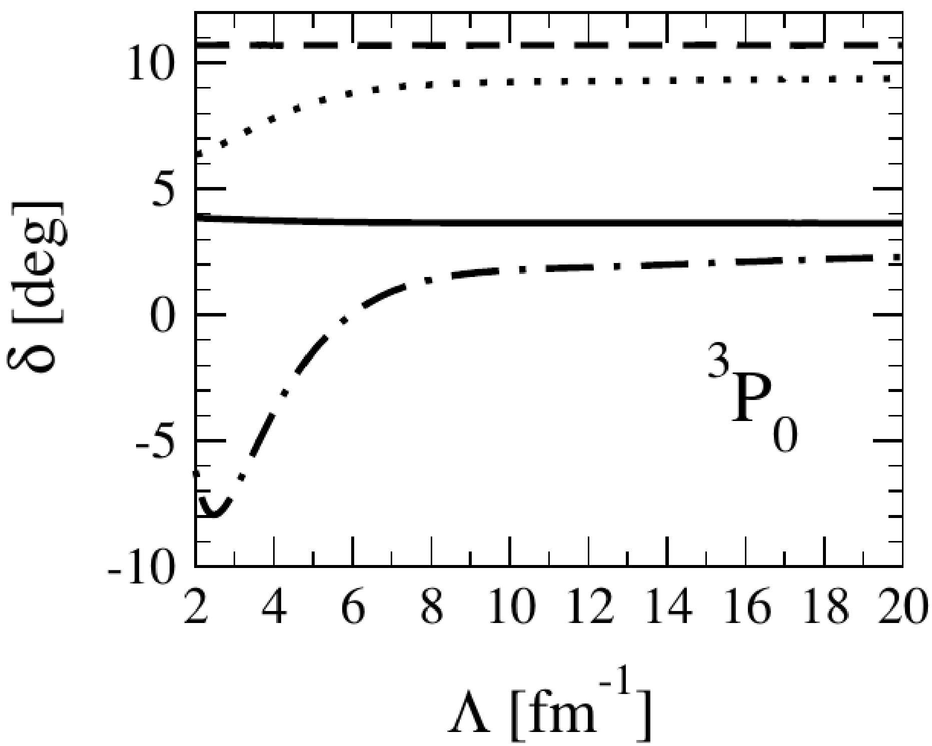
More constraints at higher orders!

Infinite cutoff ("RG-invariant") scheme. LO. $^3P_0$

$\Lambda \to \infty$: Cutoff independence for each chiral order individually!

$$V^{(0)}(p', p) = V_{1\pi}(p', p) + C_0^{(0)}(\Lambda) p' p$$

Renormalization condition: $\delta^{(0)}(E_0) = \delta_{\rm exp}(E_0)\,, \qquad E_0 = 50\,{\rm MeV}$



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U. van Kolck, **PRC72**, 054006 (2005)

Infinite cutoff scheme at NLO. $^3P_0$

B. Long, C. J. Yang, **PRC84**, 057001 (2011)

$$V^{(2)}(p',p) = V_{2\pi}(p',p) + C_0^{(2)}(\Lambda)p'p + C_2^{(2)}(\Lambda)p'p(p^2 + p'^2)$$

Perturbative NLO: $T^{(2)} = \left[\mathbb{1} + T^{(0)}G\right]V^{(2)}\left[\mathbb{1} + GT^{(0)}\right]$

Non-perturbative NLO leads to problems:
Repulsive singular two-pion-exchange potential

M. P. Valderrama, E. R. Arriola, **PRC74**, 054001 (2006)
C. Zeoli, R. Machleidt, D. R. Entem, Few Body Syst. 54, 2191 (2013)

Additional renormalization conditions:

$$\delta^{(2)}(E_0) = 0\,, \qquad E_0 = 50\text{MeV}$$

$$\delta^{(2)}(E_1) = \delta_{\text{exp}}(E_1) - \delta^{(0)}(E_1)\,, \qquad E_1 = 25\text{MeV}$$

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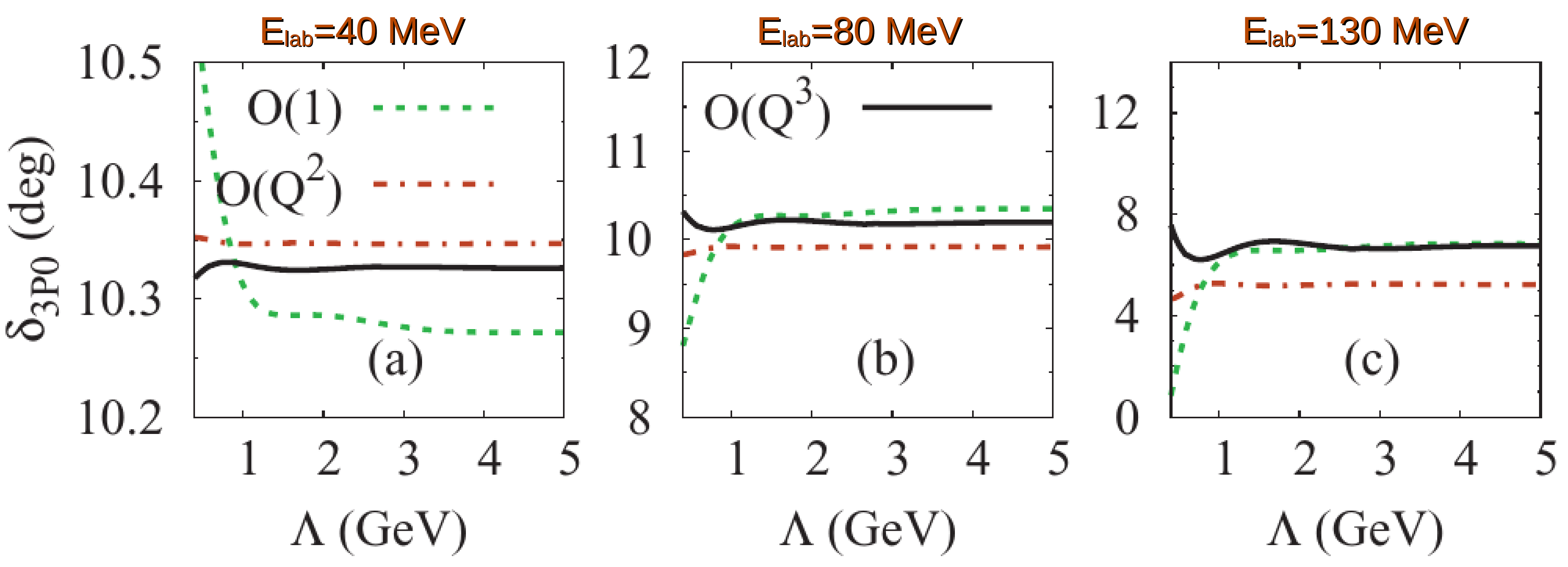
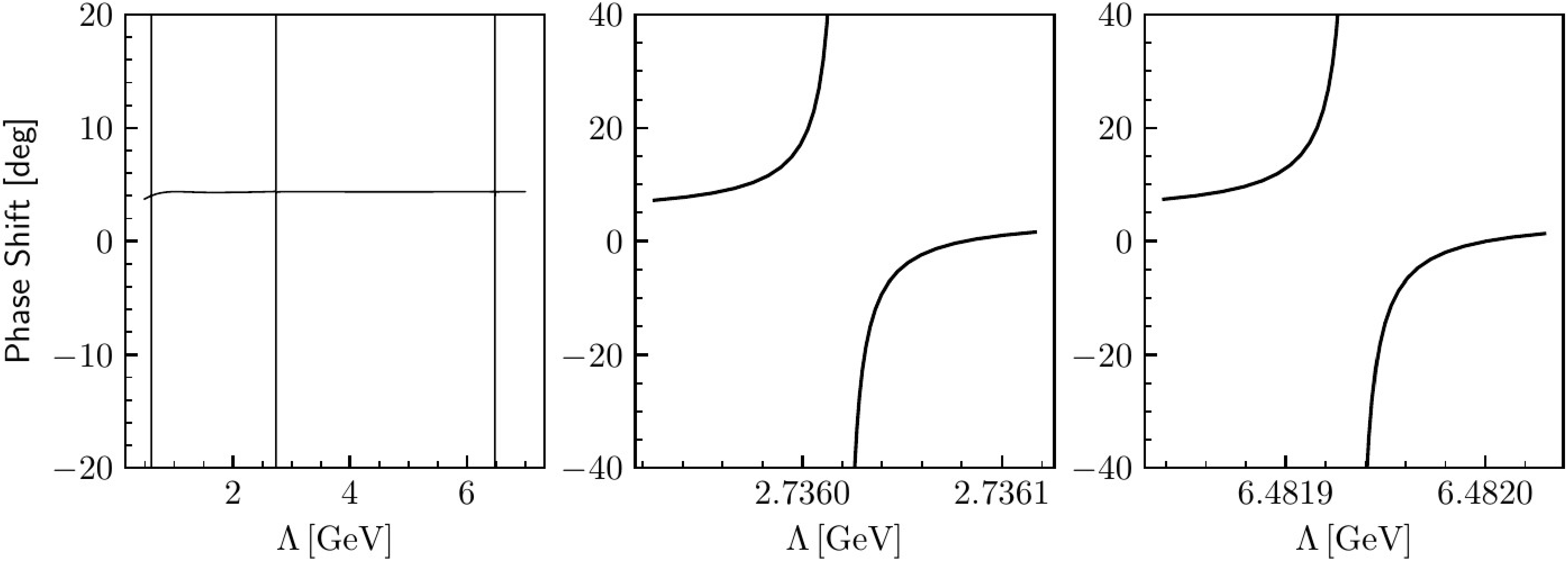


$^3P_0$ NLO phase shift at E_{lab}=130 MeV

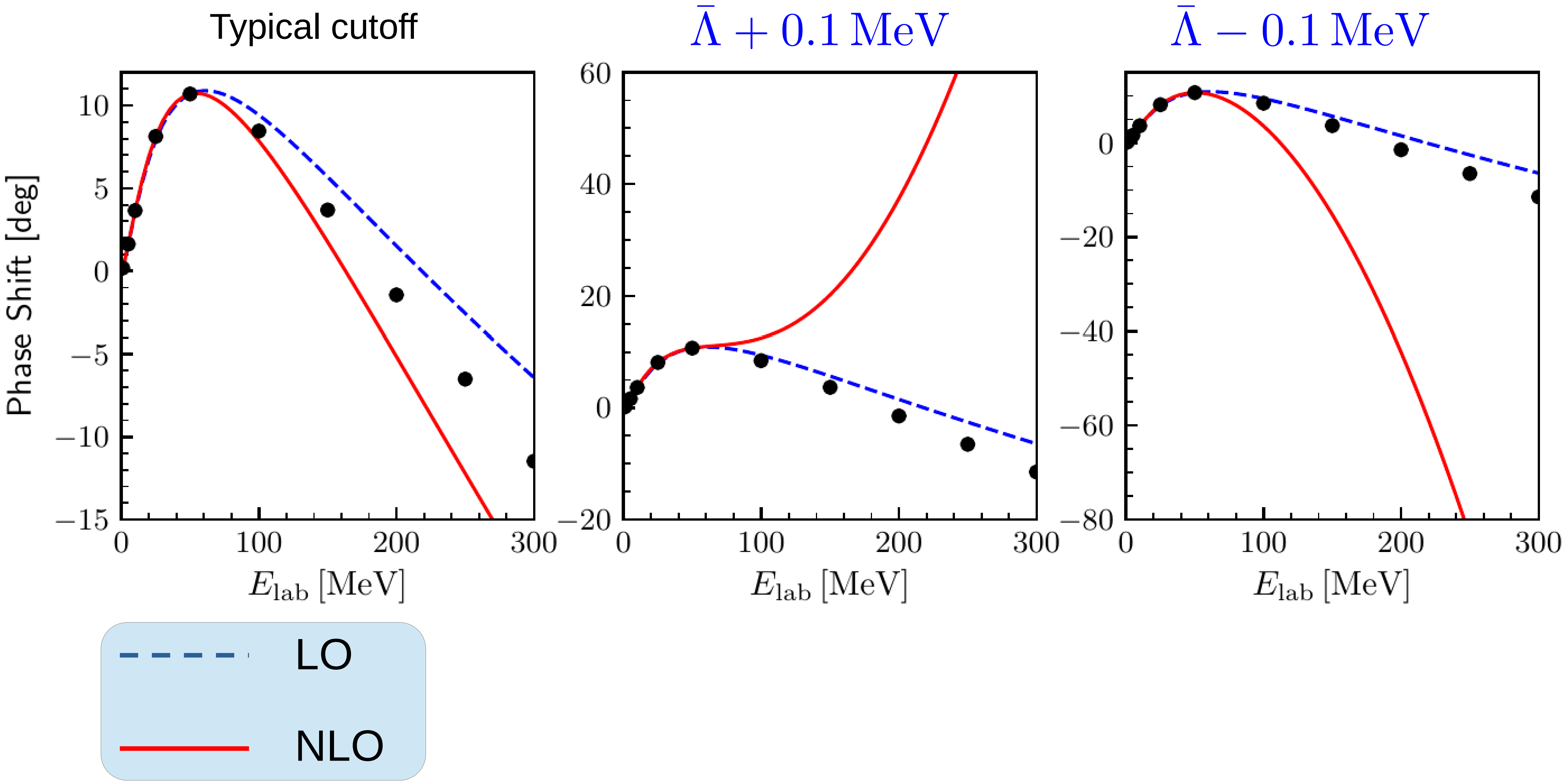
AG, E.Epelbaum, *In preparation (2022)*

“Exceptionial cutoffs”



$^3P_0$ phase shifts

"Exceptional" cutoff $\bar{\Lambda} \approx 12\,\mathrm{GeV}$



“Exceptional” cutoffs

$$T^{(2)}(E) = T_{2\pi}(E) + C_0^{(2)} T_{\mathrm{ct},0}(E) + C_2^{(2)} T_{\mathrm{ct},2}(E)$$

$$T_{\mathrm{ct},0}(E) = \psi_\Lambda(E)^2$$

$$T_{\mathrm{ct},2}(E) = 2\psi_\Lambda(p_{\mathrm{on}})\psi'_\Lambda(E),$$

Renormalization conditions:

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ψ_Λ and ψ'_Λ oscillate with Λ

$$\begin{vmatrix} \psi_{\bar\Lambda}(E_0) & \psi'_{\bar\Lambda}(E_0) \\ \psi_{\bar\Lambda}(E_1) & \psi'_{\bar\Lambda}(E_1) \end{vmatrix} = 0$$

→ The system of equations is inconsistent!

"Exceptional" cutoffs

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$$\delta^{(2)}(E_1) = \delta_{\mathrm{exp}}(E_1) - \delta^{(0)}(E_1)\,, \qquad E_1 = 25\mathrm{MeV}$$

ψ_Λ and ψ'_Λ oscillate with Λ

$$\begin{vmatrix} \psi_{\bar\Lambda}(E_0) & \psi'_{\bar\Lambda}(E_0) \\ \psi_{\bar\Lambda}(E_1) & \psi'_{\bar\Lambda}(E_1) \end{vmatrix} = 0$$

→ The system of equations is inconsistent!

"Exceptional" cutoffs

$$T^{(2)}(E) = T_{2\pi}(E) + C_0^{(2)} T_{\rm ct,0}(E) + C_2^{(2)} T_{\rm ct,2}(E)$$

$$T_{\rm ct,0}(E) = \psi_\Lambda(E)^2$$

$$T_{\rm ct,2}(E) = 2\psi_\Lambda(p_{\rm on})\psi'_\Lambda(E),$$

Renormalization conditions:

$$\delta^{(2)}(E_0) = 0\,, \qquad E_0 = 50\text{MeV}$$

$$\delta^{(2)}(E_1) = \delta_{\rm exp}(E_1) - \delta^{(0)}(E_1)\,, \qquad E_1 = 25\text{MeV}$$

ψ_Λ and ψ'_Λ oscillate with Λ

$$\begin{vmatrix} \psi_{\bar\Lambda}(E_0) & \psi'_{\bar\Lambda}(E_0) \\ \psi_{\bar\Lambda}(E_1) & \psi'_{\bar\Lambda}(E_1) \end{vmatrix} = 0$$

→ The system of equations is inconsistent!

$$\delta T_{{\rm ct},i} \sim \frac{1}{\Lambda^\alpha}$$

is multiplied with an arbitrarily large number

“Exceptional” cutoffs

“RG-invariance” requires independence of the amplitude from the form of a regulator and the value of the cutoff

For a sufficiently general regulator,
There always exist “exceptional” cutoffs

↓

Renormalization does not work

Summary

- ✔ Renormalization of NN Chiral EFT with a finite cutoff at NLO in the chiral expansion is understood
- ✔ Power-counting breaking contributions at NLO can be absorbed by the renormalization of the LO contact interactions for perturbative LO under rather general conditions
- ✔ Cutoff dependence can be studied systematically
- ✔ In the case of non-perturbative LO, the requirement of renormalizability imposes certain constraints on the LO potential
- ✔ In the infinite cutoff scheme, renormalization at NLO does not work: "exceptional" cutoffs
- ✔ Other systems (few- and many nucleon, electroweak currents) and higher orders should be possible to analyze in a similar fashion