

Symmetry Preserving Regularization of Nuclear Potentials

Hermann Krebs

Ruhr-Universität Bochum

The 4th Meeting of the LENPIC 2022

Ruhr-Universität Bochum

August 24, 2022



In collaboration with Evgeny Epelbaum

Outline

- Path-integral approach for derivation of nuclear forces
- Symmetry preserving regularization
- Status report on construction of 3N interactions & currents

Path-Integral Framework for Derivation of Nuclear Forces

Why a new Framework?

Difficulties in formulation of regularized chiral EFT

- Regularization should preserve chiral and gauge symmetries
- Regularization should not affect long-range pion physics

Pion-propagator in euclidean space: $q^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$

$$\frac{1}{q^2 + M_\pi^2} \rightarrow \frac{\exp\left(-\frac{q^2 + M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2} = \frac{1}{q^2 + M_\pi^2} - \frac{1}{\Lambda^2} + \frac{q^2 + M_\pi^2}{2\Lambda^4} + \dots$$

all $1/\Lambda$ -corrections are short-range interactions

q_0 - dependence in exponential requires second and higher order time-derivatives in pion field in the chiral Lagrangian

→ Canonical quantization of the regularized theory becomes difficult (Ostrogradski - approach, Constrains, ...)

Canonical vs Path-Integral Quantization

Canonical Quantization of QFT

Hamiltonian & Hilbert space
Creation/annihilation operators
Time-ordered perturbation theory



Path-Integral Quantization of QFT

Lagrangian & action
Summation over all classical paths
Loop expansion & Feynman rules

- Path-Integral approach was a natural choice in pionic and single-nucleon sector

Gasser, Leutwyler, *Annals Phys.* 158 (1984) 142;

Bernard, Kaiser, Meißner, *Int. J. Mod. Phys. E* 4 (1995)193

- In two - and more - nucleon sector Weinberg used canonical quantization language

Weinberg *Nucl. Phys. B* 362 (1991) 3

In using **old-fashioned perturbation theory** we must work with the Hamiltonian rather than the Lagrangian. The application of the usual rules of **canonical quantization** to the leading terms in (1) and (9) yields the total

Can we choose a formulation where we can work with the Lagrangian?

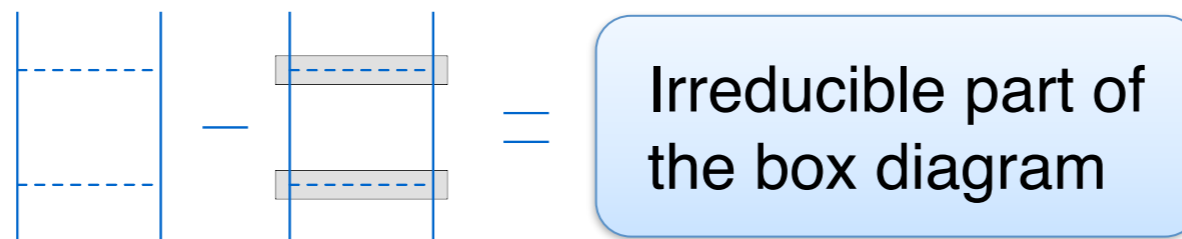
Lagrangian Formulation of Chiral EFT

Lagrangian formulation of chiral EFT so far

- Lagrangian formulation with subtractions: diagrammatic approach

Kaiser, Brockmann, Weise, Nucl. Phys. A 625 (1997) 758

➔ Less transparent in quantification of off-shell ambiguities



- Lagrangian formulation with instant subtractions: T - matrix approach

Gasparyan, Epelbaum, Phys. Rev. C 105 (2022) 2, 024001

- Nucleon-field transformation in a derivation of isospin violating nuclear forces

Friar, van Kolck, Rentmeester, Timmermans, Phys. Rev. C 70 (2004) 044001

- Path-integral formulation of chiral EFT with instant interactions on the lattice

Borasoy, Epelbaum, HK, Lee, Meißner, EPJA 31 (2007)105

- Instant interactions generate only iterative part of the NN amplitude

Path-Integral over Nucleons and Pions

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x (\mathcal{L} + \eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

Yukawa toy-model:

$$\mathcal{L} = N^\dagger \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{g}{2F} \vec{\sigma} \cdot \vec{\nabla} \pi \cdot \tau \right) N + \frac{1}{2} (\partial_\mu \pi \cdot \partial^\mu \pi - M^2 \pi^2)$$

- Perform a Gaussian path-integral over the pion fields

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN] \exp\left(i S_N + i \int d^4x (\eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

$$S_N = \int d^4x N^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N(x) - V_{NN} \leftarrow \text{Non-instant one-pion-exchange interaction}$$

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \Delta_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y)$$

with non-instant pion propagator: $\Delta_F(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq \cdot x}}{q^2 - M^2 + i\epsilon}$

Instant Interactions from Path-Integral

A general interaction $I_N = \int d^4x' d^4x d^4y' d^4y N^\dagger(x') A N(x) F(x', x, y', y) N^\dagger(y') B N(y)$ with A & B some spin-isospin matrices is called **instant** if

$$F(x', x, y', y) \sim \delta(x'_0 - x_0) \delta(y'_0 - y_0) \delta(x_0 - y_0)$$

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \Delta_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y) \text{ is } \textit{not} \text{ instant}$$

To transform V_{NN} into an instant form we rewrite a pion propagator

$$\frac{1}{q_0^2 - \omega_q^2} = -\frac{1}{\omega_q^2} + \frac{1}{q_0^2 - \omega_q^2} + \frac{1}{\omega_q^2} = -\frac{1}{\omega_q^2} + q_0^2 \frac{1}{\omega_q^2} \frac{1}{q_0^2 - \omega_q^2}, \quad \omega_q = \sqrt{\vec{q}^2 + M^2}$$

In coordinate space this corresponds to $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$ with

$$\Delta_S(x) = -\int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq \cdot x}}{\omega_q^2} = -\delta(x_0) \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{x}}}{\omega_q^2}, \quad \Delta_{FS}(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq \cdot x}}{\omega_q^2 (q_0^2 - \omega_q^2)}$$

Generalization of Static Decomposition

- The decomposition $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$ is not only valid for pion propagator

Assume that $G(x)$ is a quadrature-integrable function with its Fourier representation

$$G(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \tilde{G}(q_0^2, q^2) \text{ and } \tilde{G}(q_0^2, q^2) \text{ is differentiable at } q_0 = 0$$

If we define $G_S(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \tilde{G}(0, q^2) = \delta(x_0) \int \frac{d^3 q}{(2\pi)^3} e^{i \vec{q} \cdot \vec{x}} \tilde{G}(0, q^2)$ and

$$G_{FS}(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \frac{\tilde{G}(q_0^2, q^2) - \tilde{G}(0, q^2)}{q_0^2} \text{ is well defined due to existence}$$

of a derivative $\frac{\partial}{\partial q_0^2} \tilde{G}(q_0^2, q^2)$ at $q_0 = 0$

$$\rightarrow G(x) = G_S(x) - \frac{\partial^2}{\partial x_0^2} G_{FS}(x)$$

We can always decompose a function $G(x)$ into its instant part $G_S(x)$ and into a time-derivative of a function $G_{FS}(x)$

Instant Interactions from Path-Integral

Perform an instant decomposition of the pion propagator $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \Delta_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y)$$

$$\rightarrow V_{NN} = V_{OPE} + V_{FS}$$

$$V_{OPE} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \Delta_S(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y) \text{ is instant}$$

$$V_{FS} = \frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y) \text{ is non-instant}$$

V_{FS} is time-derivative dependent and thus can be eliminated by a non-polynomial field redefinition

$$N(x) \rightarrow N'(x) = N(x) + i \frac{g^2}{8F^2} \int d^4y [\vec{\sigma} \tau N(x)] \cdot [\vec{\nabla}_x \frac{\partial}{\partial x_0} \Delta_{FS}(x-y)] \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau N(y)]$$

$$N^\dagger(x) \rightarrow N'^\dagger(x) = N^\dagger(x) - i \frac{g^2}{8F^2} \int d^4y \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau N(y)] [\vec{\nabla}_y \frac{\partial}{\partial y_0} \Delta_{FS}(y-x)] \cdot [N^\dagger(x) \vec{\sigma} \tau]$$


Instant Interactions from Path-Integral

$$N(x) \rightarrow N'(x) = N(x) + i \frac{g^2}{8F^2} \int d^4y [\vec{\sigma} \tau N(x)] \cdot [\vec{\nabla}_x \frac{\partial}{\partial x_0} \Delta_{FS}(x-y)] \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau N(y)]$$

generate time-derivative dependent three-nucleon interactions.

These contributions can be eliminated by similar field transformations

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN] \left[\det \left(\frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)} \right) \right]^{-1} \exp \left(i S_N + i \int d^4x (\eta^\dagger(x) N'(x) + N^\dagger(x) \eta(x)) \right)$$

$$\simeq \int [DN^\dagger][DN] \left[\det \left(\frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)} \right) \right]^{-1} \exp \left(i S_N + i \int d^4x (\eta^\dagger(x) N(x) + N^\dagger(x) \eta(x)) \right)$$


Equivalence theorem: nucleon pole-structure is unaffected by the field-transf.

$$S_N = \int d^4x N^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N(x) + V_{OPE} + \mathcal{O}(g^5)$$

$$V_{OPE} = - \frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) \Delta_S(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y)$$

 Instant one-pion-exchange interaction

One-Loop Corrections to Interaction

One loop corrections to NN & NNN interaction come from functional determinant

$$\left[\det \left(\frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)} \right) \right]^{-1} = \exp \left(- \text{Tr} \log \frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)} \right)$$

Due to non-polynomial structure of field transformations $\det \left(\frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)} \right) \neq 1$

One-loop corrections to NN interaction are local but not instant

$$V_{NN}^{1\text{-loop}} = \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) L_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y) + \dots$$

Perform an instant decomposition of the loop function $L_F(x) = L_S(x) - \frac{\partial^2}{\partial x_0^2} L_{FS}(x)$

$$V_{NN}^{1\text{-loop}} = \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \tau] N(x) L_S(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \tau] N(y) + \dots \text{ is instant}$$

Non-instant part is proportional to time-derivative and can be eliminated by an appropriate field transformation

Generalization to Chiral EFT

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x (\mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \mathcal{L}_{NNN} + \eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

- Integrate over pion fields via loop-expansion of the action
 - ➔ expansion of the action around the classical pion solution
- Perform instant decomposition of the remaining interactions between nucleons
- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction
- Calculate functional determinant to get one-loop corrections to few-nucleon forces
- Perform instant decomposition of the one-loop corrections
- Perform field redefinitions to eliminate non-instant part the one-loop corrections

Connection to Unitary Transformations

Previous derivation of nuclear forces was based on unitary transformation technique

Field transformations (FT) within path-integral approach



Unitary transformations (UT) within canonical quantization approach

- Interactions generated by FT have always a form of heavy-baryon like tree-level or 4-dim loop-integrals

- Interactions generated by UT can be matched by 4-dim loop-integrals, *only* if some unitary phases are fixed

→ UT technique is more flexible

In practical calculation we do not want to explore the flexibility of UT in constructing non-renormalizable nuclear forces

- FT which don't generate interactions with time-derivatives describe off-shell ambiguities

- Allows to study unitary ambiguities of e.g. relativistic corrections

UT & FT path-integral approach lead to the same chiral EFT nuclear forces up to N⁴LO

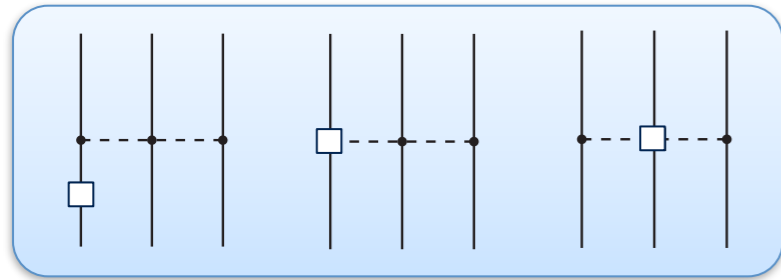
Fazit: Path-integral formulation of nuclear forces is as powerful as UT technique, however it allows consideration of a wider class of theories

Symmetry Preserving Regulator

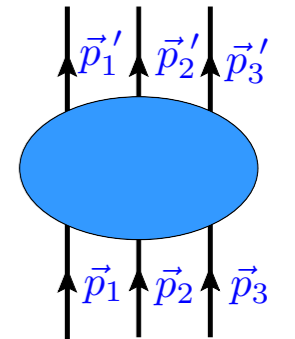
Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, *Front. in Phys.* 8 (2020) 98



← 1/m - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i \frac{g_A^2}{32mF_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)(q_3^2 + M_\pi^2)} \tau_1 \cdot (\tau_2 \times \tau_3) (2\vec{k}_1 \cdot \vec{q}_3 + 4\vec{k}_3 \cdot \vec{q}_3 + i [\vec{q}_1 \times \vec{q}_3] \cdot \vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}'_i - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2} (\vec{p}'_i + \vec{p}_i)$$

Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2} \exp\left(-\frac{q_1^2 + M_\pi^2}{\Lambda^2}\right) \exp\left(-\frac{q_3^2 + M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2} \tau_1 \cdot \tau_2 \frac{\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2}{q_1^2 + M_\pi^2} \exp\left(-\frac{q_1^2 + M_\pi^2}{\Lambda^2}\right)$$

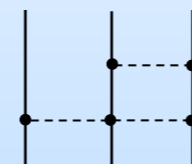
First iteration with OPE NN potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} \frac{1}{E - H_0 + i\epsilon} V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda} \frac{1}{E - H_0 + i\epsilon} V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda \frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6} (\tau_2 \cdot \tau_3 - \tau_1 \cdot \tau_3) \frac{\vec{q}_2 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3}{q_3^2 + M_\pi^2} + \dots$$

No such D-like term in chiral Lagrangian



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi} =$  $+ \dots$ is finite

Higher Derivative Lagrangian

To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + ip), \quad u = \sqrt{U}, \quad \text{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

$$\mathcal{L}_{\pi, \Lambda}^{(2)} = \mathcal{L}_\pi^{(2)} + \frac{F^2}{4} \text{Tr} \left[\text{EOM} \frac{1 - \exp\left(\frac{\text{ad}_{D_\mu} \text{ad}_{D^\mu} + \frac{1}{2} \chi_+}{\Lambda^2}\right)}{\text{ad}_{D_\mu} \text{ad}_{D^\mu} + \frac{1}{2} \chi_+} \text{EOM} \right]$$

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \text{Tr} [u_\mu u^\mu + \chi_+] \quad \text{EOM} = -[D_\mu, u^\mu] + \frac{i}{2} \chi_- - \frac{i}{4} \text{Tr} (\chi_-)$$

- ✓ Leads to regularized nuclear forces up to N⁴LO
- ✗ Leads to unregularized nuclear currents starting from N³LO

➔ We need a better formalism

Gradient-Flow Equation

Yang-Mills gradient flow in QCD: **Lüscher, JHEP 04 (2013) 123**

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \quad \text{with} \quad B_\mu|_{\tau=0} = A_\mu \quad \& \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT **(Proposed in various talks by D. Kaplan for nuclear forces)**

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying gradient-flow equation

$$\partial_\tau W = i w \text{EOM}(\tau) w \quad \text{with} \quad w = \sqrt{W} \quad \text{and} \quad \text{EOM} = [D_\mu, w_\mu] + \frac{i}{2} \chi_- - \frac{i}{4} \text{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i r_\mu)w - w(\partial_\mu - i l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w \chi^\dagger w, \quad \chi = 2B(s + ip)$$

Solution of $1/F$ - expanded gradient flow equation: $W = \exp\left(i \frac{\tau \cdot \phi}{F}\right)$

$$\phi^c = e^{-\tau(-\partial^2 + M^2)} \pi^c + \int_0^\tau ds e^{-(\tau-s)(-\partial^2 + M^2)} \left(2BFp^c - F\partial_\mu a_\mu^c \right) + \mathcal{O}(\pi^2, \pi \text{ source})$$

Regularization of Forces and Currents

To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathcal{L}_\pi^{(2)}$ & $\mathcal{L}_\pi^{(4)}$ unregularized
- Replace all pion fields in pion-nucleon Lagrangians $\mathcal{L}_{\pi N}^{(1)}, \dots, \mathcal{L}_{\pi N}^{(4)}$: $U \rightarrow W$

For $\tau = \frac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Check of chiral symmetry:

- We checked infinitesimal chiral transformation property of W up to four pions and one external source: $W \rightarrow RWL^\dagger$ if $U \rightarrow RUL^\dagger$

The check requires construction of W - field up to six pions and two sources

Status Report on 3N & Currents

Status Report on 3N & Currents

- Integration over pion-fields up to one-loop in the action in momentum space ✓

Non-zero contributions to 3N only at N⁴LO: expressed in terms of 2-dim integrals

One of two-pion-one-pion exchange contributions

$$\rightarrow \frac{3g_A^2 c_2}{F^6} \sigma_2 \cdot q_2 \sigma_3 \cdot q_1 \tau_2 \cdot \tau_3 \frac{\exp\left(-\frac{q_2^2 + M_\pi^2}{2\Lambda^2}\right)}{q_2^2 + M_\pi^2} \text{FTwoP174Str1} + \dots$$

$$\text{FTwoP174Str1} = \frac{3}{32\sqrt{2}\pi^{3/2}} \frac{\Lambda^3}{q_1} \int_0^1 dw \int_0^\infty d\lambda \exp\left(-\frac{4M_\pi^2(3 + \lambda + w(2 + \lambda)) - (1 + w)(2 + \lambda)q_1^2 + 4wq_2^2 + 4(1 - w)q_3^2}{8\Lambda^2}\right) \frac{\text{erf}\left(\frac{q_1}{2\sqrt{2}\Lambda} \frac{\lambda}{\sqrt{2 + \lambda}} \sqrt{1 + w}\right)}{(1 + w)^{5/2}(2 + \lambda)^{7/2}}$$

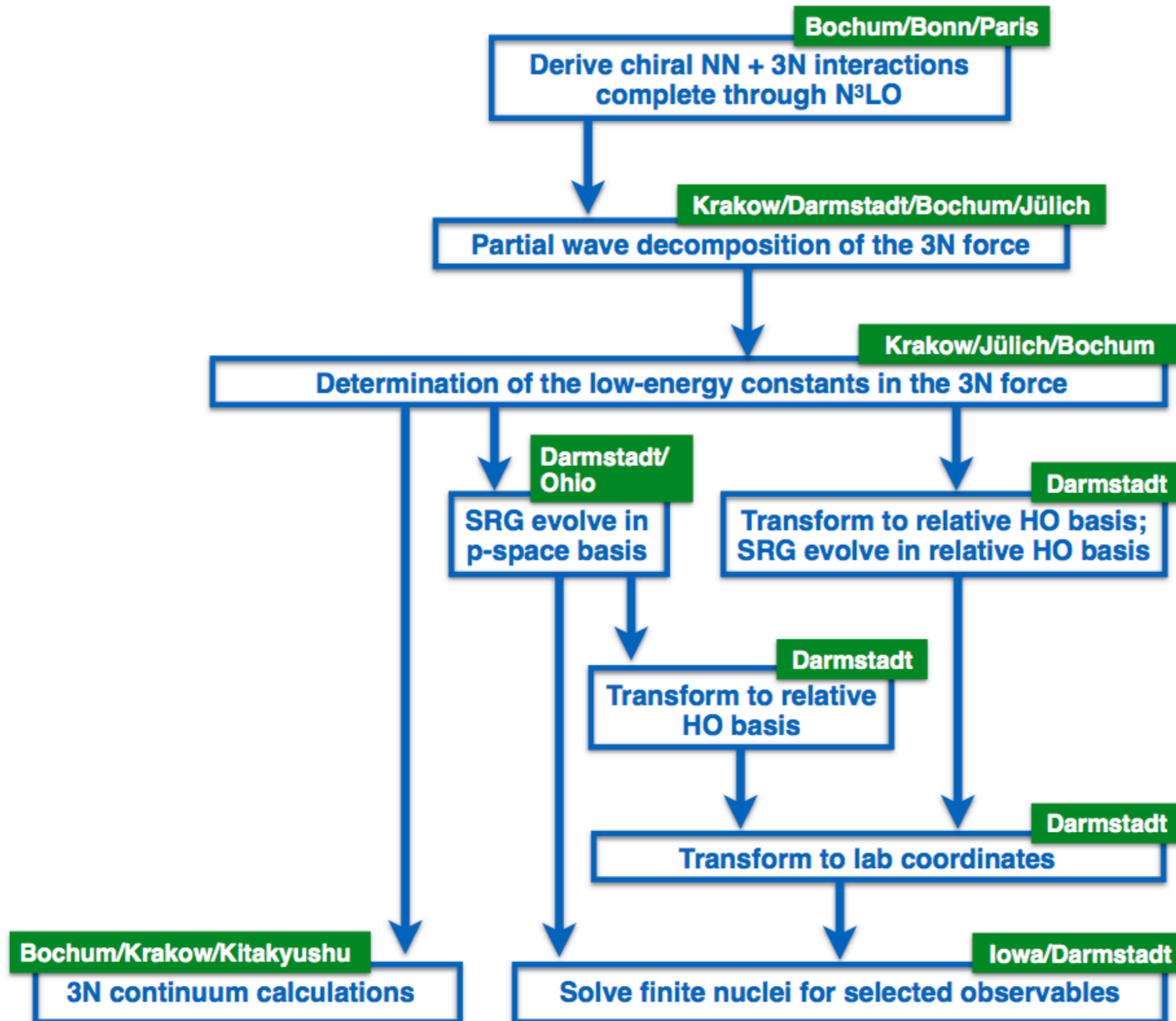
Checks: ● Large cut-off expansion: to be done

● Passarino-Veltman reduction: to be done

● Momentum space calculation of $\det\left(\frac{\delta(N^\dagger, N')}{\delta(N^\dagger, N)}\right)$: almost done

● Transformation of two-pion and two-pion-one-pion exchange contributions to coordinate space: to be done

Work Flow for Development of 3N



Summary

- Path-integral approach for derivation of nuclear forces has been developed
 - Applicable for EFT's with interactions involving second or higher number of time-derivatives
 - All results from unitary transformation technique are reproduced within path-integral approach
- Symmetry preserving regularization
 - Pion fields which couple to nucleons are smoothed within a gradient-flow equation approach
- Status report on construction of 3N interactions & currents