Universality of TMD soft-factor geometry

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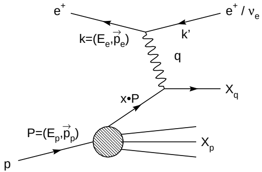
$$d\sigma(Q,q_\perp) \sim \underbrace{H\left(\frac{Q^2}{\mu^2}\right)}_{\text{Hard part}} \int dq_\perp e^{iq_\perp \xi_\perp} \underbrace{f_1(x_1,\xi_\perp) f_2(x_2,\xi_\perp)}_{\text{TMD PDF}} \underbrace{S(\xi_\perp)}_{\text{Soft factor}}$$

[Collins,Soper,Sterman,86]

- The factorization theorem for transverse momentum dependent (TMD) processes contains soft factor, needed to cure rapidity divergences (mixed UV/IR divergences).
- Soft factor consists of a set of Wilson lines and their geometrical structure can be derived.
- The geometrical structure of soft factor is universal for any subproces (Q/Q, Q/G, G/Q or G/G) but may depend on the kinematics of the process.
- The key ingredient for the derivation is to identify the convenient degrees of freedom.

Gauge invariance of usual parton distributions



$$d\sigma \sim L^{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu}(Q,p,..) = \frac{1}{4}\int d^4\xi e^{i(q\xi)} \langle p|\left[J^\mu(\xi)J^\nu(0)\right]|p\rangle,$$

$$Q^2 \gg \Lambda^2 \qquad \Rightarrow \qquad \xi^2 \sim 0$$

$$W^{\mu\nu}(Q,p,..) = \int_x^1 \frac{dy}{y} H^{\mu\nu}(Q,\frac{x}{y},\mu) f(y,\mu) + \mathcal{O}(Q^{-1}).$$

In a proper gauge (namely, light-cone gauge) and infinite momentum frame the "large" components of the quarks freely propagate along the "minus" direction and in the leading Q^{-1} order are gauge invariant. PDF resembles a free model:

$$A_+ = 0: \qquad f(x) \sim \int d\xi^- e^{ixp^+\xi^-} \langle p|\bar{q}(\xi)\gamma^+ q(0)|p\rangle\Big|_{\xi^+,\xi_\perp=0}. \tag{1}$$

Gauge invariance of usual parton distributions

From the light-cone gauge to the general case

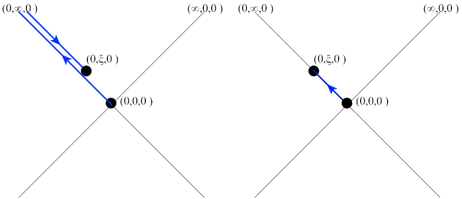
$$\text{LC:} \quad \bar{q}\partial_+\gamma^+ q \quad \xleftrightarrow[q\leftrightarrow U(A_\mu)q]{} \bar{q}(\partial_+ + igA_+)\gamma^+ q \quad \text{:no gauge fixing}$$

$$U[A_\mu](z) = W_-(z) = P\exp\left(-ig\int_0^{\pm\infty} d\sigma A_+(x+n\sigma)\right)$$

where the direction of the line depends on the boundary conditions: plus for $A_+(\infty) = 0$ (retarded), minus for $A_+(-\infty) = 0$ (advanced).

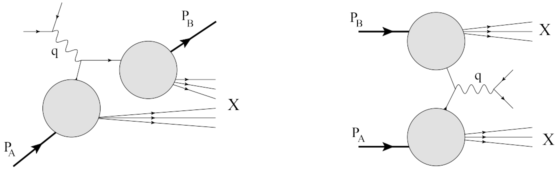
$$q(x) \to U[A_\mu](x)q(x), \qquad \bar{q}(x) \to \bar{q}(x)U^\dagger[A_\mu](x)$$

$$f(x) \sim \int d\xi^- e^{ixp^+\xi^-} \langle p|\bar{q}(\xi)W_+^\dagger(z)\gamma^+ W_+(0)q(0)|p\rangle\Big|_{\xi^+,\xi_\perp=0}$$



Transverse momentum dependence

TMD PDF are subjects of processes with two hadrons



$$Q \gg q_\perp \gg \Lambda$$

For definiteness, we keep in mind Drell-Yan kinematic.
The hadron tensor reads

$$W^{\mu\nu}(q) \quad = \quad \frac{1}{4}\sum_X (2\pi)^4\delta^{(4)}(P_A+P_B-q-P_X)\langle h_A,h_B|J^\mu(0)|X\rangle\langle X|J^\nu(0)|h_A,h_B\rangle.$$

The factorization of the TMD processes has the following form:

$$W^{\mu\nu}(q) \quad = \quad \frac{H^{\mu\nu}(Q^2/\mu^2)}{4}\int d^2\xi_\perp e^{-i(q_\perp\xi_\perp)} f_A(x_A,\xi_\perp,\mu^2) f_B(x_B,\xi_\perp,\mu^2) S(\xi_\perp) + \mathcal{O}\left(\frac{q_\perp}{Q}\right),$$

where S is a soft factor that takes care the uncompensated divergences of the interaction between final states.

The consideration of TMD processes much simplified in special frame

$$P_A^+ \sim P_B^- \sim Q$$

In such a system the hadron A mostly consists of "large" components of the quark field, whereas the hadron B mostly consists of "small" components of the quark field:

$$\begin{aligned} |h_A\rangle &\sim Q_+Q_+Q_+(1+\bar{Q}_+Q_+ + ..)|0\rangle + \mathcal{O}\left((P_A^+)^{-2}\right) \\ |h_B\rangle &\sim Q_-Q_-Q_-(1+\bar{Q}_-Q_+ - +..)|0\rangle + \mathcal{O}\left((P_B^-)^{-2}\right) \end{aligned}$$

$$Q_+(x) = \frac{\gamma^+\gamma^-}{2}q(x), \qquad Q_-(x) = \frac{\gamma^-\gamma^+}{2}q(x).$$

It further simplifies in terms of "gauge-invariant" variables

DIS experience suggests us to choose the proper gauge, but there is no privileged gauge in TMD case.
We perform a "gauge-like transformation" of variables:

$$q(z) \to W_-(z)Q_-(z) + W_+(z)Q_+(z), \qquad \bar{q}(z) \to \bar{Q}_-(z)W_-^\dagger(z) + \bar{Q}_+W_+^\dagger(z),$$

$$W_\pm(z) = P\exp\left(-ig\int_0^\infty d\sigma A_\pm\left(z+\sigma n^\pm\right)\right).$$

SCET-like variables

similar to [Bauer, et al,0202088]

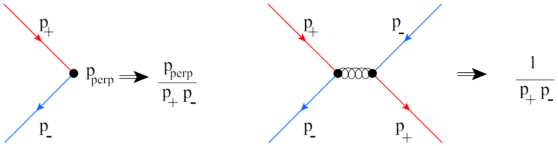
$$q(z) \to W_-(z)Q_-(z) + W_+(z)Q_+(z), \qquad \bar{q}(z) \to \bar{Q}_-(z)W_-^\dagger(z) + \bar{Q}_+(z)W_+^\dagger(z),$$

The Lagrangian of quark in new variables reads:

$$\mathcal{L}_{QCD} = \bar{Q}_-\gamma^+\partial_-Q_- + \bar{Q}_+\gamma^-\partial_+Q_+ + \underbrace{\bar{Q}_+W_+^\dagger\gamma_\perp^\mu D_\mu^\perp W_-Q_-}_{\text{non-local int.}} + \underbrace{\bar{Q}_-W_-^\dagger\gamma_\perp^\mu D_\mu^\perp W_+Q_+}_{\text{non-local int.}}$$

Mixing Q_+ and Q_- fields

Generally, the propagator of $Q_\pm$ is non-diagonal, and mixes "plus" and "minus" components of Q's. But at large p^+ for Q_+ and large p^- for Q_-, the mixture is a correction of order $\frac{k_\perp}{p^+p^-}$.

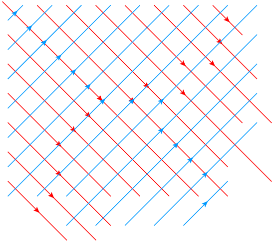


In DY kinematics

$$P_A^+ \sim P_B^- \sim Q \gg q_\perp \gg \Lambda$$

$$\mathcal{L}_{QCD} = \bar{Q}_- \gamma^+ \partial_- Q_- + \bar{Q}_+ \gamma^- \partial_+ Q_+ + \underbrace{\bar{Q}_+ W_+^\dagger \gamma_\perp^\mu D_\mu^\perp W_- Q_- + \bar{Q}_- W_-^\dagger \gamma_\perp^\mu D_\mu^\perp W_+ Q_+}_{\sim \frac{k_\perp}{Q^2} \text{ at least for } Q\text{'s from different hadrons}}$$

Suppression takes place only for Q_+ and Q_- from different hadrons, but for the same hadron one has $\frac{k_\perp^2}{p_+ p_-} \sim 1$



We finish up with two free "one dimensional" theories.

$$\langle Q_+ \bar{Q}_+ \rangle = \frac{i\gamma^+}{p^+ + i0},$$
$$\langle Q_- \bar{Q}_- \rangle = \frac{i\gamma^-}{p^- + i0},$$

here $\gamma^\pm$ are 2×2 matrices.

TMD factorization for DY

The convolution of currents contains 6 terms, but 4 of them lead to disconnected diagrams:

$$J^\mu(\xi)J_\mu(0) = \underbrace{(\bar{Q}_-\gamma^+Q_-)(\xi)(\bar{Q}_+\gamma^-Q_+)(0)}_{\text{disconnected term}} + \ldots.$$

$$\ldots. + \underbrace{(\bar{Q}_-W_-^\dagger\gamma_\perp^\mu W_+Q_+)(\xi)(\bar{Q}_-W_-^\dagger\gamma_\perp^\mu W_+Q_+)(0)}_{\text{connected term}}.$$

Operator product expansion in "free" theory

Applying Wick theorem and Fierz identities we come to

$$\langle P_A,P_B|J^\mu(\xi)J_\mu(0)|P_A,P_B\rangle \simeq$$

$$\underbrace{\langle P_A|\bar{Q}_+(\xi)\gamma^-Q_+(0)|P_A\rangle}_{\text{generic TMD PDF for } N_A}\underbrace{\langle P_B|\bar{Q}_-(0)\gamma^+Q_-(\xi)|P_B\rangle}_{\text{generic TMD PDF for } N_B}\underbrace{\frac{1}{N_c}\langle\text{tr}\left(W_-^\dagger W_+(0)W_-^\dagger W_+(\xi)\right)\rangle}_{\text{generic for soft factor}}$$

$$+\mathcal{O}\left(\frac{k_\perp}{Q}\right)$$

These expression still mix different twists.

The counting rules for derivatives (here $k_\perp$ is quark momentum inside hadron):

$$\xi^-\partial_+ Q_+ \sim 1, \qquad \xi_\perp\partial_\perp Q_+ \sim \frac{k_\perp}{q_\perp} \sim ?, \qquad \xi^+\partial_- Q_+ \sim \frac{p_-}{Q} \sim \frac{1}{Q^2}$$

$$\xi^+\partial_- Q_- \sim 1, \qquad \xi_\perp\partial_\perp Q_- \sim \frac{k_\perp}{q_\perp} \sim ?, \qquad \xi^-\partial_+ Q_- \sim \frac{p_+}{Q} \sim \frac{1}{Q^2}$$

Common assumption is that $\frac{k_\perp}{q_\perp}$ is small but not negligible.

! To my understanding the common used TMD PDF overemphasizes the $k_\perp$corrections (it is clearer in pure SCET approach where counting rules are strict $\partial_- Q_+ \sim \epsilon^2$ and $\partial_\perp Q_+ \sim \epsilon$).

$$\langle P_A|\bar{Q}_+(\xi)\gamma^- Q_+(0)|P_A\rangle = \langle P_A|\bar{Q}_+(\xi)\gamma^- Q_+(0)|P_A\rangle\Big|_{\xi^+=0} + \mathcal{O}\left(\frac{1}{Q^2}\right).$$

$$\langle P_B|\bar{Q}_-(\xi)\gamma^+ Q_-(0)|P_B\rangle = \langle P_B|\bar{Q}_-(\xi)\gamma^+ Q_-(0)|P_A\rangle\Big|_{\xi^-=0} + \mathcal{O}\left(\frac{1}{Q^2}\right).$$

Finally, we return to QCD by performing the back substitution

$$Q_-(x) = \frac{1}{2} W_-^\dagger(x)\gamma^-\gamma^+ q(x), \qquad Q_+(x) = \frac{1}{2} W_+^\dagger(x)\gamma^+\gamma^- q(x)$$

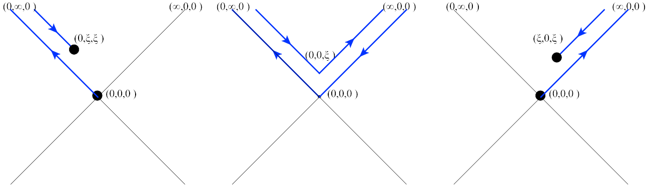
To arrive at the standard factorization expression:

$$W(q) = \frac{H(Q^2/\mu^2)}{4} \int d^2\xi_\perp e^{-i(q_\perp \xi_\perp)} f_A(x_A, \xi_\perp, \mu^2) f_B(x_B, \xi_\perp, \mu^2) S(\xi_\perp) + \mathcal{O}\left(\frac{q_\perp}{Q}\right),$$

where

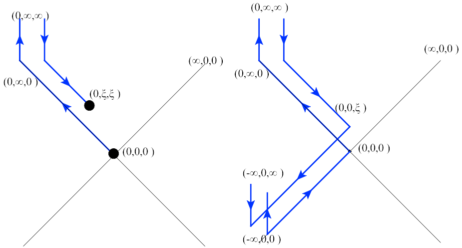
$$f(x, \xi_\perp) = \int d\xi^- e^{ixp_+\xi^-} \langle p|\bar{q}(\xi) W_+^\dagger(\xi)\gamma^+ W_+ q(0)|p\rangle \Big|_{\xi^+=0}$$

$$S(\xi_\perp) = \frac{1}{N_c} \langle \mathrm{tr}\left(W_-^+ W_+(0) W_-^\dagger W_+(\xi_\perp)\right)\rangle$$



Remarks

- Such an approach allows one to obtain the geometrical picture of TMD- processes, without consideration of diagrams.
- The passage from the DY kinematics to SIDIS can be done via crossing ($\bar{n} \to -\bar{n}$).
- In singular gauges (such as light cone gauge) one should add transverse links at infinities (these links do not change the counting rules but restore the gauge invariance of the fields Q).
- The path of soft factor ensures the color flow inside the process.
- No need for tilting the light cone (Collins&Soper).



Redefining the TMD PDF

$$F(x,k_\perp) = \int \frac{d^2\xi_\perp}{(2\pi)^3} e^{i(k_\perp \xi_\perp)} f(x,\xi_\perp) S^{-\frac{1}{2}}(\xi_\perp)$$

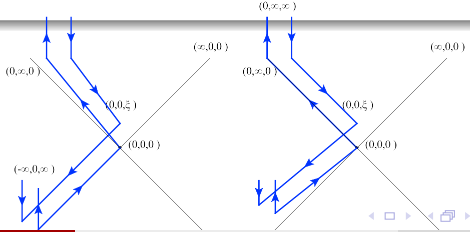
one makes the TMD factorization expression be soft-factor free.

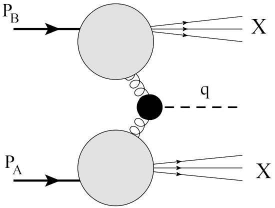
This coincides with the standard Collins definition of TMD PDF up to an auxiliary factor:

$$F_{Coll}(x,k_\perp) = \int \frac{d^2\xi_\perp}{(2\pi)^3} e^{i(k_\perp \xi_\perp)} f(x,\xi_\perp) S^{-\frac{1}{2}}(\xi_\perp) \sqrt{\frac{S_{y^+}(\xi_\perp)}{S_{y^-}(\xi_\perp)}}$$

where $S_{y\pm}(\xi)$ are soft factors for the tilted contours. These factors cancel out in the factorization formula and play only a transient in the derivation of the CS equation.

$$\ln\left(\frac{\mu^2}{(p^+)^2}\right) \to \ln\left(\frac{\mu^2}{(p^+)^2 e^{y_n}}\right)$$





The gluon TMD PDF appears (in LO) in the DY Higgs production:

$$J = \mathrm{tr}\left(F^{\mu\nu}F_{\mu\nu}\right)$$

In some proper variables (SCET consideration [Mantry,Petriello,09]):

$$J(\xi)J(0) \quad = \quad \mathrm{tr}(F^{+-}F^{+-})(\xi)\mathrm{tr}(F^{+-}F^{+-})(0) + ...$$

$$... + \mathrm{tr}\left(W_+^\dagger F^{+\mu_\perp}W_+W_-^\dagger F^{-\mu_\perp}W_-\right)(\xi)\mathrm{tr}\left(W_+^\dagger F^{+\mu_\perp}W_+W_-^\dagger F^{-\mu_\perp}W_-\right)(0)$$

Repeating the quark consideration we get the same result but with Wilson lines doubled, or in the adjoined representation:

$$W(q) \quad = \quad \frac{H(Q^2/\mu^2)}{4}\int d^2\xi_\perp e^{-i(q_\perp\xi_\perp)}f_G(x_A,\xi_\perp,\mu^2)f_G(x_B,\xi_\perp,\mu^2)S_{adj}(\xi_\perp) + \mathcal{O}\left(\frac{q_\perp}{Q}\right),$$

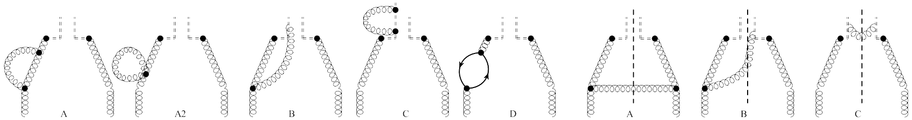
where

$$f_G(x,\xi_\perp) = \int d\xi^- e^{ixp_+\xi^-}\langle p|\bar{F}^{+\mu}(\xi)W_{+,adj}^\dagger(\xi)W_{+,adj}F^{\mu+}|p\rangle\Big|_{\xi^+=0}$$

$$S_{adj}(\xi_\perp) = \frac{1}{N_c^2-1}\langle \mathrm{tr}\left(W_{-,adj}^{+}W_{+,adj}(0)W_{-,adj}^\dagger W_{+,adj}(\xi_\perp)\right)\rangle$$

The soft factor is prescribed by the geometry color flow in the process.

One loop check

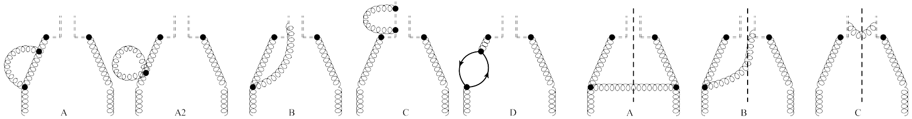


For the loop calculation one should introduce some regularization of the rapidity divergences. There are several approaches (mostly the deformation eikinal gluon propogator, see [Idilbi,et al], [Cherednikov,Stefanis])

$$\frac{1}{k^+ + i0} \to \frac{1}{k^+ + i\Delta^+}, \qquad \Delta^+ > 0$$

Our way is to change the definition of the operator by constraining the light-cone infinities at the finite points $\tau^{\pm}$.

One loop check



One can check explicitly the cancelation of the rapidity divergences in the gluon case (together with factor $-1/2$ from the square root):

$$F^{(1)}_{v.g} = \frac{C_A}{\epsilon}\left(\frac{11}{3} - 4\ln\left(\tau p^+\right)\right) + ..., \qquad S^{(1)}_{v.g} = \frac{C_A}{\epsilon}\left(\frac{1}{\epsilon} - 4\ln\left(\tau^2\mu^2/2\right)\right) + ...$$

$$F^{(1)}_{r.g} = C_A \frac{4}{\pi} \ln\left(\tau p^+\right) + ..., \qquad S^{(1)}_{r.g} = C_A \frac{4}{\pi} \ln\left(\tau^2 k_\perp^2/2\right) + ... \; .$$

The matrices of anomalous dimensions at LO read

$$\gamma = \frac{\alpha_s}{\pi}\begin{pmatrix} C_f\left(\frac{3}{2} - \ln\left(\frac{p^{+2}}{\mu^2}\right)\right) & 0 \\ 0 & \frac{11C_A}{6} - \frac{2N_f C_f}{3} - \ln\left(\frac{p^{+2}}{\mu^2}\right)\end{pmatrix}, \quad \gamma_{CS} = \frac{\alpha_s}{\pi}\begin{pmatrix} 2C_f & 0 \\ 0 & 2C_A\end{pmatrix}$$

Conclusion

- We have shown that the soft factor has (geometrically speaking) an "universal" form, in the sense that its form is process dependent but sub-process independent.
- The cancelations of rapidity divergences is shown at one-loop level for G/G TMD PDF.
- The complete matrix of anomalous dimensions at one-loop and CS anomalous dimensions are presented.

Open questions

- Overemphasize of $k_\perp$ correction in definition of the TMD PDF still unclear.
- Where is the "DGLAP part" of evolution? (anomalous dimension of TMD operator is generalized function, should one take it into account?)
- ...