Ideas to modelling nucleon GPDs with AdS / QCD models

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Lightcone 2013, Skiathos, Greece

May 21, 2013

Outline

Introduction

- Generalized parton distributions (GPDs) are important objects containing essential information about the hadronic structure.
- Unfortunately, given the non perturbative nature of these functions, it is not possible to calculate them directly from Quantum Chromodynamics, and this situation has motivated the development of other ways to access the GPDs
- Principal ways to access to GPDs are: extraction from the experimental measurements, direct calculation using lattice QCD, and different phenomenological models.
- The last procedure is based on parametrizations of the quark wave function or directly the GPDs, using constrains imposed by sum rules, which relate the parton distribution functions to nucleon electromagnetic form factors.

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- Within the phenomenological models used recently in hadronic physics, some are based on the gauge/gravity duality (for example AdS / QCD models).
- They suppose the existence of a gravity theory dual to QCD, and are divided into two classes, the top-down approach (where we start from a string theory leading to a low energy gauge theory with some QCD properties) and the bottom-up models (phenomenological approach where the geometry of an AdS space and bulk fields are specified in order to incorporate some basic properties of QCD).
- In turn, these last ones are divided into hard wall models and soft wall models, depending on the way conformal invariance in the AdS side is broken.
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Generalities about GPDs models

We start with reminding the relations between the nucleons Dirac F_1^N and Pauli F_2^N form factors, the form factors of valence quarks in nucleons (F_1^q and F_2^q, $q = u, d$) and the valence quark GPDs ($\mathcal{H}^q$ and $\mathcal{E}^q$) [1]:

$$F_i^{p(n)}(Q^2) = \frac{2}{3} F_i^{u(d)}(Q^2) - \frac{1}{3} F_i^{d(u)}(Q^2)$$

and

$$F_1^q(Q^2) = \int_0^1 dx\, \mathcal{H}^q(x, Q^2)$$

$$F_2^q(Q^2) = \int_0^1 dx\, \mathcal{E}^q(x, Q^2).$$

At $Q^2 = 0$ the GPDs $\mathcal{H}^q$ and $\mathcal{E}^q$ reduce to the valence quark $q_v(x)$ and magnetic $\mathcal{E}_q(x)$ densities

$$\mathcal{H}^q(x, 0) = q_v(x)\,, \quad \mathcal{E}^q(x, 0) = \mathcal{E}^q(x)\,,$$

[1] See for example M. Guidal, M. V. Polyakov, A. V. Radyushkin and M. Vanderhaeghen, Phys. Rev. D **72**, 054013 (2005); O. V. Selyugin and O. V. Teryaev, Phys. Rev. D **79**, 033003 (2009).

$\mathcal{H}^q(x,0)$ and $\mathcal{E}^q(x,0)$ are normalized to the number of valence u and d quarks in the proton (in case of q_v distributions) and to the anomalous magnetic moment of quark (in case of $\mathcal{E}^q$ distributions), respectively:

$$\int_0^1 dx u_v(x) = 2\,,$$

$$\int_0^1 dx\, d_v(x) = 1,$$

$$\kappa_q = \int_0^1 dx\, \mathcal{E}^q(x)\,.$$

The constants κ_q are related to the anomalous magnetic moments of nucleons $k_N = F_2^N(0)$:

$$\kappa_u = 2\kappa_p + \kappa_n = 1.673\,, \kappa_d = \kappa_p + 2\kappa_n = -2.033\,.$$

⋆ **GPDs and hadronic wave functions.**

The two body contribution to form factors can be written as

$$F^{(2)}(q^2) = \int_0^1 dx d^2 k_\perp \Psi^*(x, k_\perp + (1-x)q)\Psi(x, k_\perp).$$

If we use a gaussian ansatz for LFWF

$$\Psi(x, k_\perp) = exp\left[-\frac{k_\perp^2}{2x(1-x)\lambda^2}\right],$$

we get a nonforward parton density like

$$\mathcal{H}_G^q(x, t) = q^{(2)}(x) e^{(1-x)t/4x\lambda^2},$$

where $q^{(2)}(x)$ is the two body part in quark density q(x).

The drawback with this is that form factor F_1 decay quickly for large momentum transfer.

In order to improve results that you can obtain with the gaussian ansatz exist several parametrizations[2], thats in general looks like

$$\mathcal{H}^q(x,t) = q(x)e^{f(x)t},$$

where in general quarks densities used for $q(x)$ are parametrizations for fit data.

[2] For a brief summary see for example M. Guidal, M. V. Polyakov, A. V. Radyushkin and M. Vanderhaeghen, Phys. Rev. D **72**, 054013 (2005) [hep-ph/0410251]; O. V. Selyugin and O. V. Teryaev, Phys. Rev. D **79**, 033003 (2009) [arXiv:0901.1786 [hep-ph]].

GPDs and Holographical models

- The bottom-up soft wall models have proven to be quite useful because of their simplicity and variety of successful applications.
- Here we like to discuss in a brief way a couple of posibilities to use ideas based in holographical correspondence to calculate nucleonic GPDs
- One posibility is use calculations of form factors in AdS side and try compare with QCD expresions (using a matching procedure similar to used in LFH to get LFWF)
- Other alternative is to use a holographical wave function in some phenomenological models.

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⋆ Approach to GPDs with a matching procedure[3].

$$F_1^q(Q^2) = \int_0^1 dx\, \mathcal{H}^q(x, Q^2) \qquad F_2^q(Q^2) = \int_0^1 dx\, \mathcal{E}^q(x, Q^2).$$

The Form Factors in AdS / QCD[4].

$$F_1^p(Q^2) = C_1(Q^2) + \eta_p C_2(Q^2) \qquad F_1^n(Q^2) = \eta_p C_3(Q^2) \qquad F_2^p(Q^2) = \eta_n C_2(Q^2) \qquad F_2^n(Q^2) = \eta_n C_3(Q^2)$$

where

$$C_1(Q^2) = \int dz e^{-\Phi} \frac{V(Q,z)}{2z^3} (\psi_L^2(z) + \psi_R^2(z)) \qquad C_2(Q^2) = \int dz e^{-\Phi} \frac{V(Q,z)}{2z^2} (\psi_L^2(z) - \psi_R^2(z))$$

$$C_3(Q^2) = \int dz e^{-\Phi} \frac{2m_N V(Q,z)}{2z^3} (\psi_L^2(z) \psi_R^2(z))$$

We can compare if we use a integral representation for $V(Q,z)$ like

$$V(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\frac{\kappa^2 z^2 x}{(1-x)}}$$

[3] A. V, I. Schmidt, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **83**, 036001 (2011).

[4] Z. Abidin and C. E. Carlson, Phys. Rev. D**79**, 115003 (2009)

So the summary of Results in Soft Wall case is

$$H_v^q(x, Q^2) = q(x)x^a \quad and \quad E_v^q(x, Q^2) = e(x)x^a,$$

where

$$a = Q^2/(4\kappa^2); \quad q(x) = \alpha^q\gamma_1 + \beta^q\gamma_2; \quad e(x) = \beta^q\gamma_3,$$

and

$$\alpha^u = 2\ ,\ \alpha^d = 1\ ,\ \beta^u = 2\eta_p + \eta_n\ ,\ \beta^d = \eta_p + 2\eta_n$$

$$\gamma_1 = \frac{1}{2}(5 + 8x + 3x^2) \quad \gamma_1 = 1 - 10x + 21x^2 - 12x^3 \quad \gamma_1 = \frac{6m_N\sqrt{2}}{\kappa}(1 - x)^2$$

⋆ **Parameters involved.**

$$\kappa = 350MeV\ ,\ \eta_p = 0.224\ ,\ \eta_n = -0.239$$

fixed to reproduce mass and anomalous magnetic moment of nucleons.

⋆ Approach to GPDs in a quark-diquark model plus a LFWF[5].

We consider a model where the Dirac and Pauli quark form factors are defined in terms of LFWFs as

$$F_1^q(Q^2) = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \left[\psi_{q+}^{+\,*}(x, k'_\perp)\psi_{q+}^{+}(x, k_\perp) + \psi_{q-}^{+\,*}(x, k'_\perp)\psi_{q-}^{+}(x, k_\perp) \right]$$

$$F_2^q(Q^2) = -\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \left[\psi_{q+}^{+\,*}(x, k'_\perp)\psi_{q+}^{-}(x, k_\perp) + \psi_{q-}^{+\,*}(x, k'_\perp)\psi_{q-}^{-}(x, k_\perp) \right]$$

where M_N is the nucleon mass, $\psi_{\lambda_q q}^{\lambda_N}(x, k_\perp)$ are the LFWFs with specific helicities of nucleon $\lambda_N = \pm$ and struck quark $\lambda_q = \pm$, where plus and minus correspond to $+1/2$ and $-1/2$, respectively.

The LFWFs $\psi_{\lambda_q q}^{\lambda_N}(x, k_\perp)$ are defined as

$$\psi_{+q}^{+}(x, k_\perp) = \frac{m_{1q} + xM_N}{x}\, \varphi_q(x, k_\perp) \qquad \psi_{-q}^{+}(x, k_\perp) = -\frac{k^1 + ik^2}{x}\, (1 - x)\, \mu_q\, \varphi_q(x, k_\perp)$$

$$\psi_{+q}^{-}(x, k_\perp) = \frac{k^1 - ik^2}{x}\, (1 - x)\, \mu_q\, \varphi_q(x, k_\perp) \qquad \psi_{-q}^{-}(x, k_\perp) = \frac{m_{1q} + xM_N}{x}\, \varphi_q(x, k_\perp)$$

[5] A. V, I. Schmidt, T. Gutsche and V. E. Lyubovitskij, in preparation.

where m_{1q} is the mass of struck quark. The wave function $\varphi_q(x, k_\perp)$ is given by the product of transverse and longitudinal wave functions[6]

$$\varphi_q(x, k_\perp) = N_q \sqrt{\log(1/x)}\, x^{\beta_{1q}} (1-x)^{\beta_{2q}+1} \exp\left[-\frac{\mathcal{M}^2}{2\kappa^2}\, x \log(1/x)\right],$$

where

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \frac{k_\perp^2}{x(1-x)} = \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

is the invariant mass, and

$$N_q = \frac{4\pi}{\kappa M_N}\left[\int_0^1 dx\, x^{2\beta_{1q}} (1-x)^{3+2\beta_{2q}} R_q(x)\, e^{-\mathcal{M}_0^2/\kappa^2}\right]^{-1/2}, \qquad R_q(x) = \left(1 + \frac{m_{1q}}{xM_N}\right)^2 + \frac{\kappa^2 \mu_q^2}{M_N^2}\frac{(1-x)^3}{x^2}$$

is the normalization constant and β_{1q} and β_{2q} are the parameters depending on flavor (quark masses). The parameters μ_q ($q = u, d$) are fixed in from description of nucleon magnetic moments.

[6] S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph]; S. S. Chabysheva and J. R. Hiller, arXiv:1207.7128 [hep-ph]; T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. V, Phys. Rev. D **87**, 056001 (2013).

Our results for the quark Dirac and Pauli form factor are

$$F_1^q(Q^2) = C_q \int_0^1 dx x^{2\beta_{1q}} (1-x)^{3+2\beta_{2q}} R_q(x, Q^2) \exp\left[-\frac{Q^2}{4\kappa^2} \log(1/x)(1-x)\right] \exp\left[-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)\right],$$

$$F_2^q(Q^2) = C_q \int_0^1 \frac{2dx}{x} \mu_q \left(1 + \frac{m_{1q}}{xM_N}\right) x^{2\beta_{1q}} (1-x)^{5+2\beta_{2q}} \exp\left[-\frac{Q^2}{4\kappa^2} \log(1/x)(1-x)\right] \exp\left[-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)\right],$$

where

$$C_q = \left[\int_0^1 dx\, x^{2\beta_{1q}} (1-x)^{3+2\beta_{2q}} R_q(x)\, e^{-\mathcal{M}_0^2/\kappa^2}\right]^{-1}$$ and $$R_q(x, Q^2) = R_q(x) - \frac{Q^2}{4M_N^2} \frac{\mu_q^2 (1-x)^4}{x^2}$$

It means that the nonforward parton densities are given by

$$\mathcal{H}^q(x, Q^2) = C_q x^{2\beta_{1q}} (1-x)^{3+2\beta_{2q}} R_q(x, Q^2) \exp\left[-\frac{Q^2}{4\kappa^2} \log(1/x)(1-x)\right] \exp\left[-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)\right]$$

$$\mathcal{E}^q(x, Q^2) = C_q \frac{2}{x} \mu_q \left(1 + \frac{m_{1q}}{xM_N}\right) x^{2\beta_{1q}} (1-x)^{5+2\beta_{2q}} \exp\left[-\frac{Q^2}{4\kappa^2} \log(1/x)(1-x)\right] \exp\left[-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)\right]$$

Redefining index, quarks densities are

$$q_v(x) = C_q\, x^{\rho_{1q}}\, (1-x)^{\rho_{2q}}\, R_q(x)\, e^{-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)},$$

$$\mathcal{E}^q(x) = C_q\, x^{\rho_{1q}}\, (1-x)^{2+\rho_{2q}}\, P_q(x)\, e^{-\frac{\mathcal{M}_0^2}{\kappa^2} x \log(1/x)},$$

$$R_q(x) = \left(1 + \frac{m_{1q}}{x M_N}\right)^2 + \frac{\kappa^2 \mu_q^2}{M_N^2} \frac{(1-x)^3}{x^2}$$ and $$P_q(x) = \frac{2\mu_q}{x} \left(1 + \frac{m_{1q}}{x M_N}\right).$$

We consider three models with some usual values for quark and diquark masses.

- Model I: quarks massless.
- Model II: Struck quark with current mass.
- Model III: Struck quark with constituent mass.

Table: Parameters used for three different fits in parton distributions $q_u(x)$. In all cases we consider $\kappa = 350$ MeV.

Model	m_q [MeV]	m_D [MeV]	C_q	ρ_1	ρ_2
I	0	0	4.76	-0.18	2.56
II	7	100	189.24	1.86	6.11
III	300	600	55.86	1.14	2.50

Table: Parameters used for three different fits in parton distributions $q_d(x)$. In all cases we consider $\kappa = 350$ MeV.

Model	m_q [MeV]	m_D [MeV]	C_q	ρ_1	ρ_2
I	0	0	2.87	-0.21	3.76
II	7	100	27.94	1.91	4.66
III	300	600	28.15	1.25	3.14

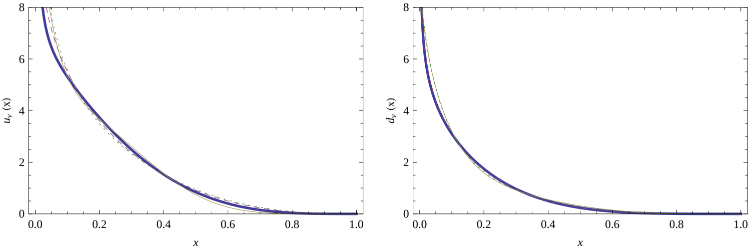

Figure: Comparison between parton distributions. The thick continuous line corresponds to the MRST global NNLO fit at the scale $\mu^2 = 1$ GeV^2, while the dashed line corresponds to the parton distributions given using Fit I, thin continuous line correspond to Fit II and dotted line is for Fit III.

And using a standard representation to $\mathcal{E}_q(x)$[7].

$$\mathcal{E}_u(x) = \frac{k_u}{N_u}(1-x)^{\kappa_1} u(x) \quad \text{and} \quad \mathcal{E}_d(x) = \frac{k_d}{N_d}(1-x)^{\kappa_2} d(x).$$

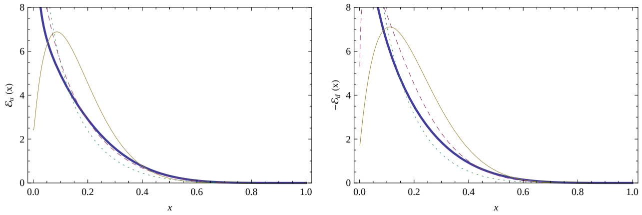

Figure: The thick continuous line corresponds to the form suggested with MRST global NNLO fit at the scale $\mu^2 = 1$ GeV^2. Dashed line corresponds to Fit I, thin continuous line correspond to Fit II and dotted line is for Fit III.

[7] M. Guidal, M. V. Polyakov, A. V. Radyushkin and M. Vanderhaeghen, Phys. Rev. D **72**, 054013 (2005).

Conclusions

- It is possible to use AdS / QCD ideas to calculate GPDs.
- Starting with a AdS / QCD form factor, we can use a matching procedure to get GPDs. Actually this approach need some improvements to reproduce quarks densities.
- Starting from a light-front quark - diquark model, and using a holographical LFWF, we derive the nucleon non forward densities and get parton densities.
- In general, works that consider parametrizations for GPDs usually pay attention to justify the Q behavior, and put by hand some popular fit to parton densities, and here we present a models that offer expressions for these functions.

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That's all Folks!