## Jet quenching in an EFT framework

Antonio Vairo

Technische Universität München



## Outline

- 1. Introduction: motivation
  - the jet quenching parameter  $\hat{q}$
  - energy scales and degrees of freedom
  - SCET
- 2. Jet broadening in covariant and light-cone gauges
- 3. Discussion: comparison with the literature
  - low-energy contributions: lattice
  - conclusions and outlook

## 1. INTRODUCTION

## Jet quenching

Jet quenching was first observed at RHIC and then confirmed at LHC.

This phenomenon happens when a very energetic quark or gluon,  $Q \gg T$ , which in vacuum would manifest itself as a jet, going through a strongly coupled plasma loses sufficient energy that few high momentum hadrons are seen in the final state.



## Jet quenching



#### p+p event





Pb+Pb event (very central)



## Jet quenching

Jet quenching manifests itself in many observables.

In particular, the hard partons produced in the collision

- lose energy;
- change direction of their momenta: transverse momentum broadening.



## Jet quenching parameter $\hat{q}$

 $P(k_{\perp})$  is the probability that after propagating through the medium for a distance L the hard parton acquires transverse momentum  $k_{\perp}$ ,

$$\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$$

 $\hat{q}$  is the jet quenching parameter, i.e. the mean square transverse momentum picked up by the hard parton per unit distance traveled,

$$\hat{q} = \frac{\langle k_{\perp}^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2 k_{\perp}}{(2\pi)^2} \, k_{\perp}^2 \, P(k_{\perp})$$

## Jet quenching: early literature

There has been a long-time effort to determine  $\hat{q}$  from QCD.

- The relation of 
   *î* to the expectation value of two Wilson lines oriented along one of the light-cone directions has been established over the last fifteen years.
  - Baier Dokshitzer Mueller Peigne Schiff NP B483 (1997) 291
     Zakharov JETPL 63 (1996) 952
     Casalderrey-Solana Salgado APP B38 (2007) 3731
     Liang Wang Zhou PR D77 (2008) 125010
- Effective field theories (EFTs) taking advantage in a systematic way of the many scales involved in the problem have been used more recently.
  - o D'Eramo Liu Rajagopal PR D84 (2011) 065015 Ovanesyan Vitev JHEP 1106 (2011) 080

#### Energy scales: kinematics and medium

A highly energetic parton, Q > 100 GeV, propagates along the light-cone direction  $\bar{n}$ .



In a medium characterized by a scale (e.g. temperature)  $T \ll Q$ :

$$\lambda \equiv \frac{T}{Q} \ll 1$$

 $\lambda$  will be the relevant expansion parameter.

#### Energy scales: collinear partons

We consider partons that undergo a transverse momentum broadening of order  $Q\lambda$ :



if q = Q(λ, 1, λ), the parton is off shell by ~ Q<sup>2</sup>λ: the parton is hard-collinear;
if q = Q(λ<sup>2</sup>, 1, λ), the parton is off shell by ~ Q<sup>2</sup>λ<sup>2</sup>: the parton is collinear.

## Energy scales: Glauber gluons

We consider the transverse momentum broadening of a collinear parton:



It may happen through

- fragmentation into collinear partons (gluons, quarks) of momentum  $q = Q(\lambda^2, 1, \lambda)$ ;
- scattering by soft gluons of momentum  $q = Q(\lambda, \lambda, \lambda)$ ;
- scattering by Glauber gluons of momentum  $q = Q(\lambda^2, \lambda, \lambda)$  or  $q = Q(\lambda^2, \lambda^2, \lambda)$ .

```
o Idilbi Majumder PR D80 (2009) 054022
```

## Degrees of freedom

The relevant degrees of freedom are:



#### Soft Collinear Effective Theory (SCET)

SCET is the suitable EFT for processes that involve a large energy transfer (larger than any other scale including masses).

B decays by weak interactions:
B → X<sub>u</sub>ℓν̄, B → Dπ, B → K\*γ, B → πℓν, B → X<sub>s</sub>γ, B → ργ, B → ργ, B → ργ, B → ρρ, B → ππ, B → D\*η', B → Kπ, B → γℓν̄, Υ → Xγ, ...
Ex. B → Dπ



 $p_{\pi}^{\mu} \approx (2.3 \,\text{GeV}, 0, 0, -2.3 \,\text{GeV}) \sim Q \bar{n}^{\mu} \sim Q(0, 1, 0)$   $Q \gg \Lambda$ , hence  $\lambda \sim \Lambda/Q$ • soft constituent (of *B* and *D*):  $p_s^{\mu} \sim (\Lambda, \Lambda, \Lambda) \sim Q(\lambda, \lambda, \lambda)$ • collinear constituent (after boost):  $p_c^{\mu} \sim (\Lambda^2/Q, Q, \Lambda) \sim Q(\lambda^2, 1, \lambda)$ 

• Bauer Fleming Pirjol Stewart PR D63 (2001) 114020

## Soft Collinear Effective Theory (SCET)





$$Q \gg \Delta$$
, hence  $\lambda \sim \Delta/Q$   
• Jet constituents:  $p^{\mu} \sim (\Delta^2/Q, Q, \Delta) \sim Q(\lambda^2, 1, \lambda)$ 

# **2. GAUGE INVARIANT** $P(k_{\perp})$

#### SCET for transverse momentum broadening

The EFT that describes the propagation of a collinear quark in the  $\bar{n}$ -direction is SCET coupled to collinear, soft, Glauber and ultrasoft gluons:

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \hbar \bar{n} \cdot D \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i D_{\perp} \frac{1}{2in \cdot D} i D_{\perp} \hbar \xi_{\bar{n}}$$

We will consider now the contribution of soft, Glauber and ultrasoft gluons only and rescale  $\bar{\xi}_{\bar{n}} \rightarrow e^{-iQx^+} \bar{\xi}_{\bar{n}}$ . The Lagrangian, organized as an expansion in  $\lambda$ , is then

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} \, i \not\! n \bar{n} \cdot D \, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \, \frac{D_{\perp}^2}{2Q} \, \not\! n \, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \, i \frac{g F_{\perp}^{\mu\nu}}{4Q} \, \gamma_{\mu} \gamma_{\nu} \, \not\! n \, \xi_{\bar{n}} \, + \dots$$

• Idilbi Majumder PR D80 (2009) 054022

#### Power-counting in covariant gauges

In a covariant gauge and in the presence of covariant soft sources, it is ( $q^+ \sim Q\lambda^2$ )

$$A^+ \sim A_\perp \sim Q\lambda^2$$

which follows from the equation of motion of a collinear parton and Lorentz symmetry.Note, however, that the fields are not homogeneous in the energy scales.

For  $\partial_{\perp} \sim \lambda$  (when acting on collinear quarks), the leading order Lagrangian in  $\lambda$  is

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not\! n \bar{n} \cdot D \,\xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \,\frac{\partial_{\perp}^2}{2Q} \,\not\! h \,\xi_{\bar{n}}$$

#### Decoupling of ultrasoft gluons

Ultrasoft gluons decouple at lowest order from collinear quarks trough

$$\xi_{\bar{n}} \to \mathrm{P} \exp\left\{ig \int_{-\infty}^{x^-} dy \,\bar{n} \cdot A_{\mathrm{us}}(x^+, y, x_{\perp})\right\} \xi_{\bar{n}}$$

- The field redefinition works for ultrasoft gluons because, at lowest order, the kinetic energy,  $\nabla_{\perp}^2/(2Q)$ , commutes with ultrasoft gluons. For the opposite reason, the field redefinition would not decouple, even at lowest order, Glauber gluons from collinear quarks.
- We may consider in the Lagrangian soft and Glauber gluons only.

• Bauer Pirjol Stewart PR D65 (2002) 054022

#### Jet broadening in covariant gauges

Only one relevant vertex



for  $k_{\perp} \neq 0$  and normalizing by the number of particles in the medium (=  $\Delta t/L$ ).

#### Glauber and soft gluons in covariant gauge

• Glauber gluon propagators may be approximated by (e.g. in Feynman gauge)

$$D_{\mu\nu}(k) = D(k^2)g_{\mu\nu} \approx D(k_{\perp}^2)g_{\mu\nu}$$

• This implies that the scattering amplitude has the form

$$\int \prod_{i} \frac{d^{4}q_{i}}{(2\pi)^{4}} \cdots \frac{iQ\hbar}{2Qq_{2}^{+} - q_{2\perp}^{2} + i\epsilon} A^{+}(q_{2} - q_{1})\hbar \frac{iQ\hbar}{2Qq_{1}^{+} - q_{1\perp}^{2} + i\epsilon} A^{+}(q_{1} - q_{0})\hbar\xi_{\bar{n}}(q_{0})$$

$$\approx \int dy^{+}d^{2}y_{\perp} \prod_{i} dy_{i}^{-} \cdots \theta(y_{3}^{-} - y_{2}^{-})A^{+}(y^{+}, y_{2}^{-}, y_{\perp}) \theta(y_{2}^{-} - y_{1}^{-})A^{+}(y^{+}, y_{1}^{-}, y_{\perp})\xi_{\bar{n}}(q_{0})$$

where  $\hbar \xi_{\bar{n}}(q_0) = 0$  and  $\xi_{\bar{n}}^{\dagger}(q_0) \xi_{\bar{n}}(q_0) = \sqrt{2}Q$ .

• The same result holds if one considers the interaction of soft gluons with collinear and hard-collinear quarks. In this case, one has to recall that the hard-collinear quark propagator reads  $\frac{i}{2q^+ + i\epsilon}\hbar$ 

Jet broadening in covariant gauges

$$P(k_{\perp}) = \frac{1}{N_c} \int d^2 x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} \left\langle \text{Tr} \left\{ W^{\dagger}[0^+, x_{\perp}] W[0^+, 0_{\perp}] \right\} \right\rangle$$



Baier et al NP B483 (1997) 291, Zakharov JETPL 63 (1996) 952
 Casalderrey-Solana Salgado APP B38 (2007) 3731
 D'Eramo Liu Rajagopal PR D84 (2011) 065015

#### Power-counting in light-cone gauge

Consider the light-cone gauge  $A^+ = 0$ :

$$D_{\mu\nu}(k) = D(k^2) \left( g_{\mu\nu} - \frac{k_{\mu}\bar{n}_{\nu} + k_{\nu}\bar{n}_{\mu}}{[k^+]} \right)$$

for Glauber gluons  $k_{\perp}/[k^+] \sim 1/\lambda$ , which leads to on enhancement of order  $\lambda$  in the singular part of the propagator.

Moreover, because of the  $k_{\perp}/[k^+]$  singularity,

 $A_{\perp}(x^{+}, x^{-}, x_{\perp}) = A_{\perp}^{\text{cov}}(x^{+}, x^{-}, x_{\perp}) + \theta(x^{-})A_{\perp}(x^{+}, \infty^{-}, x_{\perp}) + \theta(-x^{-})A_{\perp}(x^{+}, -\infty^{-}, x_{\perp})$ 

where  $A_{\perp}^{cov}$  vanishes at infinity and  $A_{\perp}(x^+, \pm \infty^-, x_{\perp}) = \partial_{\perp}\phi^{\pm}(x^+, x_{\perp})$ :

- $A_{\perp}$  does not vanish at infinity where it becomes pure gauge,
- but the field tensor does (because the energy of the gauge field is finite).

Belitsky Yuan NP B656 (2003) 165
 Garcia-Echevarria Idilbi Scimemi PR D84 (2011) 011502

## Power-counting in light-cone gauge

In the  $A^+ = 0$  light-cone gauge, the scaling of the Glauber fields appearing in the Lagrangian goes like

$$A^+ = 0 \qquad A_{\perp}^{\text{cov}} \sim Q\lambda^2 \qquad \partial_{\perp}\phi^{\pm} \sim Q\lambda$$

The leading order Lagrangian in  $\lambda$  is

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \hbar \bar{n} \cdot \partial \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \frac{(\partial_{\perp} - ig\theta(x^{-})\partial_{\perp}\phi^{+} - ig\theta(-x^{-})\partial_{\perp}\phi^{-})^{2}}{2Q} \hbar \xi_{\bar{n}}$$

The relevant vertices are two



From the vertices one constructs the amplitude (on the left of the cut)



 $G_n$  is a convolution of  $G_{n-j}^+$ , which involves only fields at  $x^- = \infty$  and  $G_j^-$ , which involves only fields at  $x^- = -\infty$ :

$$G_n(k^-, k_\perp) = \sum_{j=0}^n \int \frac{d^4q}{(2\pi)^4} G_{n-j}^+(k^-, k_\perp, q) \frac{iQ\,\hbar}{2Qq^+ - q_\perp^2 + i\epsilon} G_j^-(q)$$

The computation is done by

• solving recursively (analogously for  $G_n^+(q)$ )

$$G_n^-(q) = \int \frac{d^4q'}{(2\pi)^4} G_{n-1}^-(q') \xrightarrow{q' \quad q} + \int \frac{d^4q''}{(2\pi)^4} G_{n-2}^-(q'') \xrightarrow{q'' \quad q} + \int \frac{d^4q''}{(2\pi)^4}$$

• writing the differential amplitude as

$$\frac{1}{L^3\sqrt{2}Q} \int \frac{dk^+}{2\pi} \int \frac{dk^-}{2\pi} 2\pi Q \,\delta(2Qk_+ - k_\perp^2) \,\bar{\xi}_{\bar{n}}(q_0) \,G_m^{\dagger}(k^-, k_\perp) \bar{h} G_n(k^-, k_\perp) \,\xi_{\bar{n}}(q_0) \,G_m^{\dagger}(k^-, k_\perp) \,\bar{h} G_n(k^-, k_\perp) \,\xi_{\bar{n}}(q_0) \,G_m^{\dagger}(k^-, k_\perp) \,G_m^{\dagger}(k^-, k_\perp) \,\xi_{\bar{n}}(q_0) \,G_m^{\dagger}(k^-, k_\perp) \,G_m^{\dagger}(k^-, k_\perp) \,\xi_{\bar{n}}(q_0) \,G_m^{\dagger}(k^-, k_\perp) \,G_m^$$

• eventually summing over all m and n.

$$P(k_{\perp}) = \frac{1}{N_c} \int d^2 x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} \left\langle \text{Tr} \left\{ T^{\dagger}[0^+, -\infty^-, x_{\perp}] T[0^+, \infty^-, x_{\perp}] \right. \right. \\ \left. \times T^{\dagger}[0^+, \infty^-, 0_{\perp}] T[0^+, -\infty^-, 0_{\perp}] \right\} \right\rangle$$

where 
$$T[0^+, \pm \infty^-, x_{\perp}] = P \exp\left\{-ig \int_{-L/\sqrt{2}}^0 ds \, l_{\perp} \cdot A_{\perp}(0^+, \pm \infty^-, x_{\perp} + s l_{\perp})\right\}$$

• Benzke Brambilla Escobedo Vairo JHEP 1302 (2013) 129

The relevance of the Wilson line T in light-cone gauge SCET has been discussed in
Idilbi Scimemi PL B695 (2011) 463



## The gauge-invariant expression of $P(k_{\perp})$

The gauge invariant expression of  $P(k_{\perp})$  then reads

$$P(k_{\perp}) = \frac{1}{N_c} \int d^2 x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} \left\langle \text{Tr} \left\{ T^{\dagger}[0^+, -\infty^-, x_{\perp}] W^{\dagger}[0^+, x_{\perp}] T[0^+, \infty^-, x_{\perp}] \right. \right. \\ \left. \times T^{\dagger}[0^+, \infty^-, 0_{\perp}] W[0^+, 0_{\perp}] T[0^+, -\infty^-, 0_{\perp}] \right\} \right\rangle$$

The path ordering prescription implies that fields in the first line are anti-time ordered while fields in the second line are time ordered.

• Benzke Brambilla Escobedo Vairo JHEP 1302 (2013) 129

## The gauge-invariant expression of $P(k_{\perp})$



• Note that the Wilson lines at  $x^- = \infty$  are contiguous while those at  $x^- = -\infty$  are not. This is because the fields are not time ordered.

## The gauge-invariant expression of $P(k_{\perp})$

The expression of  $P(k_{\perp})$  may be simplified because:

- contiguous adjoint (unitary) lines cancel;
- fields separated by space-like intervals commute;
- the cyclicity of the trace.



• Note that the fields in  $(0^+, -\infty^-, 0_\perp)$  are not contiguous.

## Gauge invariance

Under a gauge transformation  $\Omega$ 

$$\operatorname{Tr}\left\{T^{\dagger}[0^{+},-\infty^{-},x_{\perp}]W^{\dagger}[0^{+},x_{\perp}]\cdots T[0^{+},-\infty^{-},0_{\perp}]\right\}$$
$$\longrightarrow \operatorname{Tr}\left\{\Omega(0^{+},-\infty^{-},-\infty l_{\perp})T^{\dagger}[0^{+},-\infty^{-},x_{\perp}]W^{\dagger}[0^{+},x_{\perp}]\cdots$$
$$\times T[0^{+},-\infty^{-},0_{\perp}]\Omega^{\dagger}(0^{+},-\infty^{-},-\infty l_{\perp})\right\}$$

$$= \operatorname{Tr} \left\{ T^{\dagger}[0^{+}, -\infty^{-}, x_{\perp}] W^{\dagger}[0^{+}, x_{\perp}] \cdots T[0^{+}, -\infty^{-}, 0_{\perp}] \right\}$$

for the fields in  $\Omega(0^+, -\infty^-, -\infty l_\perp)$  commute with all the others (space-like separations) and the cyclicity of the trace.

• Benzke Brambilla Escobedo Vairo JHEP 1302 (2013) 129

## 3. DISCUSSION

## Jet quenching: recent literature

 One can express P(k<sub>⊥</sub>) as a two particle (time ordered) Wilson loop by modifying the partition function according to the Keldysh–Schwinger contour and by identifying the anti-time ordered fields with fields on the imaginary time line of the Keldysh–Schwinger contour.

o D'Eramo Liu Rajagopal PR D84 (2011) 065015

•  $\hat{q}$  may be written in terms of field correlators as

$$\begin{split} \hat{q} &= \int^{k_{\max}} \frac{d^2 k_{\perp}}{(2\pi)^2} \int d^2 x_{\perp} \, e^{ik_{\perp} \cdot x_{\perp}} \int dx^{-} \\ &\times \frac{\sqrt{2}}{N_c} \left\langle \operatorname{Tr} \left\{ [0, x_{\perp}]_{-} U^{\dagger}_{x_{\perp}} [x^{-}, -\infty] \, gF^{+i}_{\perp}(0, x^{-}, x_{\perp}) \right. \\ &\left. \times U^{\dagger}_{x_{\perp}} [\infty, x^{-}] [x_{\perp}, 0]_{+} U_{0_{\perp}} [\infty, 0] \, gF^{+i}_{\perp}(0, 0, 0) \, U_{0_{\perp}} [0, -\infty] \right\} \right\rangle \end{split}$$

where 
$$Q\lambda \leq k_{\max} \ll Q$$
 and  $U_{x_{\perp}}[x^{-}, y^{-}] = P \exp\left[ig \int_{y^{-}}^{x^{-}} dz^{-} A^{+}(0, z^{-}, x_{\perp})\right]$ .

#### Jet quenching: recent literature

• Similar (but not gauge invariant) expressions can be found in the literature.

E.g. o Majumder arXiv:1202.5295

They seem to require  $k_{\max} \to \infty$ , which is only justified, under some circumstances, in dimensional regularization. In this case, the gauge invariant expression reads

$$\hat{q} = \int dx^{-} \frac{\sqrt{2}}{N_{c}} \left\langle \operatorname{Tr} \left\{ U_{0}_{\perp} \left[ -\infty, x^{-} \right] g F_{\perp}^{+i}(0, x^{-}, 0) \right. \\ \left. \times U_{0}_{\perp} \left[ x^{-}, 0 \right] g F_{\perp}^{+i}(0, 0, 0) U_{0}_{\perp} \left[ 0, -\infty \right] \right\} \right\rangle$$

0 -



## The contribution from the scale $g^2T$

The gauge invariant expression of  $\hat{q}$  allows for the use of lattice data.

E.g. Suppose a weakly coupled plasma characterized by the thermodynamical scales:

 $T \gg gT \gg g^2 T$  (non-perturbative)

At relative order  $g^2$ 

- one can analytically continue from Minkowski to Euclidean space-time;
- time ordering is irrelevant;
- the partition function at the energy scale  $g^2T$  is described by an EFT (MQCD) that is three-dimensional SU(3) (with coupling  $g^2T$ ).

• Caron-Huot PR D79 (2009) 065039

The contribution from the scale  $g^2T$  may be extracted from the behaviour of the Wilson loop,  $\sim e^{-LV}$ , in three-dimensional QCD.

```
• Laine EPJ C72 (2012) 2233
```

#### The contribution from the scale $g^2T$ at large distances

At large distances,  $r \gtrsim r_0 = 2.2/(g^2T)$ ,



For  $\alpha_{\rm s} \approx 0.5$  and  $T \approx 300$  MeV,  $\delta \hat{q} \approx 5$  GeV<sup>2</sup>/fm.

 Lüscher Weisz JHEP 0207 (2002) 049 (for the lattice data) see also Mykkanen JHEP 1212 (2012) 069 (for new lattice data) and Pineda Stahlhofen PR D81 (2010) 074026 (for PT)

## Conclusions and outlook

A systematic treatment of a complex phenomenon like jet quenching is possible in an EFT framework owning to the hierarchy of scales that characterize the system. These are the typical SCET scales, Q,  $Q\lambda$ ,  $Q\lambda^2$ , with  $\lambda = T/Q$ , which characterize the propagation of a very energetic parton in the medium and the thermal scales that characterize the medium itself, T,  $m_D$ , magnetic mass.

Many contributions still need to be computed both on the SCET side and on the thermal side of the theory. Work in progress includes:

- inclusion of collinear gluons;
  - o D'Eramo Liu Rajagopal JP G38 (2011) 124162
- extended perturbative analysis of the thermal bath.
  - Benzke Brambilla Escobedo Vairo TUM-EFT 32/12