Radiation reaction from lightfront QED

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CHALMERS

Vetenskapsrådet

- Anton Ilderton
- Quantum and classical radiation reaction from lightfront QED, arXiv:1304.6842 [hep-th]

Introduction

1. Classical radiation reaction (RR)
   - Problems with Abraham-Lorentz-Dirac equation (LAD)
   - Different classical equations
2. Derive RR from QED
   - Use lightfront quantisation
   - Use Hamiltonian formalism
3. Take the classical limit $\hbar \to 0$
   - **Which equations are consistent with QED?**

Classical Equations

- Most classical equations have the form

$$\ddot{x}^\mu = f^{\mu\nu}\dot{x}_\nu + \frac{2}{3}\frac{e^2}{4\pi m}R_{\mu\nu}\dot{x}^\nu$$

- Lorentz force $f = eF_{\text{ext}}/m$

Radiation Reaction	$R_{\mu\nu}$
Abraham (1905) Lorentz (1909) Dirac (1938)	$\dddot{x}\dot{x} - \dot{x}\dddot{x}$
Landau Lifshitz	$\dot{f} + (f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Eliezer (1948) Ford and O'Connell (1991)	$\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})\dot{x} - \dot{x}\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})$
Mo and Papas (1971)	$(f\ddot{x})\dot{x} - \dot{x}(f\ddot{x})$
Herrera (1977)	$(f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Sokolov (2009)	$q \mathrel{\text{ǂ}} m\dot{x}$

Classical Equations

$$\ddot{x}^\mu = f^{\mu\nu}\dot{x}_\nu + \frac{2}{3}\frac{e^2}{4\pi m}R_{\mu\nu}\dot{x}^\nu \qquad f = eF_{\text{ext}}/m$$

- We solve these equations by expansion in e^2
- The Lorentz term is treated exactly
  - Makes it easy to distinguish RR from Lorentz
  - Detect RR with high intensity lasers?

    Di Piazza et al. Rev. Mod. Phys. 84 (2012), Heinzl Int. J. Mod. Phys. A27 (2012), Harvey, Heinzl, Marklund PRD 84 (2011), Di Piazza, Hatsagortsyan, Keitel, PRL 102 (2009), Bulanov *et al.* PRE 84 (2011)
- Corresponds to the coupling expansion we will use in QED

Plane Waves

- We use a plane wave background field
- Null wavevector $n^2 = 0$
- Transverse polarisation vector $na' = 0$
- Choose coordinates so $\phi := nx = x^+$

$$eF^{\text{ext}}_{\mu\nu}(\phi) = n_\mu a'_\nu(\phi) - a'_\mu(\phi) n_\mu$$

- Depends only on ϕ
- $a'_\perp(\phi) = eE_\perp(\phi)$
  $a'(\pm\infty) = 0$

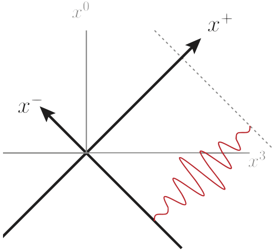


Figure : Plane wave

Talk by A. Ilderton

Classical Predictions

- Solution to zeroth order (Lorentz force equation)

$$\pi := p - a + \frac{2ap - a^2}{2np} n$$

$$a_\perp(\phi) = \int_{-\infty}^{\phi} \mathrm{d}\varphi \; eE_\perp(\varphi)$$

Classical Predictions

- Solutions to $\mathcal{O}(e^2)$ $\rightarrow$ effects due to radiation reaction

$$\Delta = \frac{2}{3}\frac{e^2}{4\pi m}\frac{np}{m}$$

- Momentum for LAD

$$q(\phi)_{\mathsf{LAD}} = \pi(\phi) + \Delta\pi'(\phi) + \frac{\Delta}{m^2}\int^{\phi} \mathrm{d}\varphi \; a'^2(\varphi)\left(\pi(\varphi) - \frac{\pi(\varphi)\pi(\phi)}{kp}k\right)$$

- Position for LAD

$$npx'_{\mathsf{LAD}} = q_{\mathsf{LAD}} - \pi\frac{\Delta}{m^2}\int a'^2$$

Classical Predictions

- Momentum

$$\left\{\begin{array}{c}\text{LAD}\\ \text{LL}\\ \text{EFO}\end{array}\right\} = \left\{\begin{array}{c}\text{MP}\\ \text{H}\\ \text{S}\end{array}\right\} + \Delta\pi'$$

- Lightfront time derivative of position

$$\left\{\begin{array}{c}\text{LAD}\\ \text{LL}\\ \text{EFO}\\ \text{S}\end{array}\right\} = \left\{\begin{array}{c}\text{MP}\\ \text{H}\end{array}\right\} + \frac{\Delta\pi'}{np}$$

- $\Delta\pi'(\infty) = 0 \rightarrow$ **Need finite time results from QED**

Lightfront quantisation

- Plane wave → lightfront quantisation
- Lightfront Hamiltonian Neville and Rohrlich PRD 8 (1971)

$$H = \frac{1}{2}\int \mathrm{dx} - A_j \partial_\perp^2 A_j + |\mathcal{D}_\perp \Phi|^2 + m^2|\Phi|^2 + ejA - e^2 A^2|\Phi|^2 + \frac{e^2}{2}\left(\frac{j_-}{\partial_-}\right)^2$$

$$\mathcal{D}_\perp = \partial_\perp + ia_\perp$$

- Treat background exactly → Furry picture
  - The background field is included in the "free" part
  - The electrons are dressed by the background field
  - Mode functions $\varphi_\mathsf{p}(x)$ in Φ are Volkov solutions

$$\mathcal{D}_\mu \varphi_\mathsf{p}(x) = \pi_\mu(\phi)\varphi_\mathsf{p}(x) \qquad \leftarrow \text{Lorentz}$$

D. Volkov, Z. Phys. 94 (1953) 250

Expectation values

- State

$$\left|\,\psi; x^+\right\rangle = \mathcal{T}_+ e^{-i\int^{x^+} H_F}\left|\,\text{in}\,\right\rangle$$

- $\langle P^e_\mu\rangle(\phi) = \langle\,\psi; x^+\,|P^e_\mu|\,\psi; x^+\,\rangle \to$ finite time RR

See also V. S. Krivitsky and V. N. Tsytovich, Sov.Phys.Usp. 34 (1991) 250, P. R. Johnson and B. L. Hu, PRD 65 (2002) 065015 and A. Higuchi and G. D. R. Martin, PRD 73 (2006) 025019.

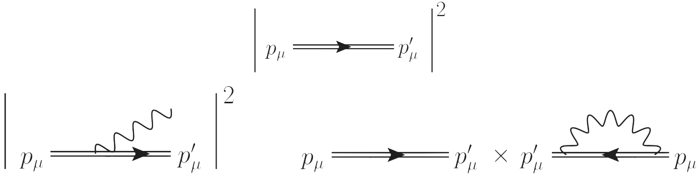


Figure : Furry-Feynman diagrams for $\mathcal{O}(e^2)$ contributions to expectation values. Lorentz force, nonlinear Compton scattering and loop

Momentum

- Momentum tensor → Electron + photon momentum

$$T_{\mu\nu} \to T^e_{\mu\nu} + T^\gamma_{\mu\nu}$$

$$T^e_{\mu\nu} = (D_\mu\Phi)^\dagger D_\nu\Phi + (D_\nu\Phi)^\dagger D_\mu\Phi - g_{\mu\nu}\left(|D\Phi|^2 - m^2|\Phi|^2\right)$$

$$D_\mu = \partial_\mu + ieA_\mu + ia_\mu$$

- Momentum operator

$$P^e_\mu = \int \mathrm{dx}\; T^e_{-\mu}$$

Momentum

- Expectation value of momentum operator

$$\langle P^e_\mu \rangle = \pi_\mu - \frac{\alpha}{2\pi\epsilon}\pi_\mu(\phi) + \frac{\alpha}{2\pi\epsilon}\frac{m^2}{np}n_\mu + e^2 \cdot \text{finite}$$

  - Transverse dim reg A. Casher, Phys. Rev. D 14 (1976) 452.
- Operator renormalisation
  - $a = 0$

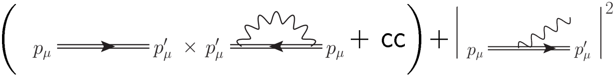
$$\langle P^e \rangle_\mu = p_\mu - \frac{\alpha}{2\pi\epsilon}p_\mu + \frac{\alpha}{2\pi\epsilon}\frac{m^2}{np}n_\mu \xrightarrow{\text{renormalise}} p_\mu$$

  - Multiplicative and mass renormalisation

    D. Mustaki, S. Pinsky, J. Shigemitsu, and K. Wilson, Phys. Rev. D 43 (1991) 3411.

Classical limit

- Classical limit $\hbar \to 0$ Brodsky & Hoyer PRD 83 (2011) 045026
- Terms proportional to $1/\hbar$ cancel



$$\lim_{\hbar\to 0} \langle P^e_\mu \rangle = q_\mu(\phi) \quad \text{as in LAD, LL, EFO}$$

Ilderton and Torgrimsson. Quantum and classical radiation reaction from lightfront QED, arXiv:1304.6842 [hep-th]

Position

- ⟨current⟩ → LAD, LL, EFO, S in the classical limit
- Position Operator

$$\mathrm{X}(\phi) = \int \mathrm{dx}\, \mathrm{x}\, j_-(\phi)$$

- Corresponds to the Newton-Wigner position operator

See also Higuchi and Martin Phys. Rev. D 73 (2006) 025019.

- No divergences → no renormalisation
- Classical limit

$$\lim_{\hbar\to 0}\langle X^\mu\rangle = x^\mu(\phi) \qquad \text{as in LAD, LL, EFO, S}$$

Conclusions

- We have derived RR directly from QED
  - Plane waves → lightfront quantisation
  - Used Hamiltonian formalism → finite time
  - Used Furry picture → exact in background field
- Need finite time to distinguish between different equations
- **LAD, LL and EFO are consistent with QED to $\mathcal{O}(e^2)$**
- Distinguish between them by extending our method to $\mathcal{O}(e^4)$
  - Many diagrams and many terms to calculate