

Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

Results

Problems

# Point-Form Calculations

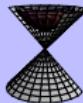
vs.

# Other Relativistic Approaches

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Theoretical Physics / Institute of Physics  
University of Graz, Austria

LC2013 Satellite Meeting  
Skiathos, May 25th, 2013



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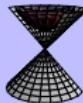
## Different relativistic approaches

to several hadron observables yield

**similar results** but not quite **the same**.

Seems to be true for:

- ▶ Hamiltonian vs. Hamiltonian approaches
- ▶ Bethe-Salpeter vs. Bethe-Salpeter approaches
- ▶ Lattice-QCD vs. lattice-QCD approaches



# Outline

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Framework of the Point-Form Approach

Spectroscopy and Mass-Operator Eigenstates  
Ground-state and excitation spectra

Rest-Frame Baryon States in 3D

Relativistic Matrix Elements  
Different relativistic approaches

Discussion of Results for Elastic E.m. Nucleon FF's

Things to Do - A Wish List



# Formalism

## Relativistic quantum mechanics

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space  $\mathcal{H}$  corresponding to a finite number of particles, not a field theory)

### Invariant mass operator

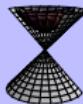
$$\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$$

### Eigenvalue equations

$$\hat{M} |P, J, \Sigma\rangle = M |P, J, \Sigma\rangle , \quad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu |P, J, \Sigma\rangle = P^\mu |P, J, \Sigma\rangle , \quad \hat{P}^\mu = \hat{M} \hat{V}^\mu$$

$P^\mu$  ... 4-momentum,  $J$  ... baryon spin,  $\Sigma$  ... spin projection



# Invariant Mass Operator

## Interacting mass operator

$$\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$$

$$\hat{M}_{\text{free}} = \sqrt{\hat{H}_{\text{free}}^2 - \hat{\vec{P}}_{\text{free}}^2}$$

$$\hat{M}_{\text{int}}^{\text{rest frame}} = \sum_{i < j}^3 \hat{V}_{ij} = \sum_{i < j} [\hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}}]$$

fulfilling the **Poincaré algebra**

$$[\hat{P}_i, \hat{P}_j] = 0, \quad [\hat{J}_i, \hat{H}] = 0, \quad [\hat{P}_i, \hat{H}] = 0,$$

$$[\hat{K}_i, \hat{H}] = -i\hat{P}_i, \quad [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k, \quad [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k,$$

$$[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, \quad [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, \quad [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}$$

$\hat{H}, \hat{P}_i$  ... time and space translations,

$\hat{J}_i$  ... rotations,  $\hat{K}_i$  ... Lorentz boosts



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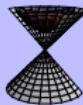
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# Baryon **Excitation Spectra**

and

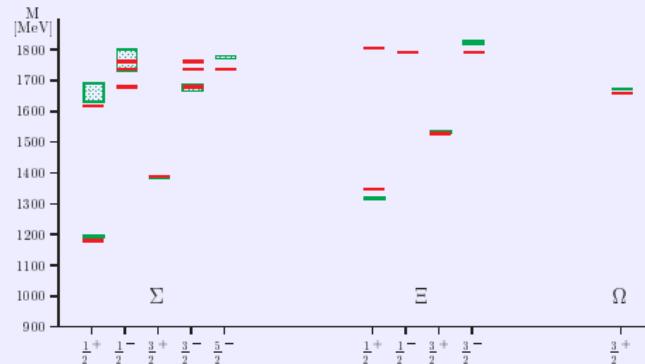
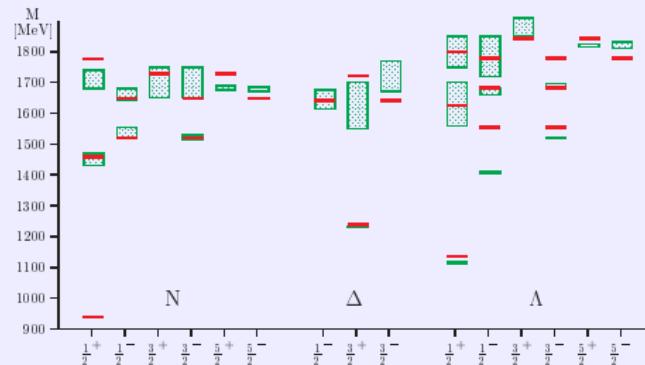
# Mass-Operator **Eigenstates**

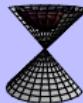


# $u, d, s$ Baryon Spectroscopy

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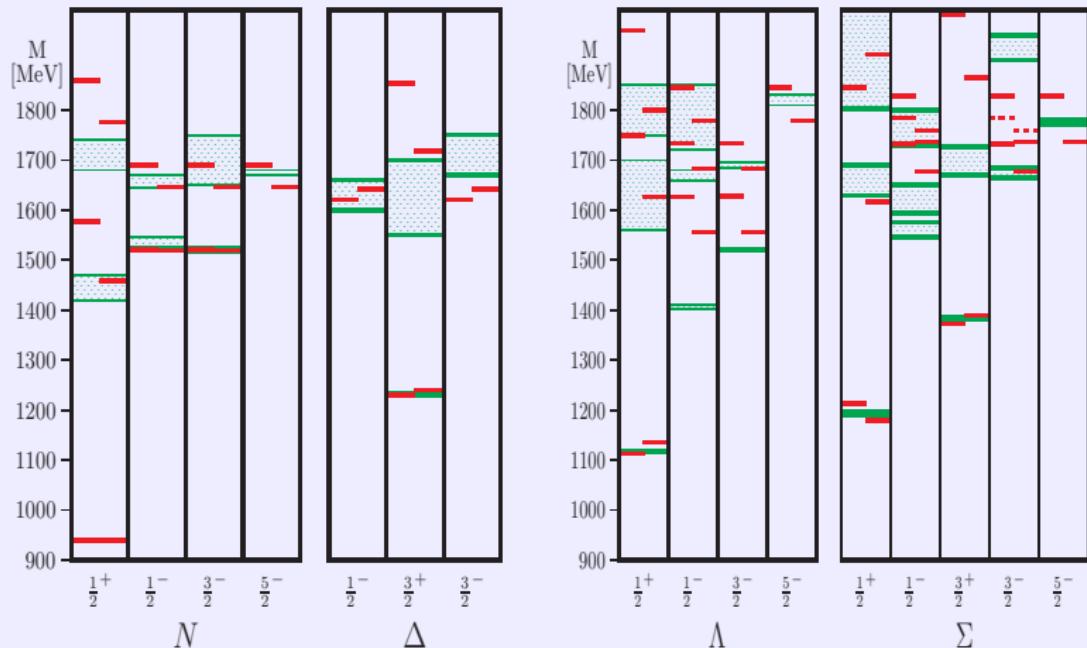
## Excitation spectra of the GBE RCQM:



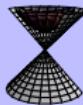


# $N$ and $\Lambda$ Excitation Spectra

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left levels: One-gluon-exchange RCQM  
right levels: Goldstone-boson-exchange RCQM



# GBE Hyperfine Interaction

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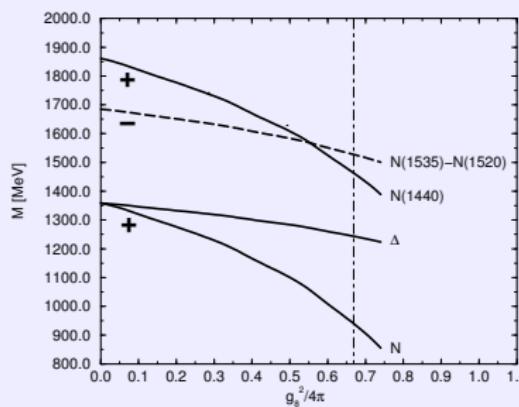
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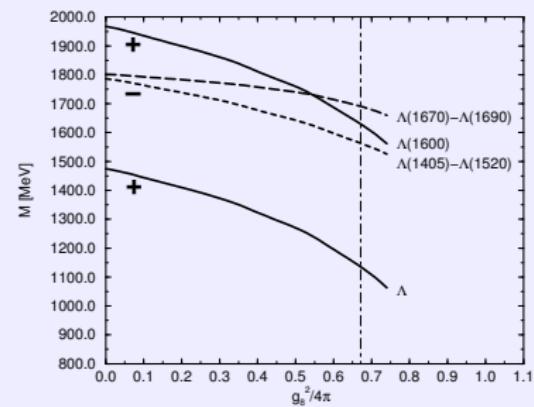
Problems

Level shifts due to hyperfine interaction:

N



A



L.Ya. Glozman, Z. Papp, W. Plessas, K. Varga, and R.F. Wagenbrunn, Phys. Rev. C 57, 3406 (1998)



# Chiral Interaction

Theory

Spectroscopy

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Eigenstates

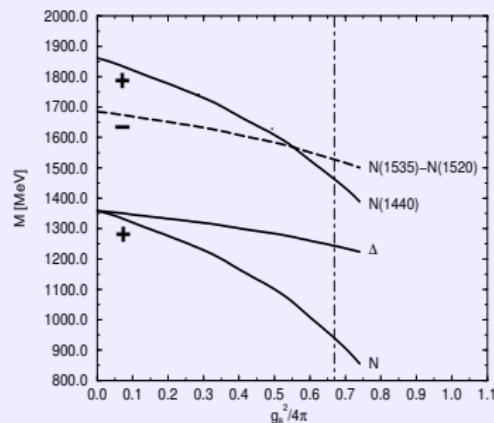
FFs

Relativistic  
Approaches

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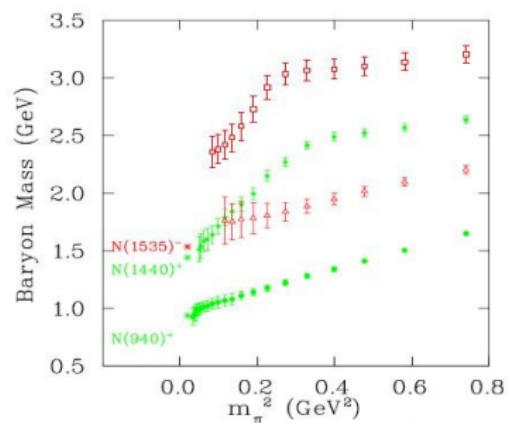
Problems

## GBE CQM

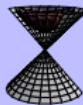


## Lattice calculation

(Kentucky Group)



K.F. Liu et al.: '*Valence QCD: Connecting QCD to the quark model*' Phys. Rev. D **59**, 112001 (1999)



# Chiral Interaction

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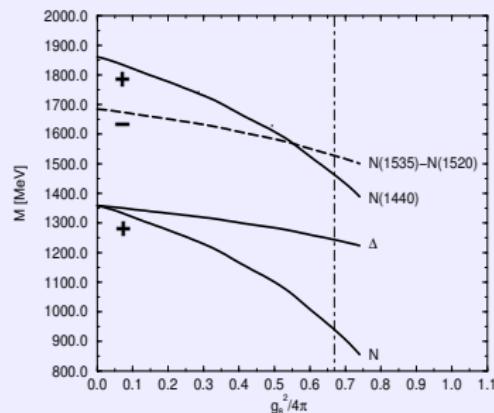
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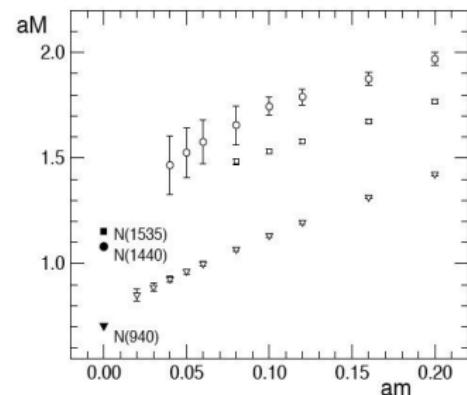
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## GBE CQM

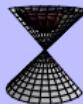


## Lattice calculation

(Graz Group)



T. Burch et al.: Phys. Rev. D 74, 014504 (2006)



# Rest-Frame Baryon States

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## Mass operator eigenstates

$$\hat{M} |P, J, \Sigma, T, M_T\rangle = M |P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

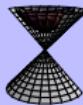
$$\langle \vec{\xi}, \vec{\eta} | P, J, \Sigma, T, M_T \rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

with  $\vec{\xi}$  and  $\vec{\eta}$  the usual Jacobi coordinates.

Picture the baryon wave functions through  
**spatial probability density distributions**

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$

$$\Psi_{PJ\Sigma TM_T}^*(\xi, \Omega_\xi, \eta, \Omega_\eta) \Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$



# Pictures of Baryons (rest frame)

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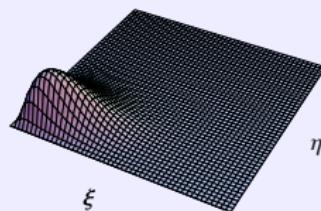
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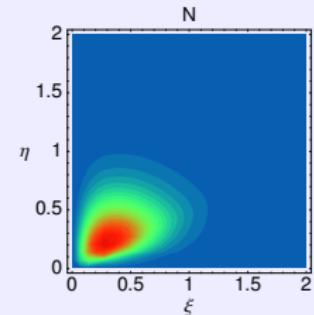
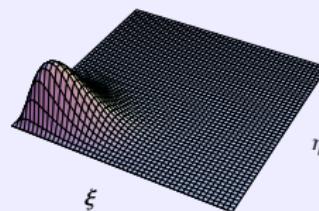
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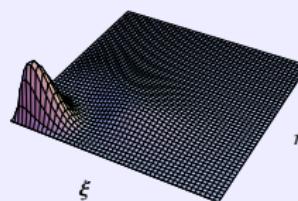
N GBE CQM



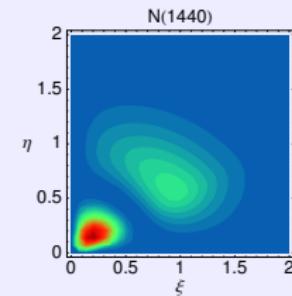
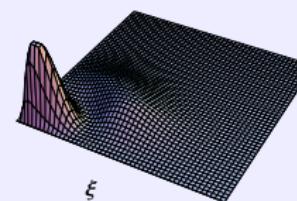
N OGE CQM

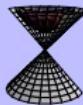


N(1440) GBE CQM



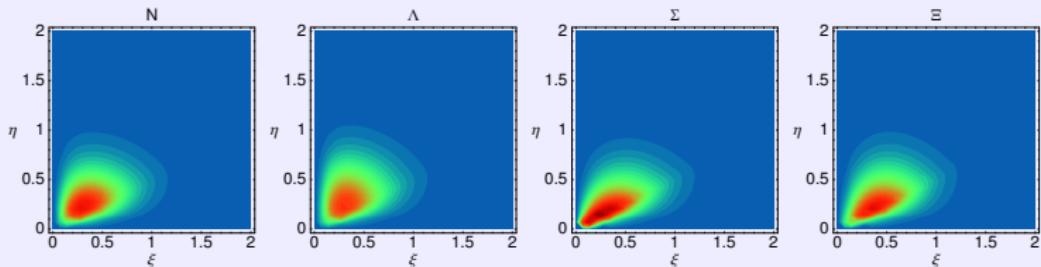
N(1440) OGE CQM





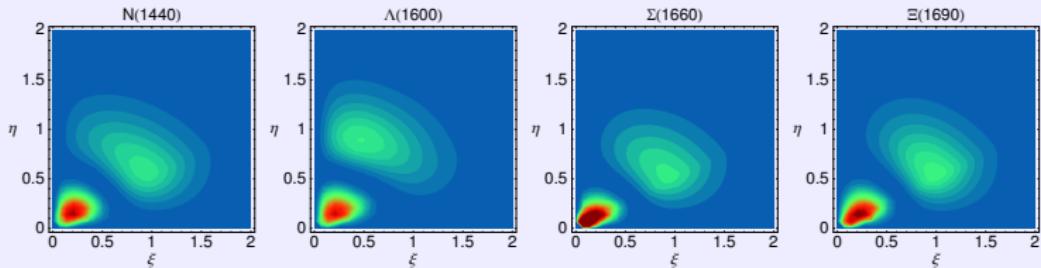
# Spatial Probability Density Distributions

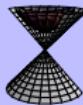
$\rho(\xi, \eta)$  for the  $\frac{1}{2}^+$  octet baryon ground states  $N(939)$ ,  $\Lambda(1116)$ ,  $\Sigma(1193)$ ,  $\Xi(1318)$ :



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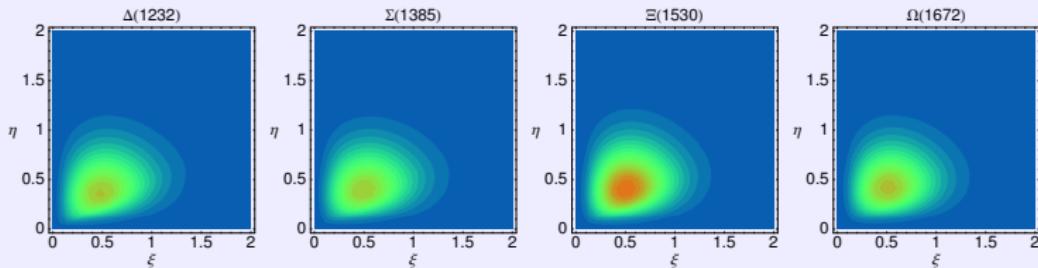
$\rho(\xi, \eta)$  for the  $\frac{1}{2}^+$  octet baryon states  $N(1440)$ ,  $\Lambda(1600)$ ,  $\Sigma(1660)$ ,  $\Xi(1690)$ :





# Spatial Probability Density Distributions

$\rho(\xi, \eta)$  for the  $\frac{3}{2}^+$  decuplet baryon states  $\Delta(1232)$ ,  $\Sigma(1385)$ ,  $\Xi(1530)$ ,  $\Omega(1672)$ :

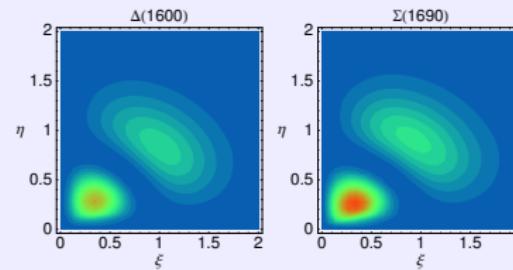


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$\rho(\xi, \eta)$  for the  $\frac{3}{2}^+$  decuplet baryon states  $\Delta(1600)$ ,  $\Sigma(1690)$ :



T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)



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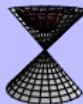
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## Relativistic

## Matrix Elements



# Matrix Elements of a Transition Operator $\hat{O}$

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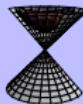
- $\hat{O} \dots \hat{J}_{\text{em}}^\mu \rightarrow \text{electromagnetic FF's}$
- $\dots \hat{A}_{\text{axial}}^\mu \rightarrow \text{axial FF's}$
- $\dots \hat{S} \rightarrow \text{scalar FF}$
- $\dots \hat{\Theta}^{\mu\nu} \rightarrow \text{gravitational FF's}$

ME's between baryon eigenstates  $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T'_3, Y' | \hat{O} | P, J, \Sigma, T, T_3, Y \rangle$$

to be calculated from microscopic three-quark ME's

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3; f_{i'_1}, f_{i'_2}, f_{i'_3} | \hat{O} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3} \rangle$$

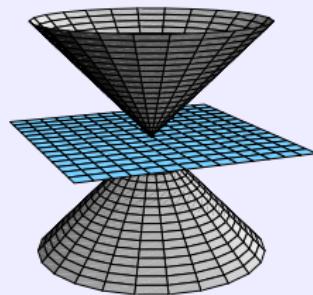


# Forms of Relativistic Dynamics

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Invariant hypersurfaces in Minkowski space:

instant

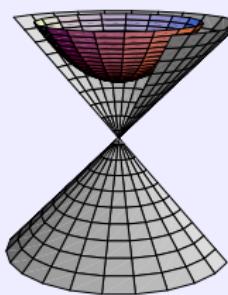


$$x^0 = 0$$

$J_i, P_i$

$P^0 = H, K_i$

point

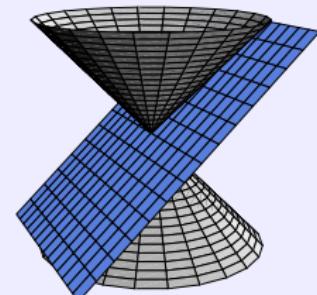


$$x^2 - a^2 = 0$$

$J_i, K_i$

$P^\mu$

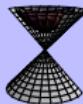
front



$$x^0 + x^3 = 0$$

$P^+, \vec{P}^\perp, E^1, E^2, J_z, K_z$   
 $P^-, F^1, F^2$

are **interaction-free** and **interaction-dependent** generators



# $e^-$ Scattering and Nucleon e.m. Form Factors

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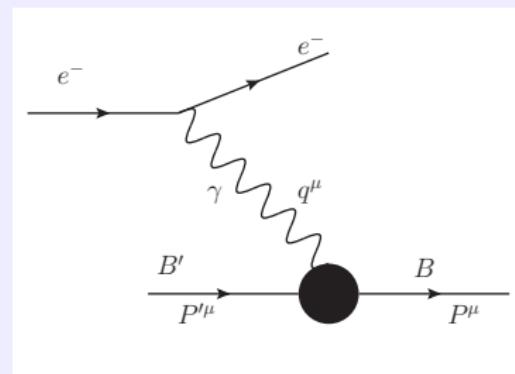
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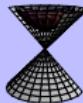
Problems



## Invariant form factors:

$$F_{\Sigma'\Sigma}^\mu(Q^2) = \langle P', J, \Sigma', T, T_3 | \hat{J}_{\text{em}}^\mu | P, J, \Sigma, T, T_3 \rangle$$

$$\text{with } Q^2 = -q^2; \quad q^\nu = P^\nu - P'^\nu$$



# Transition Matrix Elements in Point Form

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Incoming baryon state:  $|V, M, J, \Sigma\rangle$   $\hat{O} = \hat{\mathbf{J}}_{\text{em}}^\mu$   $\hat{O} \hat{O}^\dagger \hat{O}^\dagger = \hat{O}$   
 Outgoing baryon state:  $|V', M', J', \Sigma'\rangle$   $\hat{O}^\dagger = \hat{O}$   
 Transition operator:  $\hat{O} = \hat{\mathbf{J}}_{\text{em}}^\mu$

$$\begin{aligned} & \langle V', M', J', \Sigma' | \hat{\mathbf{J}}_{\text{em}}^\mu | V, M, J, \Sigma \rangle = \\ & \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3 \\ & \times \sqrt{\frac{(\sum_i \omega'_i)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{\star \frac{1}{2}} \{ R_W [k'_i; B(V')] \} \Psi_{M' J' \Sigma'}^* (\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3) \\ & \times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{\mathbf{J}}_{\text{rd}}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\ & \times \sqrt{\frac{(\sum_i \omega_i)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}} \{ R_W [k_i; B(V)] \} \Psi_{MJ\Sigma} (\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3) \\ & \times 2MV_0 \delta^3 (M \vec{V} - M' \vec{V}' - \vec{q}) \end{aligned}$$

where  $p_i = B_c(V) k_i$ ,  $p'_i = B_c(V') k'_i$ , and  $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$



# Point-Form Spectator Model (PFSM) Currents

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## Electromagnetic current

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{J}_{\text{rd}}^{\mu} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle =$$

$$3\mathcal{N} \langle p'_1, \sigma'_1 | \hat{J}_{\text{spec}}^{\mu} | p_1, \sigma_1 \rangle 2p_{20}\delta(\vec{p}_2 - \vec{p}'_2) 2p_{30}\delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

with

$$\langle p'_1, \sigma'_1 | \hat{J}_{\text{spec}}^{\mu} | p_1, \sigma_1 \rangle =$$

$$e_1 \bar{u}(p'_1, \sigma'_1) \left[ f_1(\tilde{Q}^2) \gamma^{\mu} + \frac{i}{2m_1} f_2(\tilde{Q}^2) \sigma^{\mu\nu} \tilde{q}_{\nu} \right] u(p_1, \sigma_1)$$

## Axial current:

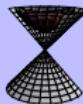
$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{A}_{\text{a,rd}}^{\mu} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle =$$

$$3\mathcal{N} \langle p'_1, \sigma'_1 | \hat{A}_{\text{a,spec}}^{\mu} | p_1, \sigma_1 \rangle 2p_{20}\delta(\vec{p}_2 - \vec{p}'_2) 2p_{30}\delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

with

$$\langle p'_1, \sigma'_1 | \hat{A}_{\text{a,spec}}^{\mu} | p_1, \sigma_1 \rangle =$$

$$\bar{u}(p'_1, \sigma'_1) \left[ g_A^q \gamma^{\mu} + \frac{2f_{\pi}}{\tilde{Q}^2 + m_{\pi}^2} g_{qq\pi} \tilde{q}^{\mu} \right] \gamma_5 \frac{1}{2} \tau_a u(p_1, \sigma_1)$$



# Peculiarities of the PF Spectator Model

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Point-like constituent quarks:

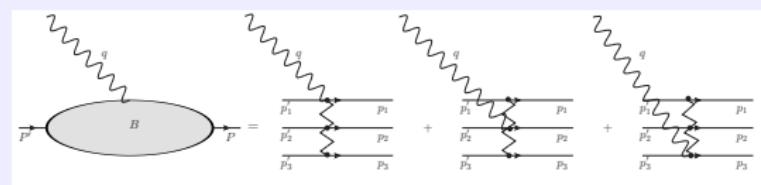
$$f_1(\tilde{Q}^2) = 1, \quad f_2(\tilde{Q}^2) = 0; \quad \tilde{Q}^2 = -\tilde{q}^\mu \tilde{q}_\mu$$

$$\Rightarrow \langle p'_i, \sigma'_i | \hat{\mathbf{J}}_{\text{spec}}^\mu | p_i, \sigma_i \rangle = e_i \bar{u}(p'_i, \sigma'_i) \gamma^\mu u(p_i, \sigma_i)$$

with

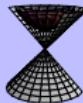
$$p'^\mu_i - p^\mu_i = \tilde{q}^\mu \neq Q^\mu = P'^\mu - P^\mu; \quad \tilde{q}^\mu = \xi Q^\mu$$

$$\mathcal{N} = \left( \frac{M}{\sum_i \omega_i} \right)^{\frac{3}{2}} \left( \frac{M'}{\sum_i \omega'_i} \right)^{\frac{3}{2}}$$



The **PFSM** current operator ME is **manifestly covariant** (has the **same form in any reference frame**)!

It is an **effective many-body operator** through the appearance of  $\tilde{q}$  and  $\mathcal{N}$ , which are both completely determined, however.



# Transition Matrix Elements in Instant Form

Theory  
 Spectroscopy  
 Spectra  
 Eigenstates  
 FFs  
 Relativistic Approaches  
 Results  
 Problems

Incoming baryon state:  $|P, J, \Sigma\rangle$   $\hat{=}|V, M, J, \Sigma\rangle$   
 Outgoing baryon state:  $|P', J', \Sigma'\rangle$   $\hat{=}|V', M', J', \Sigma'\rangle$   
 Transition operator:  $\hat{O} = \hat{\mathcal{J}}_{\text{em}}^\mu$

$$\begin{aligned} & \langle P', J', \Sigma' | \hat{\mathcal{J}}_{\text{em}}^\mu | P, J, \Sigma \rangle = \\ & 2\sqrt{EE'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3 \frac{1}{\sqrt{E_{\text{free}} E'_{\text{free}}}} \\ & \times \sqrt{\frac{(\sum_i \omega'_i)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{*\frac{1}{2}} \{R_W [k'_i; B(V')]\} \Psi_{M' J' \Sigma'}^* (\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3) \\ & \times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{\mathcal{J}}_{\text{rd}}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\ & \times \sqrt{\frac{(\sum_i \omega_i)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}} \{R_W [k_i; B(V)]\} \Psi_{MJ\Sigma} (\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3) \end{aligned}$$

where  $p_i = B_c(V)k_i$ ,  $p'_i = B_c(V')k'_i$ , and  $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$



# Instant-Form Spectator Model (IFSM) Currents

Theory  
Spectroscopy  
Spectra  
Eigenstates  
FFs  
Relativistic  
Approaches  
Results  
Problems

## Electromagnetic current

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{J}_{\text{rd}}^{\mu} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \\ 3\mathcal{N} \langle p'_1, \sigma'_1 | \hat{J}_{\text{spec}}^{\mu} | p_1, \sigma_1 \rangle 2p_{20}\delta(\vec{p}_2 - \vec{p}'_2) 2p_{30}\delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

with

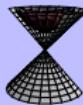
$$\langle p'_1, \sigma'_1 | \hat{J}_{\text{spec}}^{\mu} | p_1, \sigma_1 \rangle = \\ e_1 \bar{u}(p'_1, \sigma'_1) \left[ f_1(\bar{Q}^2)\gamma^{\mu} + \frac{i}{2m_1} f_2(\bar{Q}^2)\sigma^{\mu\nu}\bar{q}_{\nu} \right] u(p_1, \sigma_1)$$

## Axial current:

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{A}_{a,\text{rd}}^{\mu} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \\ 3\mathcal{N} \langle p'_1, \sigma'_1 | \hat{A}_{a,\text{spec}}^{\mu} | p_1, \sigma_1 \rangle 2p_{20}\delta(\vec{p}_2 - \vec{p}'_2) 2p_{30}\delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

with

$$\langle p'_1, \sigma'_1 | \hat{A}_{a,\text{spec}}^{\mu} | p_1, \sigma_1 \rangle = \\ \bar{u}(p'_1, \sigma'_1) \left[ g_A^q \gamma^{\mu} + \frac{2f_{\pi}}{\bar{Q}^2 + m_{\pi}^2} g_{qq\pi} \bar{q}^{\mu} \right] \gamma_5 \frac{1}{2} \tau_a u(p_1, \sigma_1)$$



# Peculiarities of the IF Spectator Model

Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

Results

Problems

Point-like constituent quarks:

$$f_1(\bar{Q}^2) = 1, \quad f_2(\bar{Q}^2) = 0$$

$$\Rightarrow \langle p'_i, \sigma'_i | \hat{J}_{\text{spec}}^\mu | p_i, \sigma_i \rangle = e_i \bar{u}(p'_i, \sigma'_i) \gamma^\mu u(p_i, \sigma_i)$$

with

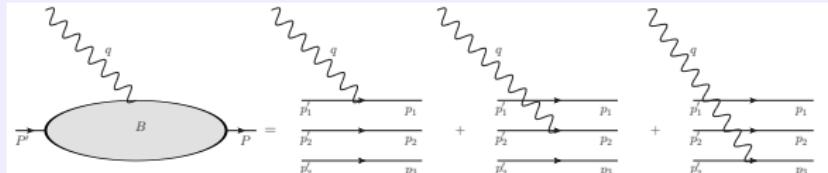
$$\vec{p}'_i - \vec{p}_i = \vec{q} = \vec{Q} = \vec{P}' - \vec{P}$$

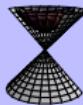
but

$$p'^0_i - p^0_i = \bar{q}^0 \neq Q^0 = P'^0 - P^0; \quad \bar{q}^0 = \zeta Q^0$$

Since in the **IF** the 3-momenta are interaction-free, while the Hamiltonian  $P^0$  is dynamical!

The whole 3-momentum  $\vec{Q}$  carried by the photon is transferred to quark 1 but only part of its energy  $Q^0$ .  
 $\bar{q}^0$ , however, is uniquely determined by overall momentum conservation.





# Peculiarities of the FF Spectator Model

Theory

Spectroscopy

Spectra

Eigenstates

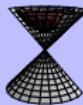
FFs

Relativistic  
Approaches

Results

Problems

?



# Elastic Sachs Form Factors

Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

Results

Problems

## Spin- $\frac{1}{2}$ baryons:

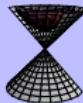
$$G_E^B(Q^2) = \frac{1}{2M} F_{\frac{1}{2} \frac{1}{2}}^{\nu=0}(Q^2)$$

$$G_M^B(Q^2) = \frac{1}{Q} F_{\frac{1}{2} - \frac{1}{2}}^{\nu=1}(Q^2)$$

## Spin- $\frac{3}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{4M} [F_{\frac{1}{2} \frac{1}{2}}^{\nu=0}(Q^2) + F_{\frac{3}{2} \frac{3}{2}}^{\nu=0}(Q^2)]$$

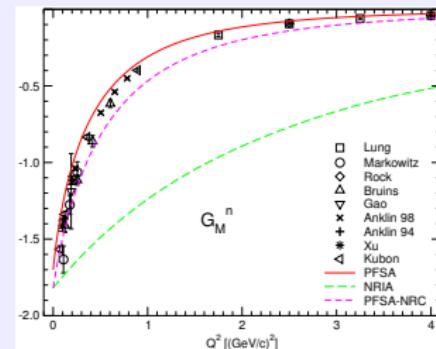
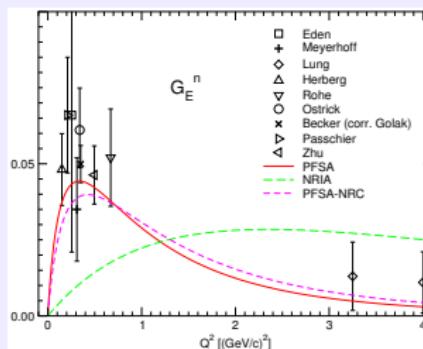
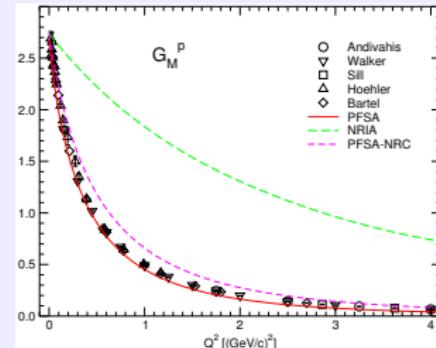
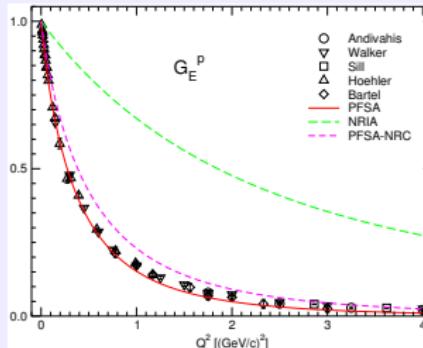
$$G_M^B(Q^2) = \frac{3}{5Q} [F_{\frac{1}{2} - \frac{1}{2}}^{\nu=1}(Q^2) + \sqrt{3} F_{\frac{3}{2} \frac{1}{2}}^{\nu=1}(Q^2)]$$

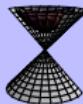


# Electromagnetic Nucleon Form Factors

Theory  
Spectroscopy  
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Eigenstates  
FFs  
Relativistic  
Approaches  
Results  
Problems

## Covariant predictions of the GBE CQM:





# Nucleon Electric Radii and Magnetic Moments

Electric radii  $r_E^2$  [fm $^2$ ]

Baryon	GBE PFSM	Experiment
$p$	0.82	$0.7692 \pm 0.0123^{1)}$
		$0.70870 \pm 0.00113^{2)}$
$n$	-0.13	$-0.1161 \pm 0.0022$

1) CODATA value (PDG)

2) Pohl et al.: Nature **466** (2010) 213

Theory

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Approaches

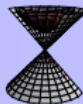
Results

Problems

Magnetic moments  $\mu$  [n.m.]

Baryon	GBE PFSM	Experiment
$p$	2.70	2.792847356
	$-1.70$	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)



# Comparison of PFSM to IFSM

Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

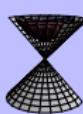
Results

Problems

## Point Form vs. Instant Form Calculations

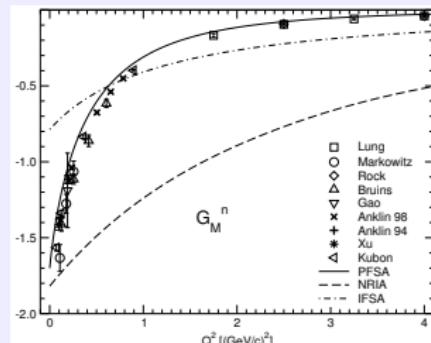
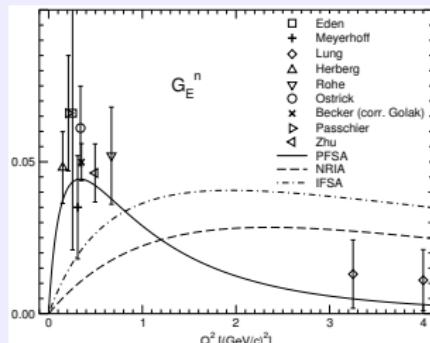
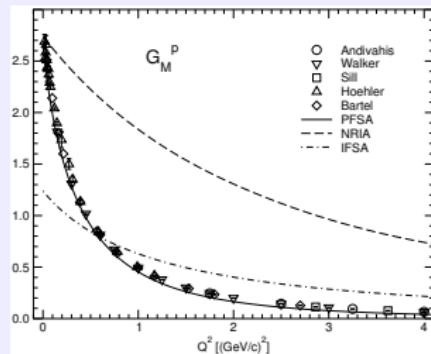
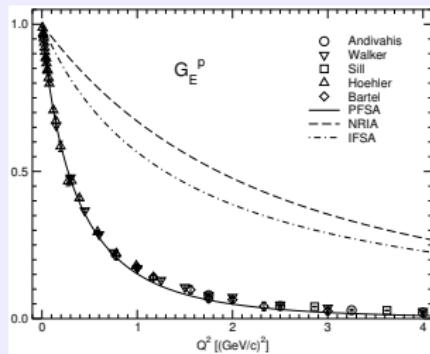
of

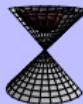
## Nucleon Electromagnetic Form Factors



# Electromagnetic Form Factors of the Nucleons

Theory  
Spectroscopy  
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FFs  
Relativistic  
Approaches  
Results  
Problems





# Comparison of Different RCQM Predictions

Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

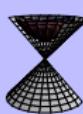
Results

Problems

**GBE RCQM      vs.      OGE RCQM**

**vs.**

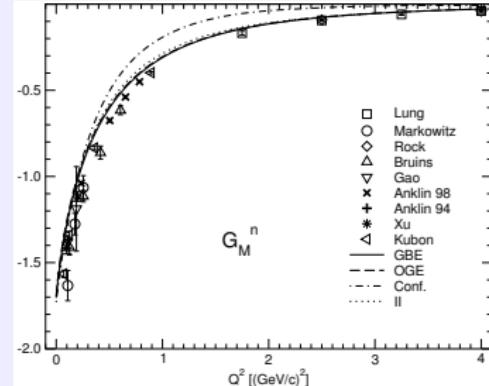
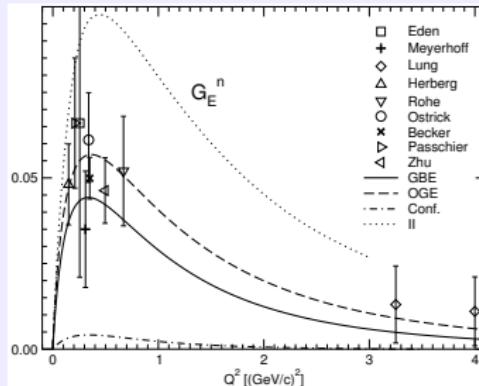
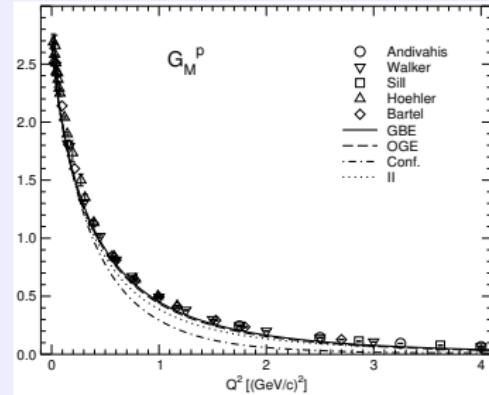
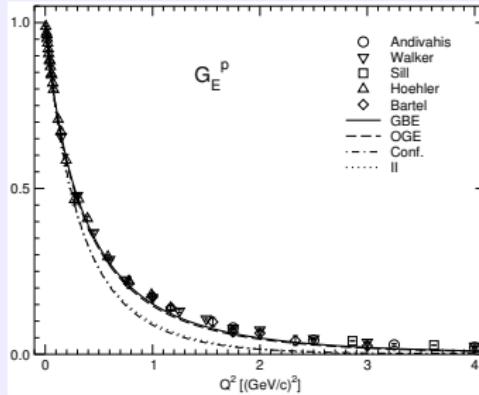
**Instanton-Induced RCQM**  
**(Salpeter Equation - Bonn Group)**



# Electromagnetic Form Factors of the Nucleons

Theory  
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FFs  
Relativistic  
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Problems

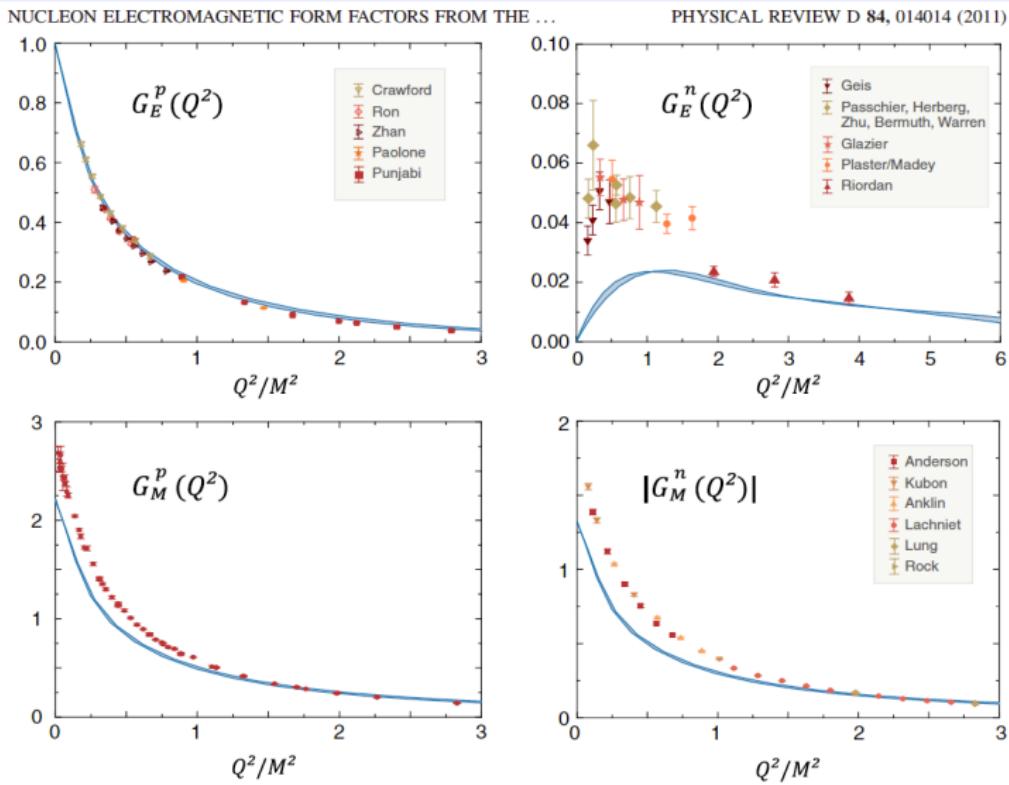
## Different Quark-Model Predictions:

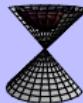




# DSE/Faddeev Result from Graz (G. Eichmann)

Theory  
Spectroscopy  
Spectra  
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FFs  
Relativistic  
Approaches  
Results  
Problems

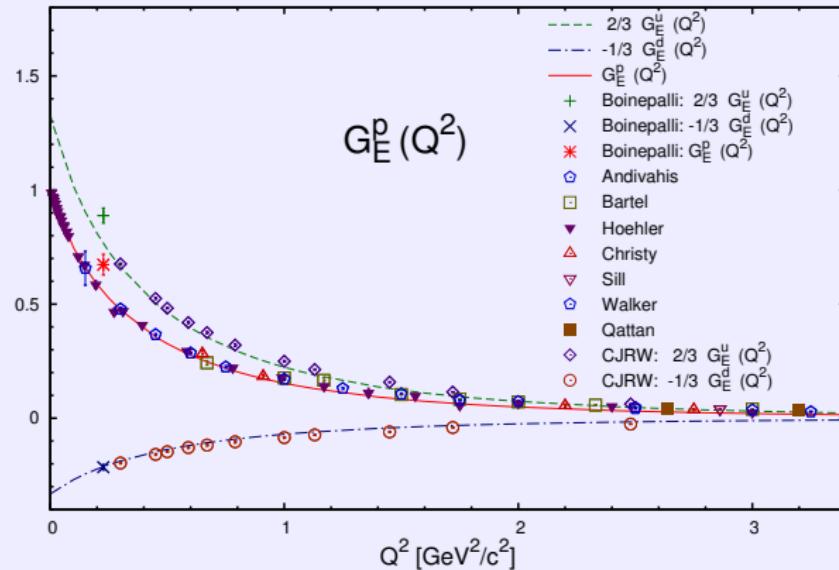




# Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$

Theory  
Spectroscopy  
Spectra  
Eigenstates  
FFs  
Relativistic  
Approaches  
Results  
Problems



GBE RCQM prediction:

M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Lattice QCD (FLIC):

S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

Phenomenology (CJRW):

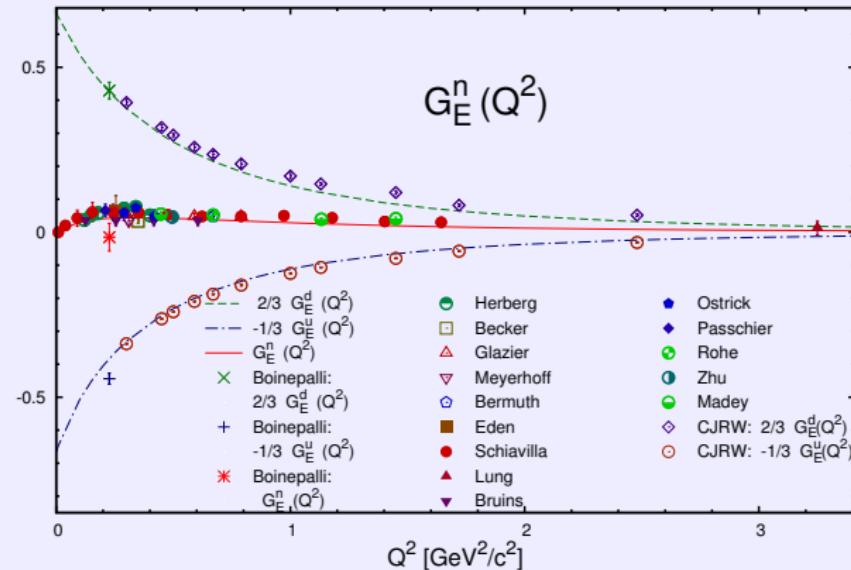
G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)



# Neutron Electric Form Factor

$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$

Theory  
Spectroscopy  
Spectra  
Eigenstates  
FFs  
Relativistic  
Approaches  
Results  
Problems



GBE RCQM prediction:

M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

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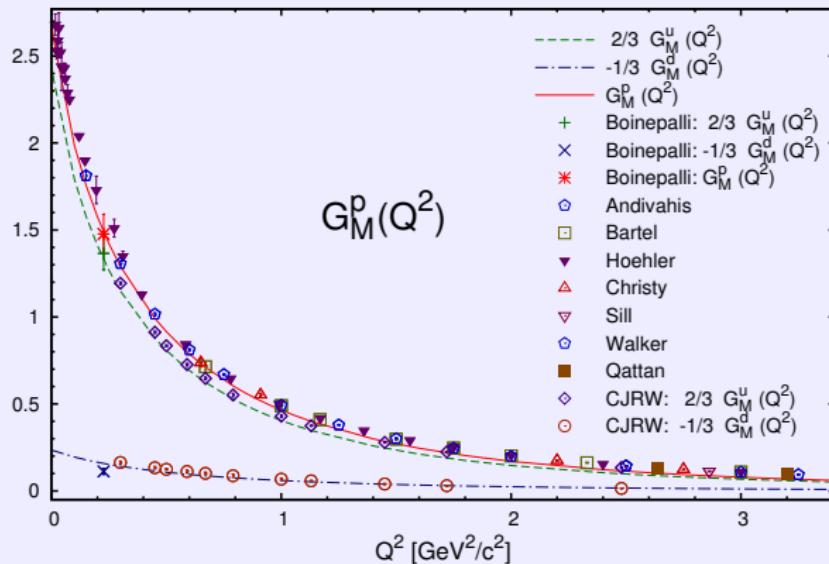
G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)



# Proton Magnetic Form Factor

$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$

Theory  
Spectroscopy  
Spectra  
Eigenstates  
FFs  
Relativistic  
Approaches  
Results  
Problems



GBE RCQM prediction:

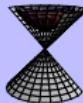
M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

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Phenomenology (CJRW):

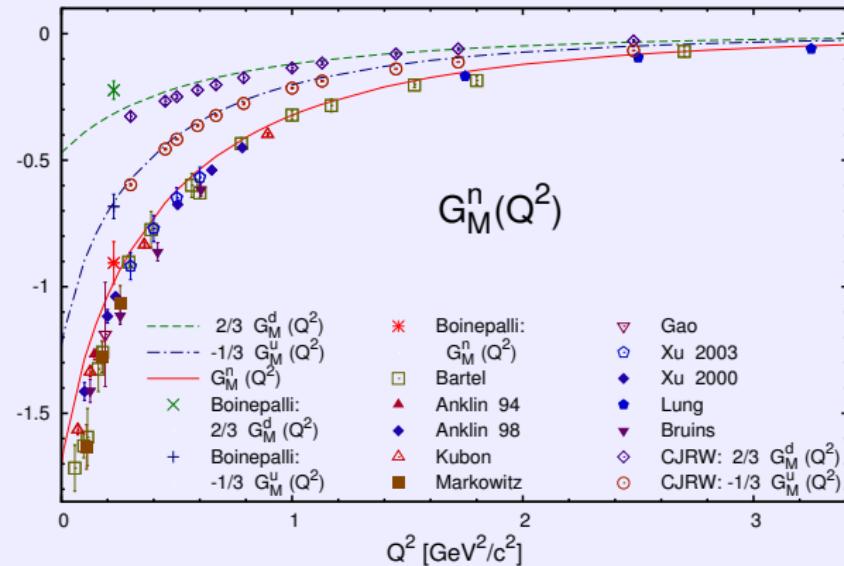
G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)



# Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$

Theory  
Spectroscopy  
Spectra  
Eigenstates  
FFs  
Relativistic  
Approaches  
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Problems



GBE RCQM prediction:

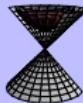
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Lattice QCD (FLIC):

S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

Phenomenology (CJRW):

G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)



# Wish List

Theory

Spectroscopy  
Spectra

Eigenstates

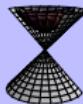
FFs

Relativistic  
Approaches

Results

Problems

- ▶ Results from **different relativistic** approaches differ to a lesser or larger extent.
- ▶ It is not all **completely transparent**, how relativity is included in different approaches.
- ▶ Certainly, **Lorentz invariance** is of utmost importance.
- ▶ **Suggestion** (at least for Hamiltonian approaches):  
Take the same rest frame-wave function (e.g., of the GBE RCQM) and go ahead with calculating a certain reaction (e.g., elastic  $e^-$  scattering on the nucleon) along the spectator-model construction of the e.m. current operator.



# Collaborators

## Graz

K. Berger, J.P. Day, K.-S. Choi, L. Glozman, T. Melde,  
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## Padova

L. Canton  
(INFN, Sezione di Padova)

## Iowa City

W. Klink  
(Department of Physics, University of Iowa, USA)



Theory

Spectroscopy

Spectra

Eigenstates

FFs

Relativistic  
Approaches

Results

Problems

Thank you very much  
for  
your attention!