Rho-meson Distribution Amplitudes

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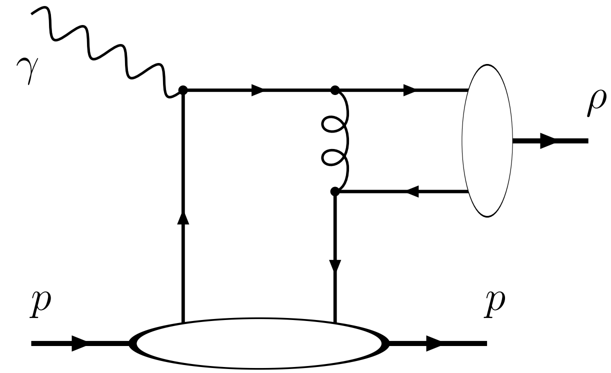
Outline:

- Motivations
- QCD Sum Rule
- Nonlocal condensates
- Longitudinal ρ-meson DA
- Transversal ρ-meson DA
- Conclusions

Motivations:



- B-meson decays: $B \to \rho l \nu$, $\overline{B}^0 \to \rho^0 \gamma$
- CKM matrix constrains
- Diffractive ρ-meson photoproduction

Leading twist ρ-meson DA definitions:

$$\langle 0 \,|\, \bar{d}(z)\gamma_\mu u(0) \,|\, \rho(P,\lambda)\rangle = f_\rho P_\mu \int_0^1 dx\; e^{ix(zp)}\, \varphi_\rho^{\mathsf{L}}(x) + \text{higher twists}$$

$$\langle 0 \,|\, \bar{d}(z)\sigma_{\mu\nu} u(0) \,|\, \rho(P,\lambda)\rangle = i f_\rho^{\mathsf{T}} (\varepsilon_\mu^{(\lambda)} P_\nu - \varepsilon_\nu^{(\lambda)} P_\mu) \int_0^1 dx\; e^{ix(zp)}\, \varphi_\rho^{\mathsf{T}}(x) + \ldots$$

- Typical characteristic used to describe observables:
  $\int_0^1 dx\;\; \varphi_\rho^{\mathsf{L,T}}(x)/x$ (Inverse moment) and $\int_\epsilon^1 dx\;\; \varphi_\rho^{\mathsf{L,T}}(x) f(x)$ with $\epsilon \simeq 0.1 - 0.5$.
- DA evolution with μ^2, according to ERBL equation **[79-80]**.

Gegenbauer expansion of pion DA: $\varphi_\pi(x,\mu^2) \Leftrightarrow a_2, a_4, ..., a_n$

$$\varphi_\pi(x,\mu^2) = 6x\bar{x}(1 + a_2(\mu^2) C_2^{3/2}(x-\bar{x}) + a_4(\mu^2) C_4^{3/2}(x-\bar{x}) + \ldots)$$

QCD SR Approach

Determination of spectrum parameters from requirement of agreement between two ways for correlators **[SVZ 79]**: $\Pi(-q^2) = \int d^4x e^{-iqx} \langle 0 | J_1(0) J_2(x) | 0 \rangle$.

- Dispersion relation: decay constants f_h and masses m_h,

$$\Pi_{\text{had}}\left(Q^2\right) = \int_0^\infty \frac{\rho_{\text{had}}(s)\ ds}{s + Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\text{had}}(s) = f_h^2\, \delta\left(s - m_h^2\right) + \rho_{\text{pert}}(s)\, \theta\left(s - s_0\right)$.

- Operator product expansion:

$$\Pi_{\text{OPE}}\left(Q^2\right) = \Pi_{\text{pert}}\left(Q^2\right) + \sum_n C_n \frac{\langle 0| : O_n : |0\rangle}{Q^{2n}}\,.$$

  - Condensates $\langle 0| : O_n : |0\rangle \equiv \langle O_n \rangle \neq 0$ (next slides).

QCD SR reads:

$$\Pi_{\text{had}}\left(Q^2, m_h, f_h\right) = \Pi_{\text{OPE}}\left(Q^2\right)\,.$$

Introducing condensates in QCD calculations

$$\langle 0|T\left(\bar{q}_B(0)q_A(x)\right)|0\rangle = \boxed{\langle 0|:\bar{q}_B(0)q_A(x):|0\rangle} - i\hat{S}_{AB}(x)$$

QCD PT

$$\langle 0|:\bar{q}q:|0\rangle \stackrel{\text{def}}{=} 0$$

QCD SR

$$\langle 0|:\bar{q}_A(0)q_A(x):|0\rangle =$$

$$\langle \bar{q}q\rangle = \text{CONST} \neq 0$$

[SVZ'79]

Condensate

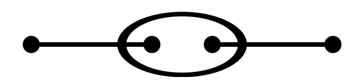
Decay constants,

masses of hadrons

NLC QCD SR

$$\langle 0|:\bar{q}(0)q(x):|0\rangle =$$

$$F_S(x^2) + \hat{x}F_V(x^2)$$



M&R '86

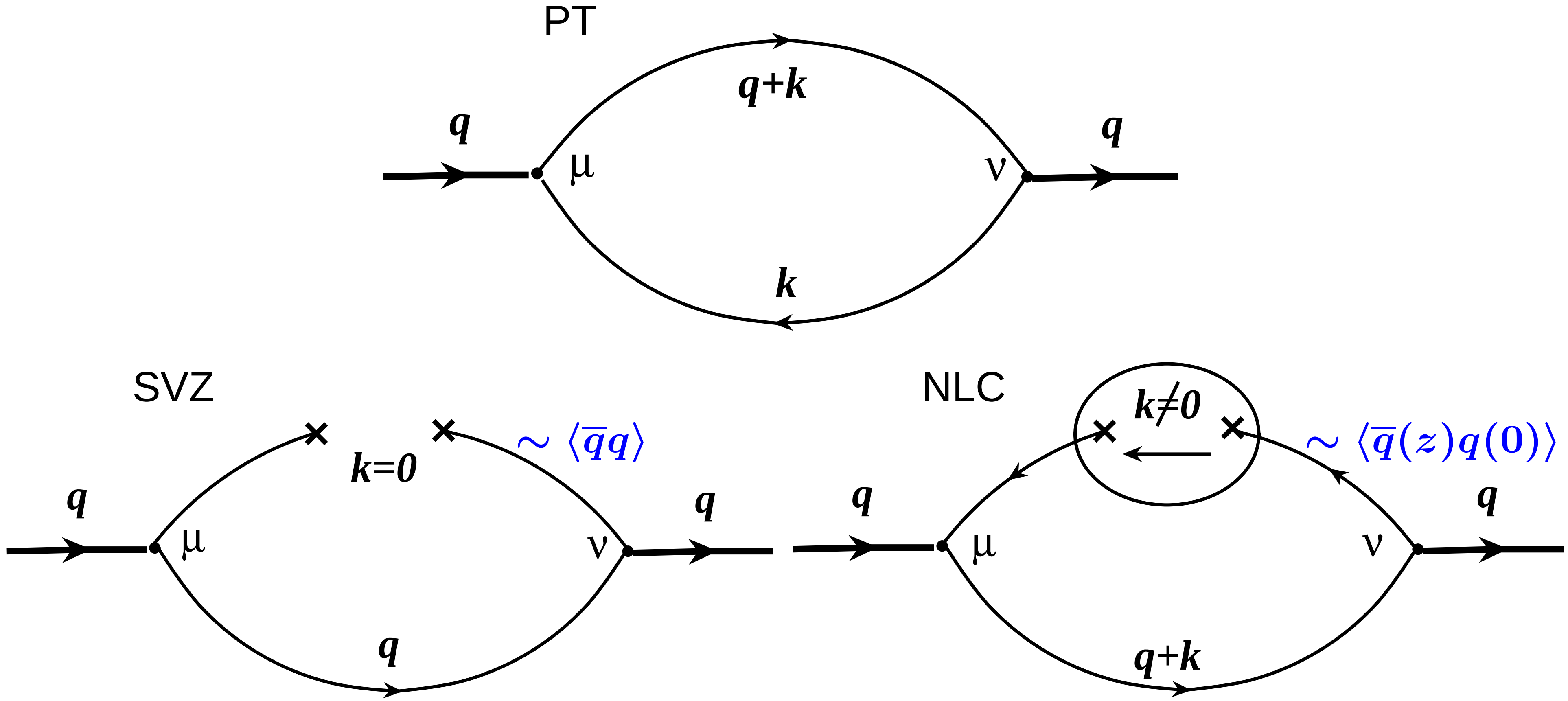
Nonlocal condensate

Distribution Amplitudes (DA),

Form Factors

$$\langle \bar{q}_B(0)\,q_A(x)\rangle = \frac{\delta_{AB}}{4}\left[\langle \bar{q}q\rangle + \frac{x^2}{4}\frac{\langle \bar{q}D^2q\rangle}{2} + \ldots\right] + i\frac{\widehat{x}_{AB}}{4}\frac{x^2}{4}\left[\frac{2\alpha_s\pi\langle \bar{q}q\rangle^2}{81} + \ldots\right].$$

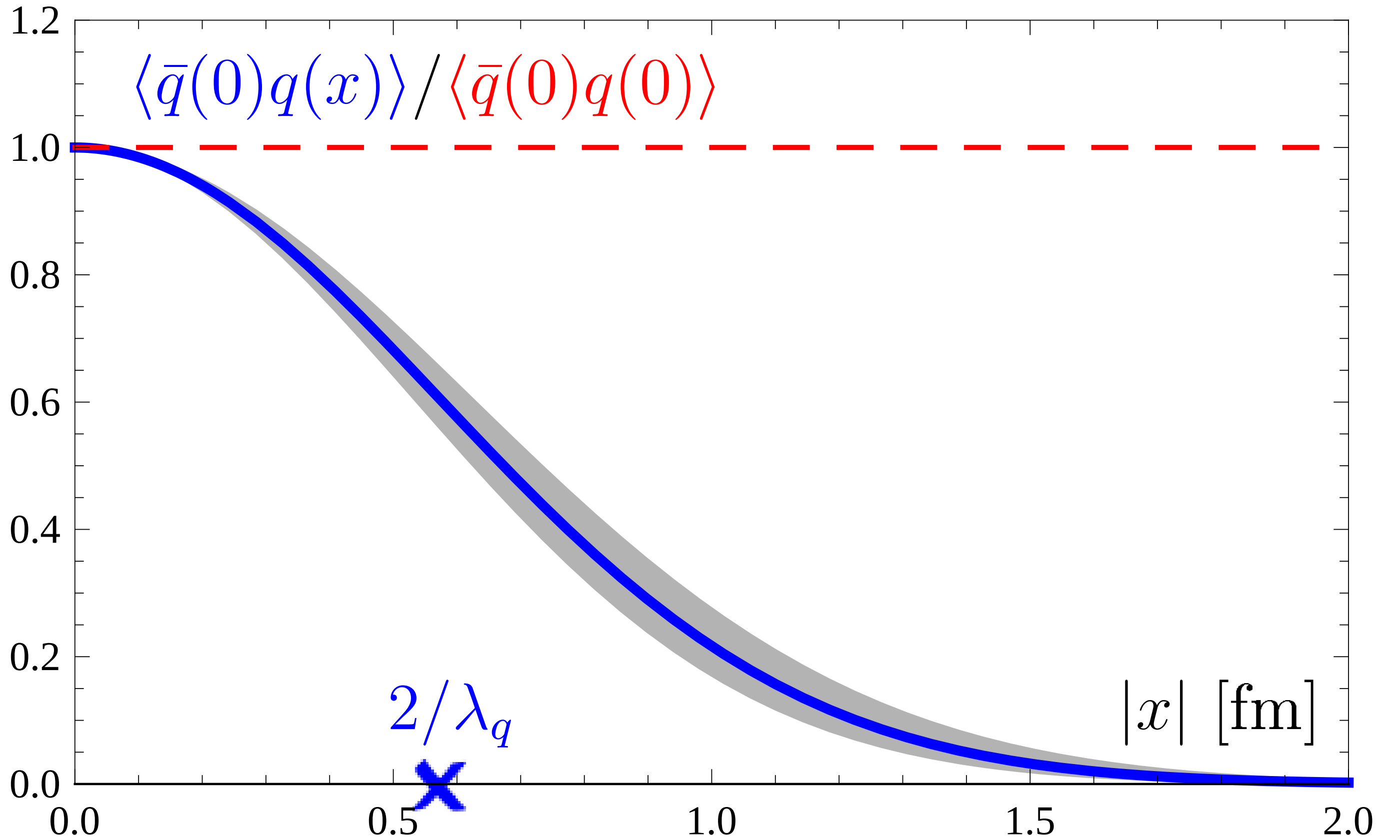
Diagrams for $\langle T\left(J_\nu(z)J_\mu(0)\right)\rangle$



- Quarks run through vacuum with nonzero momentum $k \neq 0$ with average k^2:

$$2\langle k^2 \rangle = \frac{\langle \bar{q} D^2 q \rangle}{\langle \bar{q} q \rangle} = \lambda_q^2 = 0.40(5)\,\text{GeV}^2$$

Nonlocality from Lattice data



(Bakulev&Mikhailov, PRD(2002))

Nonlocality of quark condensates $\lambda_q^2 = 0.42(8)$ GeV^2 from lattice data of Pisa group **(Di Giacomo et.al.,PRD(1999))** in comparison with **local limit**.

- Even at $|z| \simeq 0.5$ fm nonlocality is quite important!

Coordinate dependence of condensates

Parametrization for scalar condensate was suggested in works of Bakulev, Mikhailov and Radyushkin:

$$\langle : \bar{q}_A(0) q_A(x) : \rangle \;=\; \langle \bar{q} q \rangle \int\limits_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,, \text{ where } x^2 < 0.$$

- First approximation which takes into account finite width of quark distribution in vacuum: $f_S(\alpha) = \delta\left(\alpha - \lambda_q^2/2\right),\;\; \lambda_q^2 = \langle \bar{q} D^2 q \rangle / \langle \bar{q} q \rangle$.
- Such representation corresponds to Gaussian form $\sim \mathbf{exp}\left(\lambda_q^2 x^2/8\right)$ of NLC in coordinate representation.
- The heavy-quark effective theory **(Radyushkin 91)** tells us that the scalar condensate decreases exponentially at large distances x^2.
- The **smooth model** $f_S(\alpha) \sim \alpha^{n-1} \mathbf{exp}\left(-\Lambda^2/\alpha - \sigma^2 \alpha\right)$ has a sensible asymptotic form $\langle \bar{q}(0) q(x) \rangle \Big|_{x^2 \to \infty} \sim \mathbf{exp}\left(-\Lambda |x|\right)$ in x-representation. $\Lambda \approx 0.45$ GeV - is the lowest energy level of heavy-light mesons in HQET.
- $\Lambda \approx 0.15 - 0.45$ GeV from Gauge/String Duality **(O. Andreev PRD82)**.

QCD SR for ρ-meson DA

QCD SR technique for correlator of two vector current leads to SR for ρ-DA $\varphi_\rho^L(x)$:

$$f_\rho^2\,\varphi_\rho(x)e^{-m_\rho^2/M^2} + f_{\rho'}^2\,\varphi_{\rho'}(x)\,e^{-m_{\rho'}^2/M^2} = \int_0^{s_0} \rho_{\text{pert}}(s,x)\,e^{-s/M^2}ds + \Phi_\rho(x,M^2)\,,$$

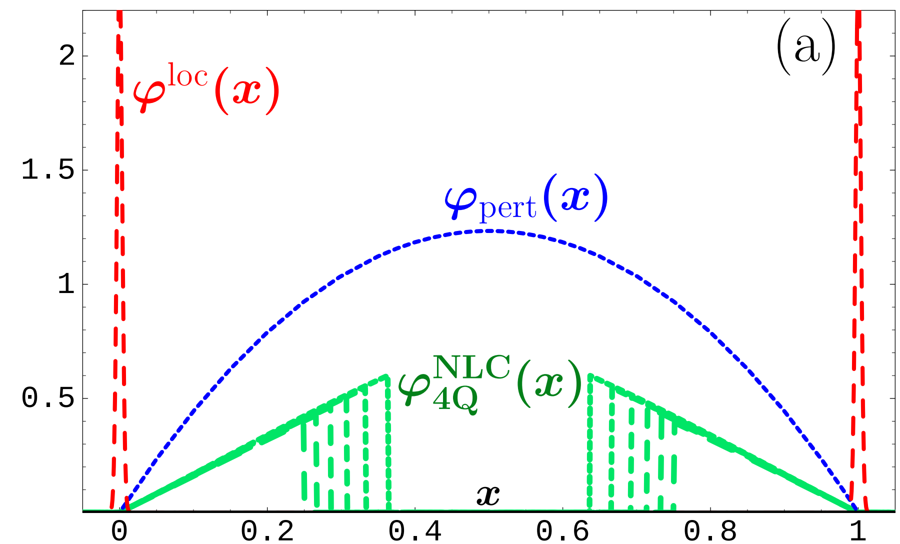
where

$$\Phi_\rho(x,M^2) = -\Phi_{\text{4Q}} + \Phi_{\text{T}} + \Phi_{\text{V}} + \Phi_{\text{G}}\,,$$
$$\Phi_\pi(x,M^2) = +\Phi_{\text{4Q}} + \Phi_{\text{T}} + \Phi_{\text{V}} + \Phi_{\text{G}}\,.$$

The largest nonperturbative term:

$$\Phi_{\text{4Q}} \sim x\theta(\Delta - x) \xrightarrow{\text{loc. lim}} \Phi_{\text{4Q}}^{\text{loc}} \sim \delta(x)\,,$$

is defined by scalar quark condensate, where $\Delta = \lambda_q^2/M^2 \in [0.1, 0.3]$.



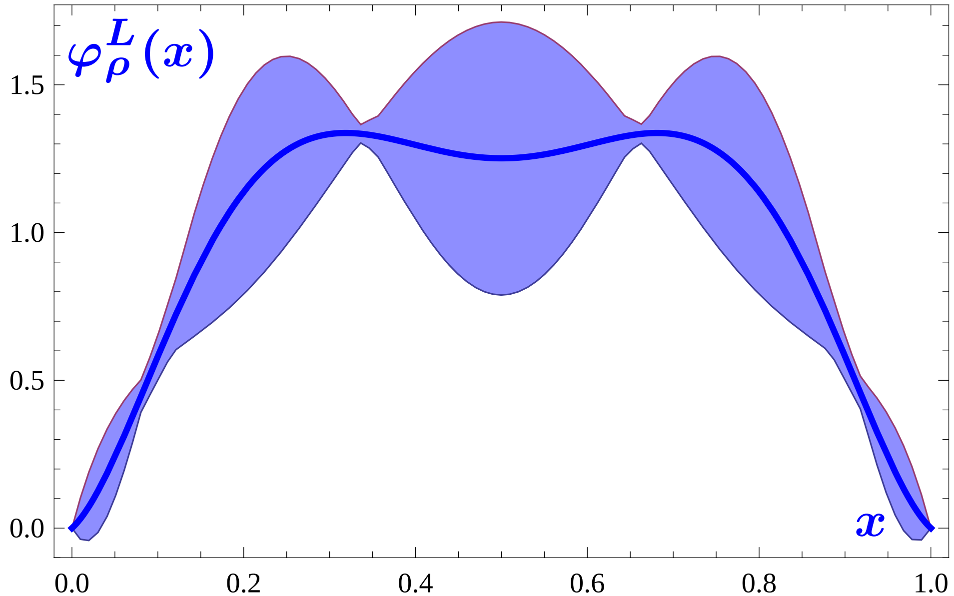
- Nonperturbative contribution has **singularities**.

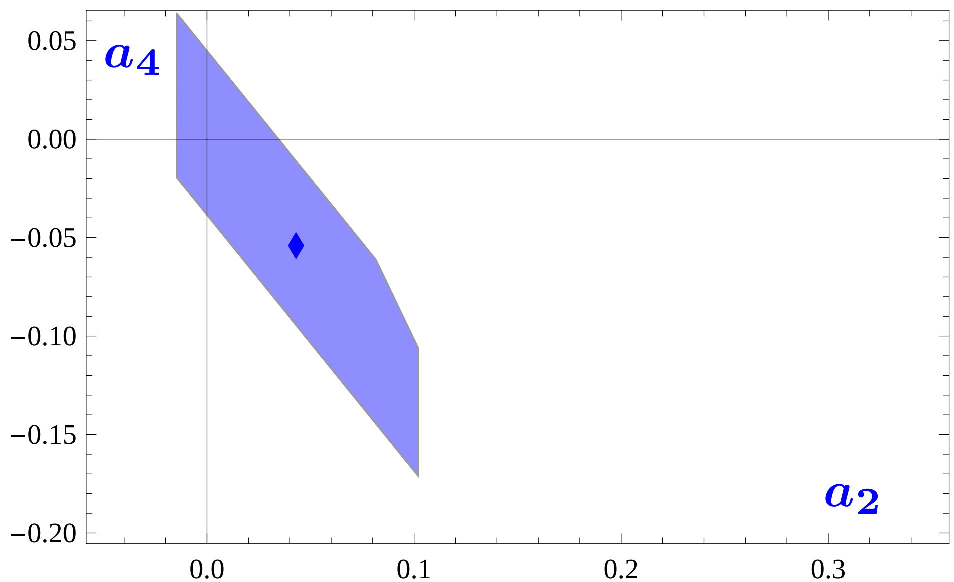
SRs for integral characteristics ($\int_0^1 dx\,\varphi(x)(1-2x)^N$, $\int_0^1 dx\,\varphi(x)/x$) take into account all condensates and have less model dependence.

- In end-point region only 4-quark condensate Φ_{4Q} contributes without any singularities. That allows us to study slope $\varphi'(0)$ and $\int_0^1 dx\,\varphi(x)/x$.

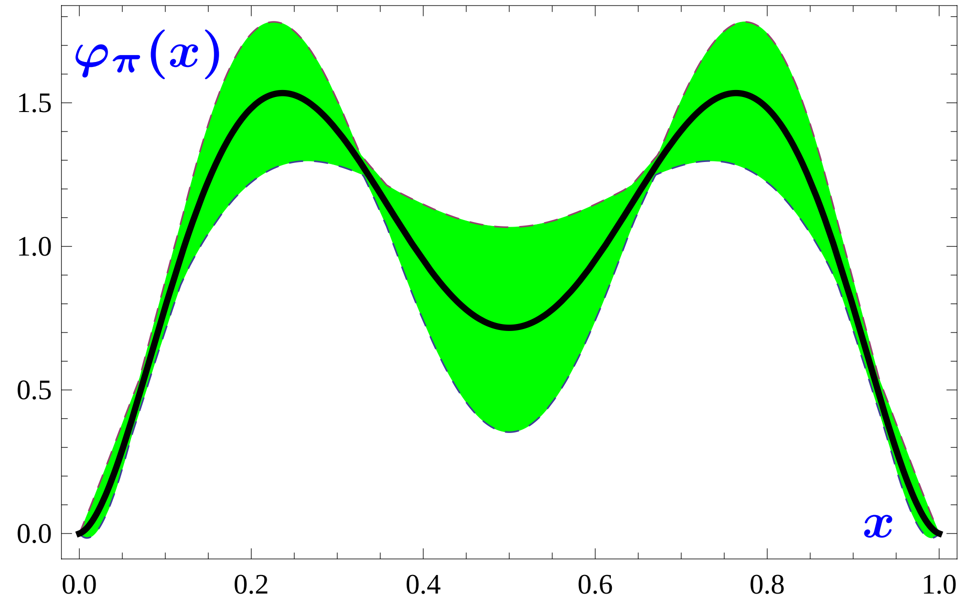
ρ vs π DA$_{\mu=1\ GeV}$ from NLC QCD SR

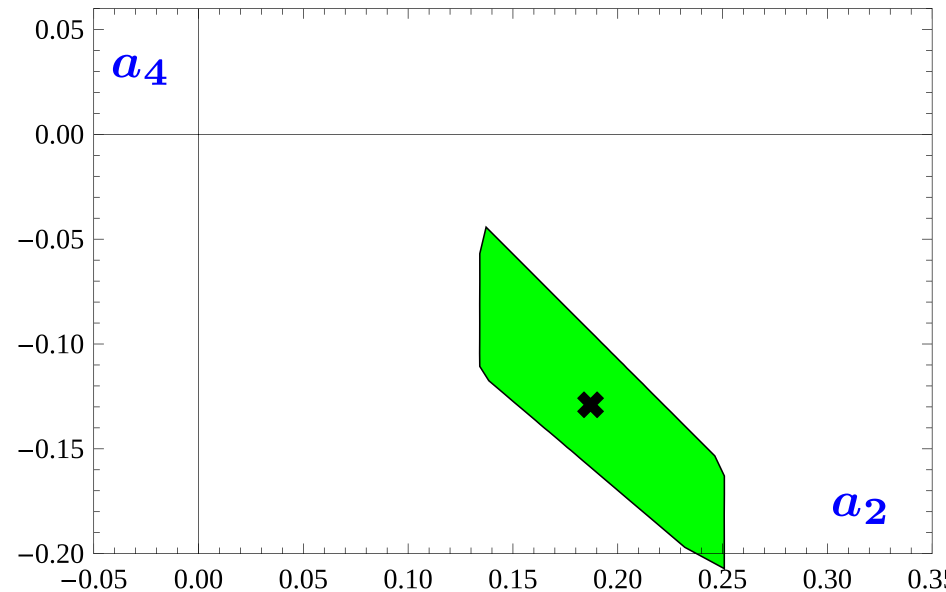
ρ (here)



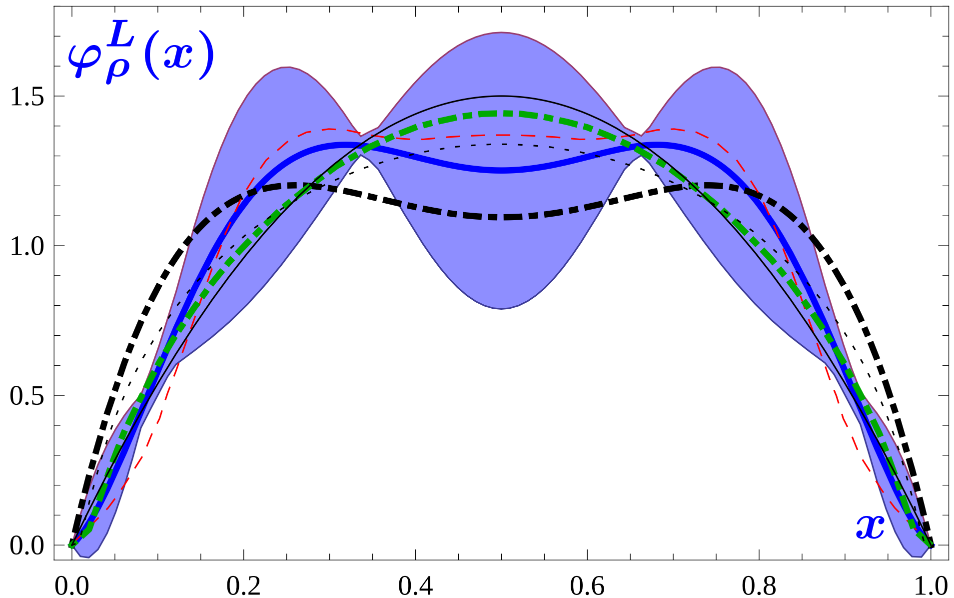


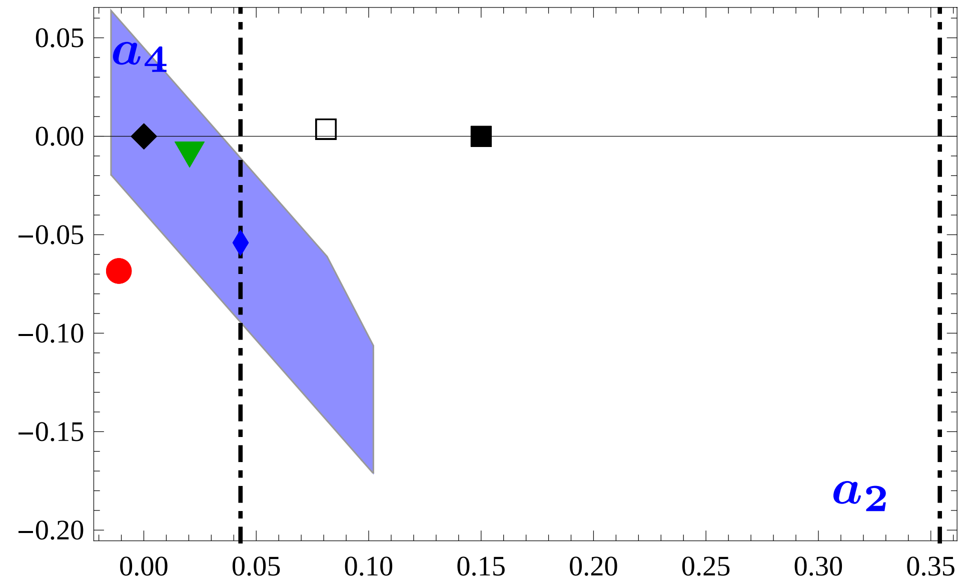
π (BMS,Bakulev et.al., PLB508(2001))

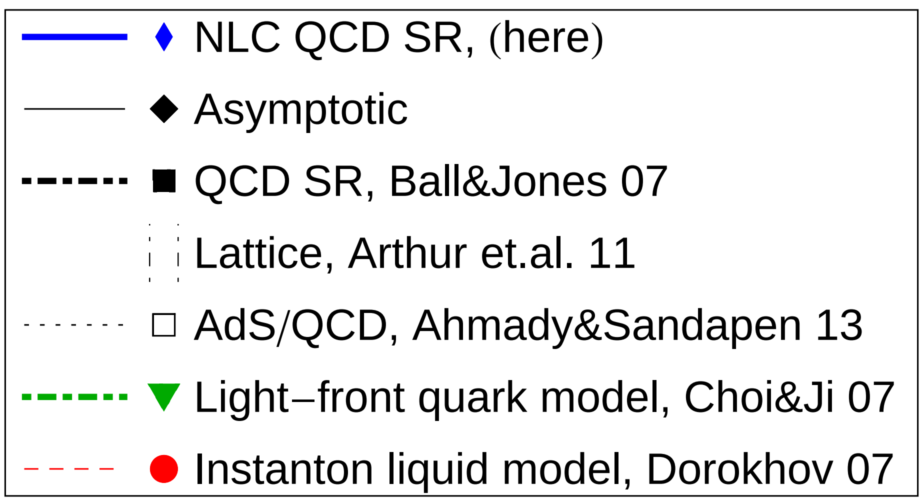




φ_ρ^L DA from QCD SR with NLC







$\mu = 1\ GeV$

- DA model and bunch were obtained using Gaussian condensate model with single nonlocality parameter $\lambda_q^2 = 0.4\,\text{GeV}^2$.
- Higher Gegenbauer coefficients $a_{n\geq 6} = 0$ are assumed to be equal 0 — this does not contradict QCD SR.

SR for transverse DA $\varphi_\rho^{\mathrm{T}}$

$$\Pi_N^{\mu\nu;\alpha\beta}(q) \equiv \int d^4x\, e^{iqx} \langle 0 | T(J_{\mu\nu}^N(x) J_{\alpha\beta}^0(0)) | 0 \rangle = \sum_{i=1}^{6} C_i P_i^{\mu\nu;\alpha\beta}$$

$$J_{\mu\nu}^N(x) \equiv \bar{u}(x) \sigma_{\mu\nu} (z\nabla)^N d(x)$$

Mixed SR, Ball&Braun 96: $\rho + b_1 \sim \Pi_N^{\mu\nu;\alpha\beta}(q) g_{\mu\alpha} n_\nu n_\beta \sim C_1 - C_2$

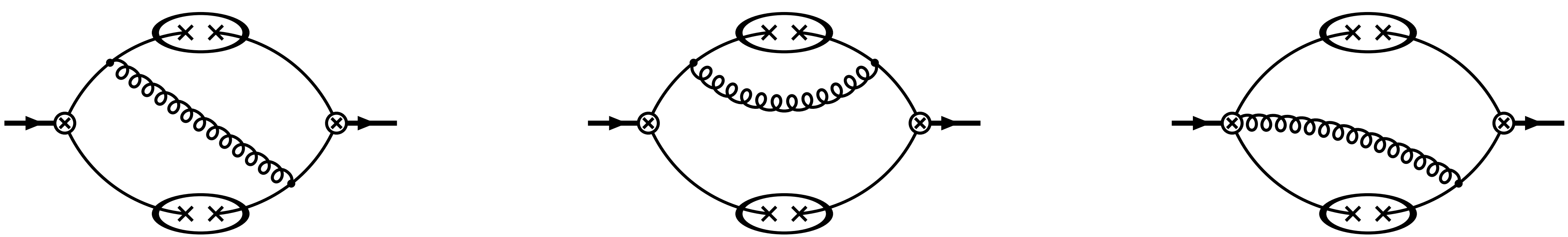
$\rho +$ higher twists $\sim C_1$

$b_1 +$ higher twists $\sim C_2$

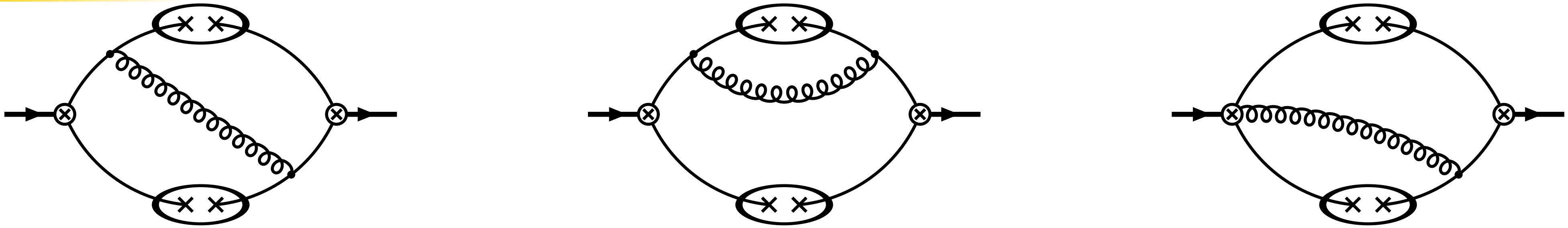
Pure SR, Bakulev&Mikhailov 01: $\rho \sim C_1 + C_4 = \frac{1}{2}(C_1 - C_2) + \Delta_{4\mathrm{Q}}\Phi$

$b_1 \sim -(C_2 + C_4) = \frac{1}{2}(C_1 - C_2) - \Delta_{4\mathrm{Q}}\Phi$

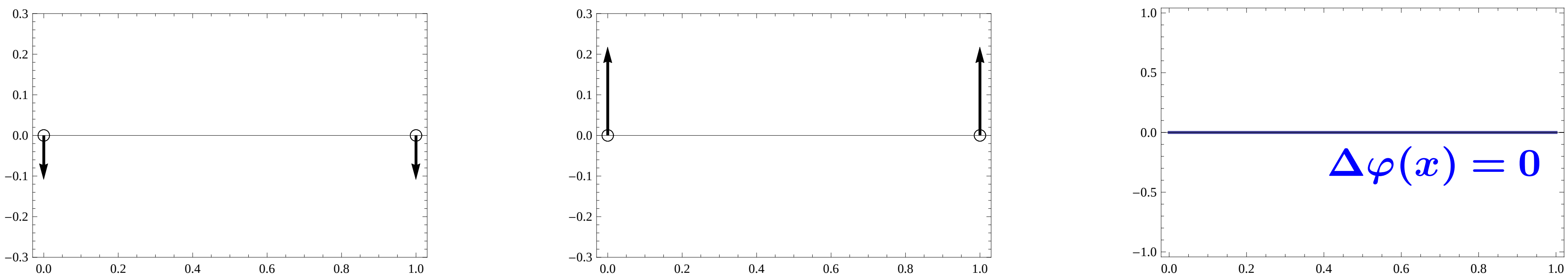
Where $\Delta_{4\mathrm{Q}}\Phi$ is total 4-quark condensate contribution defined by following diagrams:



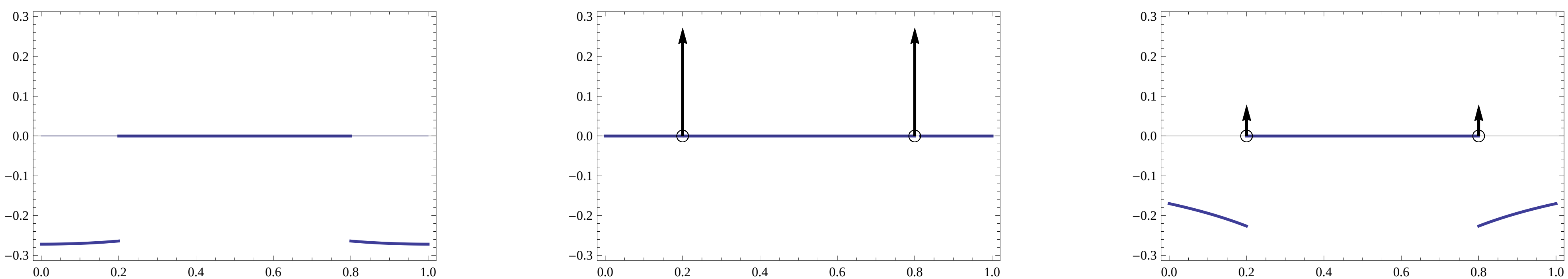
4-quark condensate terms to SR for φ_ρ^T-DA



Local Condensates ($\lambda_q^2 \to 0$)



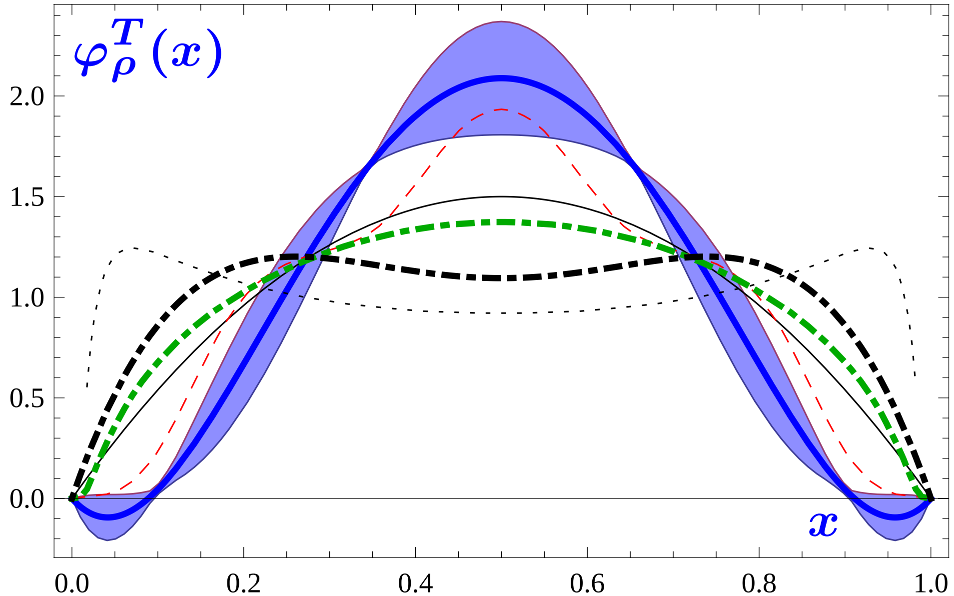
NonLocal Condensates ($\lambda_q^2 = 0.4$ GeV^2)

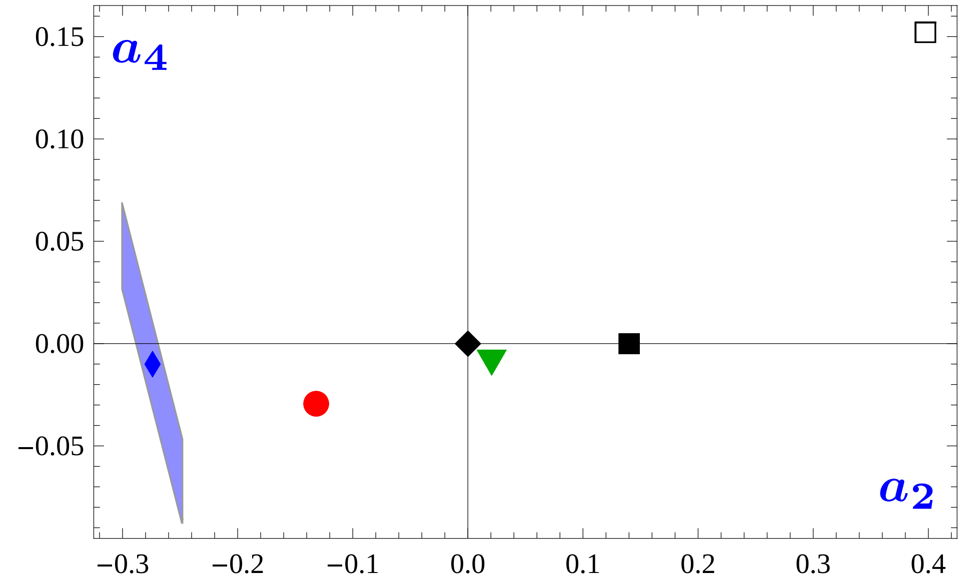


For typical value of Borel parameter $M^2 = 1$GeV^2

φ_ρ^T DA from QCD SR with NLC

Preliminary result:





- ♦ NLC QCD SR, (here)
- ◆ Asymptotic
- ■ QCD SR, Ball&Jones 07
- □ AdS/QCD, Ahmady&Sandapen 13
- ▼ Light–front quark model, Choi&Ji 07
- ● Instanton liquid model, Dorokhov 07

- We take into account only first two Gegenbauer harmonics.
- The inclusion of higher harmonics is not reliable for this case.

Parameters of ρ and π

M	f_M	a_2	a_4	$\varphi'(0)$	$\int_0^1 dx \varphi(x)/x$
Asy	1	0	0	6	3
π (BMS)	$0.137(8)$	$0.187(60)$	$-0.129(40)$	2 ± 6	$3.2(1)$
ρ_L (here)	$0.21(2)$	$0.04(6)$	$-0.05(12)$	3 ± 8	$3.0(2)$
ρ_T	$0.167(15)$	$-0.274(27)$	$-0.01(8)$	-5 ± 6	$2.2(2)$

$\mu{=}1\ GeV$

Conclusions

- Using QCD SR with nonlocal condensate we recalculated the leading twist longitudinal and transverse ρ-meson DAs.
- The longitudinal ρ-meson DA was found to be close to AdS/QCD result and the asymptotic form.
- Calculated 4-quark condensate contribution to transverse ρ-meson DA appeared to be non-zero in the end-point x-region.
- While our preliminary result on transverse ρ-meson DA is controversial, integral characteristics are compatible with other approaches.