
Photo: courtesy of Skiathos Palace

Lightcone 2013+

Venturing off the lightcone • local vs. global features

Topics

Wilson lines off the lightcone • Lightcone field theory • AdS/CFT theory and applications
Hadron structure in QCD • Effective theories • Lattice simulations

May 20-24, 2013, Skiathos, Greece

Wigner Distributions
in
Light-Cone Quark Models

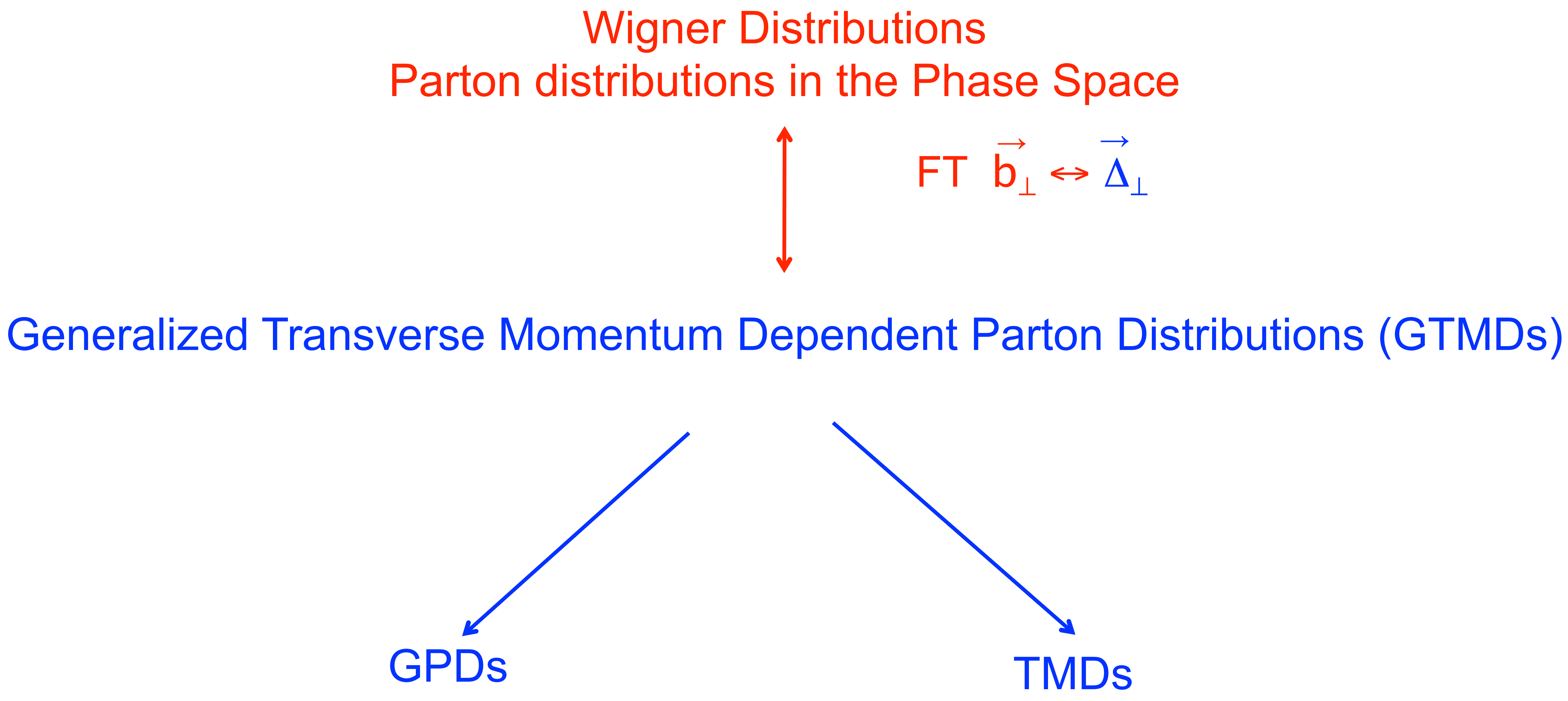
Barbara Pasquini

Pavia U. & INFN, Pavia (Italy)

INFN
Istituto Nazionale
di Fisica Nucleare

UNIVERSITÀ
DI PAVIA

Outline



Wigner Distributions
Parton distributions in the Phase Space

FT $\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$

Generalized Transverse Momentum Dependent Parton Distributions (GTMDs)

GPDs

TMDs

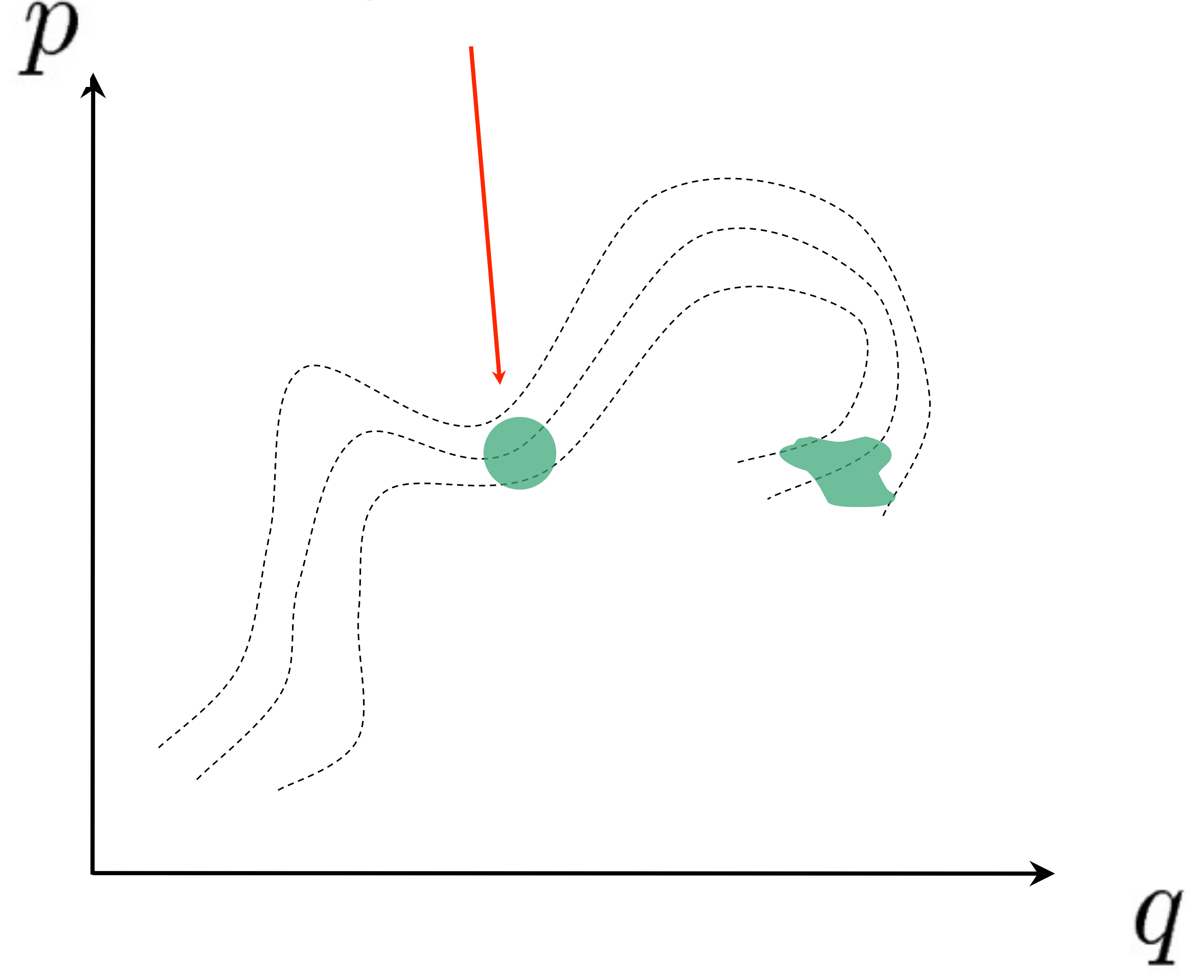
spin and orbital angular momentum structure of the nucleon

insights from quark model calculations

Phase-space distribution

Quantum Mechanics

[Wigner (1932)]
[Moyal (1949)]



Wigner distribution

$$P_W(p,q) = \int \frac{\mathrm{d}z}{2\pi}\, e^{-ipz}\, \psi^*(q-\tfrac{z}{2})\, \psi(q+\tfrac{z}{2})$$

$$= \int \frac{\mathrm{d}\Delta}{(2\pi)^2}\, e^{-iq\Delta}\, \varphi^*(p+\tfrac{\Delta}{2})\, \varphi(p-\tfrac{\Delta}{2})$$

Position-space density

$$|\psi(q)|^2 = \int \mathrm{d}p\, P_W(p,q)$$

Momentum-space density

$$|\varphi(p)|^2 = 2\pi \int \mathrm{d}q\, P_W(p,q)$$

Quantum average

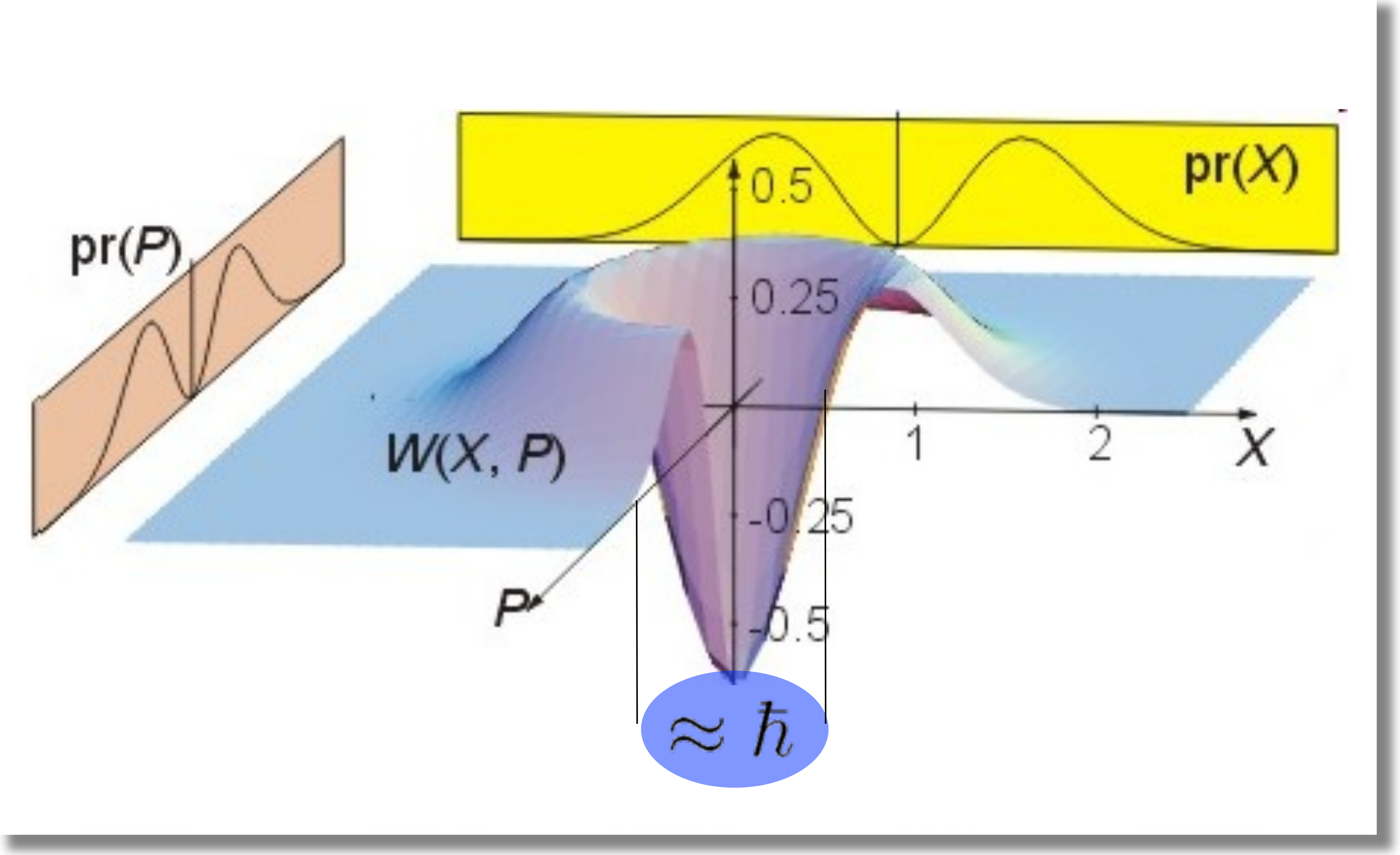
$$\langle \hat{O} \rangle = \int \mathrm{d}q\, \mathrm{d}q\, O(p,q)\, P_W(p,q)$$

Phase-space distribution

Wigner distribution

Numerous applications in

- Nuclear physics
- Quantum chemistry
- Quantum molecular dynamics
- Quantum information
- Quantum optics
- Classical optics
- Signal analysis
- Image processing
- Heavy ion collisions
- ...



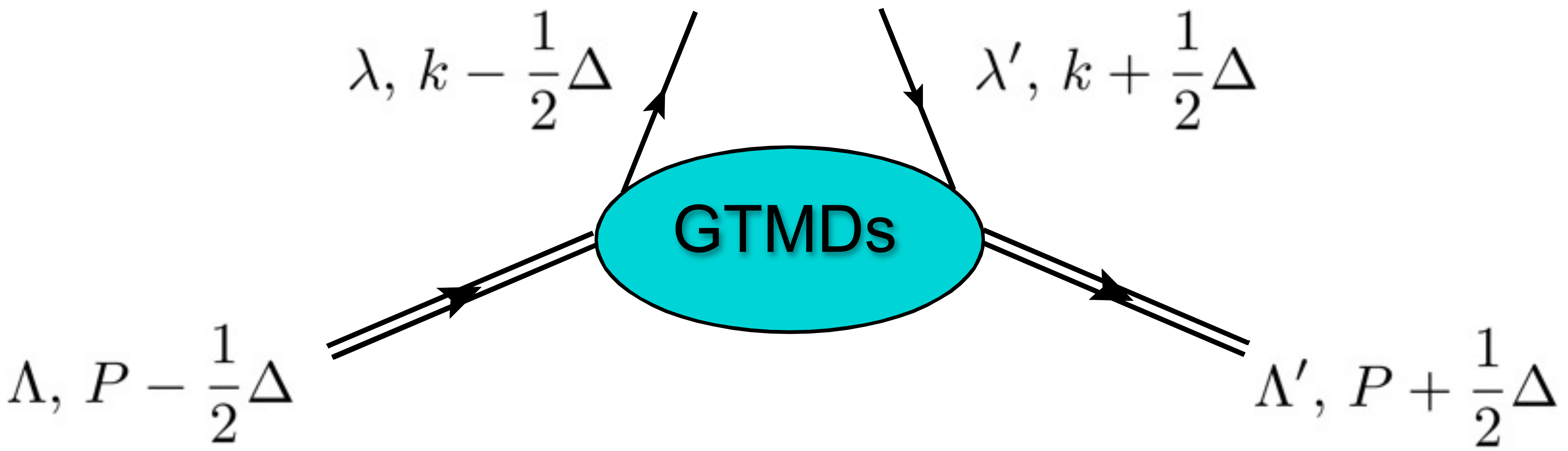
[Antonov et al. (1980-1989)]

Heisenberg's uncertainty relation

→ **Quasi-probabilistic interpretation** $\xrightarrow[\hbar \to 0]{}$ **classical density**

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P',\Lambda'|\bar\psi^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}$$

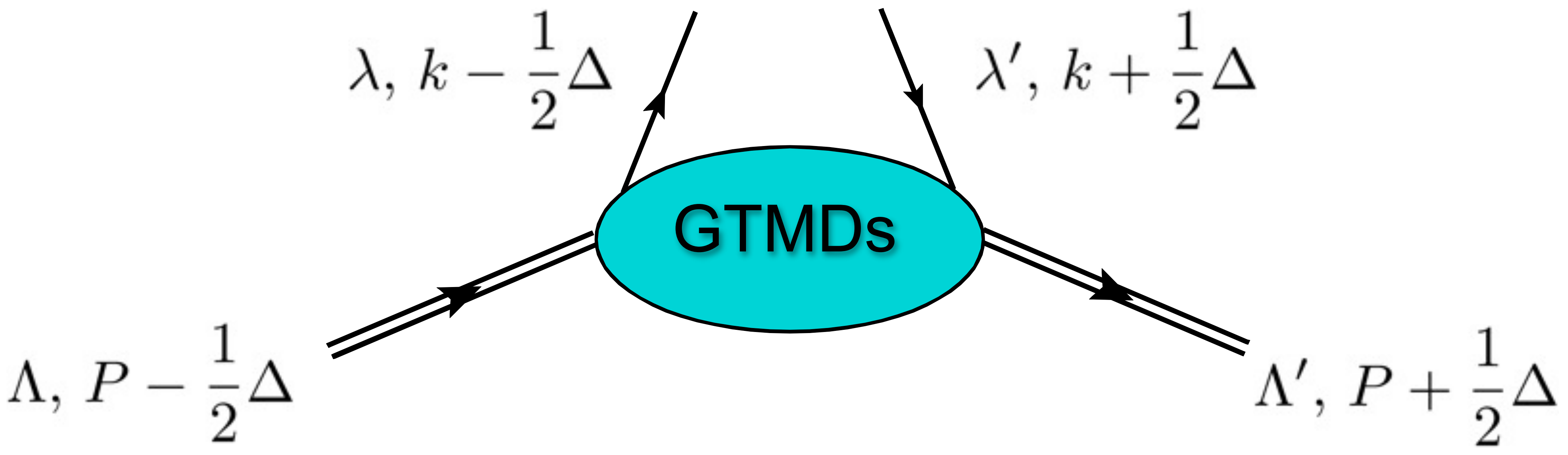
Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+ z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

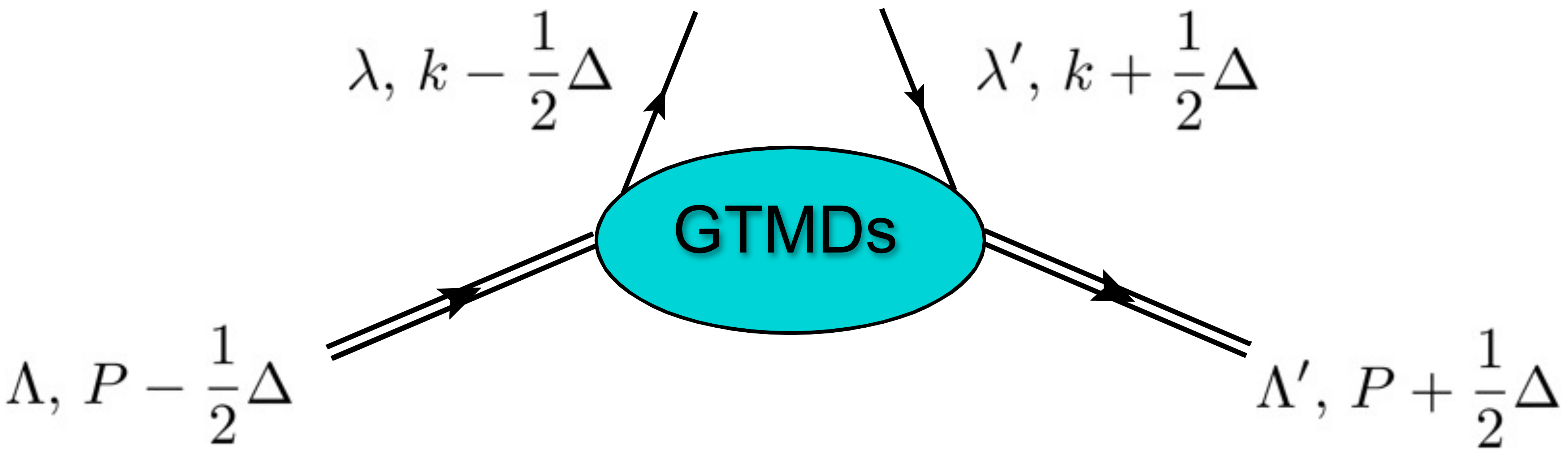
ξ: fraction of longitudinal momentum transfer

$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse-momentum transfer

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

$$\Big\downarrow \quad \text{Fourier transform } \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$$

$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp)$ 16 real Wigner distributions

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

●

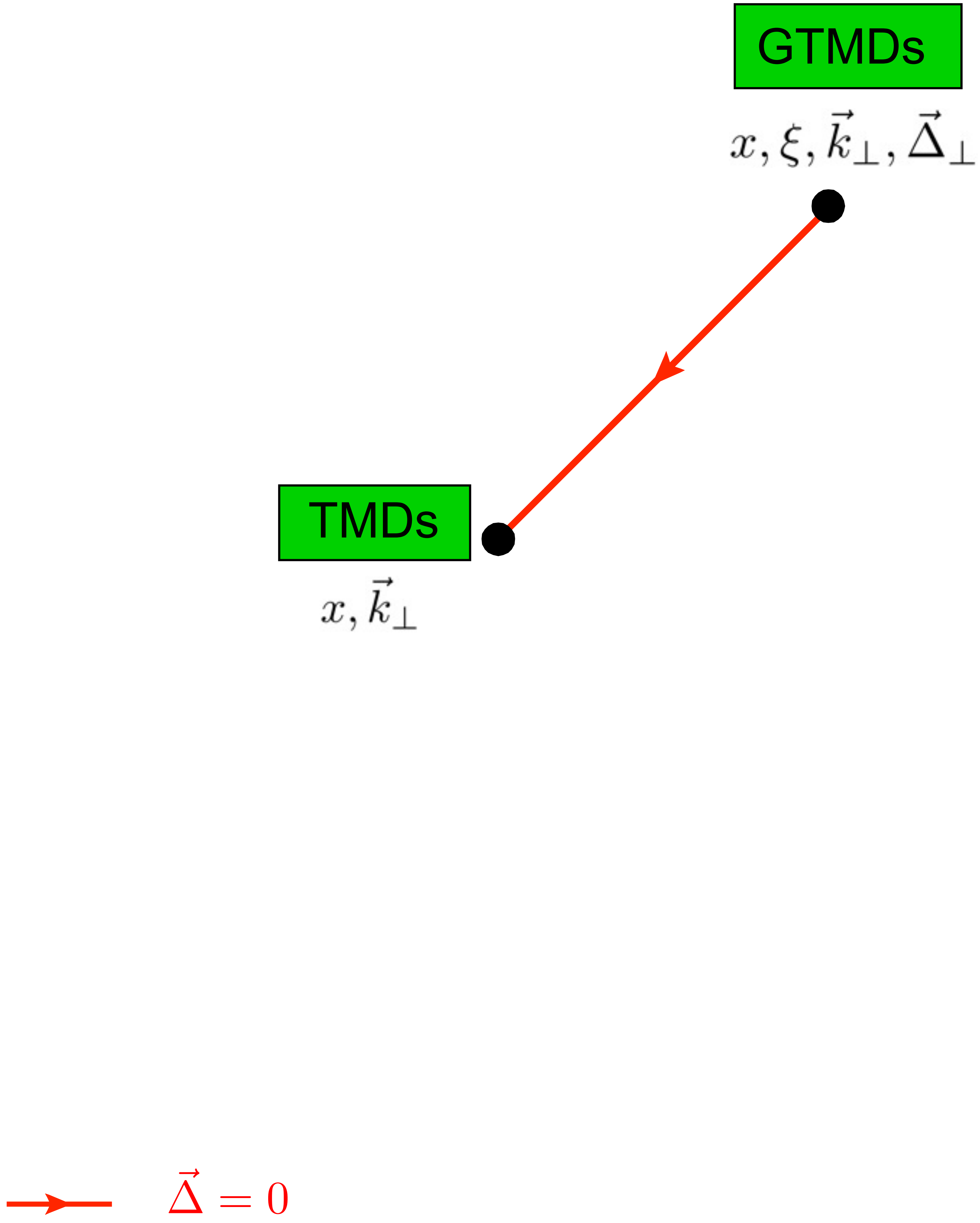
2D Fourier
transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

$\vec{\Delta} = 0$

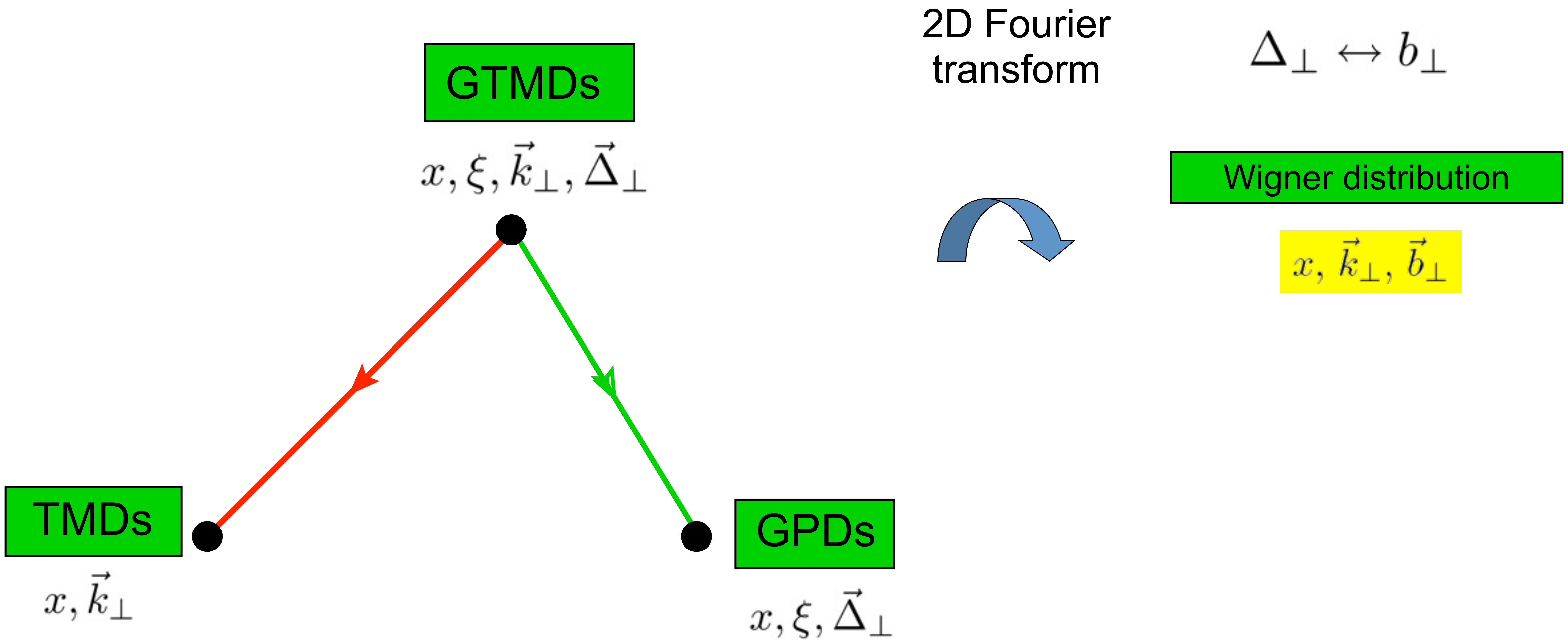
2D Fourier
transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

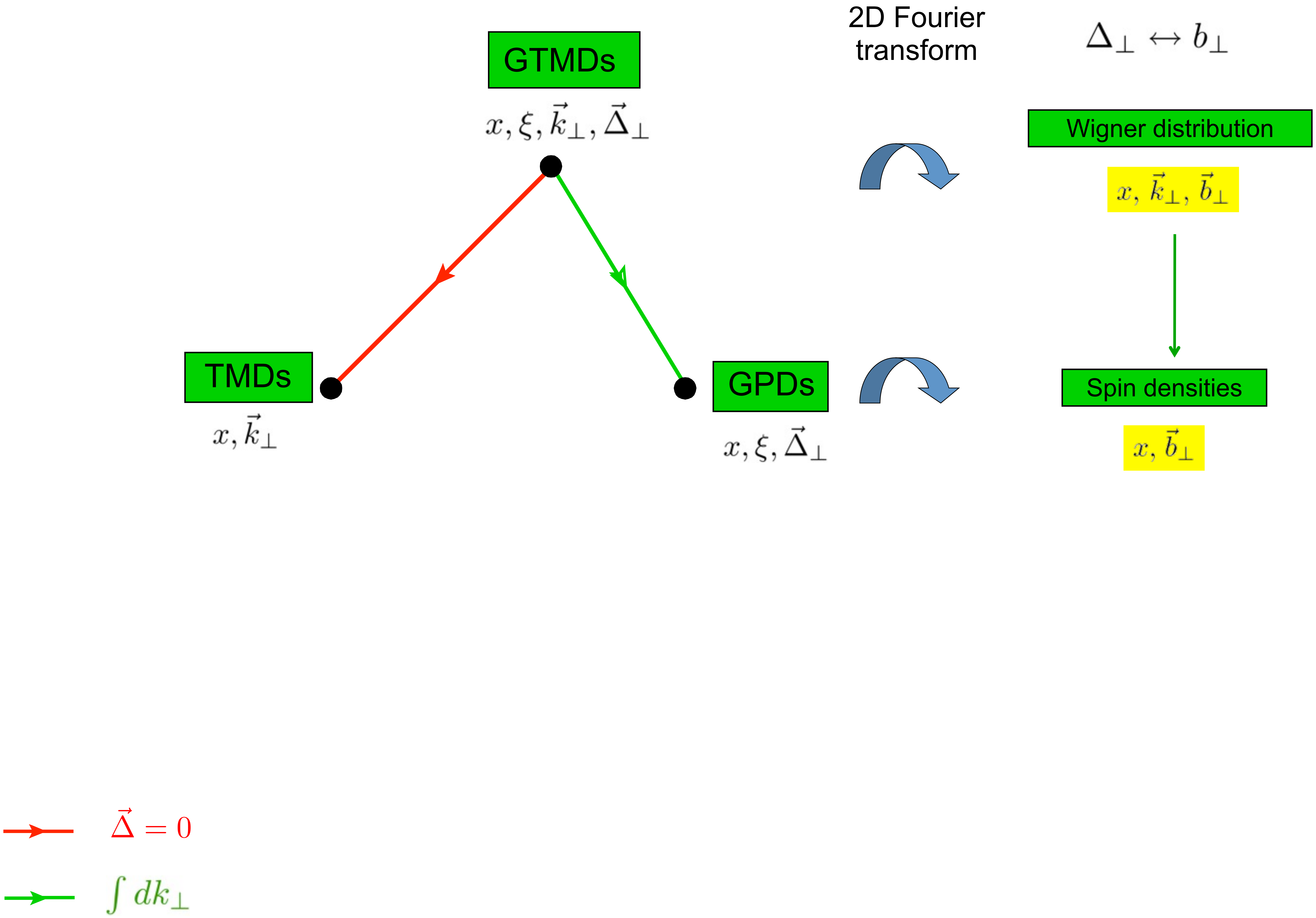
Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

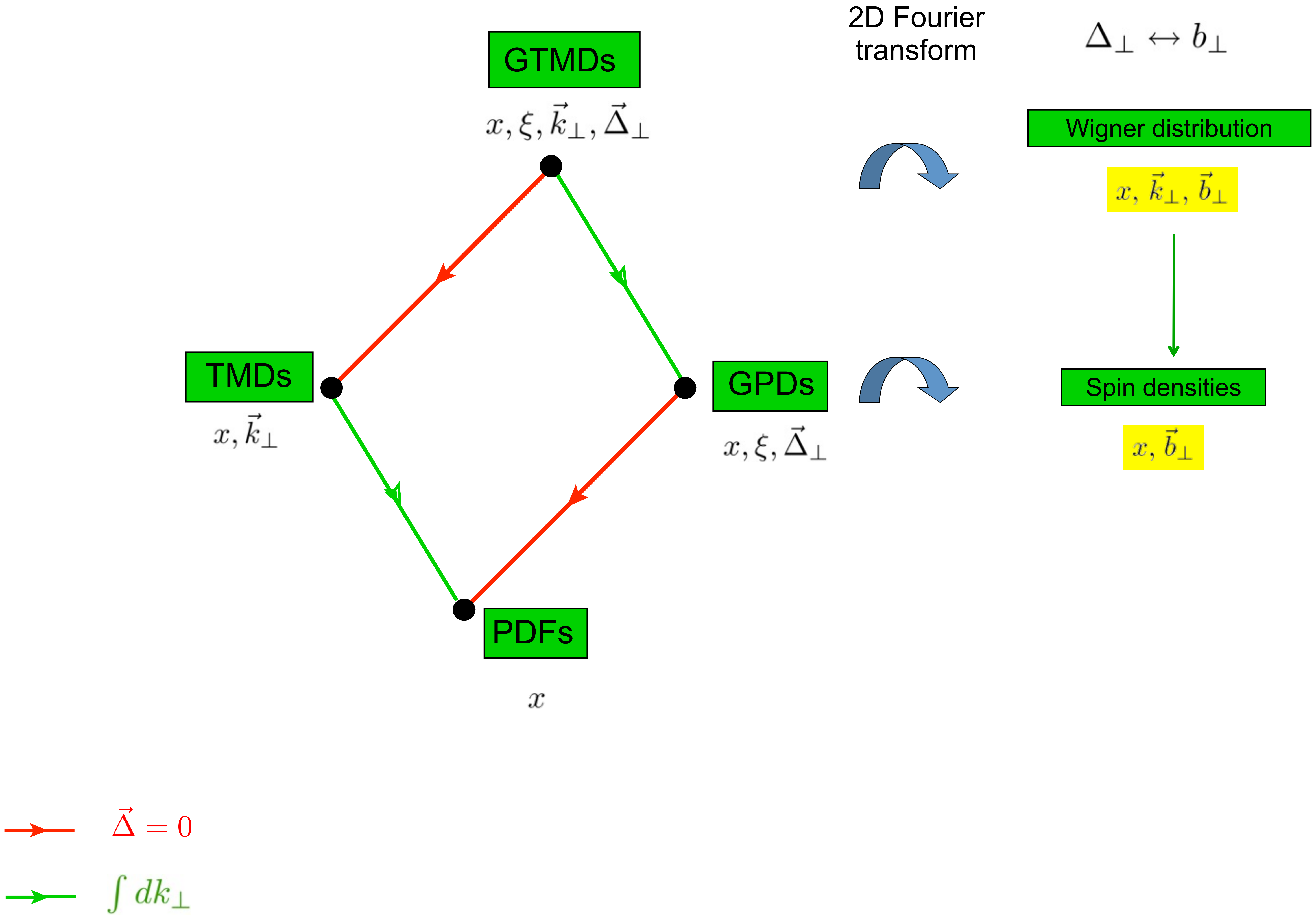
Spin densities

$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

Spin densities

$x, \vec{b}_\perp$

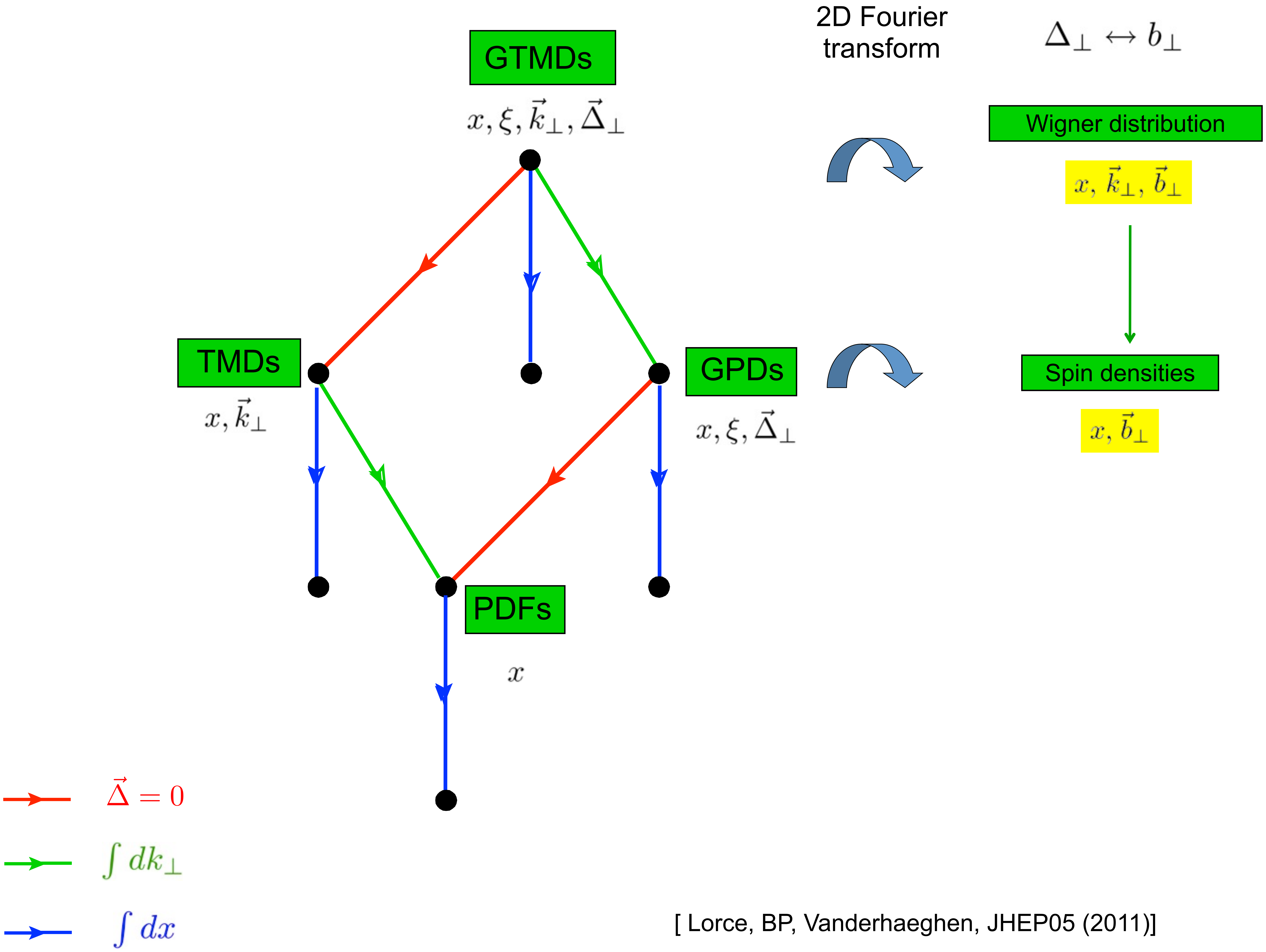
PDFs

x

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

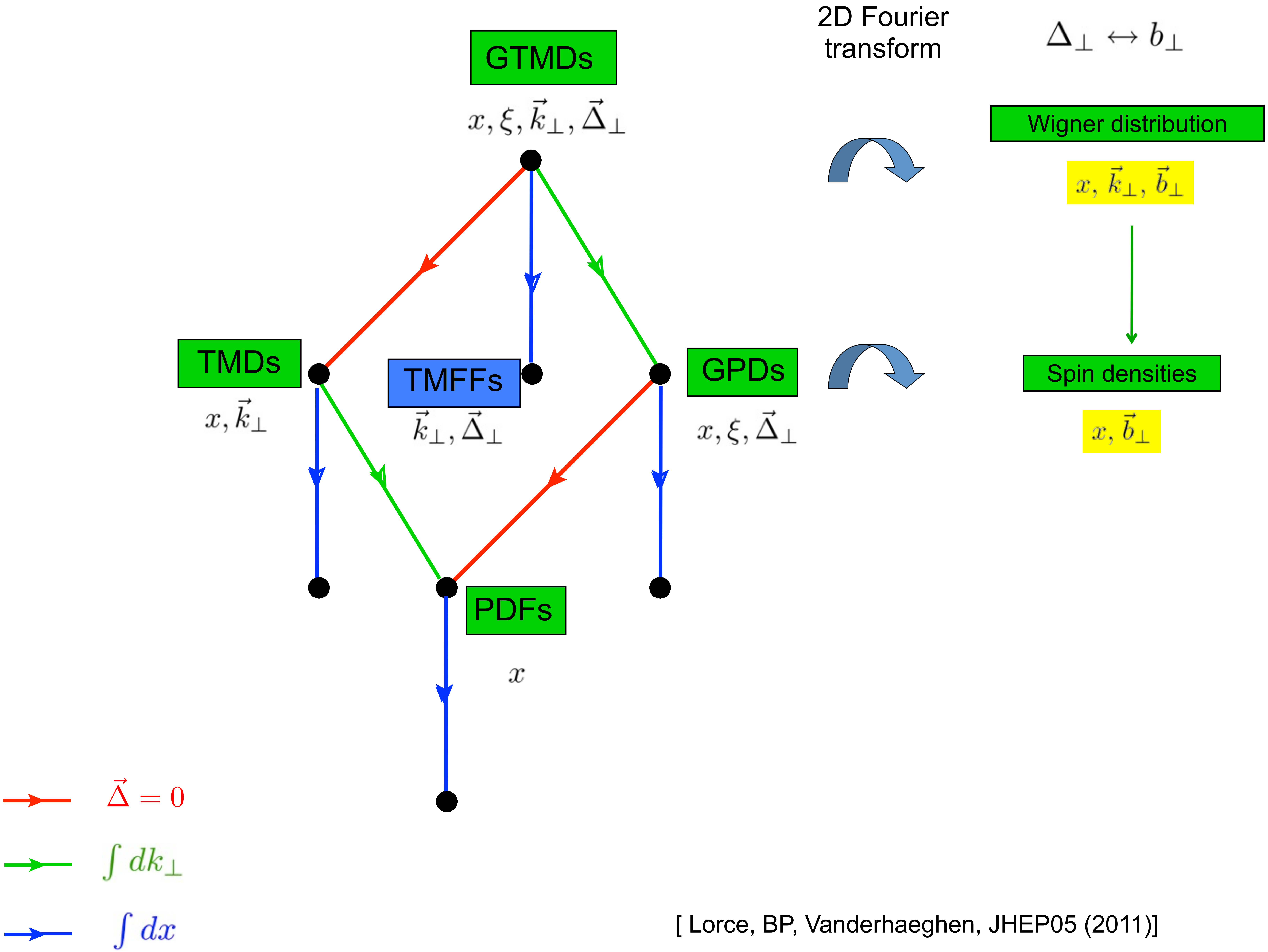
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

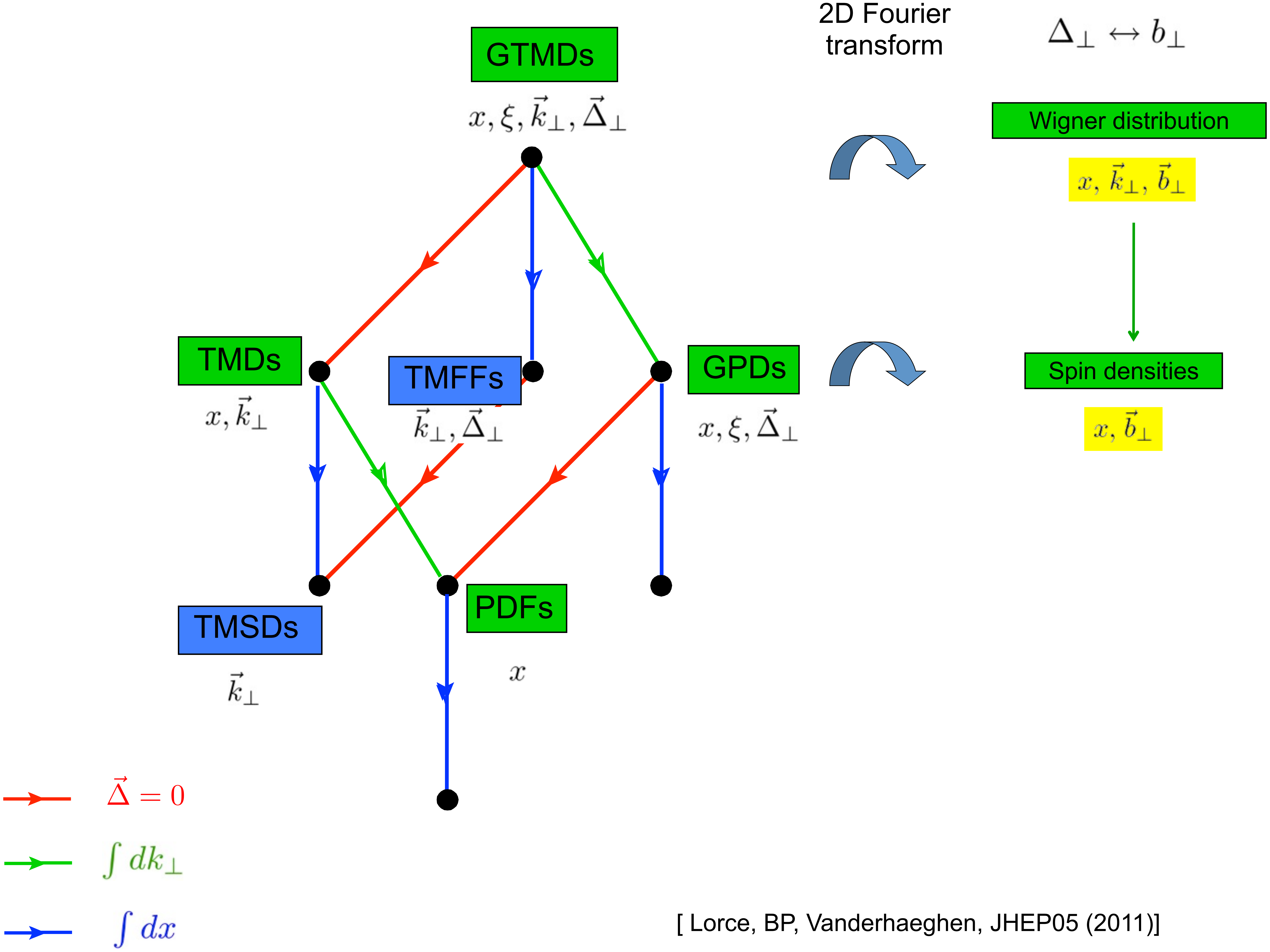
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform $\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

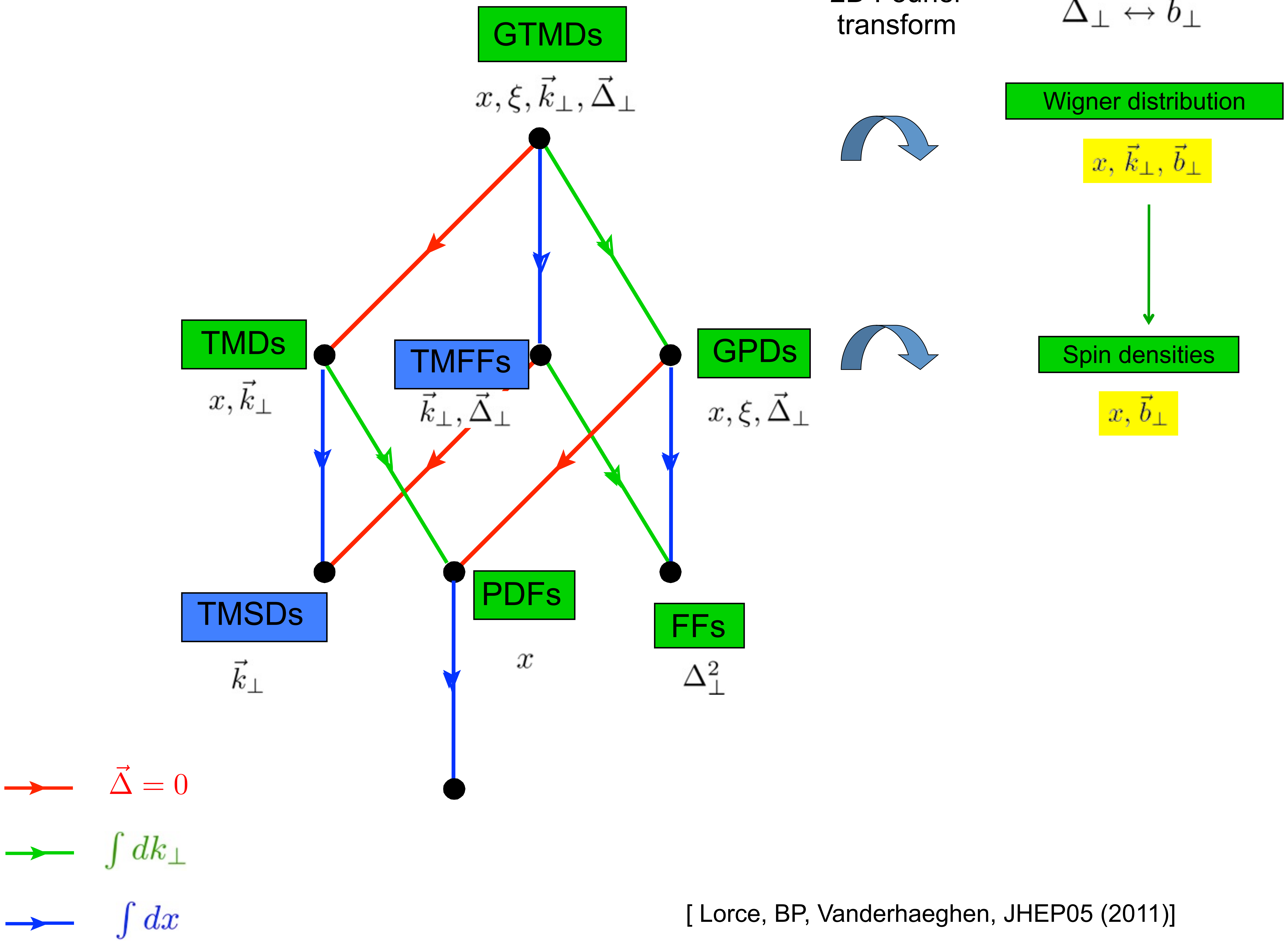
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

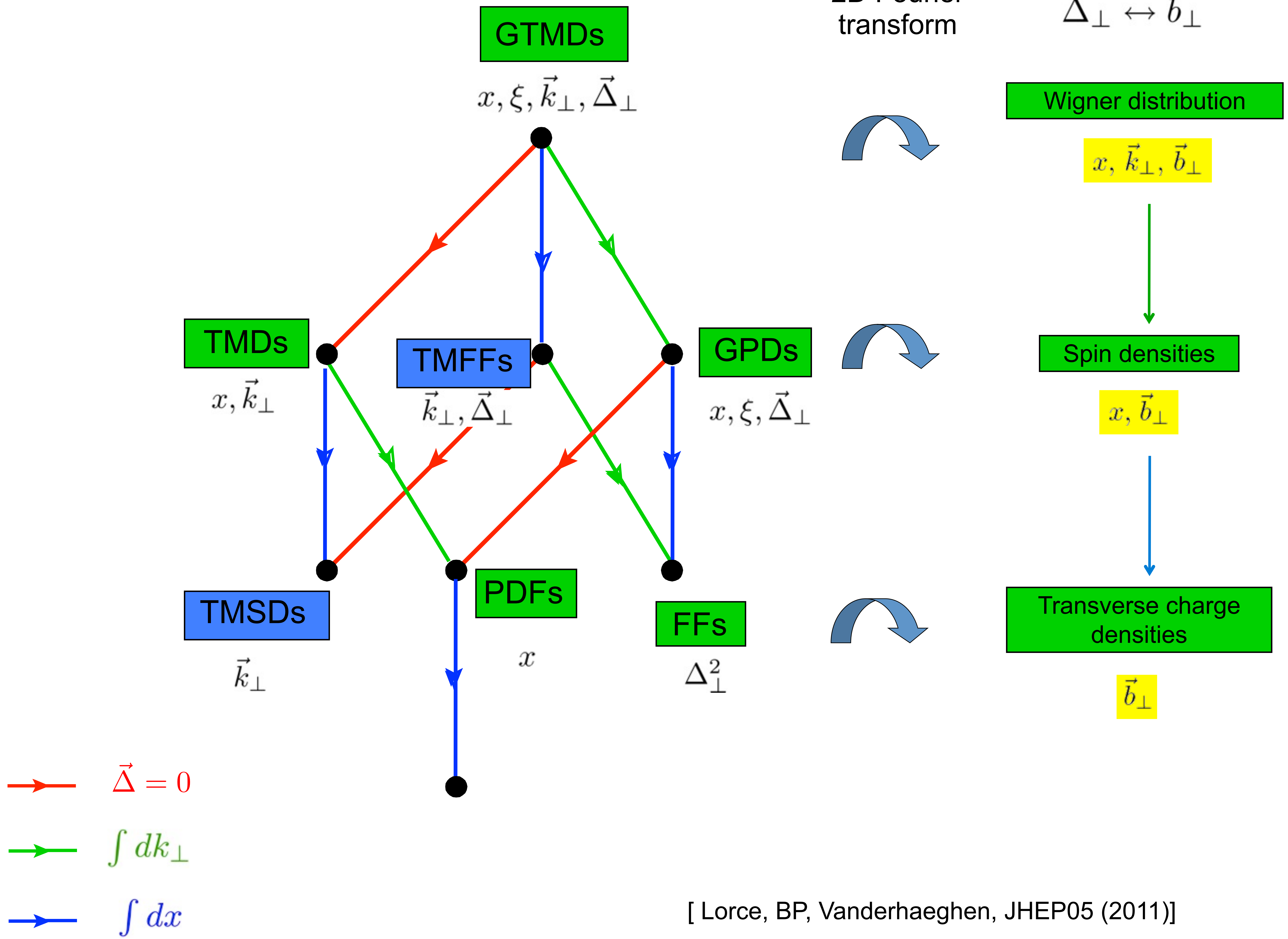
$x, \vec{b}_\perp$

🟥 $\vec{\Delta} = 0$

🟩 $\int dk_\perp$

🟦 $\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

$x, \vec{b}_\perp$

Transverse charge densities

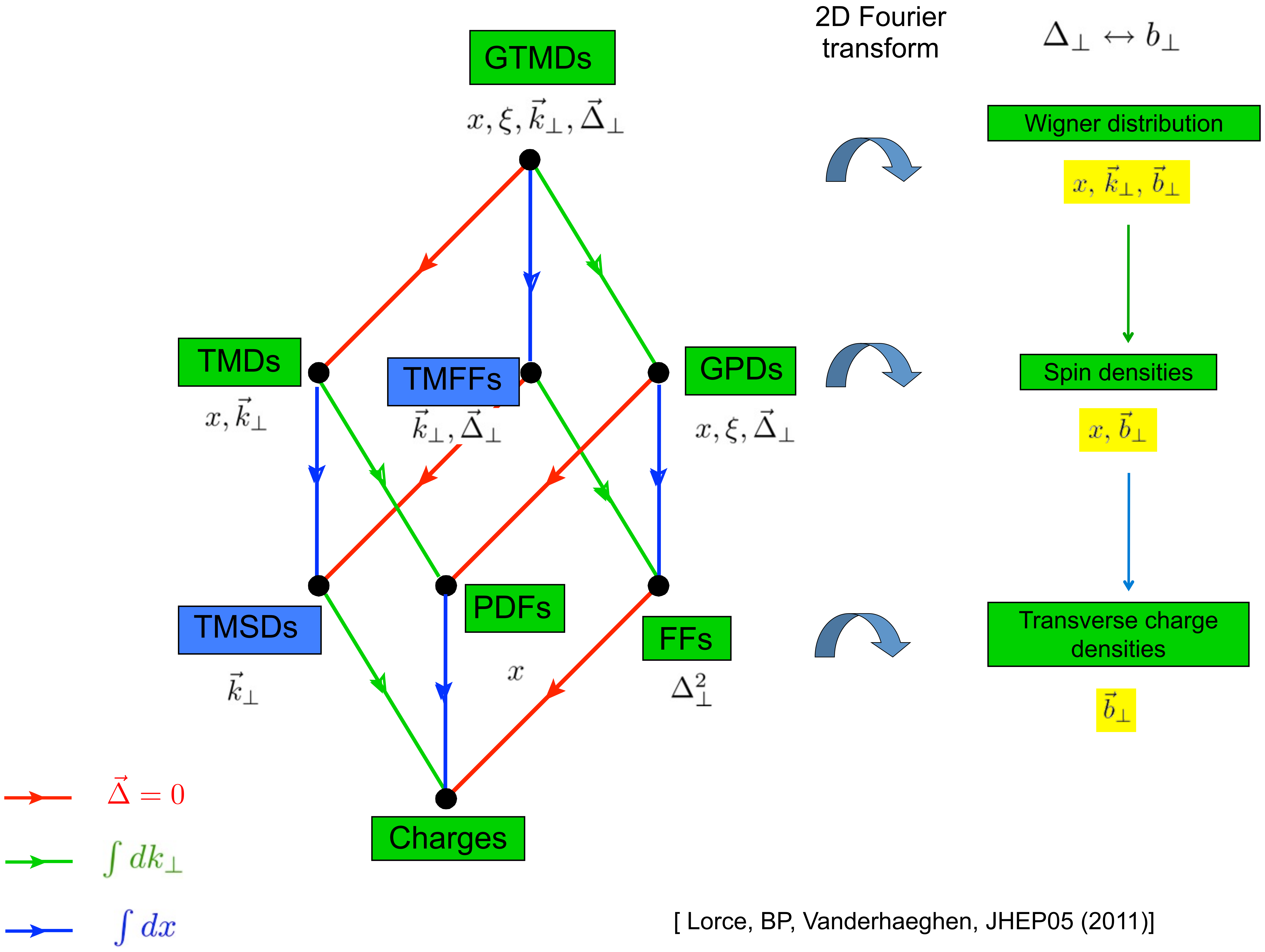
$\vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

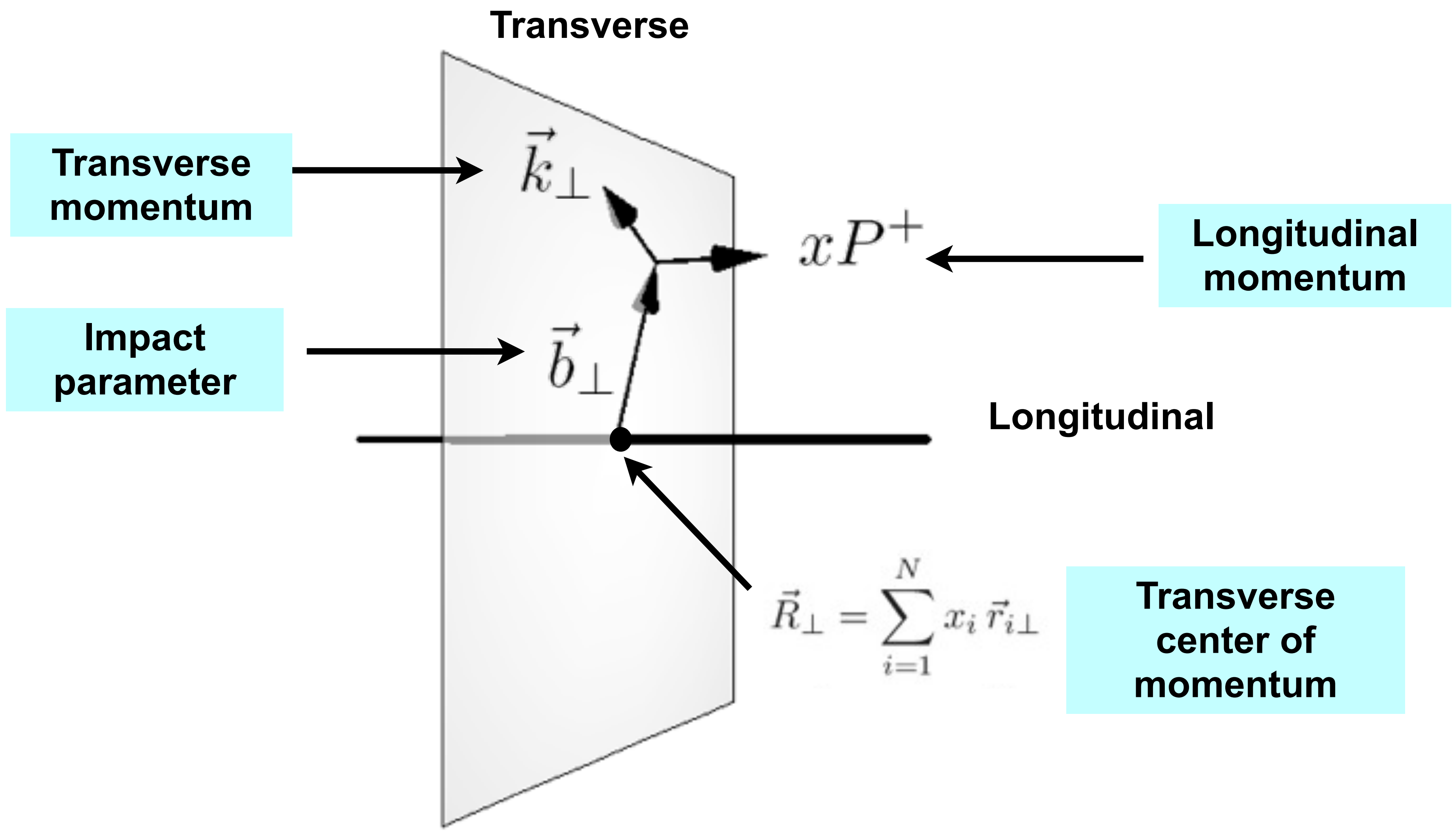
$x, \vec{b}_\perp$

Transverse charge densities

$\vec{b}_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]

Wigner Distributions



Transverse

Transverse momentum

$\vec{k}_\perp$

xP^+

Longitudinal momentum

Impact parameter

$\vec{b}_\perp$

Longitudinal

$\vec{R}_\perp = \sum_{i=1}^{N} x_i \, \vec{r}_{i\perp}$

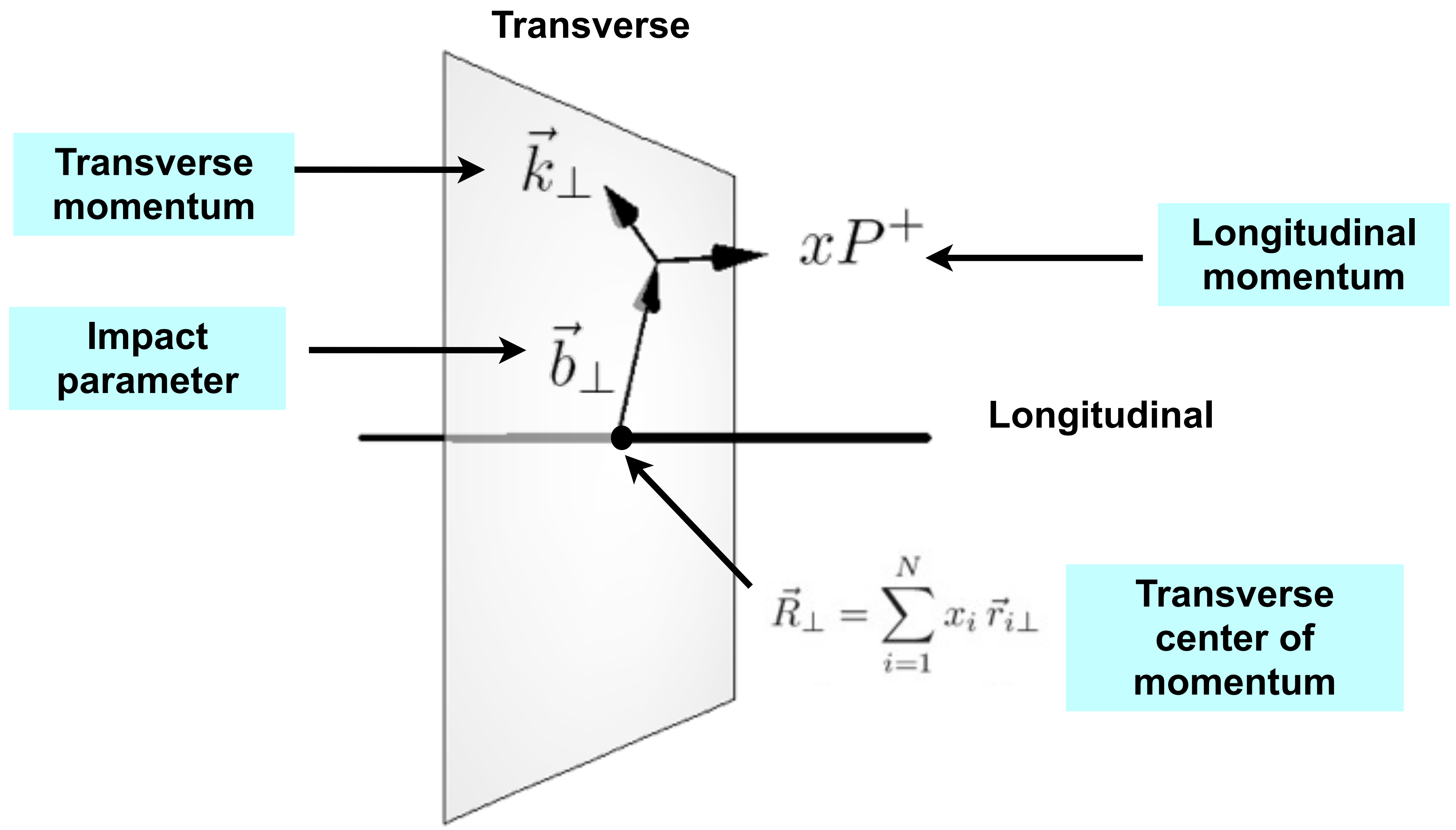
Transverse center of momentum

Heisenberg’s uncertainty relations

↓

Quasi-probabilistic

Wigner Distributions



Heisenberg's uncertainty relations

↓

Quasi-probabilistic

- ❖ real functions, but in general not-positive definite

  ➡ correlations of quark momentum and position in the transverse plane as function of quark and nucleon polarizations

- ❖ quantum-mechanical analogous of classical density in the phase space

- ❖ not directly measurable in experiments

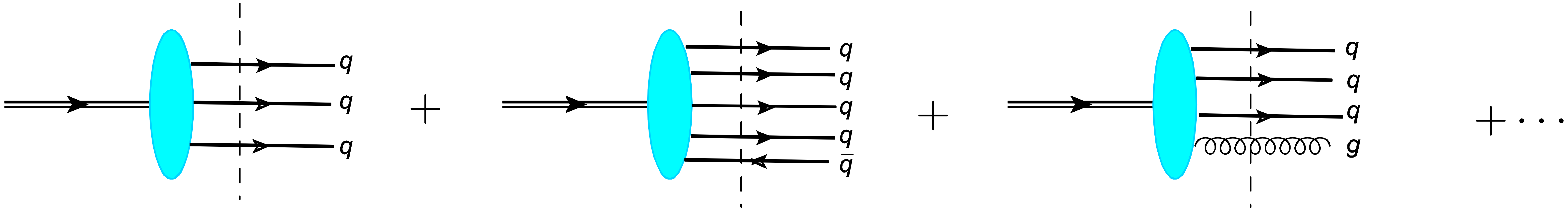
  ➡ needs phenomenological models with input from experiments on GPDs and TMDs

Light-Front Wave Function

- Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$

fixed light-cone time (x^+=0)

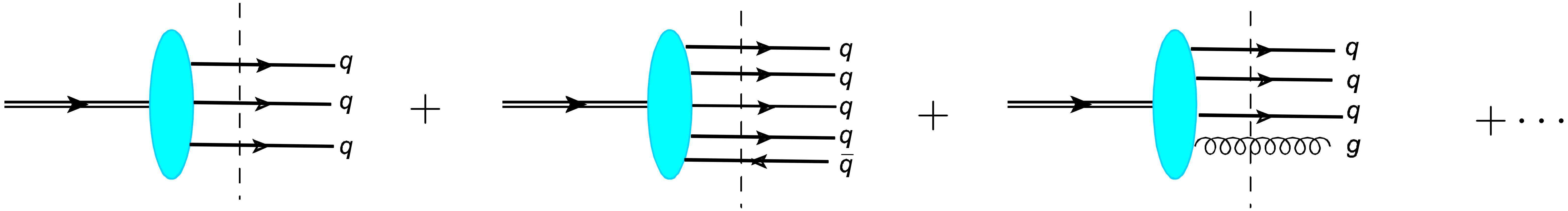


Light-Front Wave Function

- ✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$

fixed light-cone time ($x^+=0$)



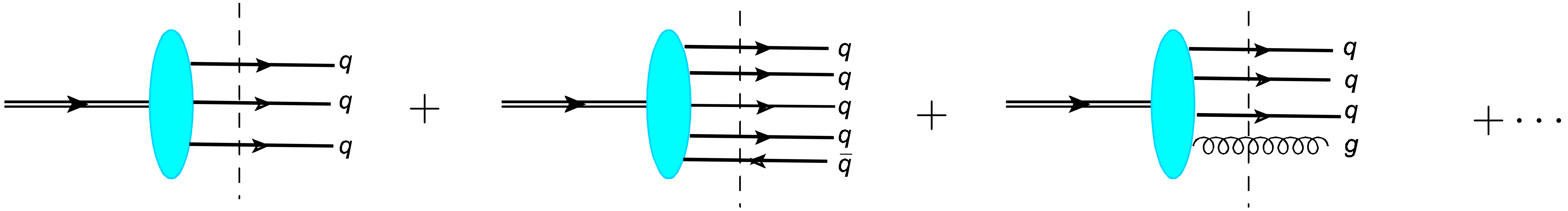
- ✦ Eigenstates of momentum $P^+ = \sum_{i=1}^{N} k_i^+ \qquad \vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\,\perp} = \vec{0}_\perp$

Light-Front Wave Function

- ✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$

fixed light-cone time (x⁺=0)



- ✦ Eigenstates of momentum $$P^+ = \sum_{i=1}^N k_i^+ \qquad \vec{P}_\perp = \sum_{i=1}^N \vec{k}_{i\,\perp} = \vec{0}_\perp$$

- ✦ Eigenstates of parton light-front helicity $$\hat{S}_{iz}\,\Psi^\Lambda_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^\Lambda_{\lambda_1\lambda_2\cdots\lambda_N}$$

$$\boxed{\Lambda = \sum_{i=1}^N \lambda_i + l_z}$$

- ✦ Eigenstates of total orbital angular momentum $$\hat{L}_z\,\Psi^\Lambda_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^\Lambda_{\lambda_1\lambda_2\cdots\lambda_N}$$

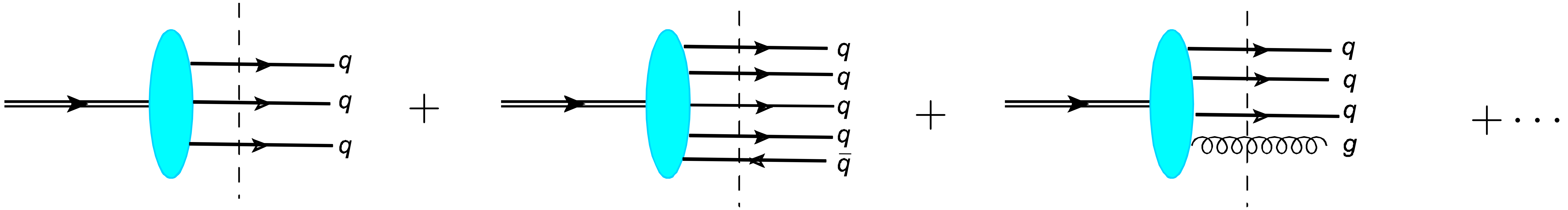
⚠ $A^+ = 0$ gauge

Light-Front Wave Function

✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$

fixed light-cone time ($x^+=0$)



✦ Eigenstates of momentum $\quad P^+ = \sum_{i=1}^N k_i^+ \qquad \vec{P}_\perp = \sum_{i=1}^N \vec{k}_{i\,\perp} = \vec{0}_\perp$

✦ Eigenstates of parton light-front helicity $\quad \hat{S}_{iz}\,\Psi^\Lambda_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^\Lambda_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^N \lambda_i + l_z}$$

✦ Eigenstates of total orbital angular momentum $\quad \hat{L}_z\,\Psi^\Lambda_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^\Lambda_{\lambda_1\lambda_2\cdots\lambda_N}$

⚠ $A^+ = 0$ gauge

✦ Probability to find N partons in the nucleon $\quad \rho^\Lambda_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^\Lambda_{\lambda_1\cdots\lambda_N}|^2$

normalization $\quad \sum_{N,\beta} \rho^\Lambda_{N,\beta} = 1$

total helicity

$$s_z = \langle \hat{S}_z \rangle = \sum_{N,\beta} \sum_{i=1}^N \lambda_i\,\rho^\Lambda_{N,\beta}$$

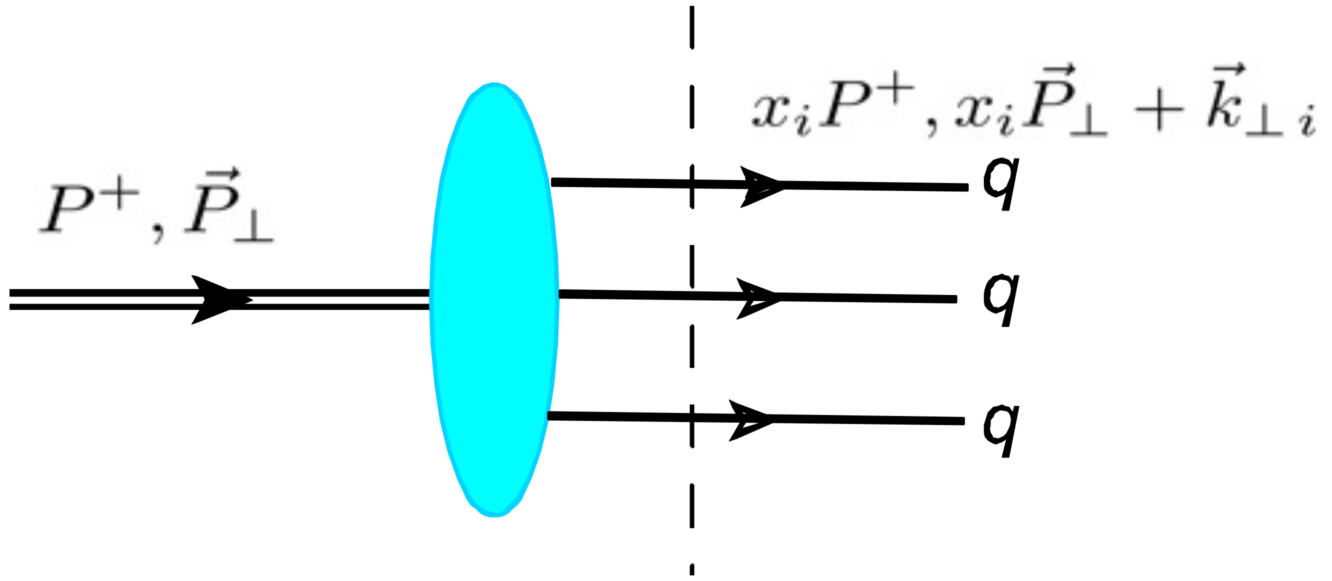
total OAM

$$l_z = \langle \hat{L}_z \rangle = \sum_{N,\beta} \sum_{i=1}^N l_z\,\rho^\Lambda_{N,\beta}$$

nucleon helicity

$$\Lambda = s_z + l_z$$

LFWF Overlap representation



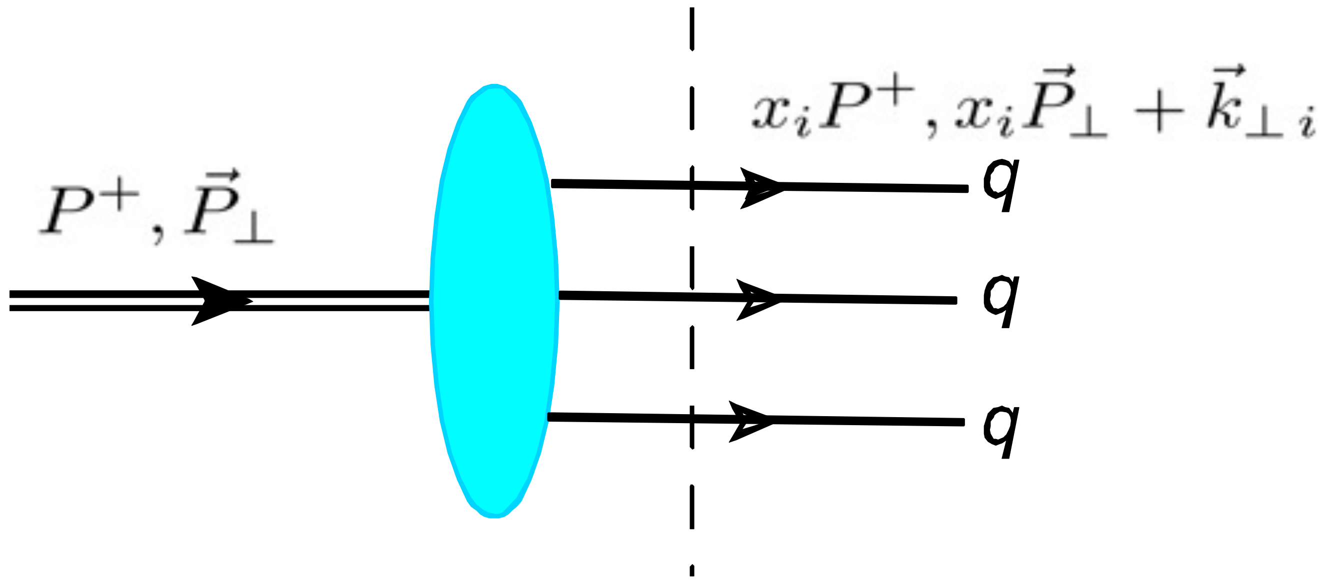
3q LFWF: $\Psi^{\Lambda; q_1 q_2 q_3}_{\lambda_1 \lambda_2 \lambda_3}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P^μ

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp\, i} = \vec{0}_\perp$

[Brodsky, Pauli, Pinsky, ’98]

LFWF Overlap representation



3q LFWF: $\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P^μ

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp\, i} = \vec{0}_\perp$

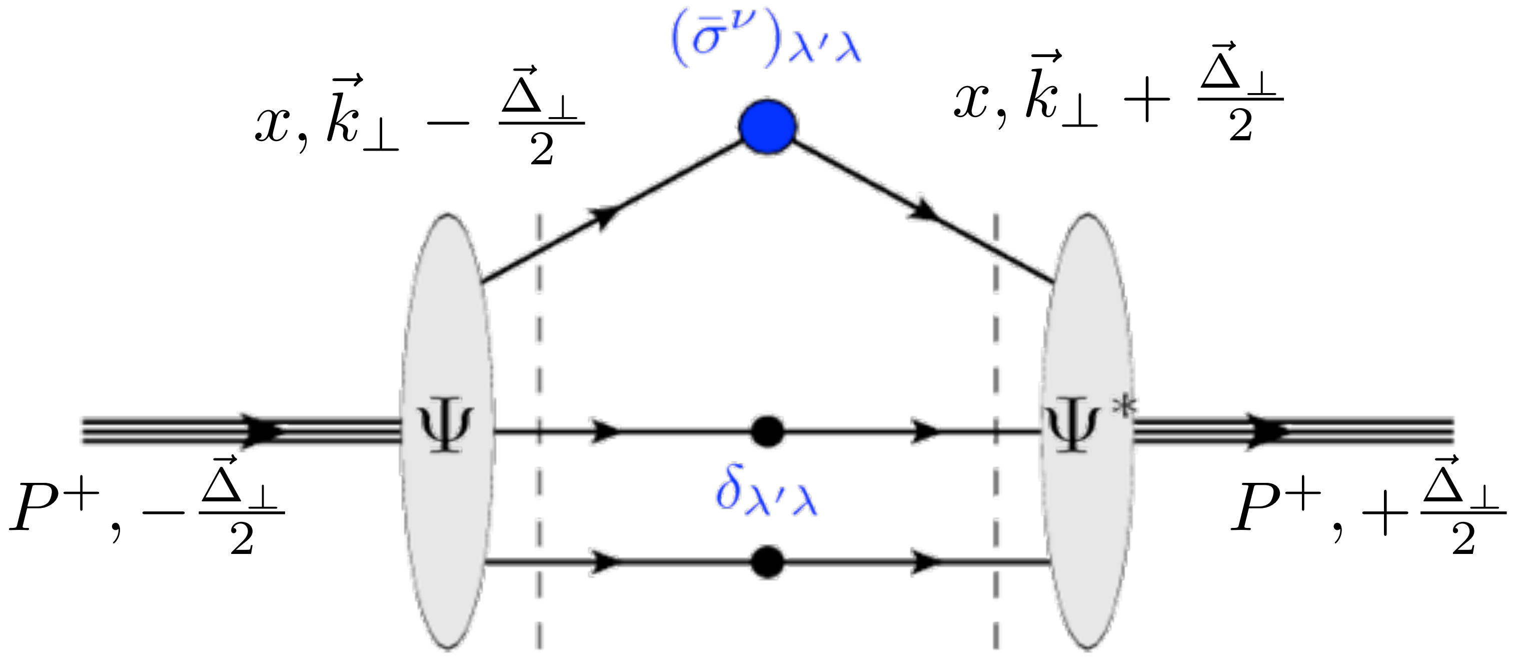
[Brodsky, Pauli, Pinsky, '98]

$A^+=0$ ⇒ Wilson line equal to unit

$\Delta^+=0$ ⇒ diagonal in the Fock-space

quark-quark correlator

($\Delta^+ =0$)



General formalism valid for

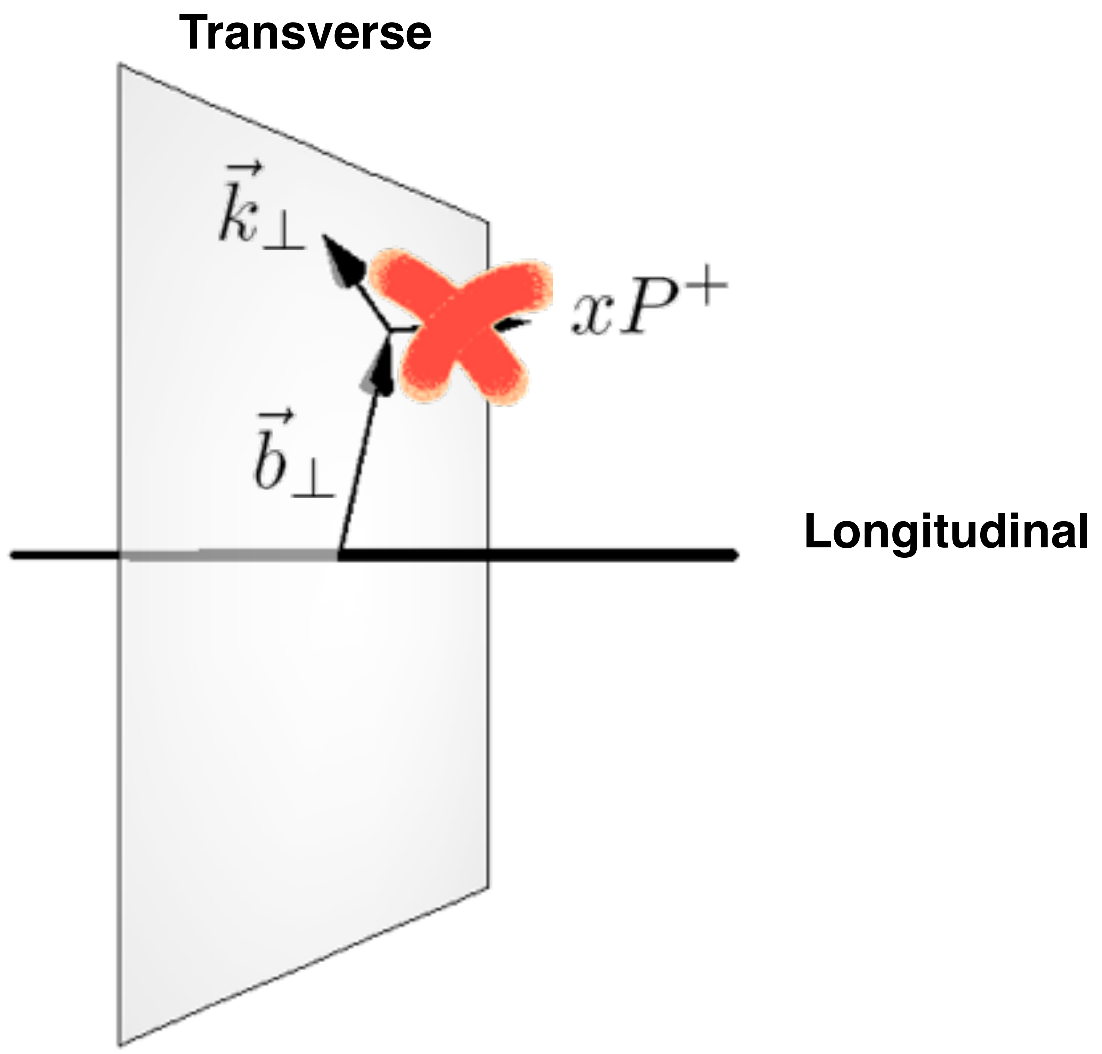
Bag Model, LFχQSM, LFCQM, Quark-Diquark, Covariant Parton Models

Common assumptions :

- No gluons
- Independent quarks

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]

Quark Wigner Distributions



$$\rho(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x\, \rho(x, \vec{k}_\perp, \vec{b}_\perp)$$

at fixed $\vec{k}_\perp$ → two-dimensional distributions in impact-parameter space

★ **Twist-2 ~ LO in P$^+$**

$$\boxed{\Gamma_{\text{twist-2}} = \gamma^+, \gamma^+\gamma_5, i\sigma^{j+}\gamma_5}$$

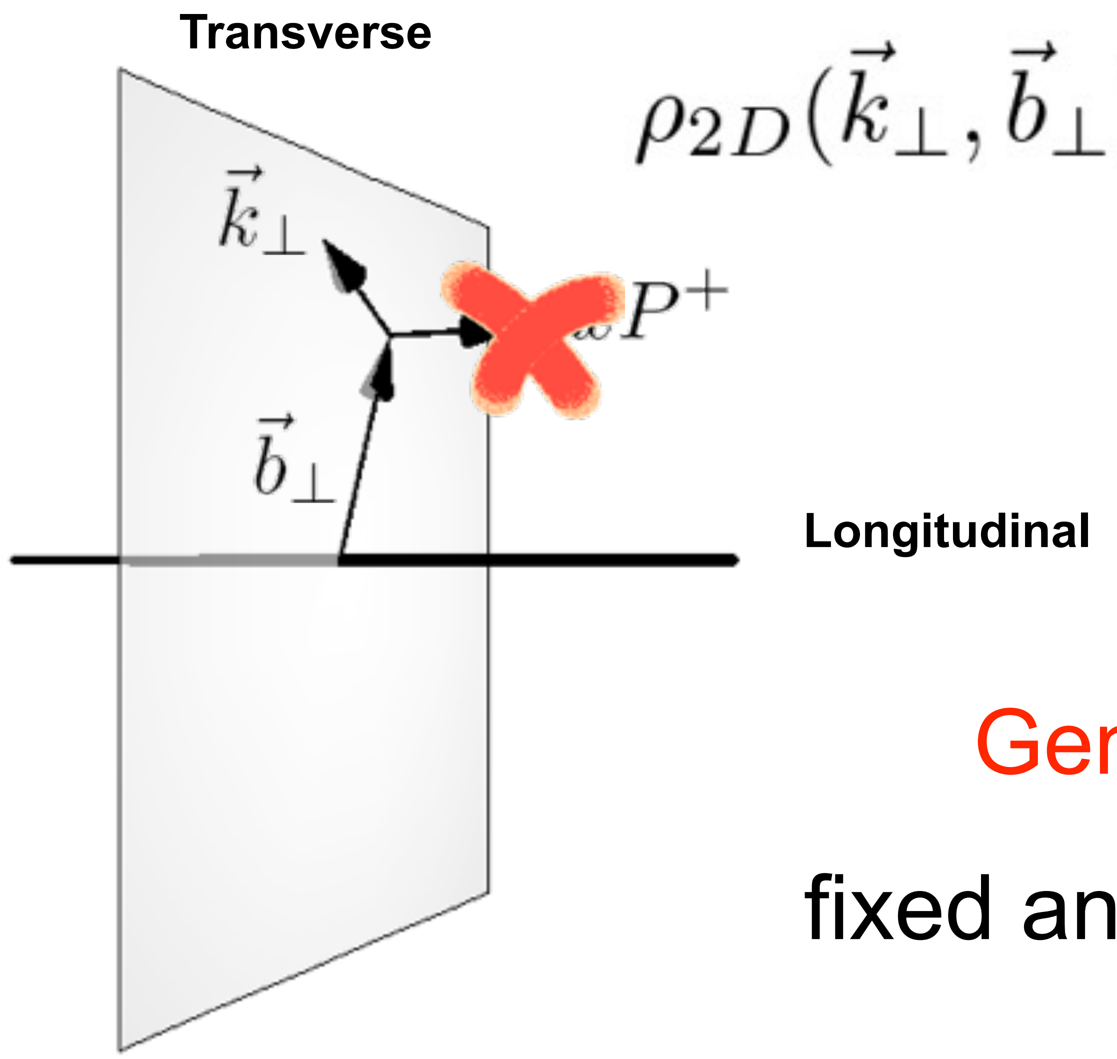
quark polarization ⟶ **U** **L** **T**

★ **Nucleon polarization: U L T**

→ 16 independent Wigner distributions

Unpol. up Quark in Unpol. Proton

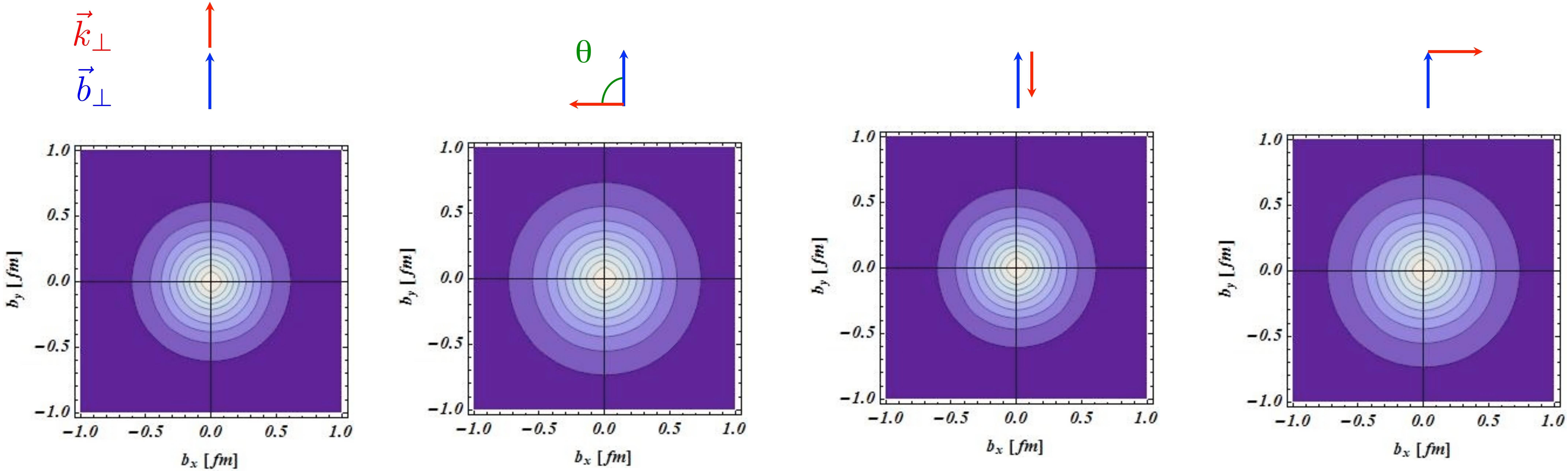
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

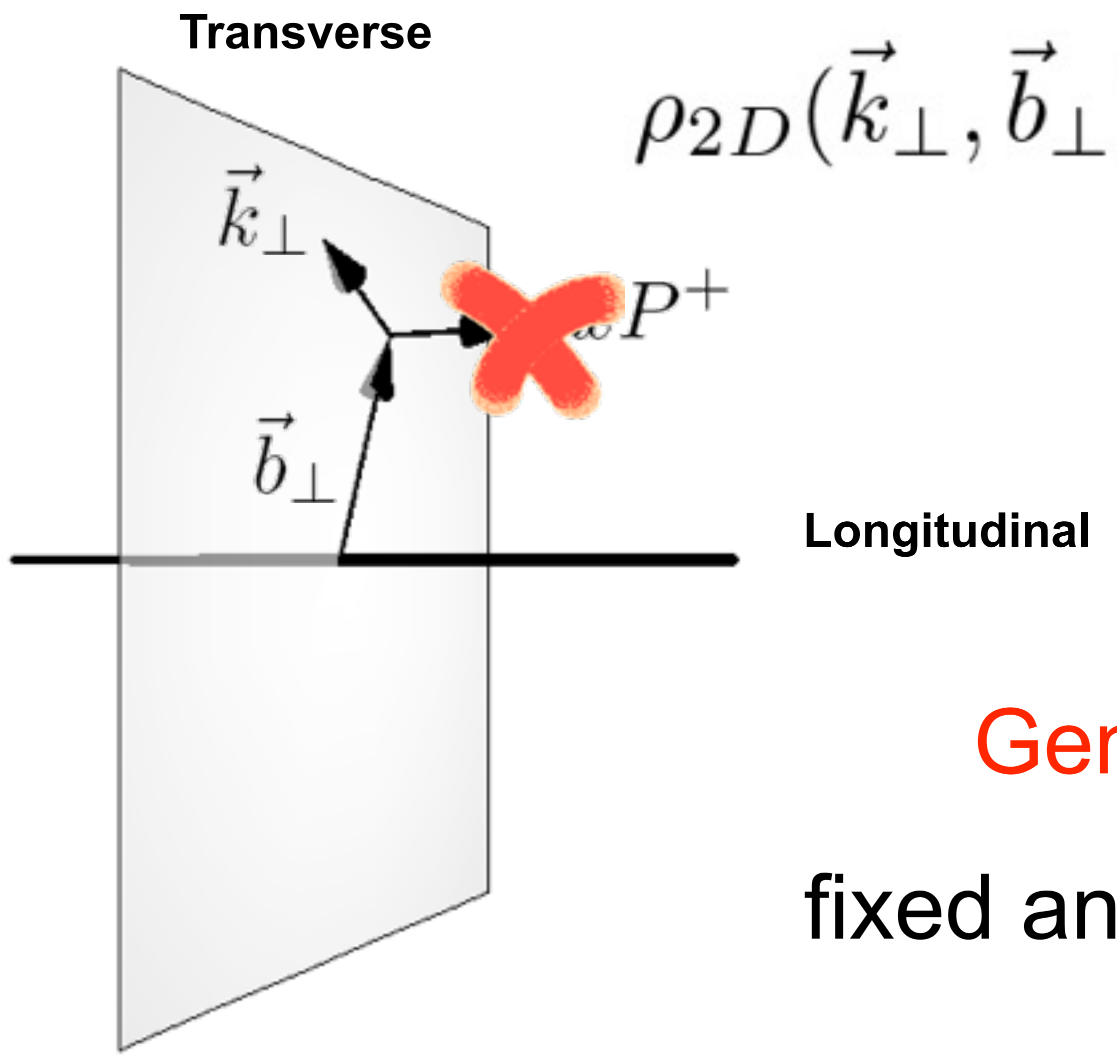
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

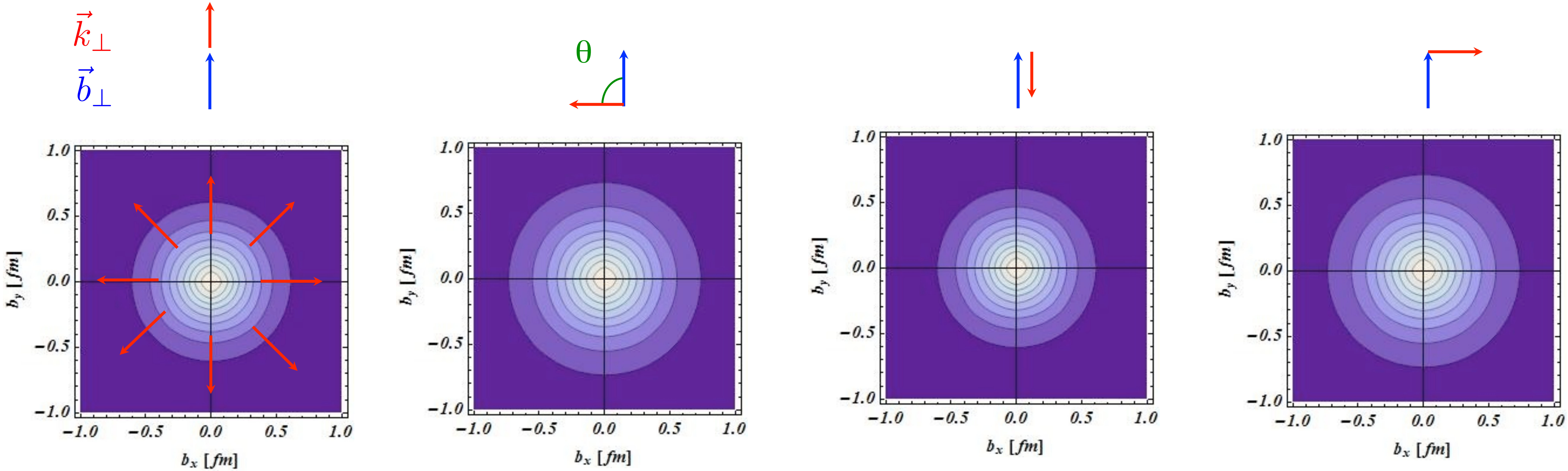
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

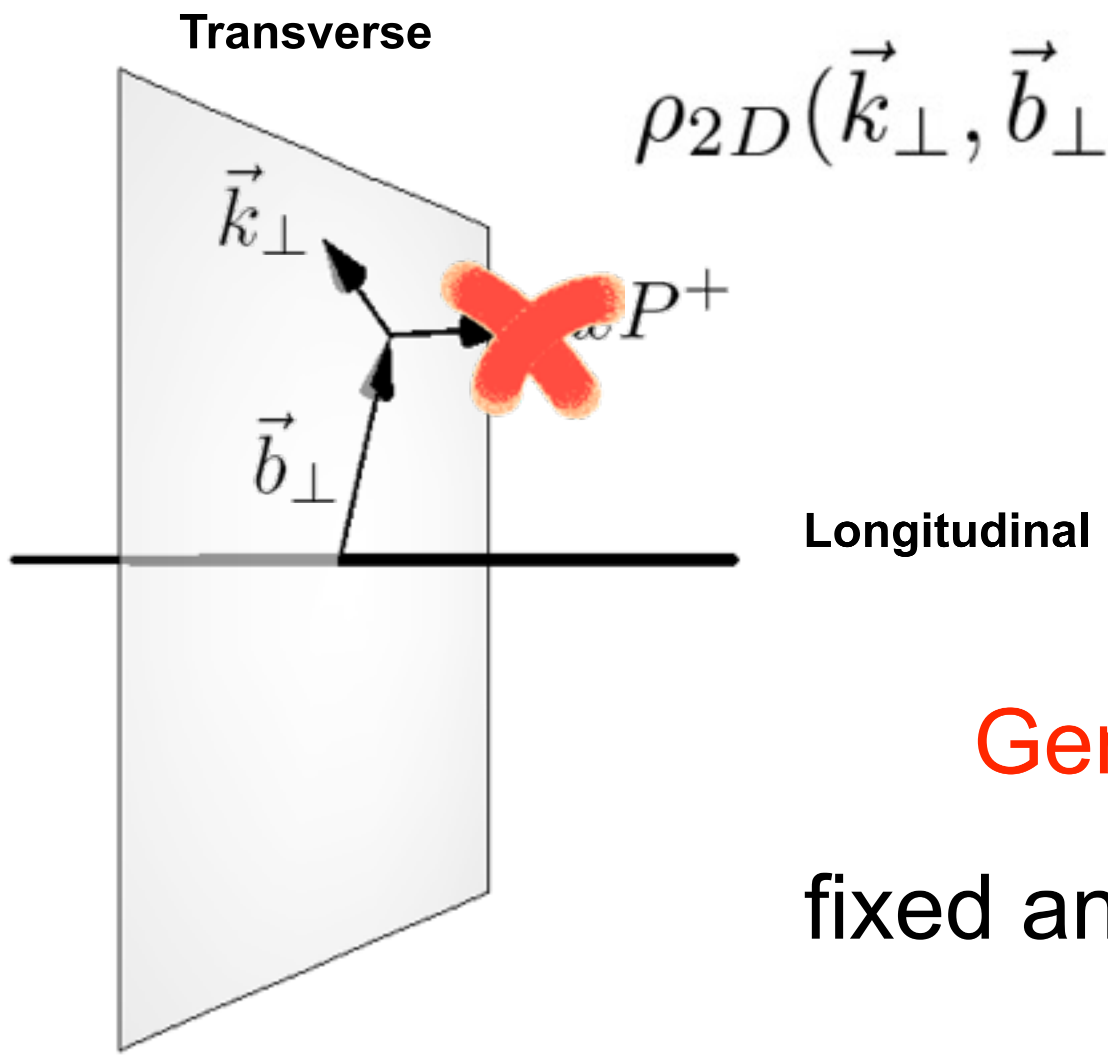
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

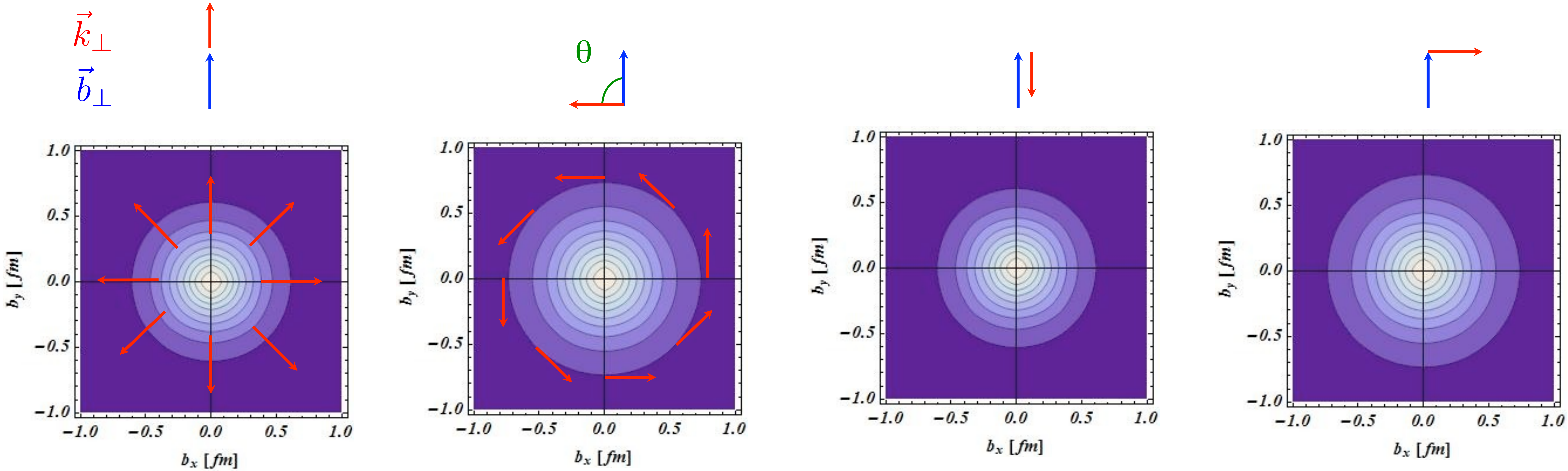
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

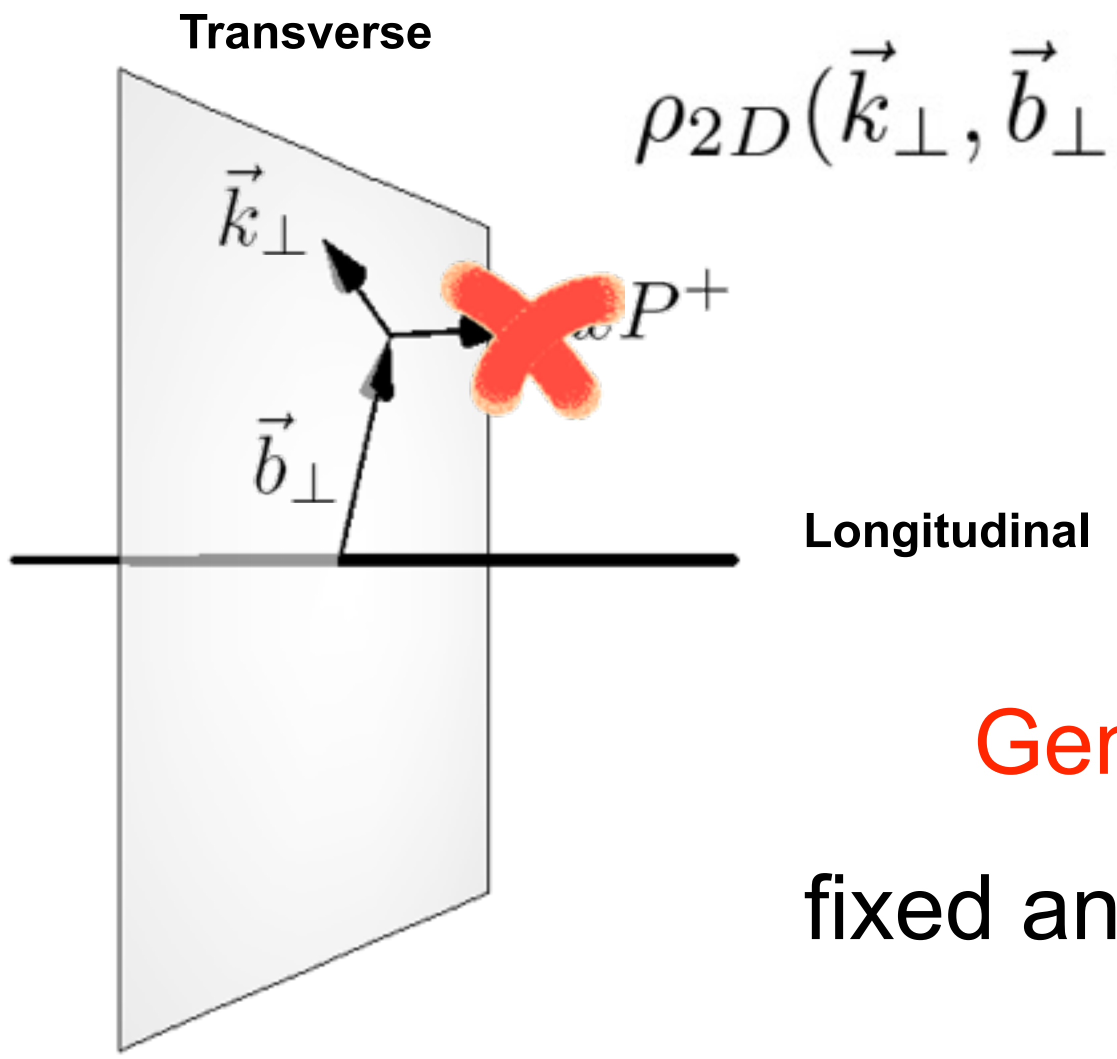
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

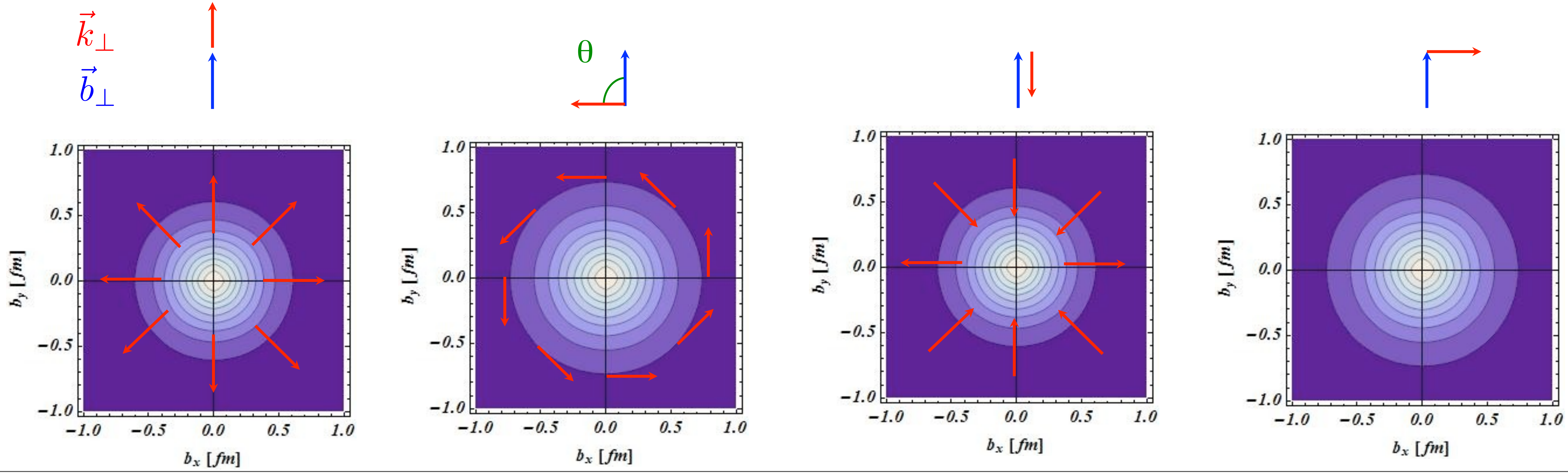
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

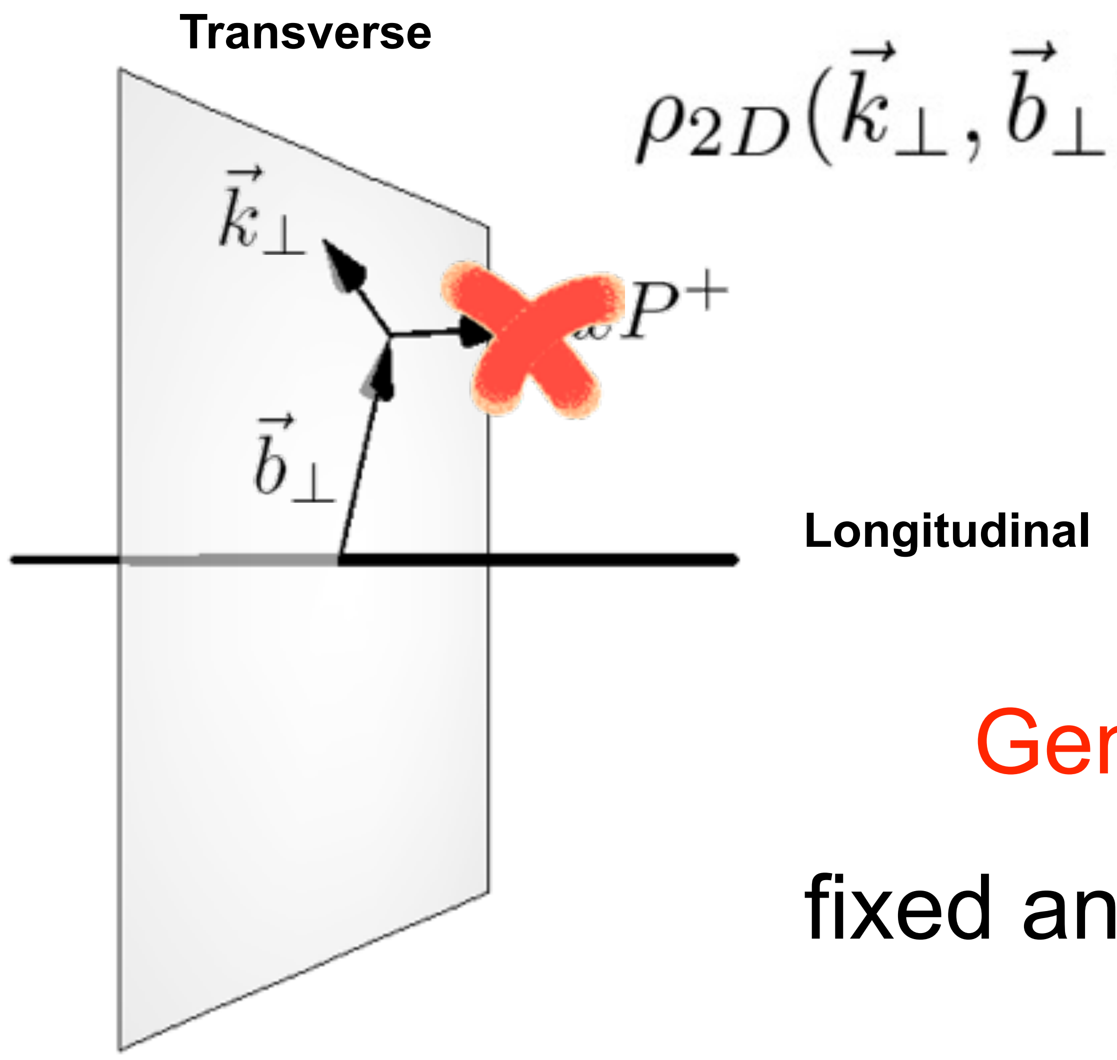
Generalized Transverse Charge Density

fixed angle between $\vec{\mathrm{k}}_\perp$ and $\vec{\mathrm{b}}_\perp$ and fixed value of $|\vec{\mathrm{k}}_\perp|$



Unpol. up Quark in Unpol. Proton

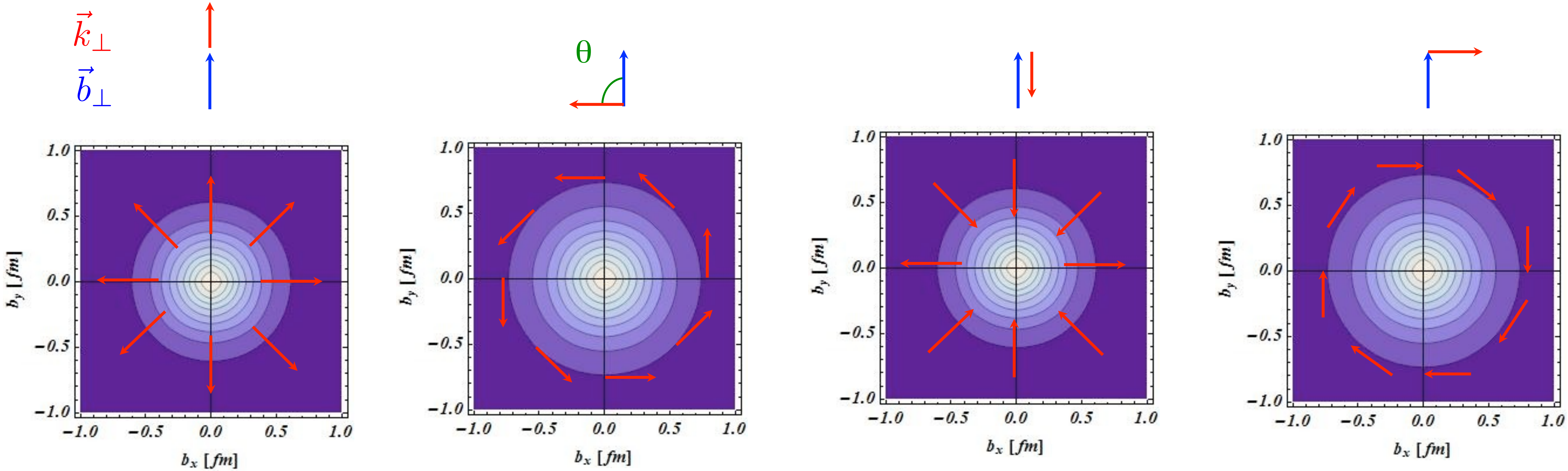
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

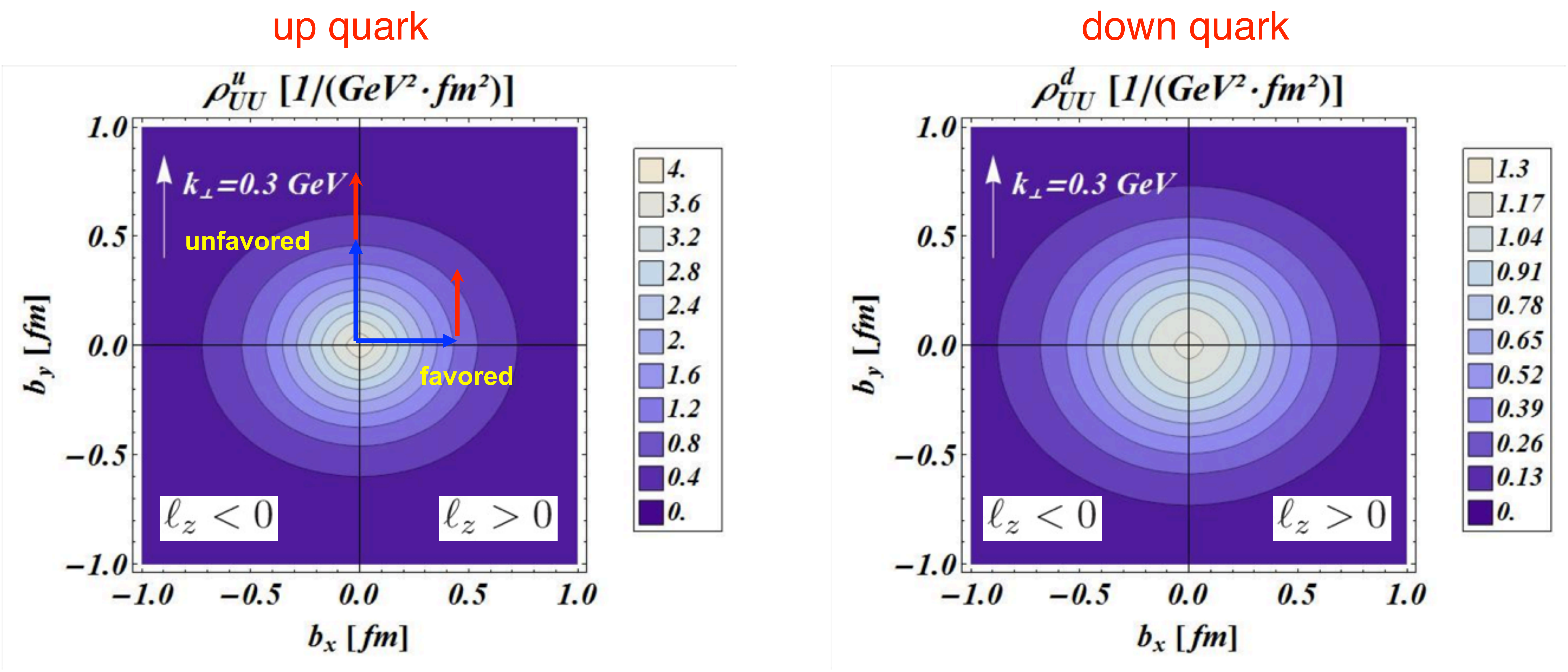
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

fixed $\vec{k}_\perp$: ↑


up quark

down quark

Distortion due to correlations between $\vec{k}_\perp$ and $\vec{b}_\perp$

↳ absent in GPDs and TMDs !

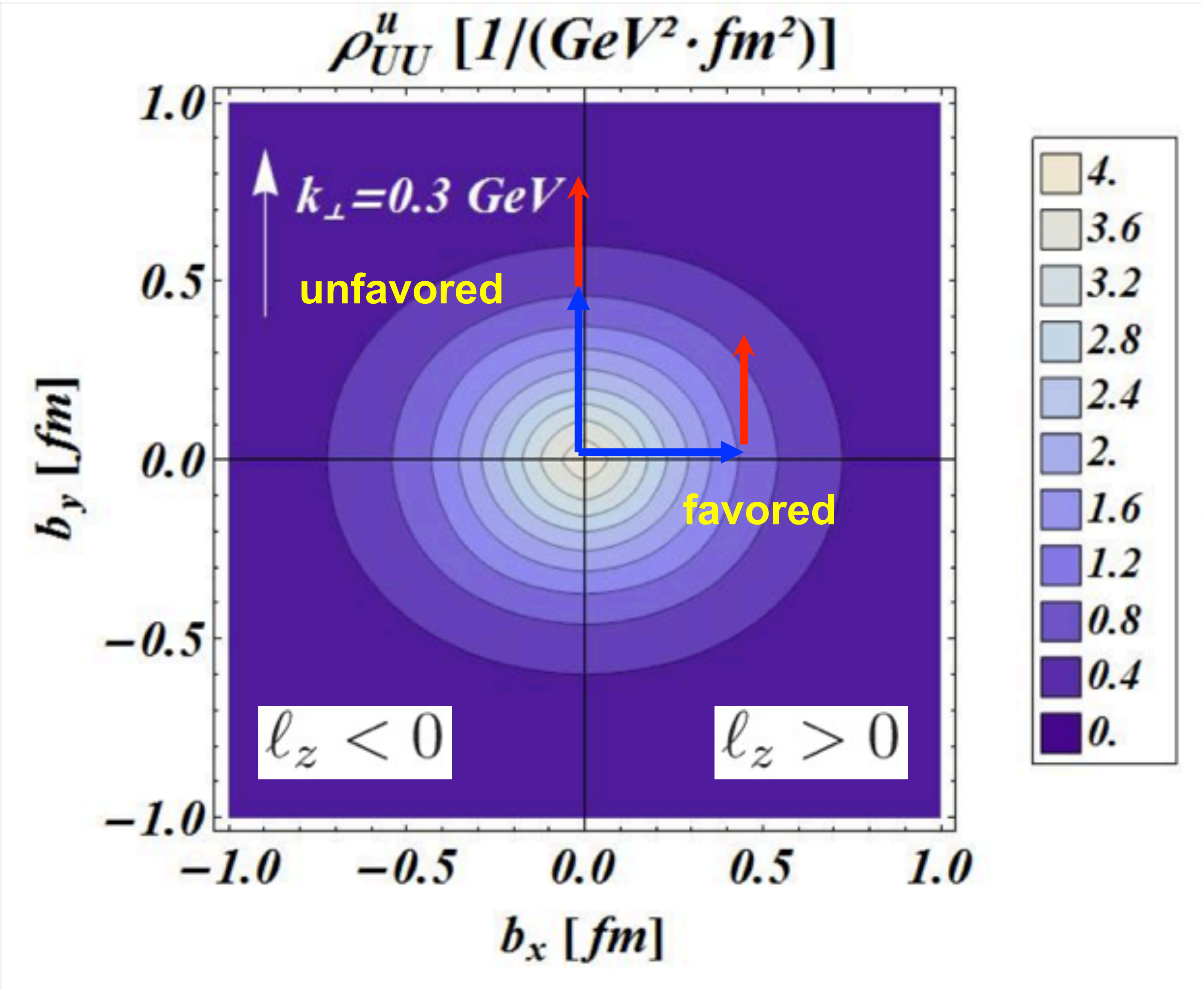
Left-right symmetry → no net quark OAM

[Lorce', Pasquini (2011)]

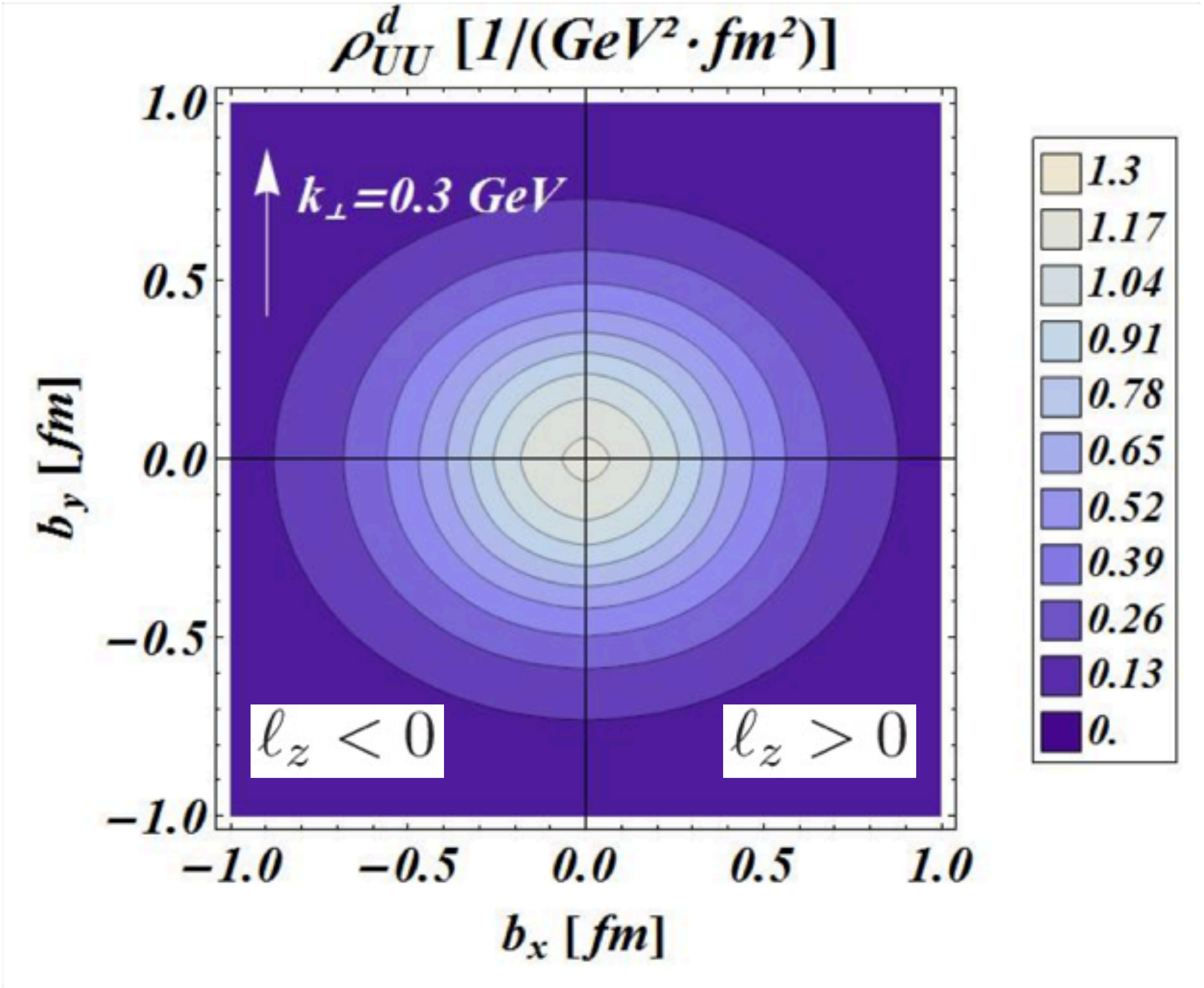
Unpol. up Quark in Unpol. Proton

fixed $\vec{k}_\perp$: $\uparrow$

up quark



down quark



✦ integrating over $\vec{b}_\perp$ ⟹ transverse-momentum density

$$f_1^q(k_\perp^2) = \int \mathrm{d}x f_1^q(x, k_\perp^2)$$

✦ integrating over $\vec{k}_\perp$ ⟹ charge density in the transverse plane $\vec{b}_\perp$

[Miller (2007); Burkardt (2007)]

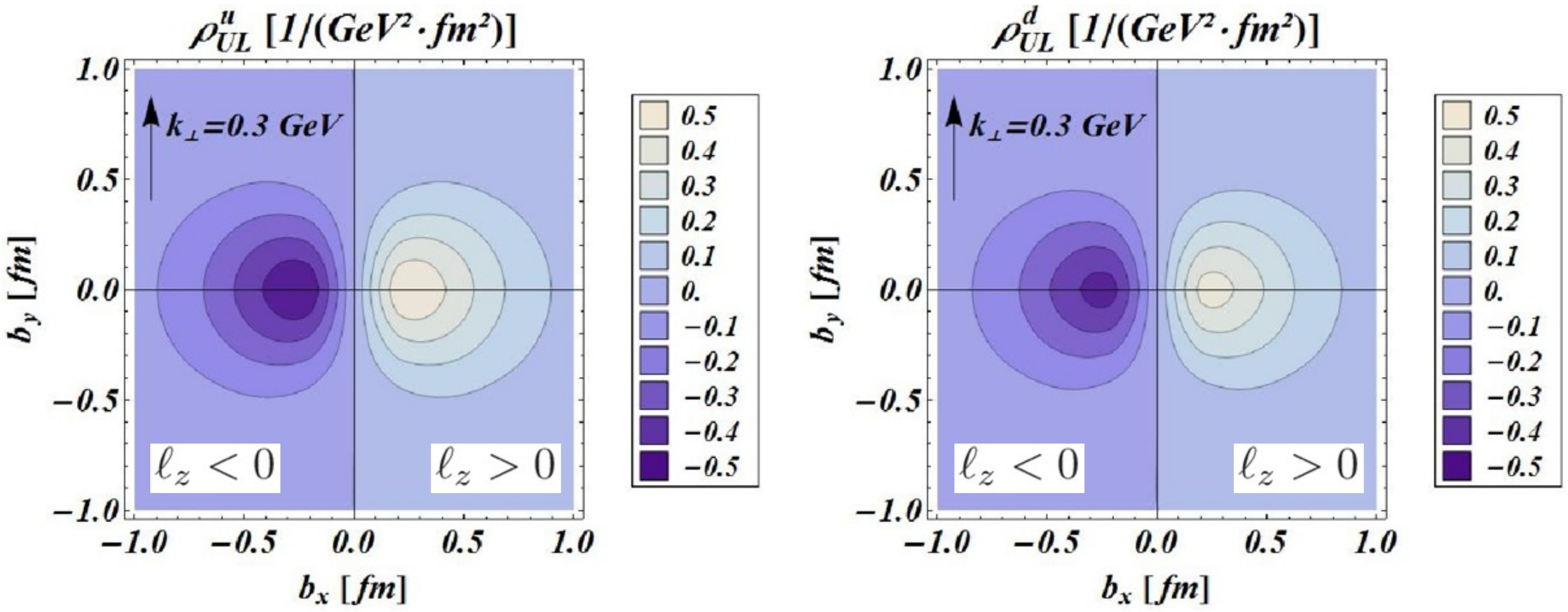
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

Monopole

Distributions

Long. pol. quark in Unp. proton

fixed $\vec{k}_\perp$ $\uparrow$



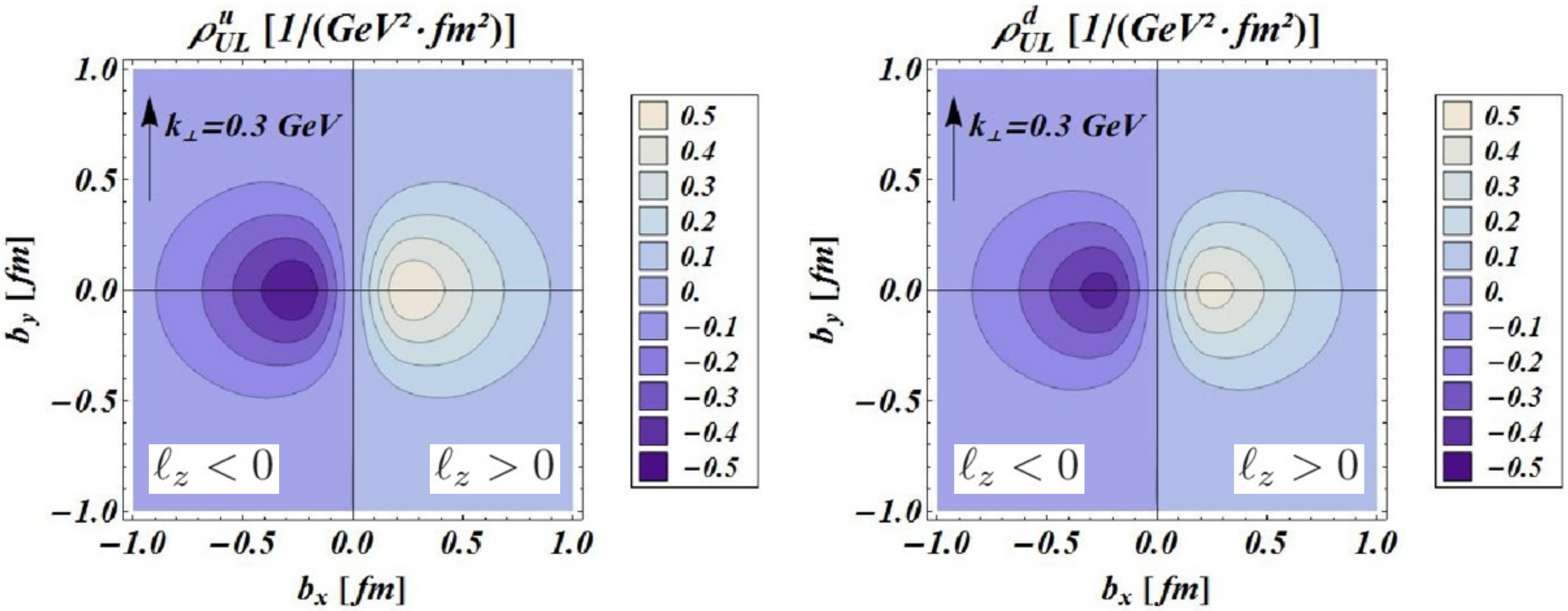
- ✦ projection to GPD and TMD is vanishing

➡ unique information on OAM from Wigner distributions

[Lorce', Pasquini (2011)]

Long. pol. quark in Unp. proton

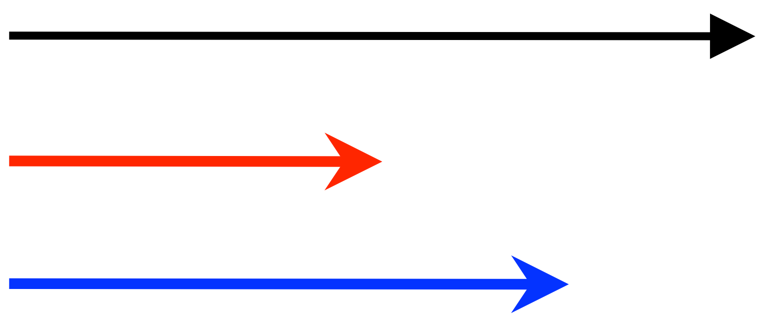
fixed $\vec{k}_\perp \uparrow$



correlation between quark spin and quark OAM

$$\boxed{C_z^q = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \mathrm{d}\vec{b}_\perp\, \left(\vec{b}_\perp \times \vec{k}_\perp\right) \rho^q_{UL}(x, \vec{k}_\perp, \vec{b}_\perp)}$$

	u-quark	d-quark
C_z^q	0.23	0.19



Quark spin

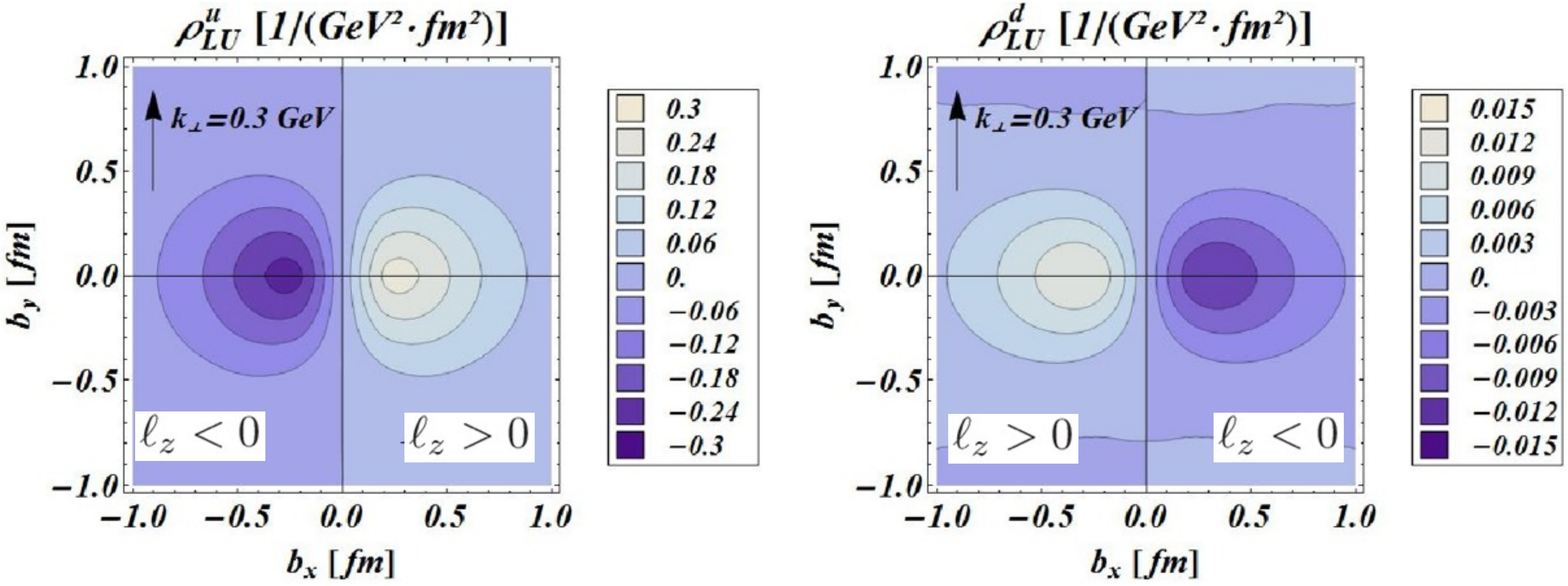
u-quark OAM

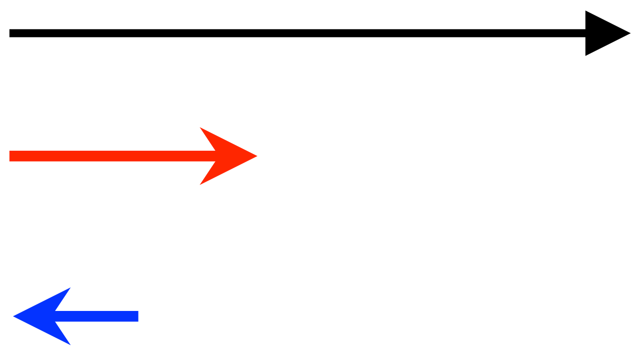
d-quark OAM

[Lorce', Pasquini (2011)]

Unpol. quark in long. pol. proton

fixed $\vec{k}_\perp$ ↑





Proton spin

u-quark OAM

d-quark OAM

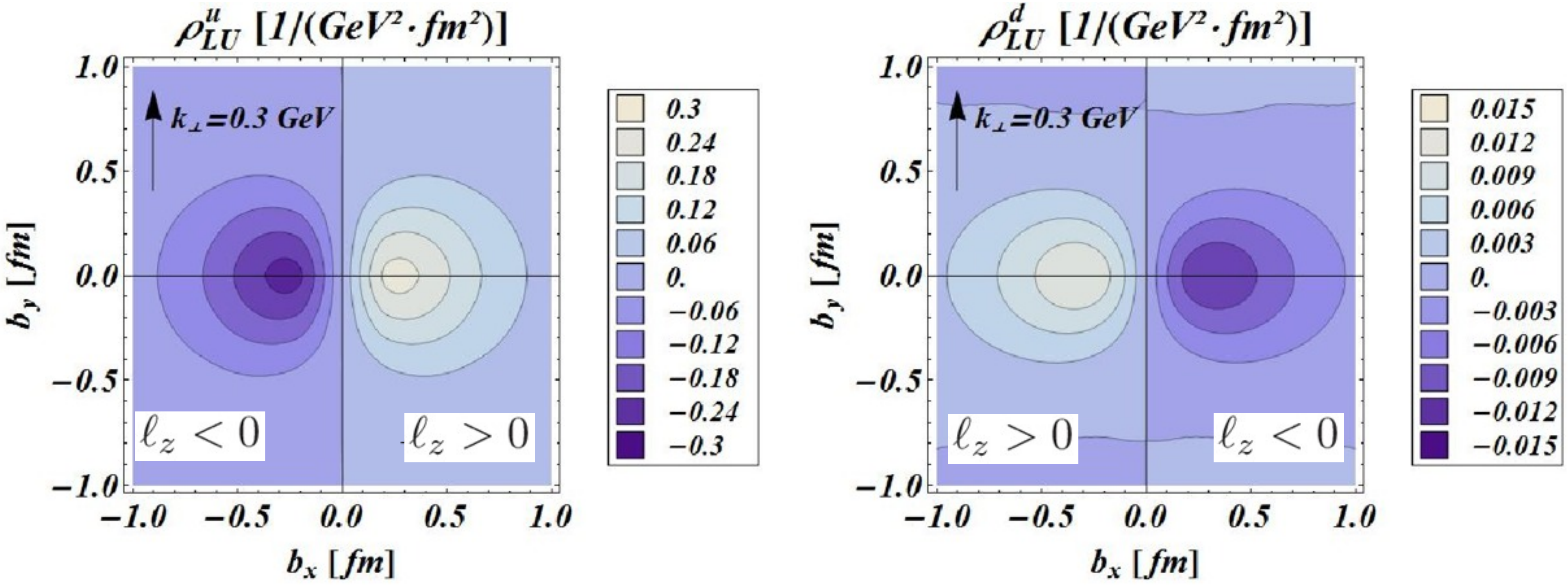
✦ projection to GPD and TMD is vanishing

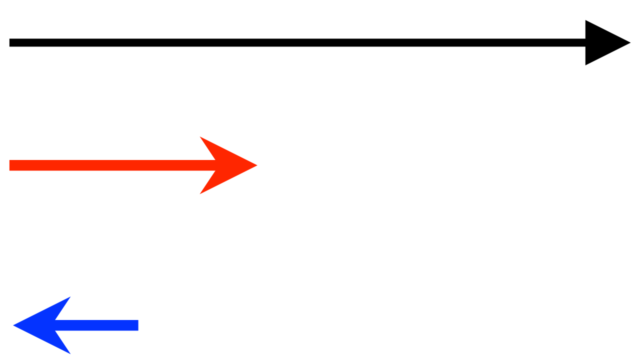
➡ unique information on OAM from Wigner distributions

[Lorce', Pasquini (2011)]

Unpol. quark in long. pol. proton

fixed $\vec{k}_\perp$ ↑





Proton spin

u-quark OAM

d-quark OAM

[Lorce', Pasquini (2011)]

Quark Orbital Angular Momentum

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)

$$\mathcal{L}_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

⬇

Wigner distribution
for Unpolarized quark in a Longitudinally pol. nucleon

Quark Orbital Angular Momentum

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)

$$\mathcal{L}_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

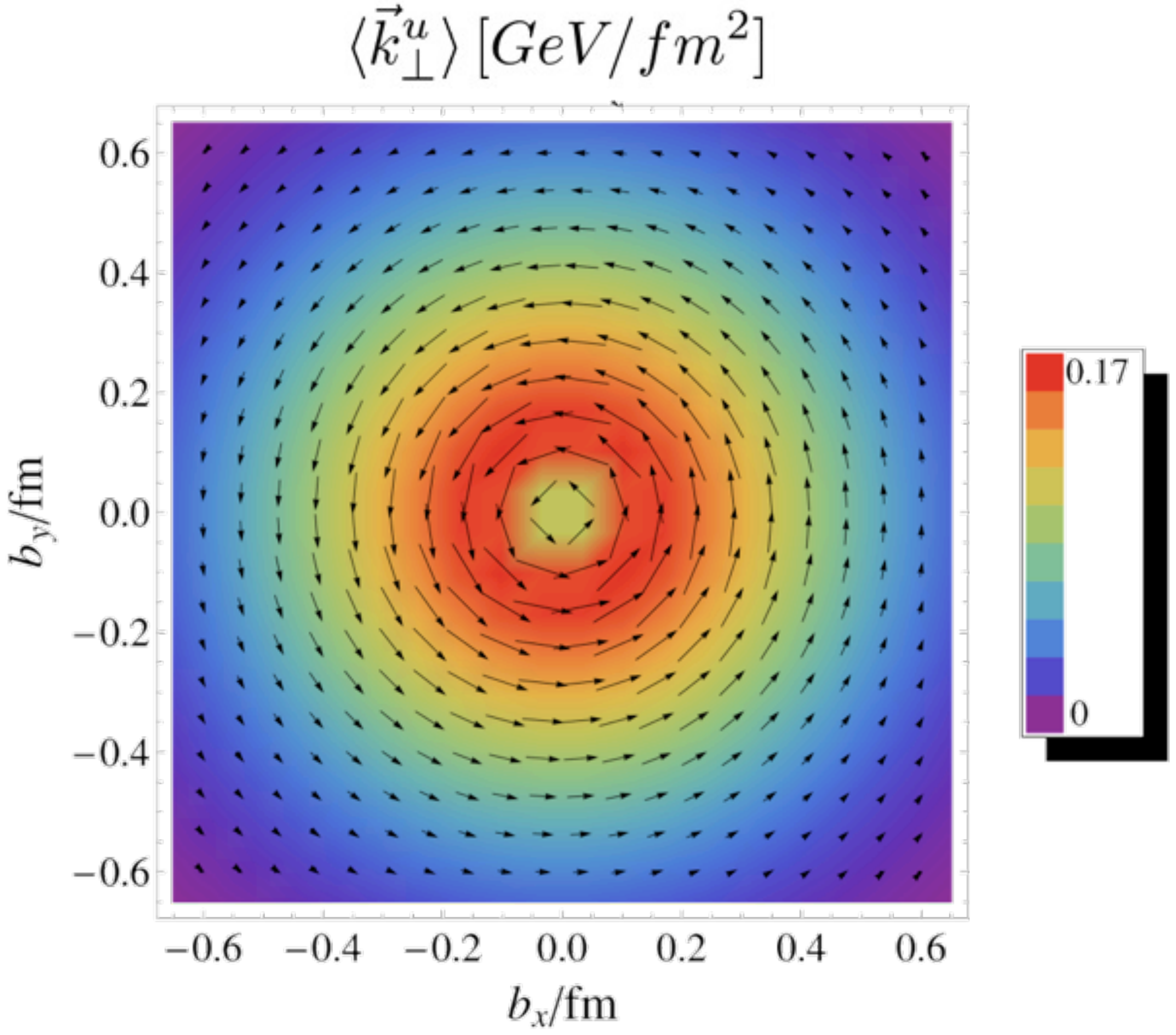
$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x \mathrm{d}\vec{k}_\perp \, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

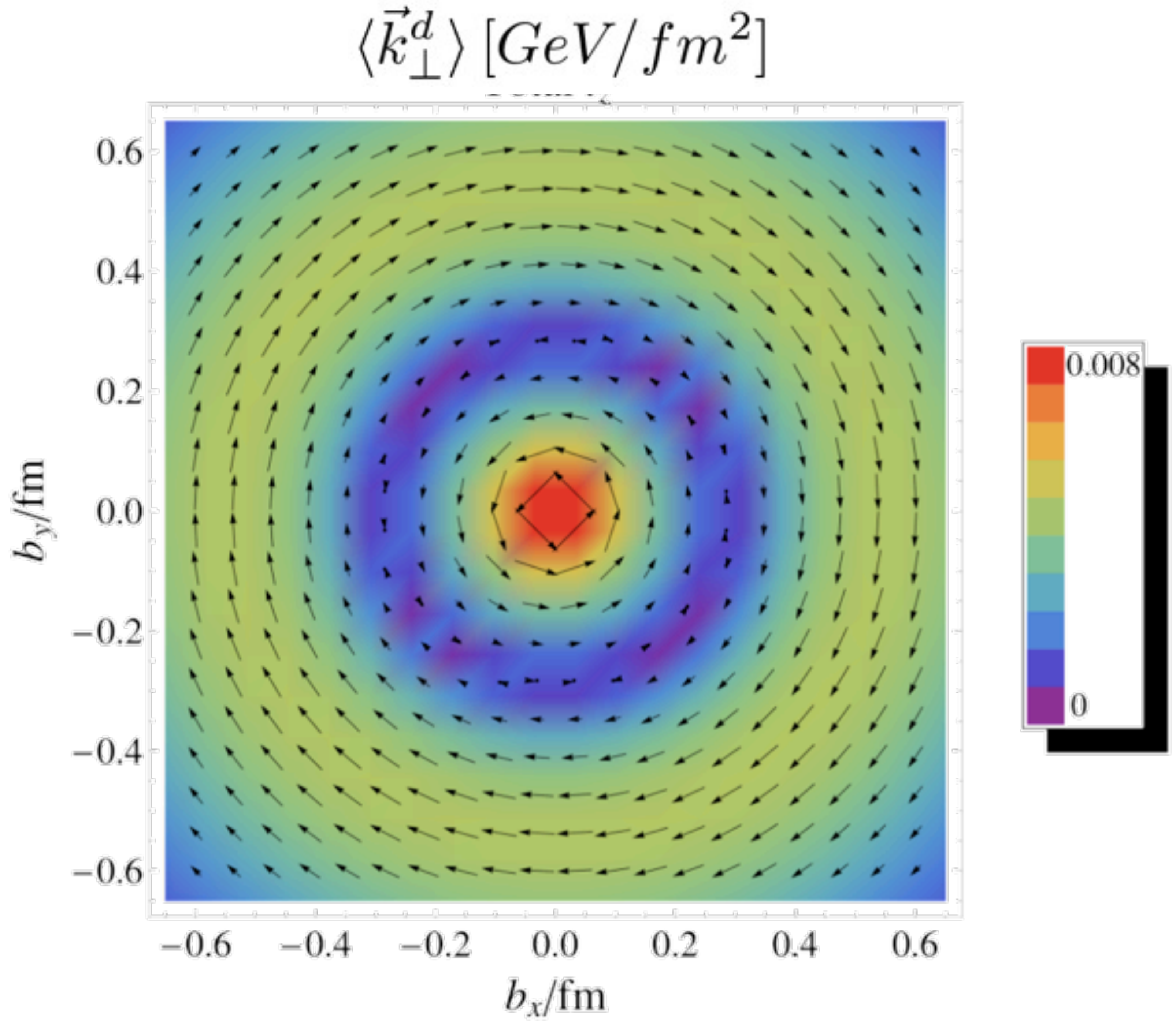
Quark Orbital Angular Momentum

$$\mathcal{L}_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp,\vec{k}_\perp,x)$$

$$= \int \mathrm{d}^2\vec{b}_\perp\vec{b}_\perp \times \langle\vec{k}^q_\perp\rangle \longrightarrow \langle\vec{k}^q_\perp\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\,\vec{k}_\perp\rho^q_{LU}(\vec{b}_\perp,\vec{k}_\perp,x)$$

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)





$\longrightarrow$ Proton spin

$\longrightarrow$ *u*-quark OAM

$\longleftarrow$ *d*-quark OAM

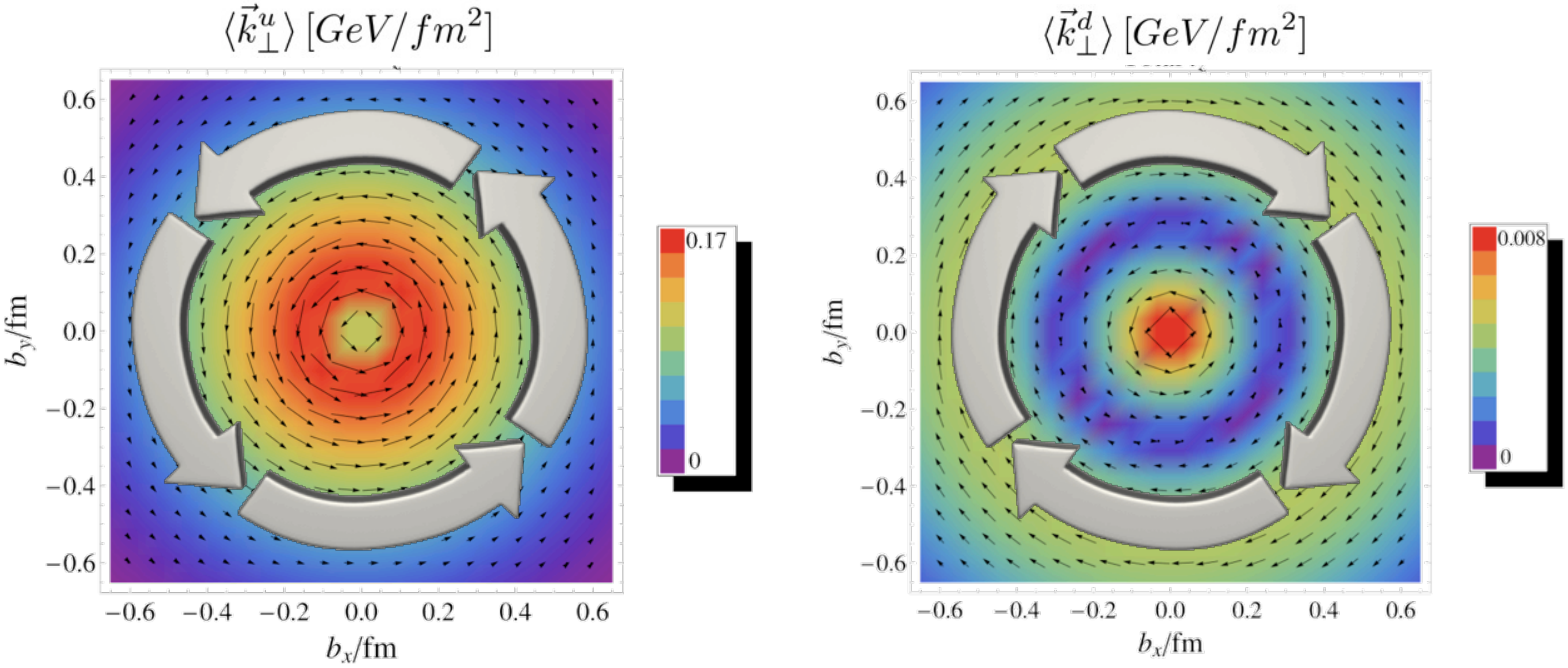
Results in a light-front constituent quark model:
Lorce', BP, Xiong, Yuan, PRD85 (2012)

Quark Orbital Angular Momentum

$$\mathcal{L}_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x \mathrm{d}\vec{k}_\perp \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)



⟶ Proton spin

⟶ *u*-quark OAM

← *d*-quark OAM

Results in a light-front constituent quark model:
Lorce', BP, Xiong, Yuan, PRD85 (2012)

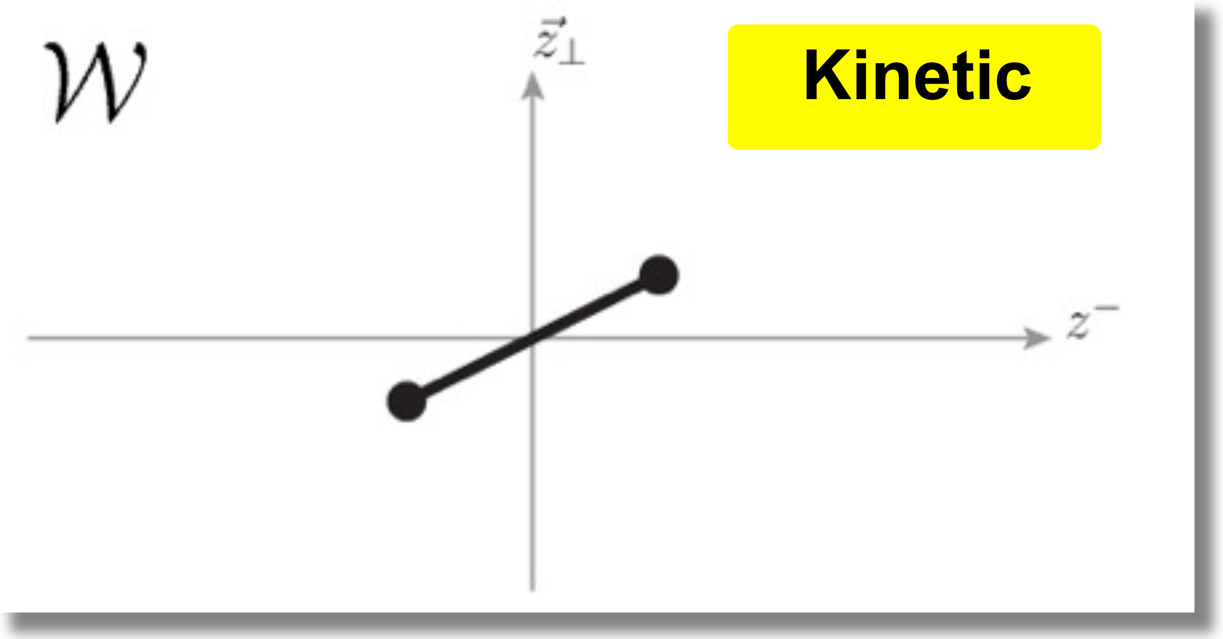
$$\boxed{\mathcal{L}_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) W(\vec{b}_\perp, \vec{k}_\perp, x)}$$

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan(2011)]

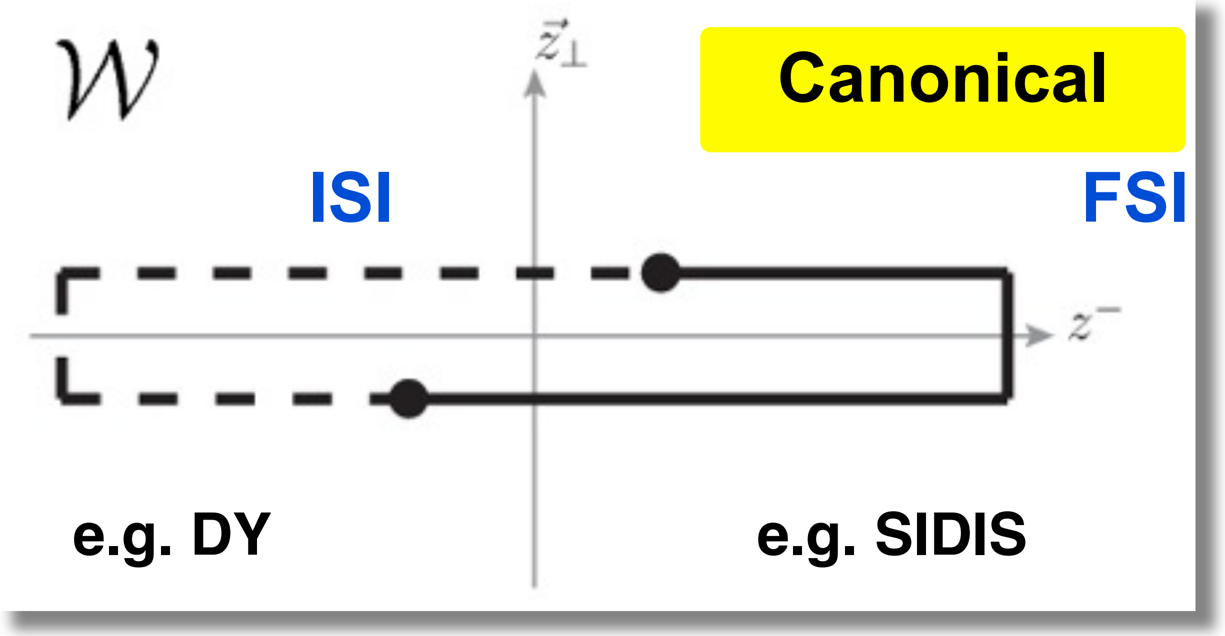
Light-cone gauge $A^+ = 0$
not gauge invariant, but with simple partonic interpretation

Gauge-invariant extension

$$W \to W^{\mathcal{W}}$$

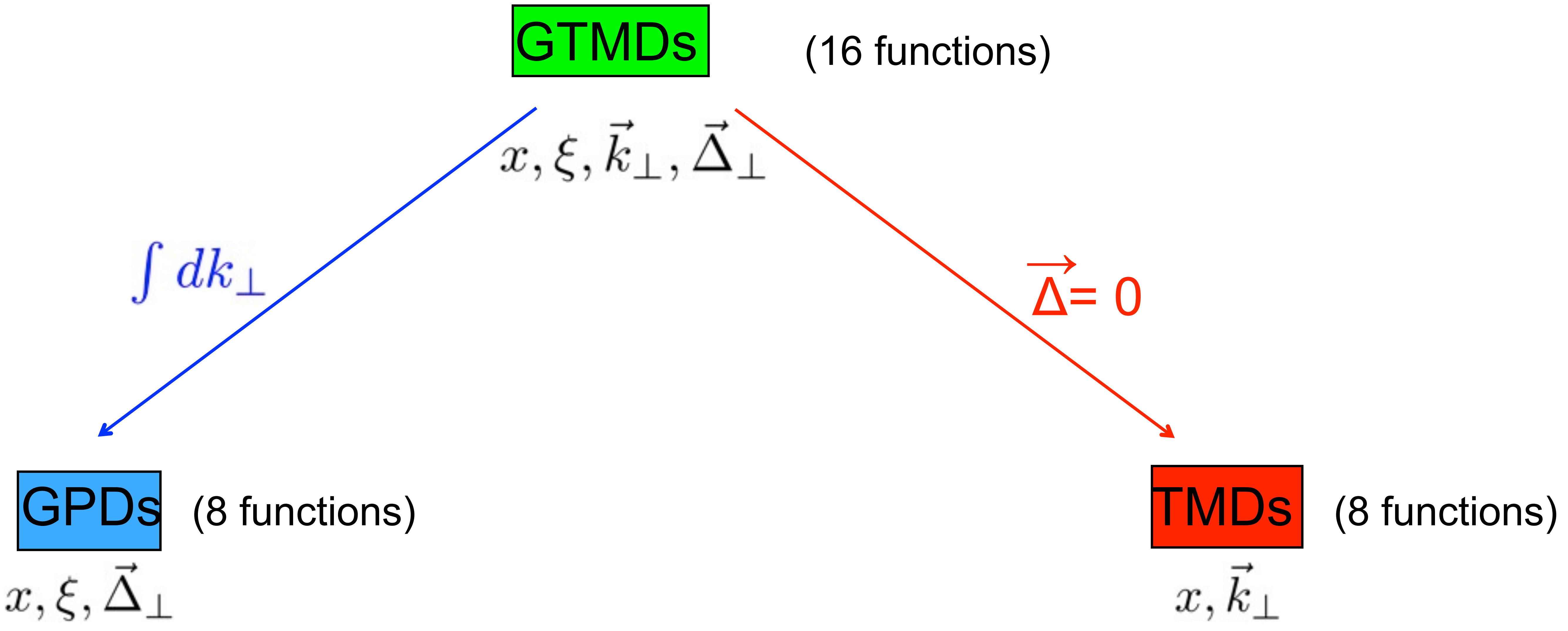


[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]



[Hatta (2012)]

relations between the two gauge-invariant definitions
⟶ talk Burkardt



GTMDs (16 functions)

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\int dk_\perp$

$\vec{\Delta} = 0$

GPDs (8 functions)

$x, \xi, \vec{\Delta}_\perp$

TMDs (8 functions)

$x, \vec{k}_\perp$

Quark polarization

Nucleon polarization

	U	T	L
U	H	$\mathcal{E}_T$	
T	E	$H_T, \tilde{H}_T$	$\tilde{E}$
L		$\tilde{E}_T$	$\tilde{H}$

Quark polarization

Nucleon polarization

	U	T	L
U	f_1	$h_1^\perp$	
T	$f_{1T}^\perp$	$h_1, h_{1T}^\perp$	g_{1T}
L		$h_{1L}^\perp$	g_{1L}

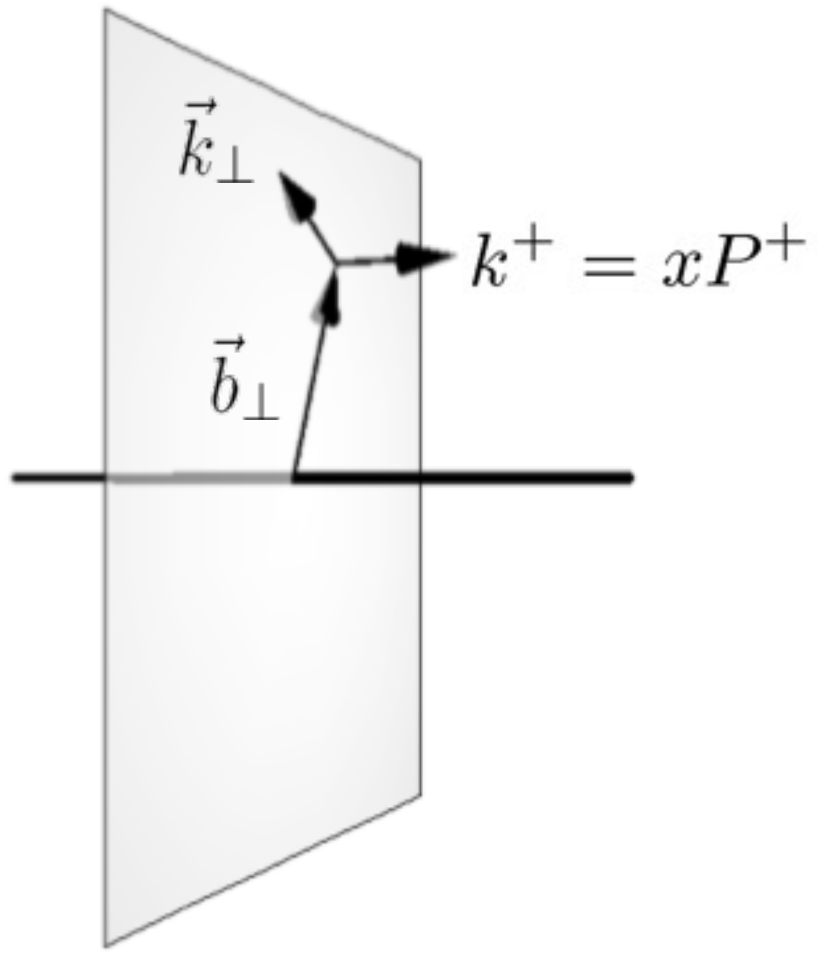
✦ almost all distributions (in red) vanish if there is no quark orbital angular momentum

✦ quark GPDs (at ξ=0) and TMDs given by the same overlap of LCWFs but in different kinematics
⇒ each distribution contains unique information

⇒ no model-independent relations between GPDs and TMDs

Wigner Distributions

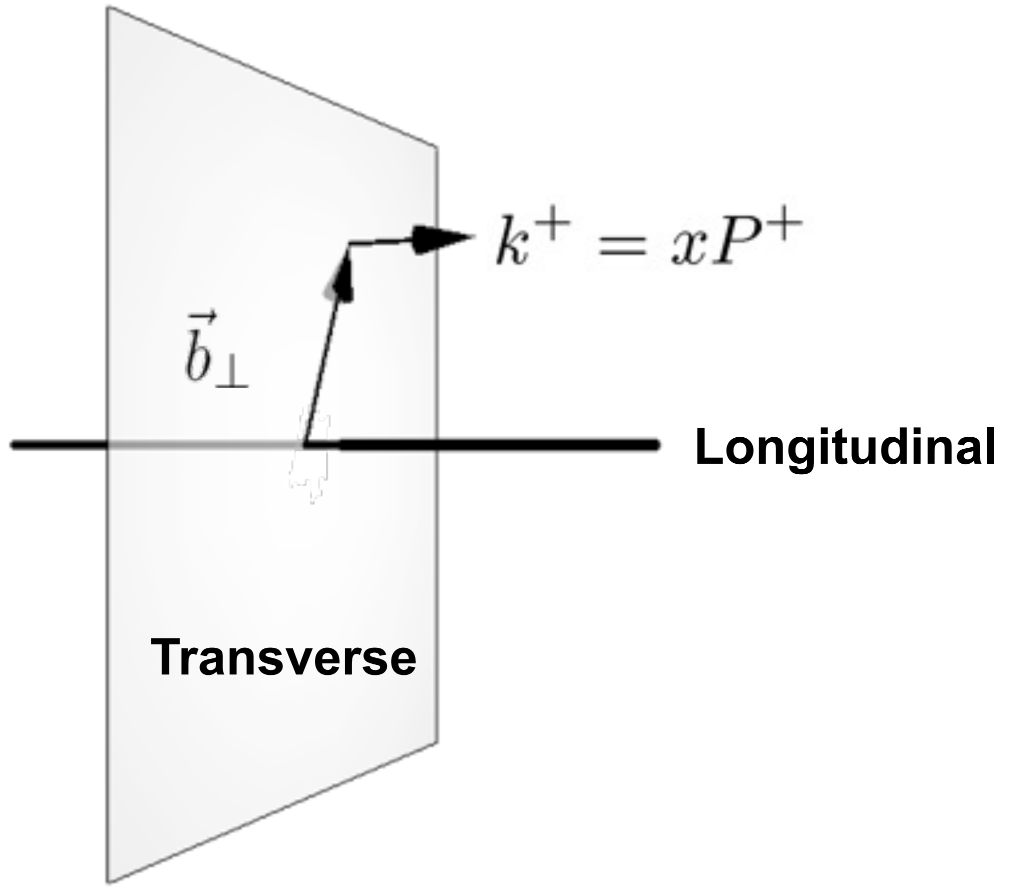
$$\rho(x,\vec{b}_\perp,\vec{k}_\perp)$$



quasi-probabilistic interpretation

GPDs

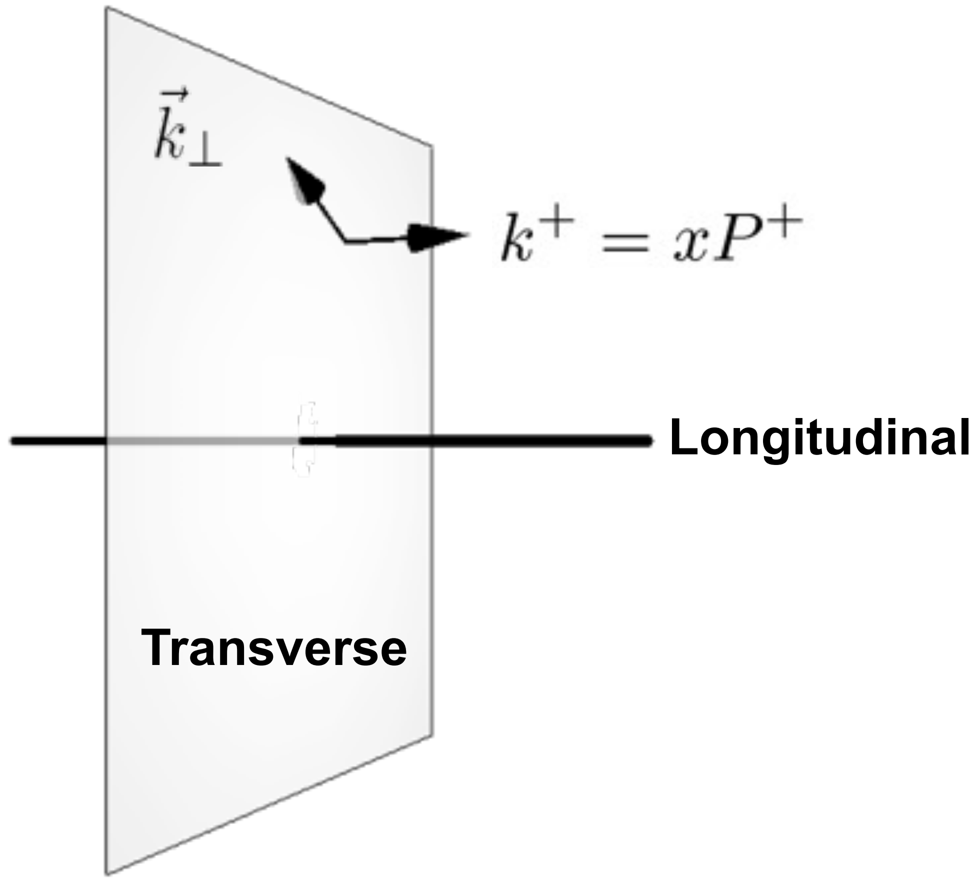
$$\rho(x,\vec{b}_\perp)$$



probabilistic interpretation

TMDs

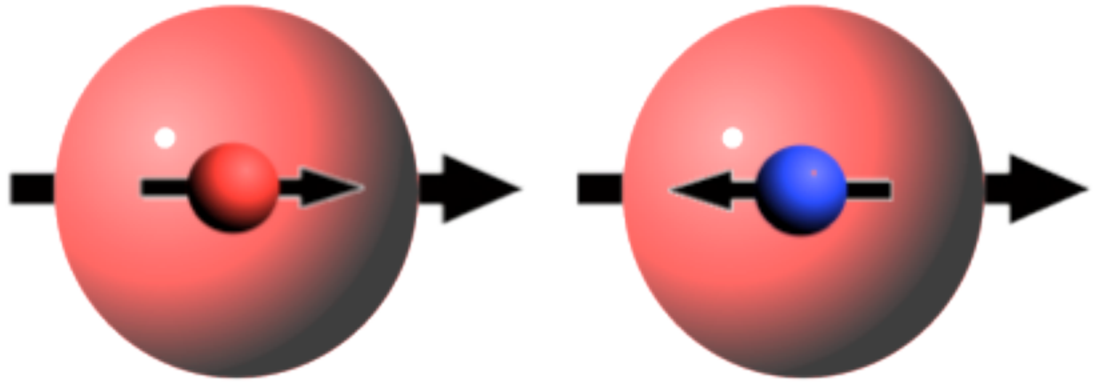
$$\rho(x,\vec{k}_\perp)$$



Longitudinal

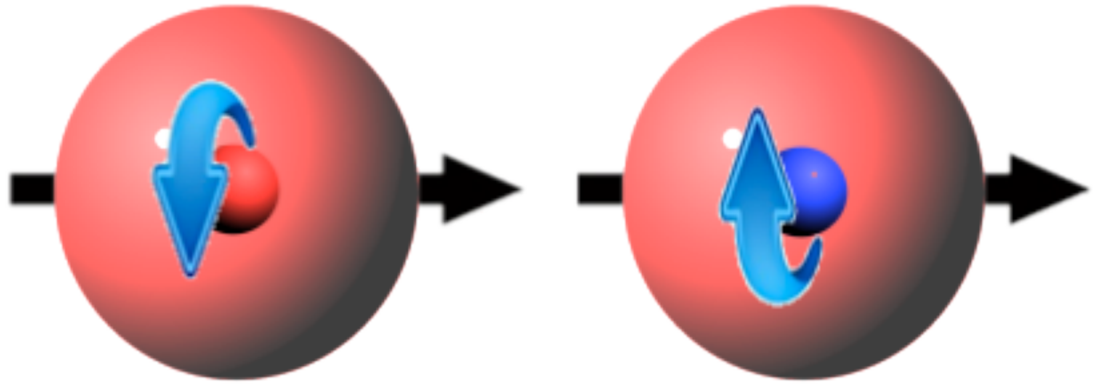
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$g_{1L}^q \leftrightarrow \tilde{\mathcal{H}}^q$



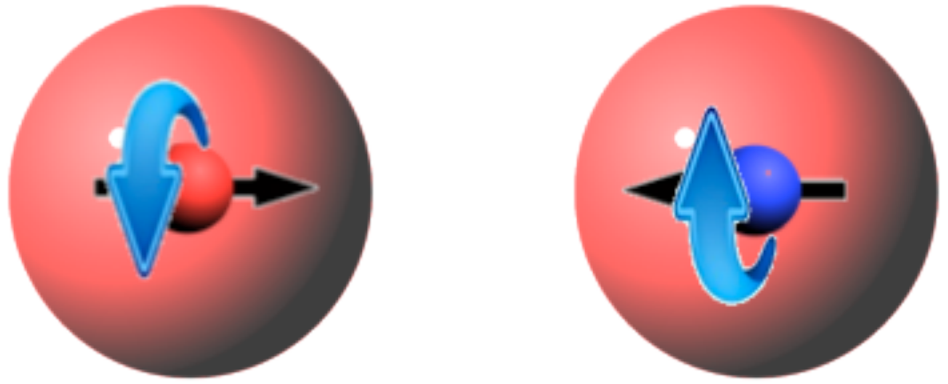
$\vec{b}_\perp, \vec{k}_\perp$

$\ell_z^q \leftrightarrow \tilde{\mathcal{F}}_{14}^q$



$\vec{b}_\perp, \vec{k}_\perp$

$C_z^q \leftrightarrow \tilde{\mathcal{G}}_{11}^q$

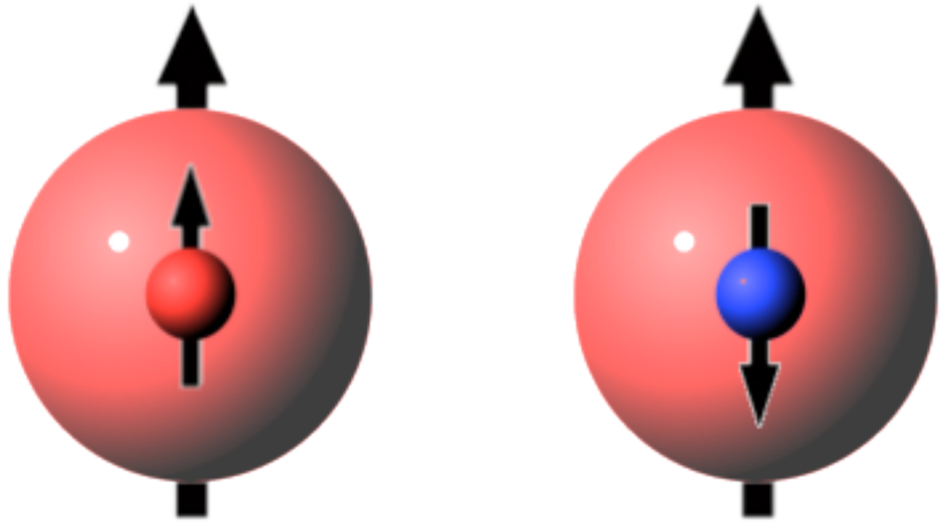


[C.L., Pasquini (2011)]

Transverse

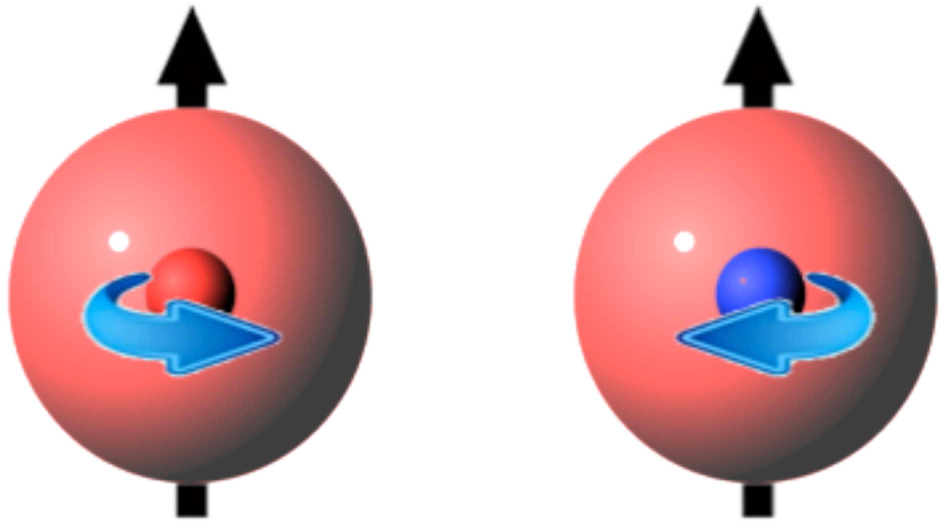
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$h_1^q \leftrightarrow \mathcal{H}_T^q$



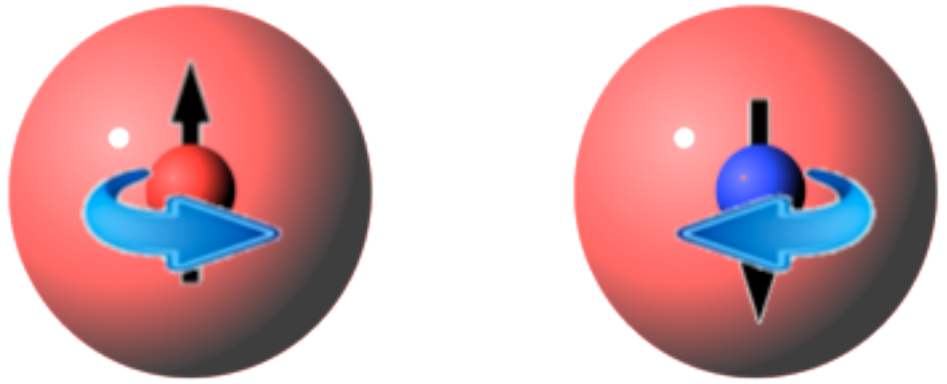
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$f_{1T}^{\perp q} \leftrightarrow \mathcal{E}^q$



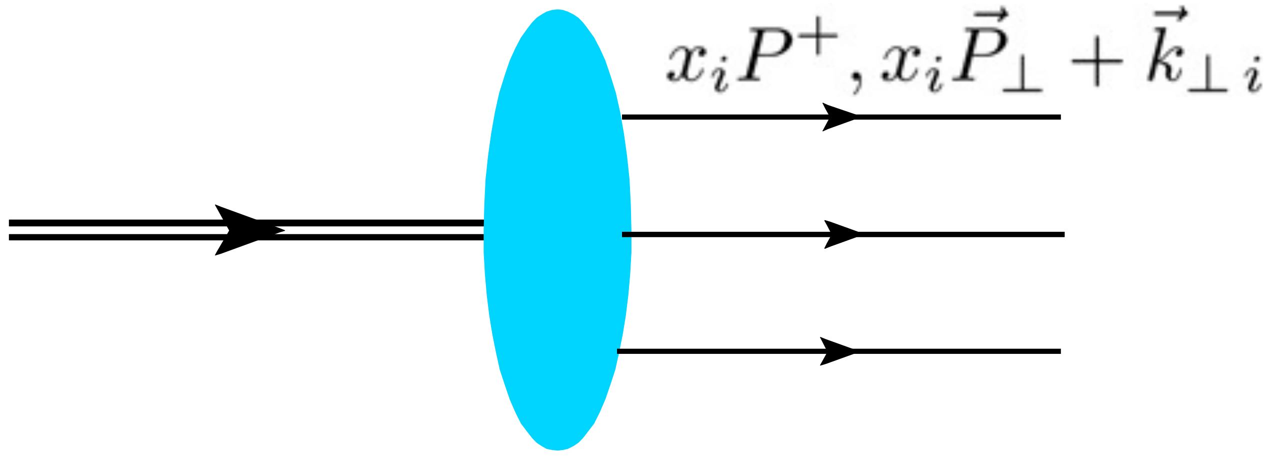
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$h_1^{\perp q} \leftrightarrow \mathcal{E}_T^q$



[Burkardt (2005)]
[Barone et al. (2008)]

Quark OAM: Partial-Wave Decomposition



$$| P, \Lambda \rangle = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \; \Psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i}) \; \frac{\varepsilon^{ijk}}{\sqrt{6}} \, u^{\dagger}_{i\lambda_1}(1) u^{\dagger}_{j\lambda_2}(2) d^{\dagger}_{k\lambda_3}(3) | \, 0 \rangle$$

↓

LCWF: eigenstate of OAM

(gauge A⁺=0 ⟶ Jaffe-Manohar)

J^q_z	→	$(\uparrow\uparrow\uparrow)_{LC} = \frac{3}{2}$	$(\uparrow\uparrow\downarrow)_{LC} = \frac{1}{2}$	$(\uparrow\downarrow\downarrow)_{LC} = -\frac{1}{2}$	$(\downarrow\downarrow\downarrow)_{LC} = -\frac{3}{2}$
$L^q_z = \frac{1}{2} - J^q_z$	→	$L^q_z = -1$	$L^q_z = 0$	$L^q_z = 1$	$L^q_z = 2$

${}^{L_z}\langle P, \uparrow | P, \uparrow \rangle^{L_z}$:probability to find the proton in a state with eigenvalue of OAM L_z

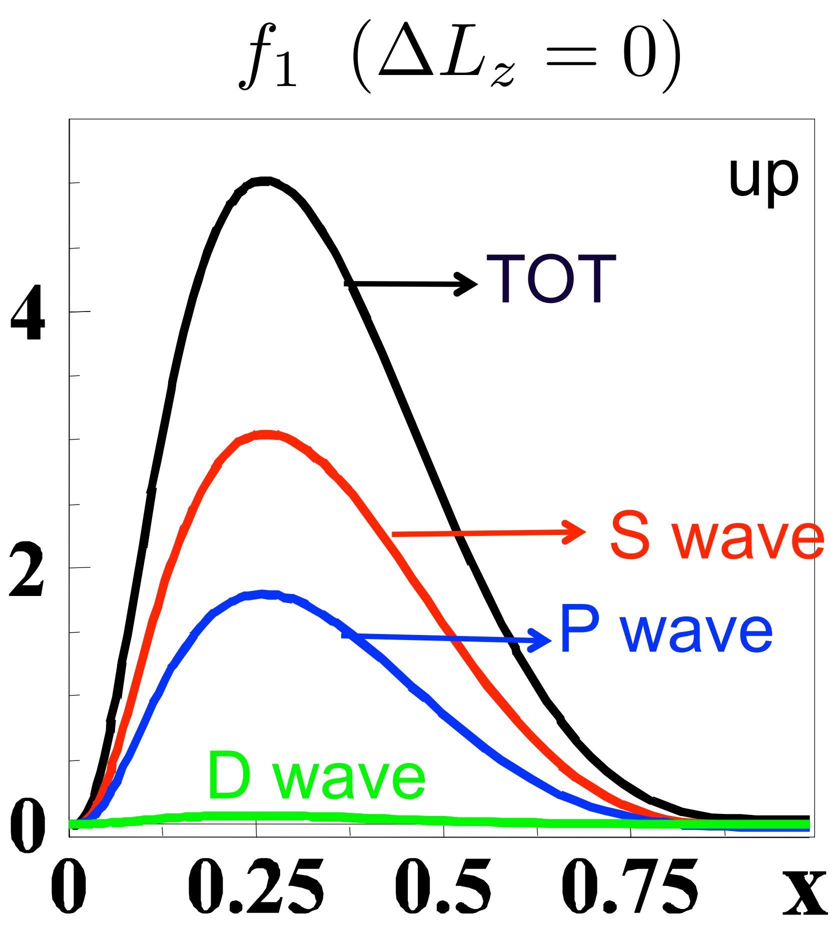
⬇

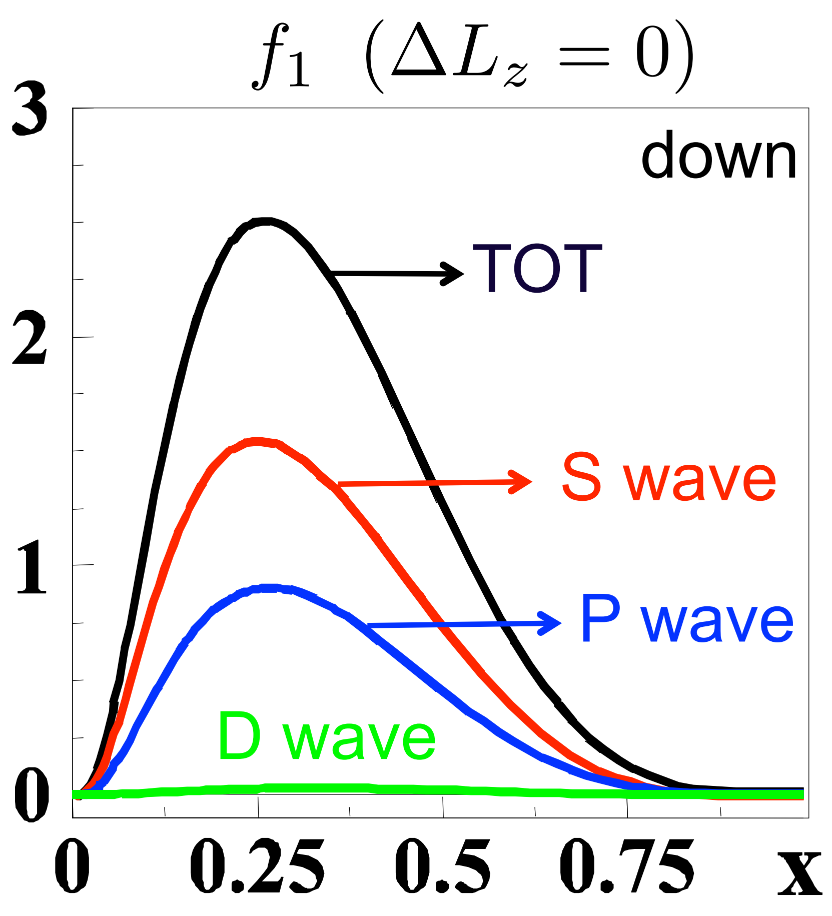
$$\mathcal{L}_z = \sum_{L_z} L_z \; {}^{L_z}\langle P, \uparrow | P, \uparrow \rangle^{L_z}$$

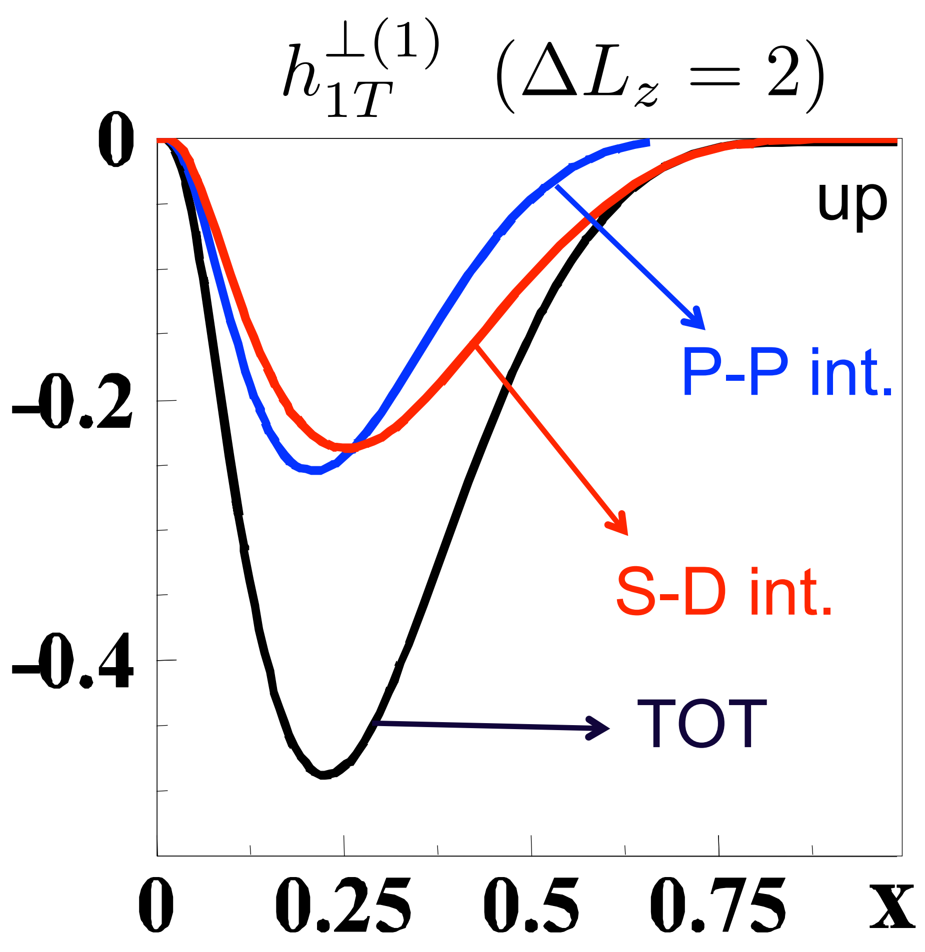
↓

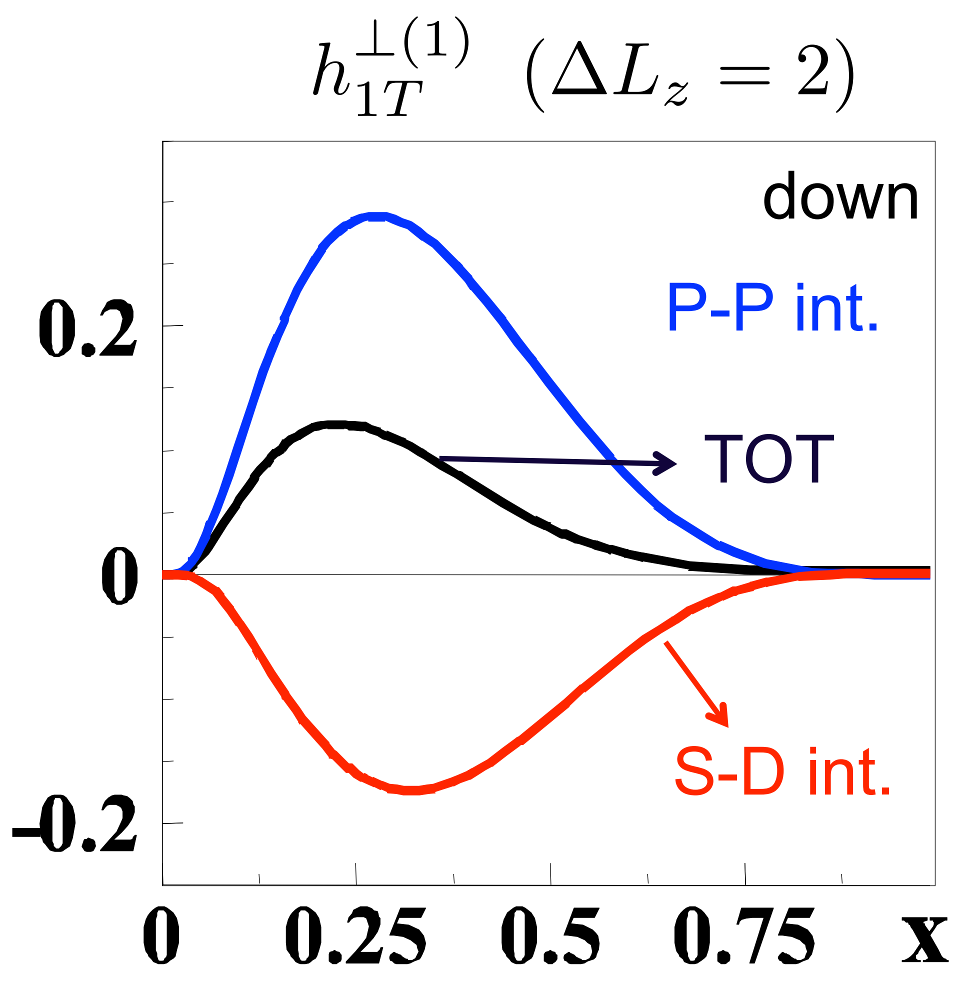
squared of LCWFs

✦ Orbital angular momentum content of TMDs (light-front constituent quark model)



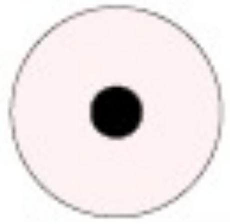




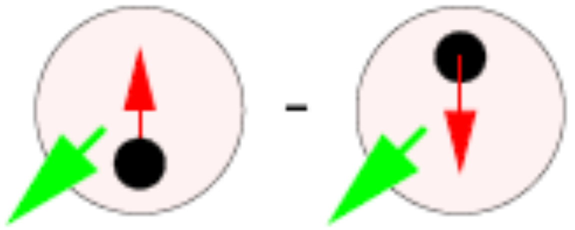


“pretzelosity”

$f_1 =$

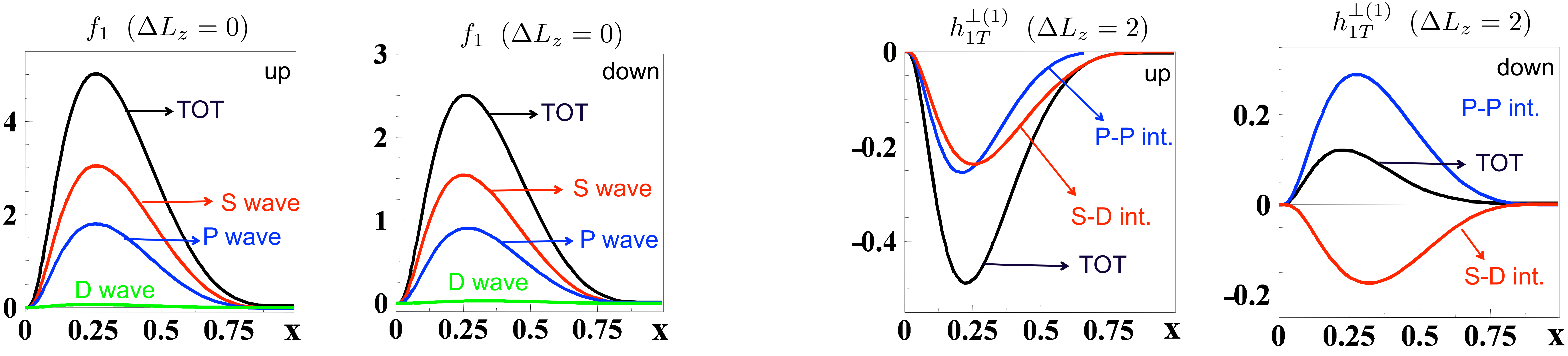


$h_{1T}^{\perp} =$



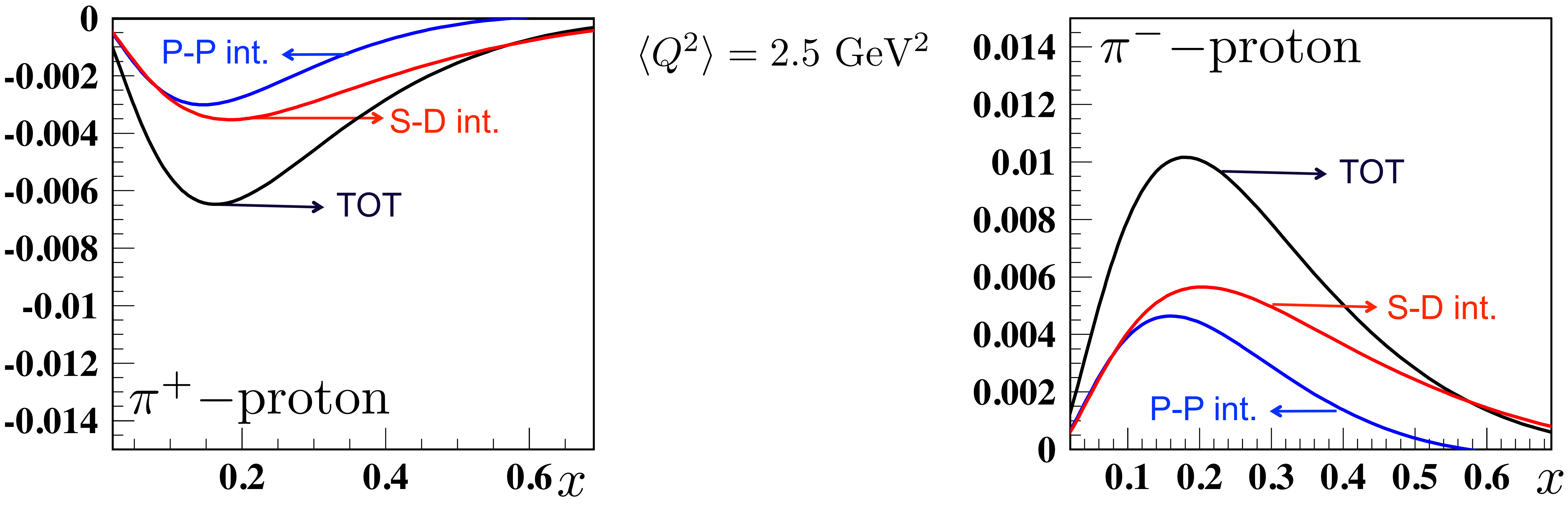
→ Efremov’s lecture on TMDs

✦ Orbital angular momentum content of TMDs (light-front constituent quark model)



✦ Effects on SIDIS observables

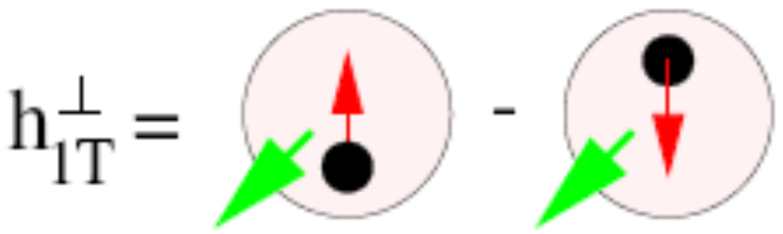
$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



Boffi, Efremov, BP, Schweitzer, PRD79(2009)

⟶ Efremov's lecture on TMDs

Quark OAM from Pretzelosity



“pretzelosity”

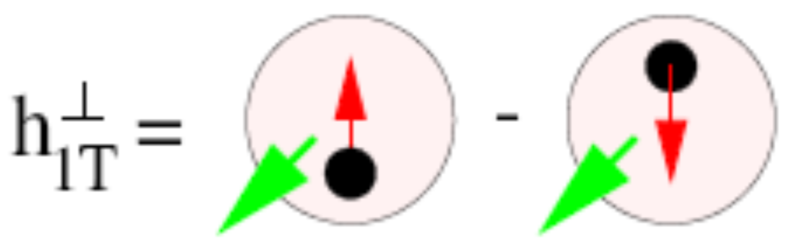
model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LC-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

Quark OAM from Pretzelosity



"pretzelosity"

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LC-diquark model and bag model

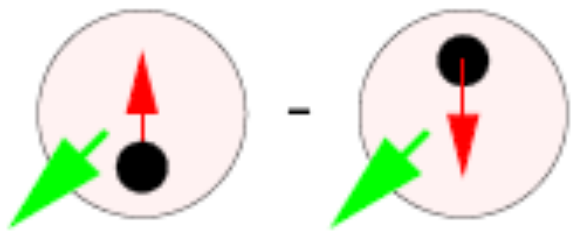
[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert\Delta L_z\rvert = 2$

no operator identity
relation at level of matrix elements of operators

Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$



“pretzelosity”

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

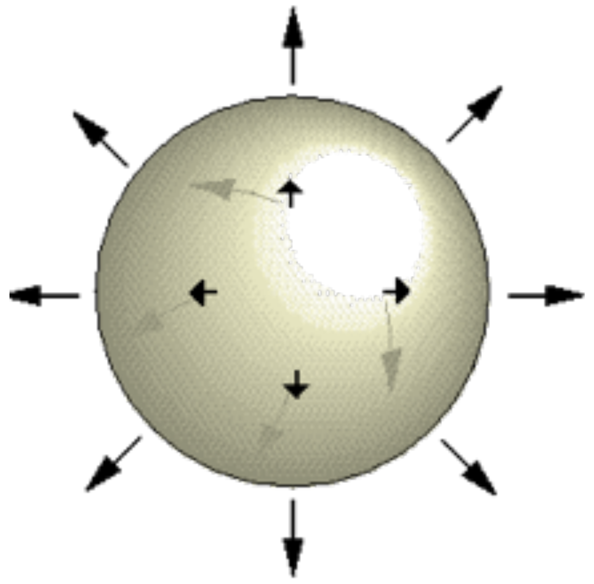
first derived in LC-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert \Delta L_z \rvert = 2$

no operator identity
relation at level of matrix elements of operators

$\Downarrow$



valid in all quark models with spherical symmetry in the rest frame

[Lorce’, BP, PLB (2012)]

→ Efremov’s lecture on TMDs

Quark spin and OAM

GTMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, G_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

polarized PDF
inclusive DIS

$$\ell_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, F_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorce, BP(2011)]
[Hatta (2011)]
[Lorce',BP, et al. (2012)]

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp}\times\vec{k}_{i\perp}$$

TMDs

Quark spin

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, g_{1L}^q(x,\vec{k}_\perp)$$

polarized PDF
inclusive DIS

$$\mathcal{L}_z^q(x,\vec{k}_\perp) = -\frac{\vec{k}_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x,\vec{k}_\perp^2)$$

[Burkardt (2007)]
[Efremov et al. (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian et al. (2010)]
[Lorce', BP (2011)]

⚠
- **Model-dependent**
- **Not intrinsic!**

$$\mathcal{L}_{iz} = \vec{r}_{i\perp}\times\vec{k}_{i\perp}$$

Quark spin and OAM

GTMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, G_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

polarized PDF
inclusive DIS

$$\ell_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorce, BP(2011)]
[Hatta (2011)]
[Lorce',BP, et al. (2012)]

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp}\times\vec{k}_{i\perp}$$

TMDs

Quark spin

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, g_{1L}^q(x,\vec{k}_\perp)$$

polarized PDF
inclusive DIS

$$\mathcal{L}_z^q(x,\vec{k}_\perp) = -\frac{\vec{k}_\perp^2}{2M^2}\,h_{1T}^{\perp q}(x,\vec{k}_\perp^2)$$

[Burkardt (2007)]
[Efremov et al. (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian et al. (2010)]
[Lorce', BP (2011)]

⚠

- **Model-dependent**
- **Not intrinsic!**

$$\mathcal{L}_{iz} = \vec{r}_{i\perp}\times\vec{k}_{i\perp}$$

GPDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\tilde{H}^q(x,0,0)$$

polarized PDF
inclusive DIS

Ji sum rule

$$J^q = \tfrac{1}{2}\int \mathrm{d}x\,x\,[H^q(x,0,0)+E^q(x,0,0)]$$

$$L^q = J^q - S_z^q$$

[Ji (1997)]

Twist-3

$$L_z^q + 2S_z^q = -\int \mathrm{d}x\,x\,\tilde{E}_{2T}^q(x,0,0)$$

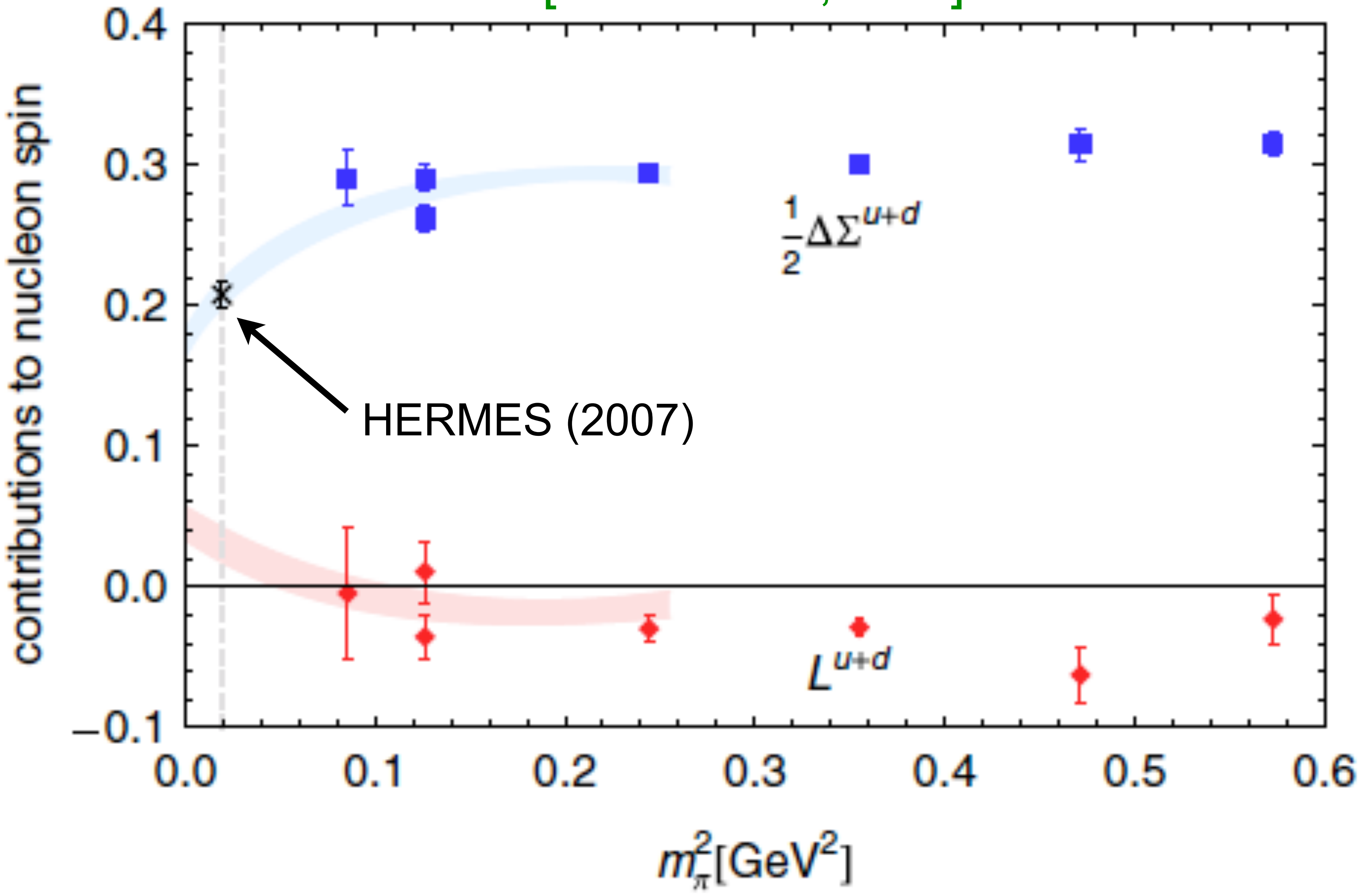
$$L_z^q = -\int \mathrm{d}x\,x\,G_2^q(x,0,0)$$

Pure twist-3!

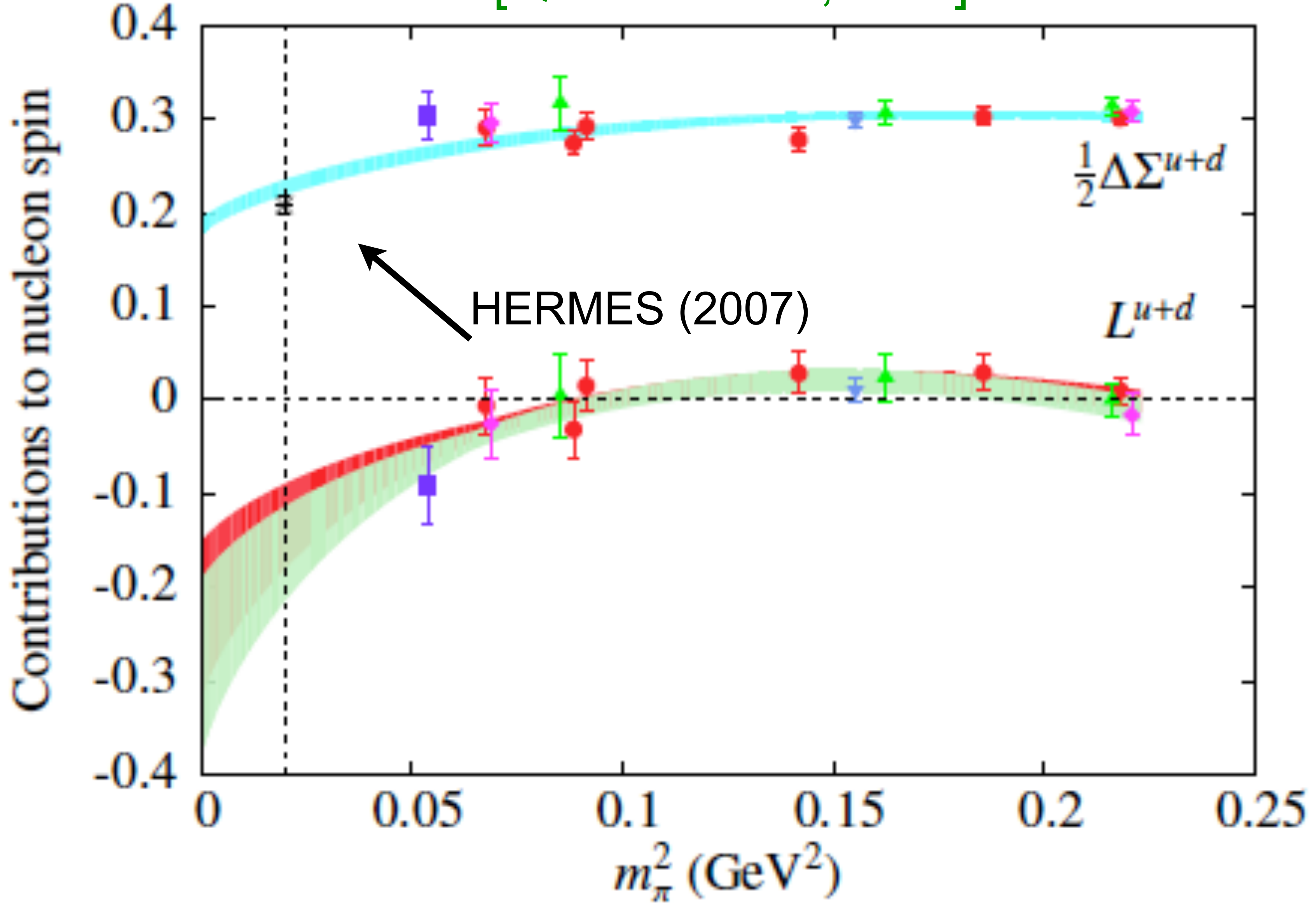
[Penttinen et al. (2000)]

Lattice Results

[LHPC Coll.,2010]



[QCDSF Coll.,2013]



- “disconnected diagrams” not included
- Error bands: chiral extrapolation in m_π and extrapolation to t=0

Lattice results (μ = 2 GeV)

J^{u+d}=0.264(6) $\Delta\ \Sigma^{u+d}$ /2= 0.208(10) L^{u+d} =0.056(11) [LHPC Coll.,2010]

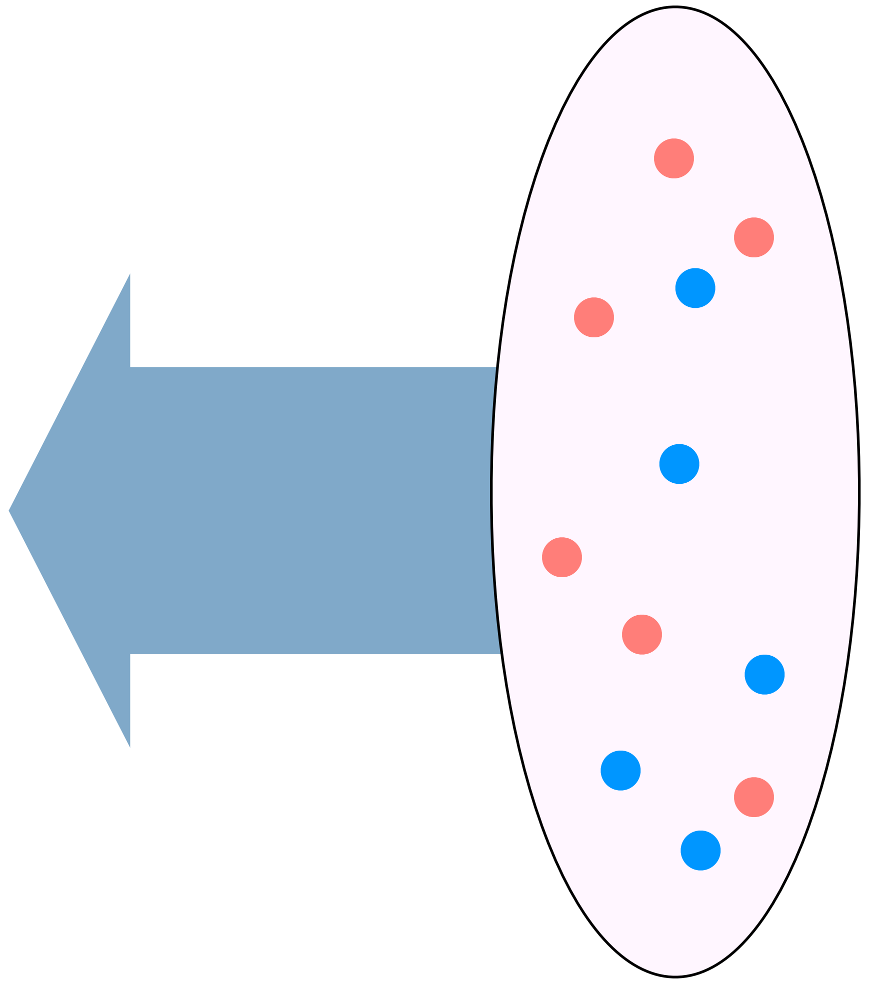
J^{u+d}=0.168(42) $\Delta\ \Sigma^{u+d}$ /2= 0.225(8) L^{u+d} =-0.141(27) [QCDSF Coll.,2013]

↓

cancelation between L^u <0 and L^d >0

Constraining quark OAM with Sivers function

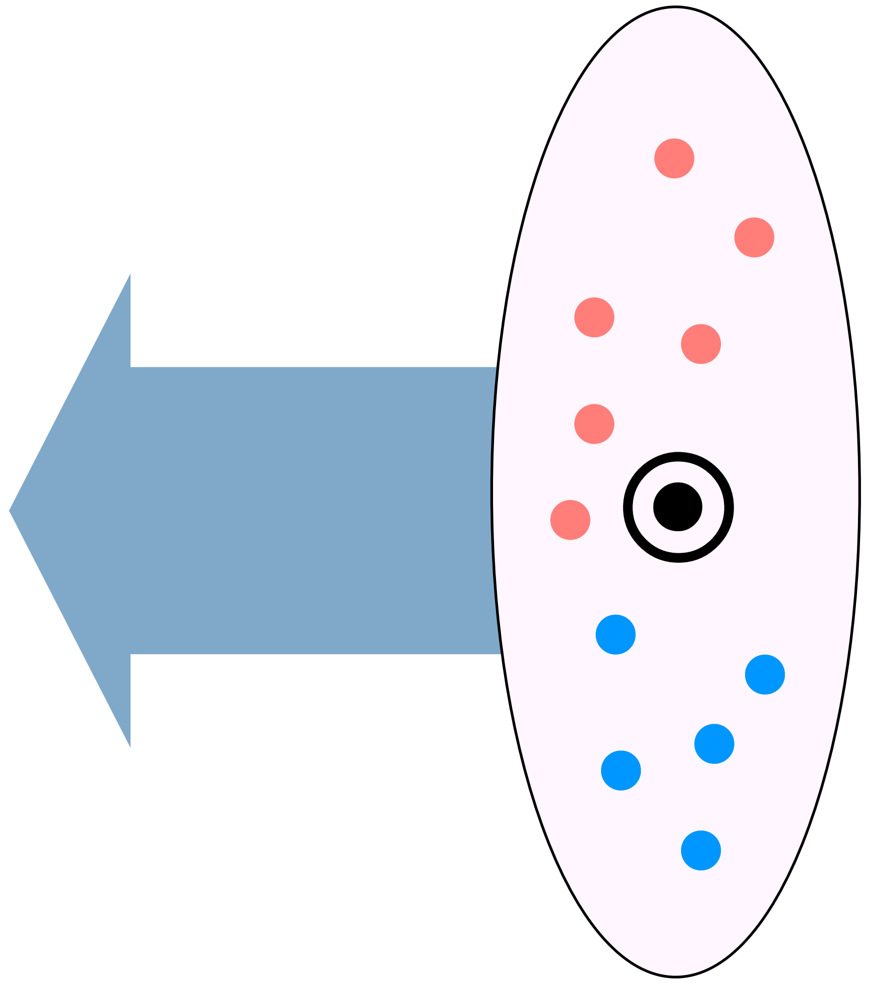
unpolarized quark in unpolarized nucleon



Burkardt, PRD66 (02)

Constraining quark OAM with Sivers function

unpolarized quark in transversely pol. nucleon

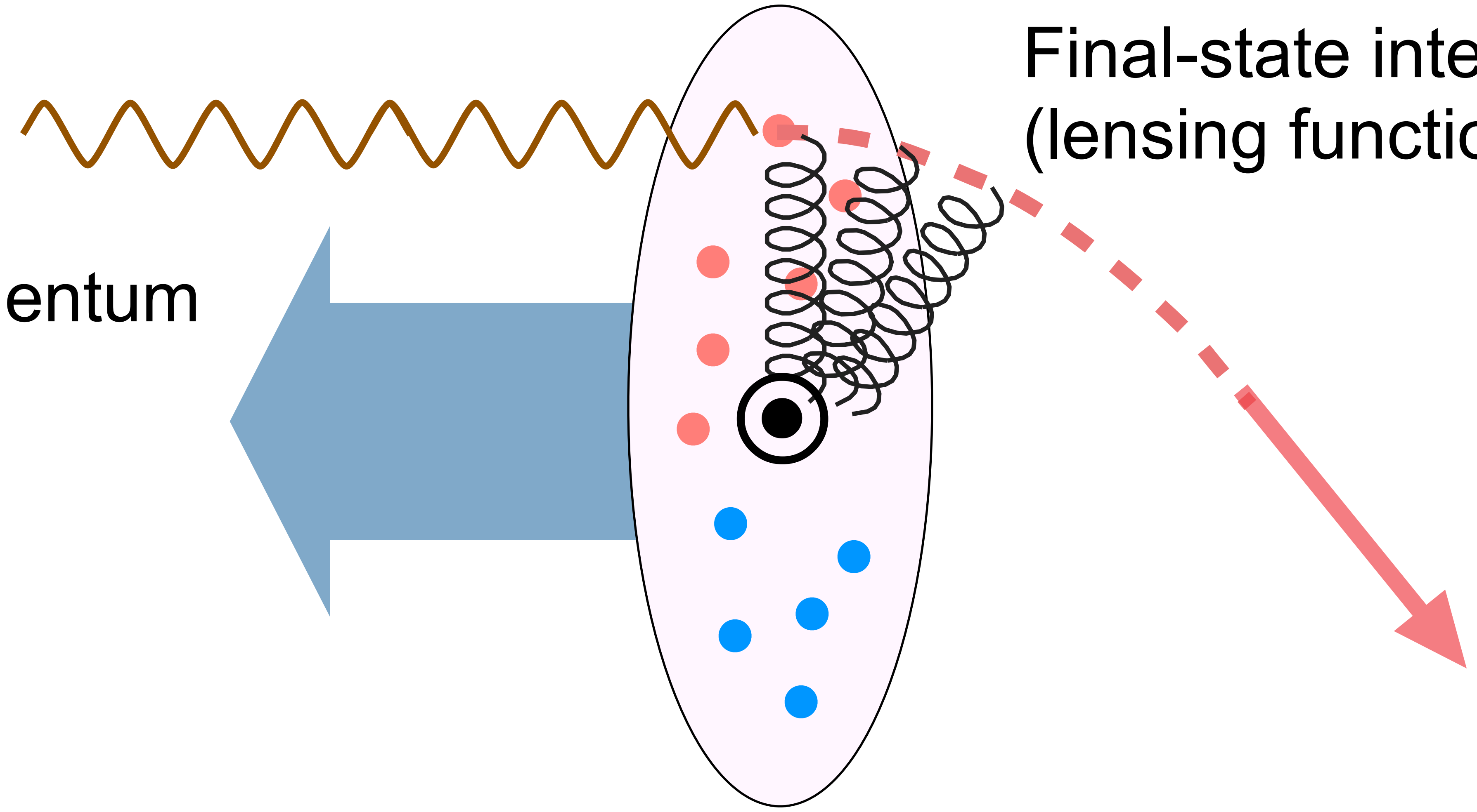


Distortion in impact parameter
(related to GPD E)

Burkardt, PRD66 (02)

Constraining quark OAM with Sivers function

unpolarized quark in transversely pol. nucleon



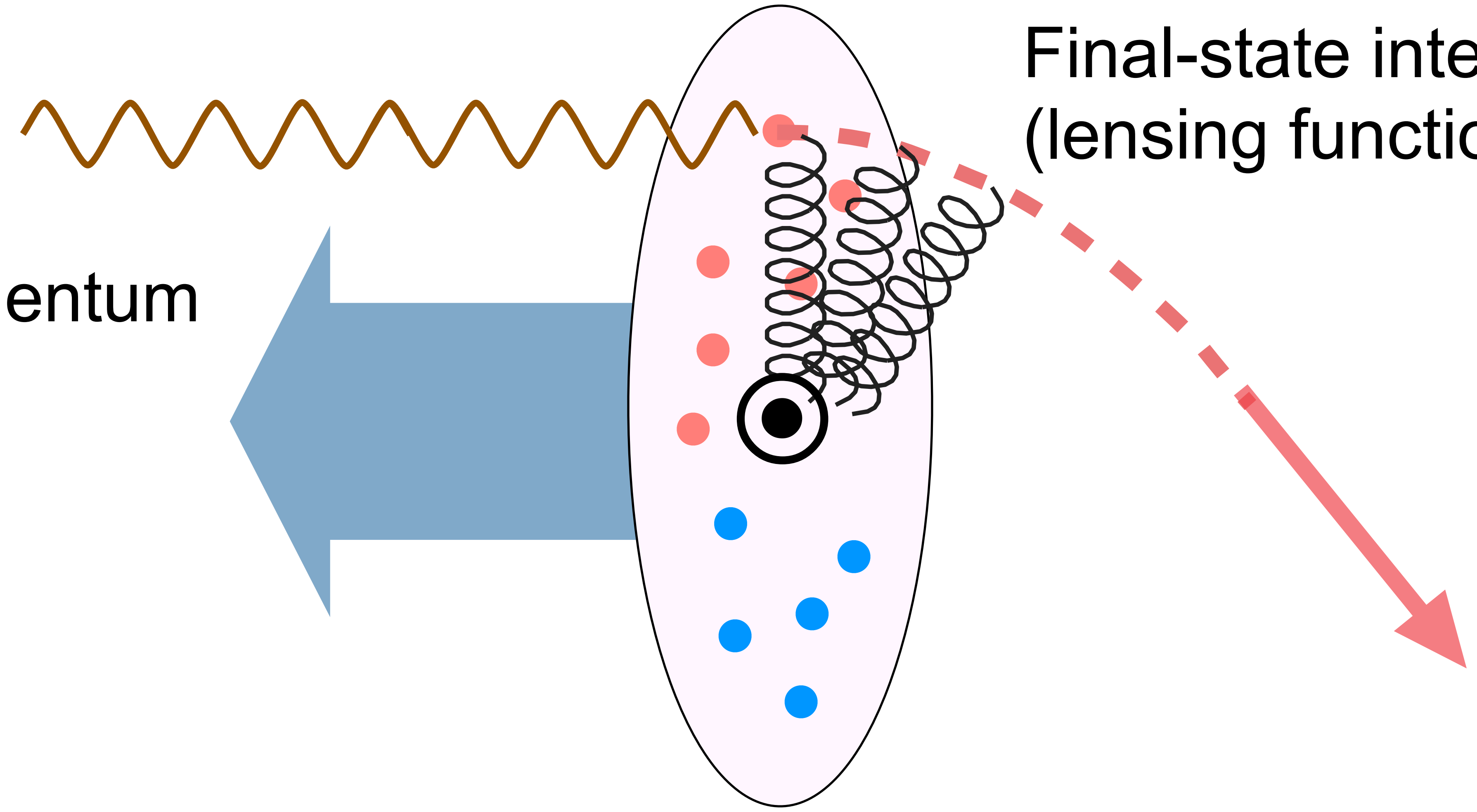
Final-state interaction
(lensing function)

Distortion in transverse momentum
(related to Sivers function)

Burkardt, PRD66 (02)

Constraining quark OAM with Sivers function

unpolarized quark in transversely pol. nucleon



Final-state interaction
(lensing function)

Distortion in transverse momentum
(related to Sivers function)

Burkardt, PRD66 (02)

$$-\int d^2\vec{k}_T\, k_T^i \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} f_{1T}^{\perp q}(x,\vec{k}_T^2) \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\frac{\epsilon_T^{jk} b_T^j S_T^k}{M}\left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

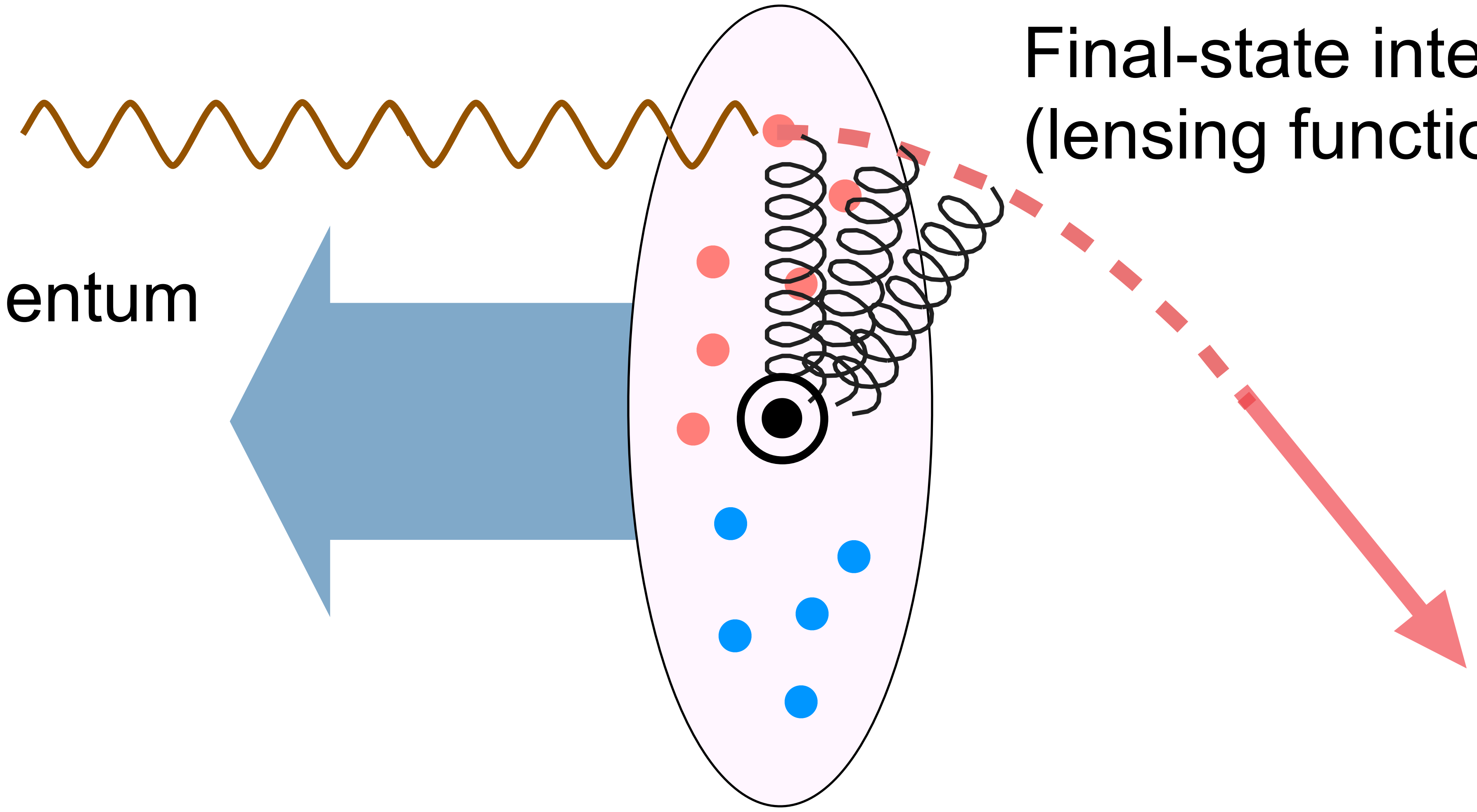
Sivers function

Lensing function

F.T. of E(x,0,t)

Constraining quark OAM with Sivers function

unpolarized quark in transversely pol. nucleon



Final-state interaction
(lensing function)

Distortion in transverse momentum
(related to Sivers function)

Burkardt, PRD66 (02)

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \;\simeq\; \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

F.T. of E(x,0,t)

inspired from model results

Bacchetta, Radici, PRL107(2011)

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\,E^a(x,0,0;Q_L^2)$$

fitted to SIDIS data
(COMPASS,HERMES,JLab)

flavor independent
$\frac{K}{(1-x)^\eta}$

first moment constrained
from anomalous magnetic moment

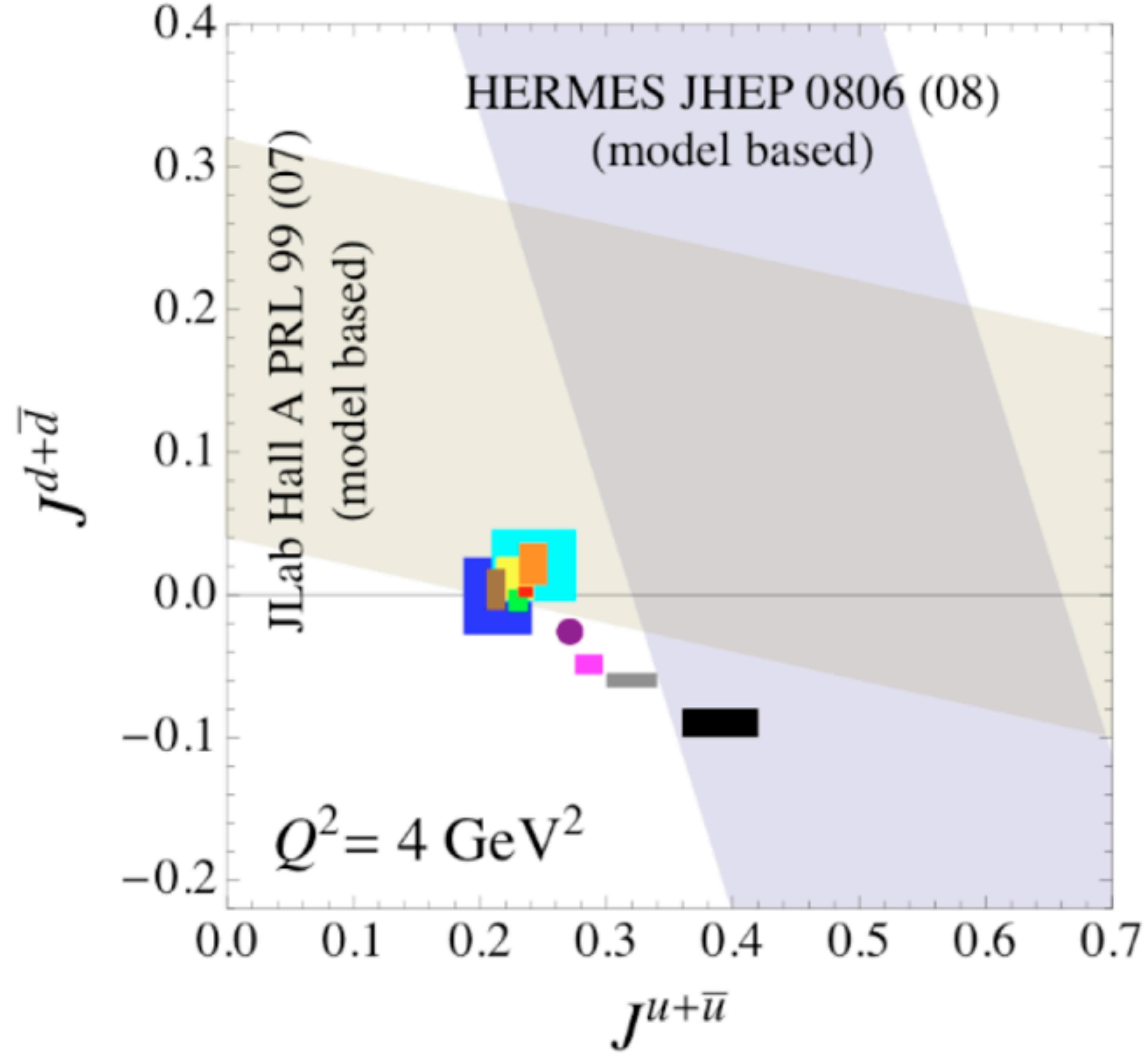
- Results from Sivers ⇦ lensing ⇨ GPD

Bacchetta, Radici, PRL107(2011)

$$J^u = 0.229 \pm 0.002^{+0.008}_{-0.012}, \qquad J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000},$$
$$J^d = -0.007 \pm 0.003^{+0.020}_{-0.005}, \qquad J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000},$$
$$J^s = 0.006^{+0.002}_{-0.006}, \qquad J^{\bar{s}} = 0.006^{+0.000}_{-0.005}.$$

$(Q^2 = 4\ \mathrm{GeV}^2)$

- Comparing with GPD models and Lattice calculations



- 🟦 Goloskokov and Kroll, EPJC59, (09) 509
- 🟫 Diehl at al, EPJC39 (05)1
- 🟪 Guidal et al, PRD72 (05) 054013
- 🟪 Liuti et al., PRD84 (11) 034007
- 🟧 Bacchetta and Radici, PRL 107 (11) 211801
- 🟦 LHPC-1, PRD 77 (08) 094502
- 🟥 LHPC-2, PRD 82 (10) 094502
- 🟩 QCDSF, PoS LAT2007 (2007) 158
- 🟨 Wakamatsu, EPJ A44, (10)297
- ⬛ Thomas, PRL 101 (08) 102003
- ◼ Thomas, INT 2012 workshop

Summary

- GTMDs Wigner Distributions
  - the most complete information on partonic structure of the nucleon

Summary

- GTMDs Wigner Distributions
  - the most complete information on partonic structure of the nucleon

- Results for Wigner distributions in the transverse plane
  - non-trivial correlations between $\vec{b}_\perp$ and $\vec{k}_\perp$ due to orbital angular momentum

Summary

- ❖ GTMDs Wigner Distributions
  - the most complete information on partonic structure of the nucleon

- ❖ Results for Wigner distributions in the transverse plane
  - non-trivial correlations between $\vec{b}_\perp$ and $\vec{k}_\perp$ due to orbital angular momentum

- ❖ Orbital Angular Momentum from phase-space average with Wigner distributions

Summary

- GTMDs Wigner Distributions
  - the most complete information on partonic structure of the nucleon
- Results for Wigner distributions in the transverse plane
  - non-trivial correlations between $\vec{b}_\perp$ and $\vec{k}_\perp$ due to orbital angular momentum
- Orbital Angular Momentum from phase-space average with Wigner distributions
- GPDs and TMDs probe the same overlap of 3-quark LCWF in different kinematics
  - no model-independent relations between GPDs and TMDs
  - give complementary information useful to reconstruct the nucleon wf

Summary

- GTMDs Wigner Distributions
  - the most complete information on partonic structure of the nucleon

- Results for Wigner distributions in the transverse plane
  - non-trivial correlations between $\vec{b}_\perp$ and $\vec{k}_\perp$ due to orbital angular momentum

- Orbital Angular Momentum from phase-space average with Wigner distributions

- GPDs and TMDs probe the same overlap of 3-quark LCWF in different kinematics
  - no model-independent relations between GPDs and TMDs
  - give complementary information useful to reconstruct the nucleon wf

- No direct connection between TMDs and OAM ➡ need to use model-inspired connections
  - use LCWF (eigenstate of quark OAM) to quantify amount of OAM in different observables
  - model relation between pretzelosity and OAM
  - OAM from model relation between Sivers function and GPD E