Generalized Parton Distributions

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- *Introductory to GPDs*
- *Status of Theory and Phenomenology*
- *The concept of effective LCWF*
- *Conclusions*

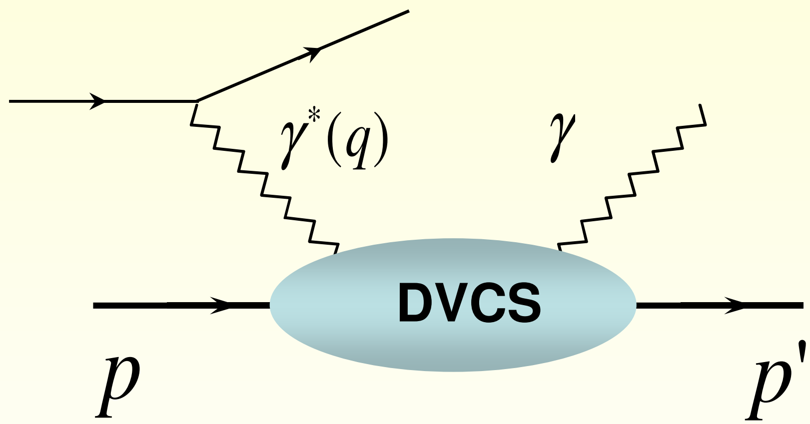
K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray
K. Passek-Kumerički (KP-K), T. Lautenschlager, A. Schäfer; M. Meskauskas
A. Belitsky, Y. Ji; V. Braun, A. Manashov, B. Pirnay
D.S. Hwang

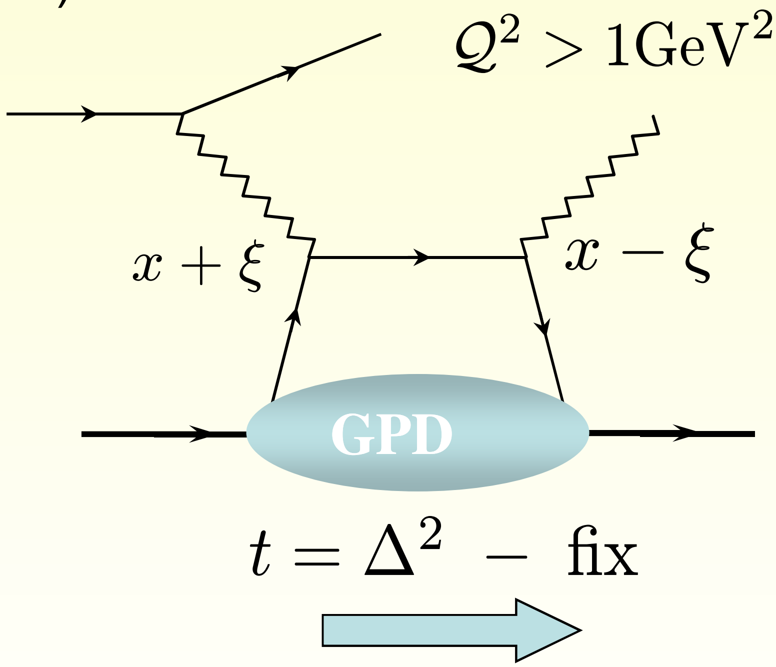
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

[DM et. al (91/94)
Radyushkin (96)
Ji (96)]

e.g., hard electroproduction of photons (DVCS)





$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

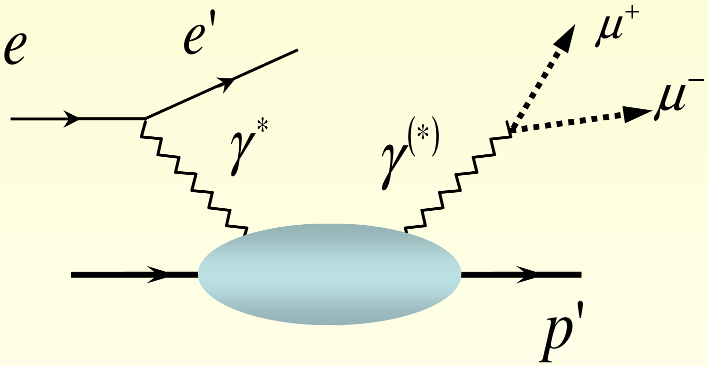
GPD
universal
(conventional)

higher twist
depends on
approximation

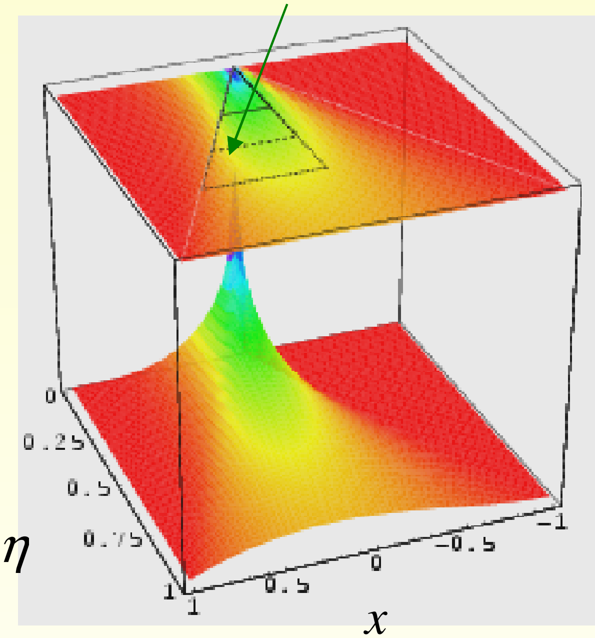
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$



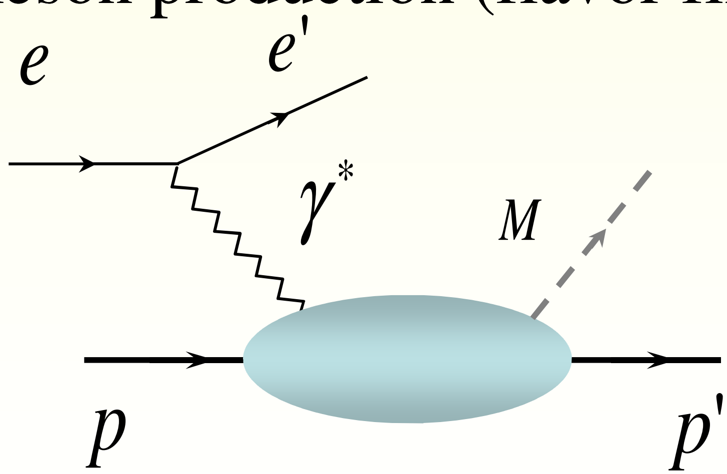
scanned area of the surface as a functions of lepton energy



$ep \to e'\,p'\,\mu^+\mu^-$

- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



twist-two observables:

longitudinal cross sections

transverse target spin asymmetries

- etc.

factorization proof for longitudinal cross sections

[Collins, Frankfurt, Strikman (96)]

Field theoretical GPD definition

GPDs are defined as matrix elements of **renormalized** ***light-ray*** operators:

DM, Robaschik, Geyer, Dittes, Hořejši (94)

$$F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa\, x\, n\cdot P} \langle P_2 | \mathcal{R}T : \phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n) : |P_1\rangle, \;\; n^2 = 0$$

momentum fraction x , skewness $\eta = \frac{n\cdot\Delta}{n\cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\bar\psi_i \gamma_+ \psi_i \quad \Rightarrow \quad {}^{i}q^{V} = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar\psi_i \gamma_+ \gamma_5 \psi_i \quad \Rightarrow \quad {}^{i}q^{A} = \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1) \widetilde H_i + \bar U(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde E_i$$

shorthands:

chiral even GPDs: $F = \{H, E, \widetilde H, \widetilde E\}$ & CFFs: $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T = \{H_T, E_T, \widetilde H_T, \widetilde E_T\}$ $\mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \widetilde{\mathcal{H}}_T, \widetilde{\mathcal{E}}_T\}$

GPD properties (from definition)

- *polynomiality arises from Lorentz covariance*
  (but GPDs are not Lorentz invariant or covariant)

$$\int_{-1}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

- *satisfied within double distribution representation*

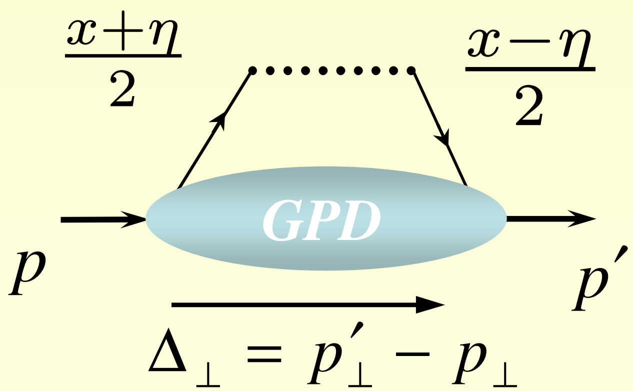
$$F(x,\eta,t) = (1-x)^p \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta) f(y,z,t), \;\; p=\{0,1\}$$

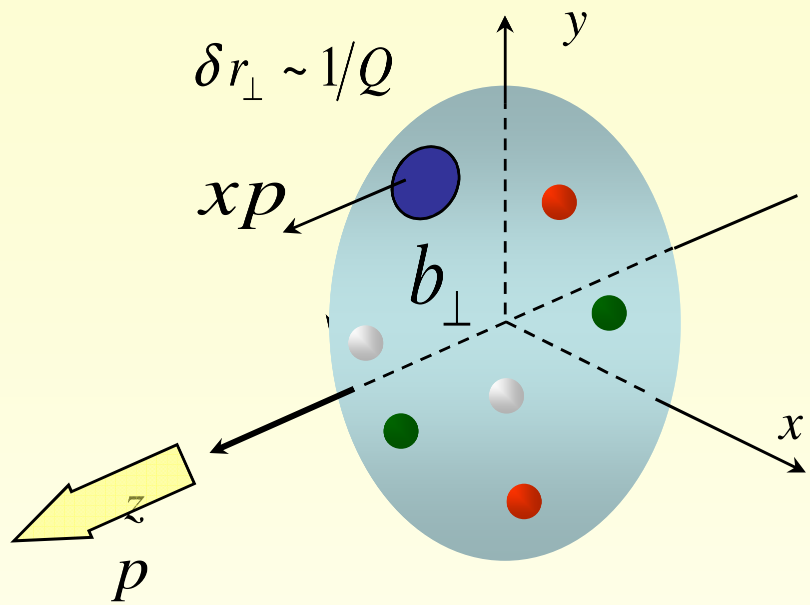
- *lowest moment: partonic form factor – related to observables*
- *first moment: expectation value of energy-momentum tensor*
- *reduction to parton densities (PDFs)* ***[Ji (96)]***

$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

- *positivity constraints (requirement on GPD and scheme)* ***[Pobylitsa(00,02)]***
  are automatically satisfied in the overlap representation

Partonic interpretation of GPDs

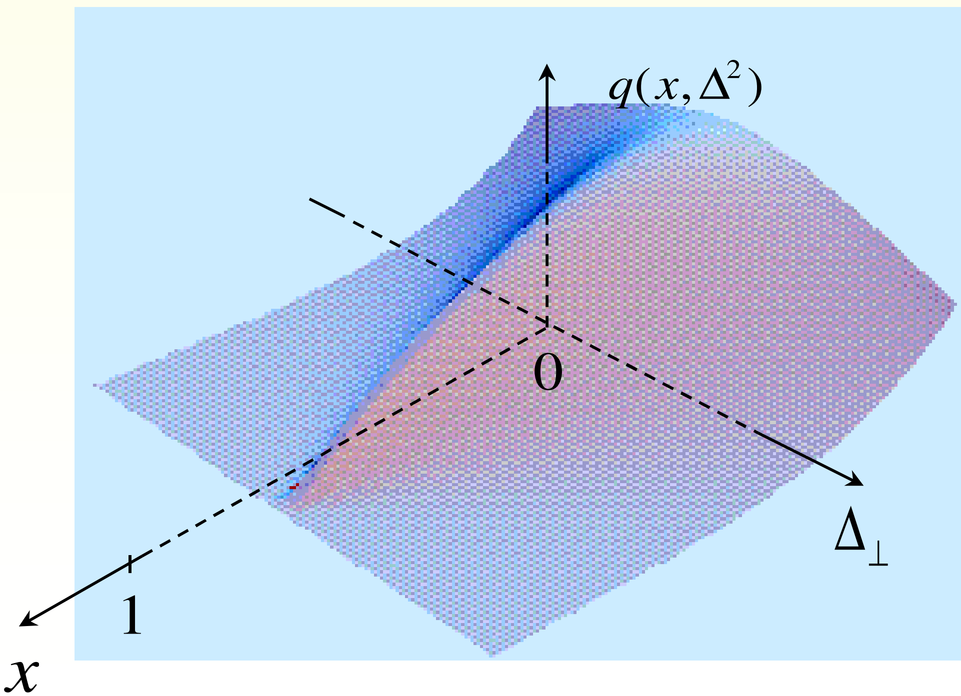




GPDs simultaneously carry information on *longitudinal* and *transverse* distribution of partons in a proton

for η=0 they have a probabilistic interpretation (infinite momentum frame) **[Burkhardt (00)]**

$$b_\perp = \sqrt{4 \frac{d}{dt} \ln H(x,0,t)}\Big|_{t=0}$$



GPDs contain also information on partonic angular momentum **[X. Ji (96)]**

$$\frac{1}{2} = \sum_{a=q,G} J_a^z$$

$$J_a^z = \lim_{\Delta \to 0} \frac{1}{2} \int_{-1}^{1} dx\, x\, (H_a + E_a)(x,\eta,\Delta^2)$$

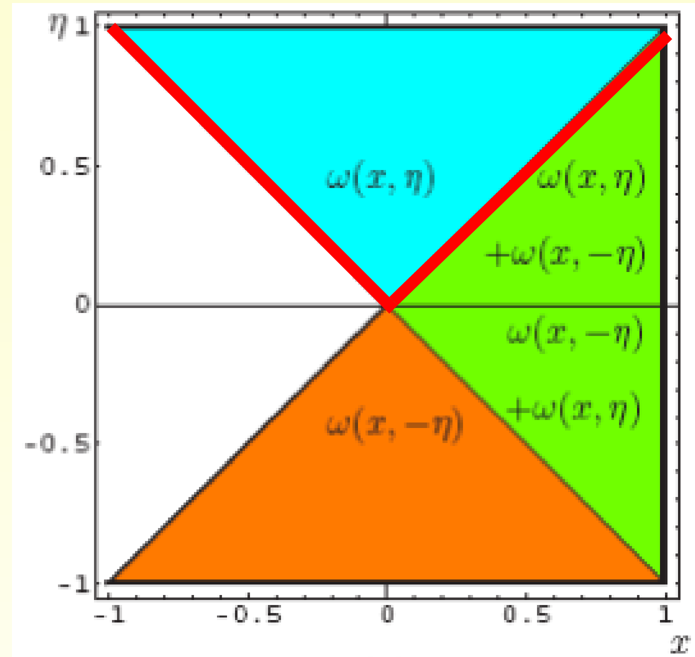
A partonic duality interpretation

quark GPD (anti-quark $x \to -x$):

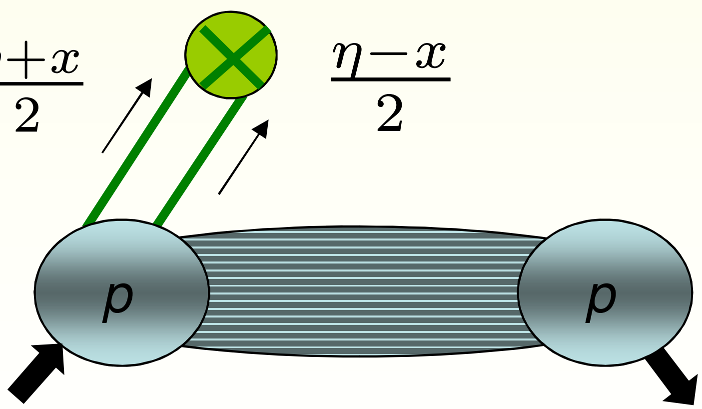
$$F(x,\eta,t) \quad =$$

$$\theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y,(x-y)/\eta,t)$$

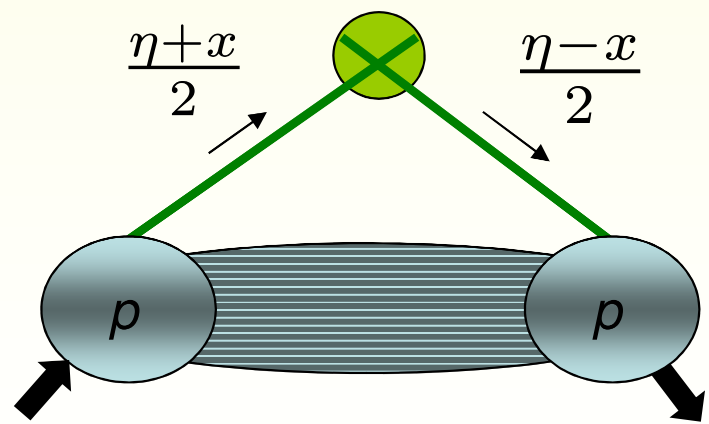


dual interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region - η < x < η

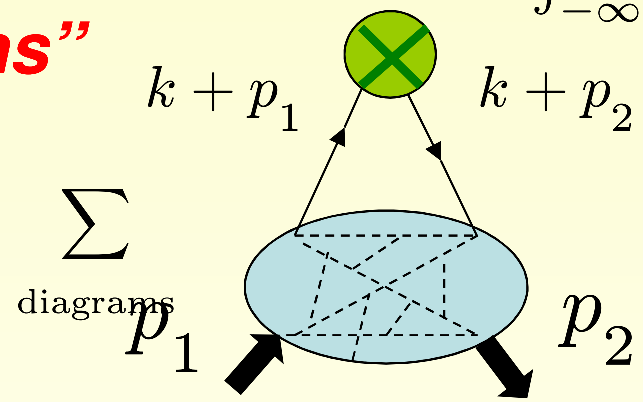
mesonic exchange in *t*-channel

outer region η < x

partonic exchange in *s*-channel

Overview: GPD representations

$$\equiv \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi}\, e^{i\kappa(xP_+ - P_+ - 2k_+)}$$



``light-ray spectral functions"

diagrammatic α-representation

DM, Robaschik, Geyer, Dittes, Hořejši (88 (92) 94)

called ***double distributions***

A. Radyushkin (96)

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann, Jakob, Kroll (98,00)

Diehl, Brodsky, Hwang (00)

SL(2,R) (conformal) expansion

(series of local operators)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05); Manashov, Kirch, Schäfer (05) ….

one version is called Shuvaev transformation, used in `dual' (*t*-channel) GPD parameterization

Shuvaev (99,02); Noritzsch (00) Polyakov (02,07)

⇒ each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out)**

Can one `measure' GPDs?

- ***CFF*** given as ***GPD*** **convolution**:

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\stackrel{\mathrm{LO}}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \mathrm{PV}\int_{0}^{1} dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$

- $H(x,x,t,\mathcal{Q}^2)$ viewed as **"spectral function"** (s-channel cut):

$$H^{-}(x,x,t,Q^2) \equiv H(x,x,t,Q^2) - H(-x,x,t,Q^2) \stackrel{\mathrm{LO}}{=} \frac{1}{\pi}\Im\mathrm{m}\mathcal{F}(\xi=x,t,Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- ***CFFs*** satisfy **`dispersion relations'**
(not the physical ones, threshold ξ_0 set to 1)

$$\Re\mathrm{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\mathrm{PV}\int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \Im\mathrm{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

[Terayev (05)]

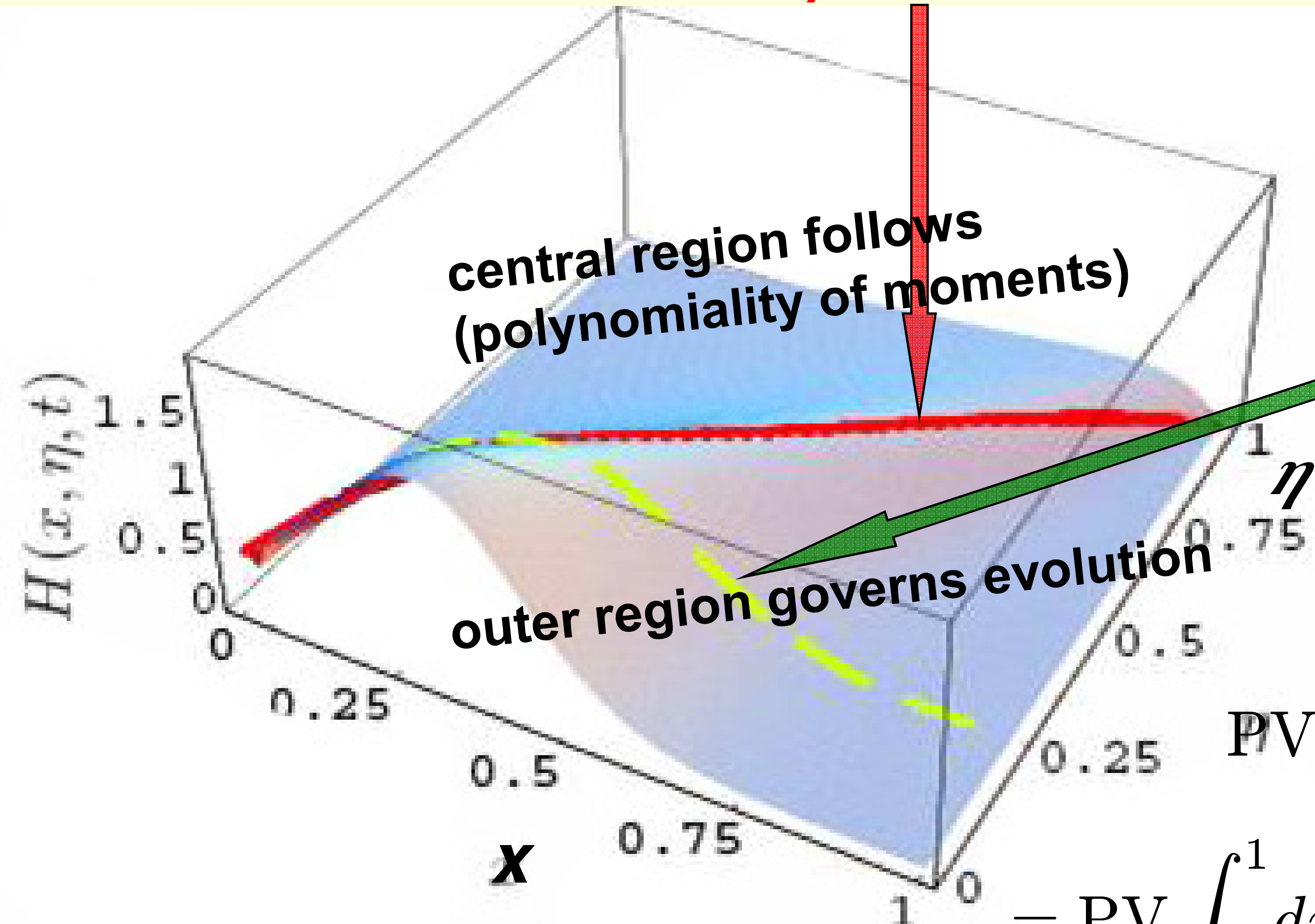
⇒ ***access*** to the ***GPD*** on the **cross-over line** $\eta = x$ (at LO)

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)



net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV} \int_0^1 dx \, \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$

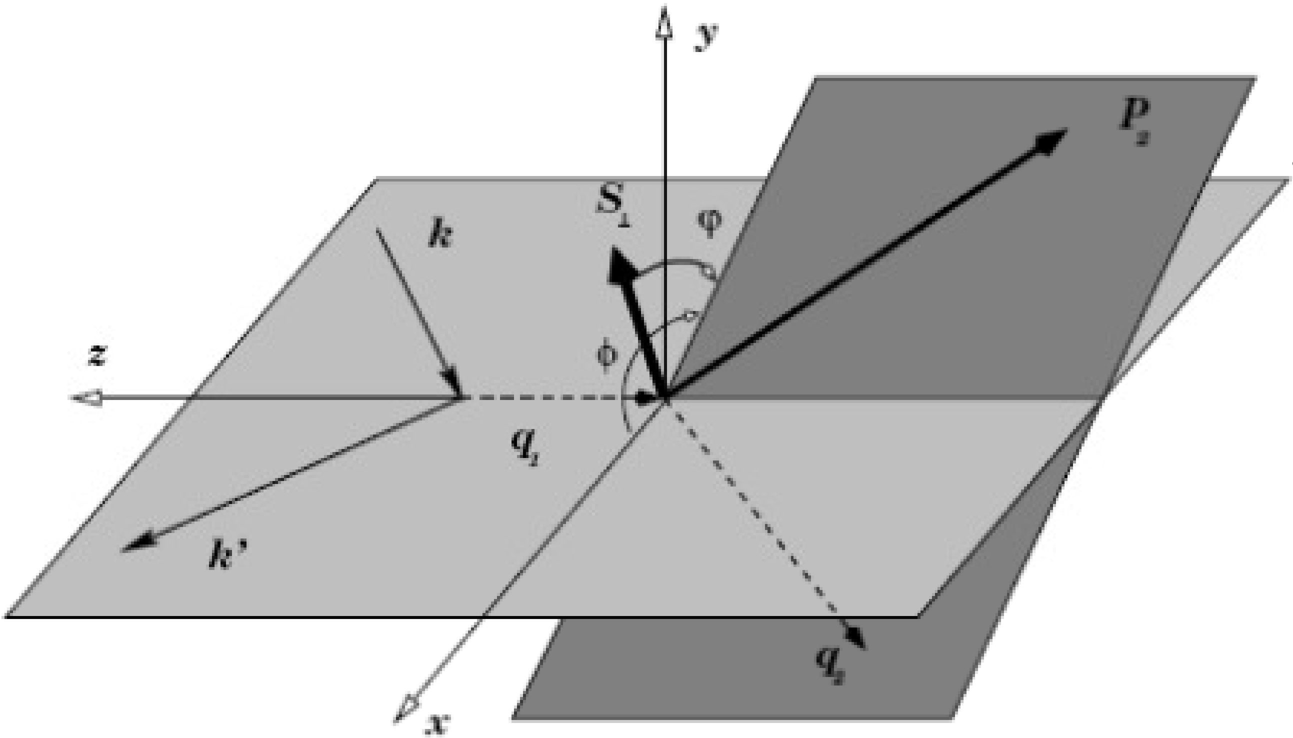
$$= \mathrm{PV} \int_0^1 dx \, \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + C(t)$$

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

planed at ***COMPASS, JLAB@12GeV***, perhaps at ?? EIC,

$$\frac{d\sigma}{dx_{\rm Bj}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\rm Bj} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



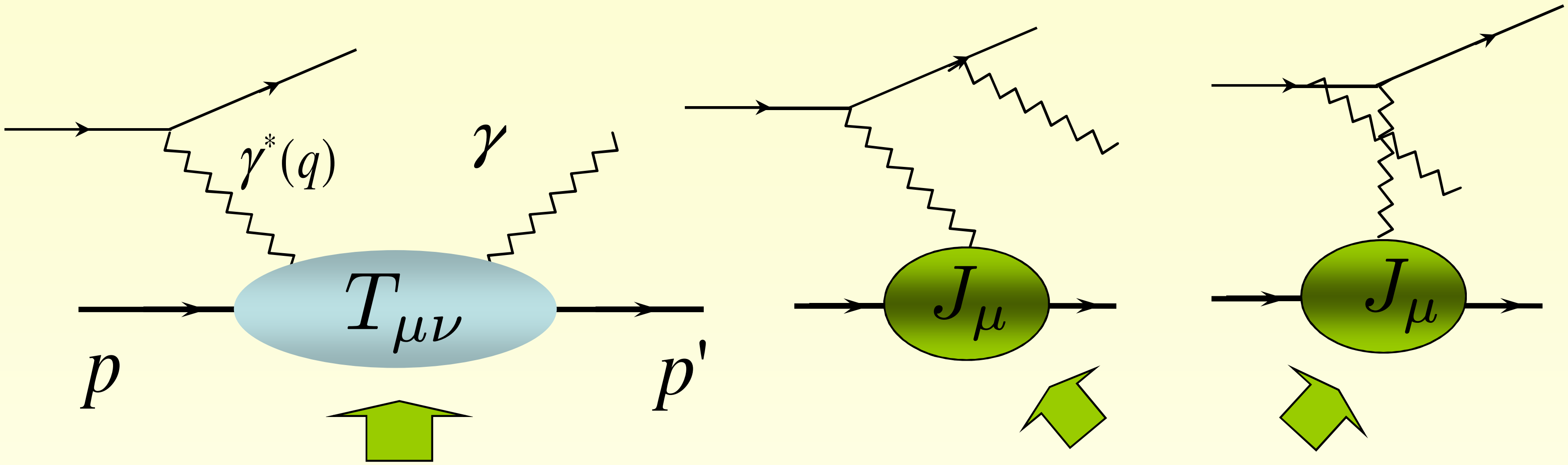
$$x_{\rm Bj} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \;\; (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \;\; (> 1\text{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors (helicity amplitudes) $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \ldots$ elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics ⇕ **1:1** helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics ⇕ **1:1** helicity ampl.

relations among **harmonics** and **GPDs** are not more based on $1/\mathcal{Q}$ expansion: (all harmonics are expressed by twist-2 and -3 GPDs) [Diehl et. al (97) Belitsky, DM, Kirchner (01)] Belitsky, DM, Ji (12)]

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \text{ tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{ tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{ tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{\mathcal{Q}} (\text{tw-2})^{\text{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\text{CS}} \propto (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\text{CS}} \propto \frac{\Delta}{Q} (\text{tw-2}) (\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\text{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\text{GT}}$$

setting up the **perturbative framework:** [Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (97/98); Pire, Szymanowski, Wagner (11) DM et al. (12)]

- ✓ ***twist-two*** coefficient functions at ***next-to-leading*** order
- ✓ anomalous dimensions and evolution kernels at ***next-to-leading*** order [Belitsky, DM (98) + Freund (01)]
- ✓ ***next-to-next-to-leading*** order in a specific conformal subtraction scheme [KMP-K & Schaefer 06]
- ✓ ***twist-three*** including quark-gluon-quark correlation at LO [Anikin,Teryaev, Pire (00); Belitsky DM (00); Kivel et. al]
- ✓ partially, ***twist-three*** sector at ***next-to-leading*** order [Kivel, Mankiewicz (03)]

? \`target mass corrections' (not understood) [Belitsky DM (01)]

✓ **kinematical twist-four** corrections [Braun, Manashov, (11)]

DVMP

- observable set is completed if polarization of final state proton is measured

$$\frac{d\sigma^{\gamma_L^* N\to M\, N}}{dt d\varphi} = \frac{2\pi\alpha_{\rm em}}{Q^4\sqrt{1+\epsilon^2}}\frac{x_B^2}{1-x_B}\frac{1}{2}\Big\{\mathcal{C}_{\rm unp}(\mathcal{F}_M,\mathcal{F}_M^*|s_2) + \Lambda\cos(\theta)\mathcal{C}_{\rm LP}(\mathcal{F}_M,\mathcal{F}_M^*|s_2)$$
$$+\Lambda\cos(\varphi)\sin(\theta)\,\mathcal{C}_{\rm TP+}(\mathcal{F}_M,\mathcal{F}_M^*|s_2) + \Lambda\sin(\varphi)\sin(\theta)\,\mathcal{C}_{\rm TP-}(\mathcal{F}_M,\mathcal{F}_M^*|s_2)\Big\}\,.$$

at least in principle on can measure the imaginary and real part of two TFFs (e.g., at small *x* for vector meson production)

$$\mathcal{C}_{\rm unp}(\mathcal{F},\mathcal{F}^*) \simeq |\mathcal{H}|^2 - \frac{t}{4M^2}|\mathcal{E}|^2\,, \qquad \mathcal{C}_{\rm LP}(\mathcal{F},\mathcal{F}^*|s_2^{\parallel}) \simeq \sqrt{1-\frac{t}{M^2}}\left[|\mathcal{H}|^2 + \frac{t\,|2\mathcal{H}+\mathcal{E}|^2}{4\,(M^2-t)}\right],$$

$$\mathcal{C}_{\rm TP-}(\mathcal{F},\mathcal{F}^*) \simeq -\frac{\sqrt{-t}}{M}\Im{\rm m}\mathcal{H}\mathcal{E}^*\,, \qquad \mathcal{C}_{\rm TP+}(\mathcal{F},\mathcal{F}^*|s_2^{\parallel}) \simeq \frac{\sqrt{-t}\sqrt{1-\frac{t}{M^2}}}{4M}\left[|\mathcal{E}|^2 - \frac{|2\mathcal{H}+\mathcal{E}|^2}{1-\frac{t}{M^2}}\right].$$

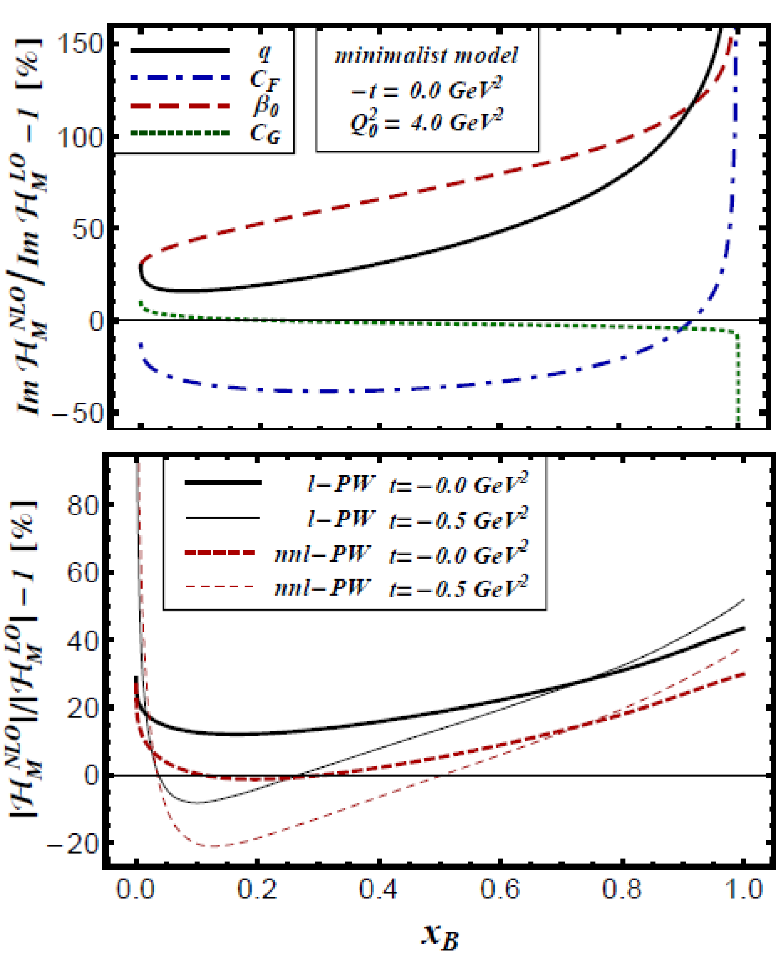
- do not forget, we need distribution amplitude, too, e.g. LO formulae

$$\mathfrak{F}_{\rm M}^{q(\pm)}(x_B,t,\mathcal{Q}^2) \stackrel{\rm LO}{=} \frac{C_F f_{\rm M}\,\alpha_s(\mu_{\rm R})}{N_c\mathcal{Q}}\int_{-1}^{1}dx\,\frac{F^{q(\pm)}(x,\xi,t,\mu_{\rm F}^2)}{\xi-x-i\epsilon}\int_0^1 dv\,\frac{\varphi_{\rm M}(v,\mu_\varphi^2)}{\overline{v}}\,,$$

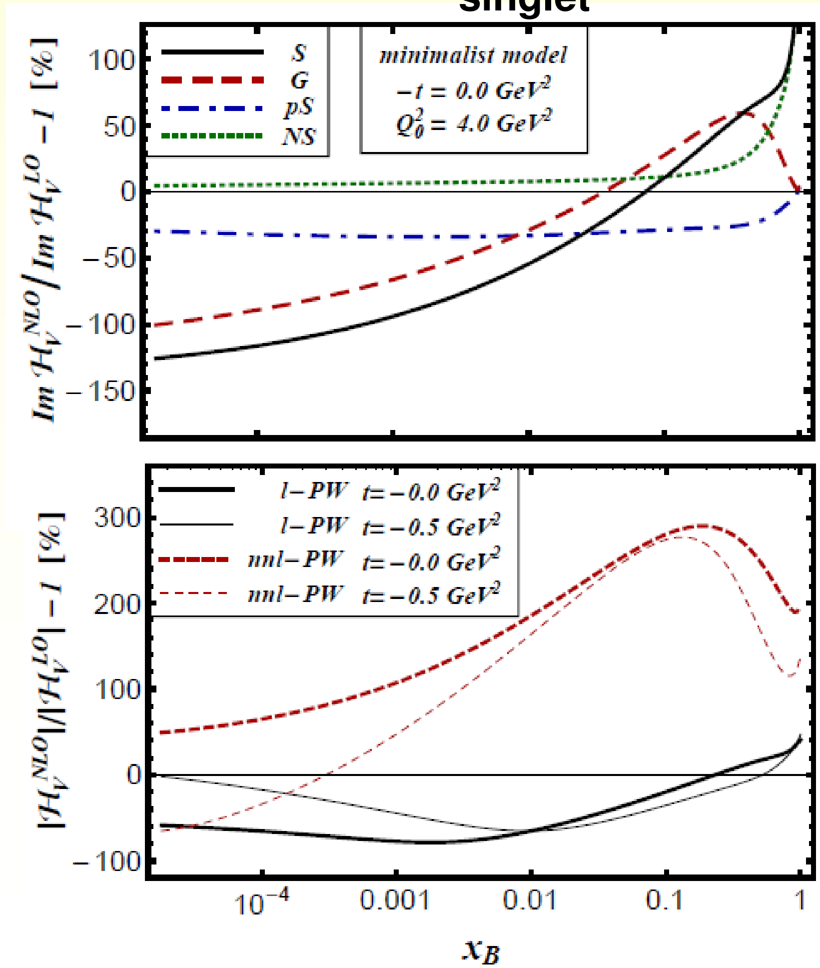
$$\mathfrak{F}_{\rm V^0}^{\rm G}(x_B,t,\mathcal{Q}^2) \stackrel{\rm LO}{=} \frac{f_{\rm V^0}\,\alpha_s(\mu_{\rm R})}{N_c\mathcal{Q}}\int_{-1}^{1}dx\,\frac{F^{\rm G}(x,\xi,t,\mu_{\rm F}^2)}{\xi(\xi-x-i\epsilon)}\int_0^1 dv\,\frac{\varphi_{\rm V^0}(v,\mu_\varphi^2)}{\overline{v}}$$

- theory is worked out at NLO in momentum fraction representation **[Belitsky, DM (02) Ivanov et al. (04)]**
- and in terms of (conformal) moments **[DM, KK-P, Lautenschlager, Schaefer (soon)]**
- NLO corrections are `universal': flavor non-singlet with signature, singlet
- NLO corrections are considered to be large/huge **[Diehl, Kugler (07)]**
- generic and model dependent features of NLO corrections are well understood (btw. also for DVCS)

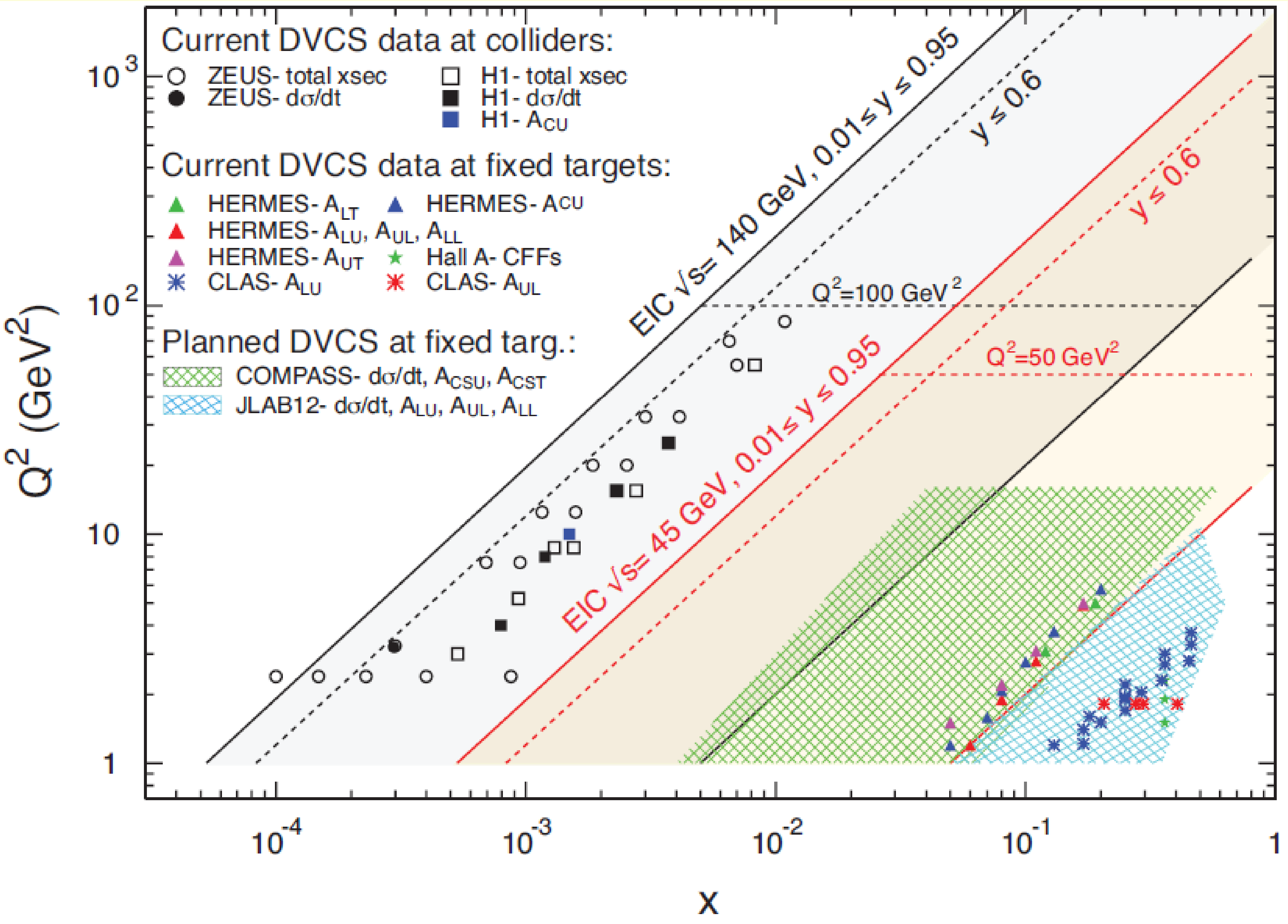
non-singlet



singlet



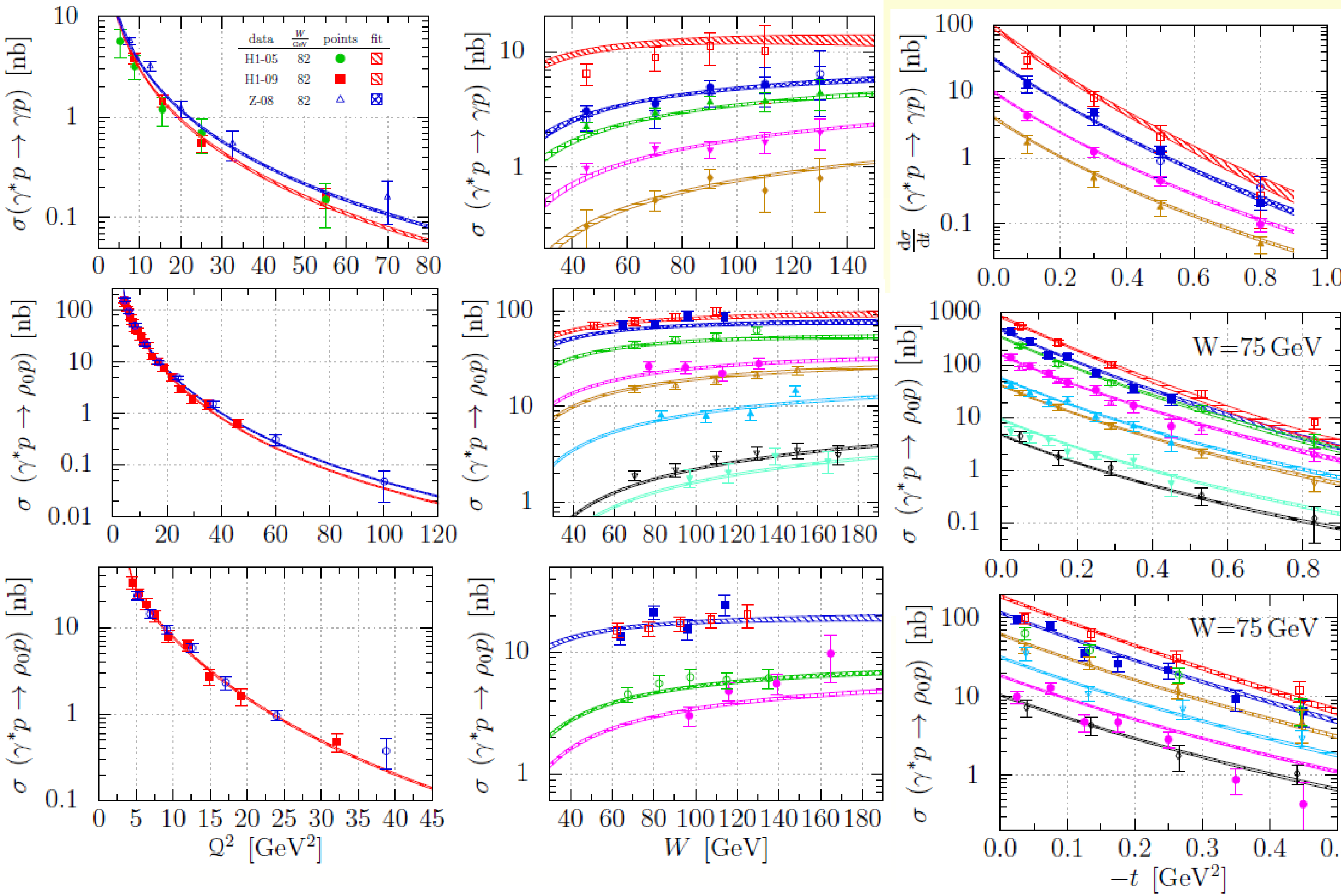
DVCS (and DVMP) world data set



DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (soon)]**

works at NLO ($Q^2 > 4\ GeV^2$), done with Bayes theorem (probability distribution function)



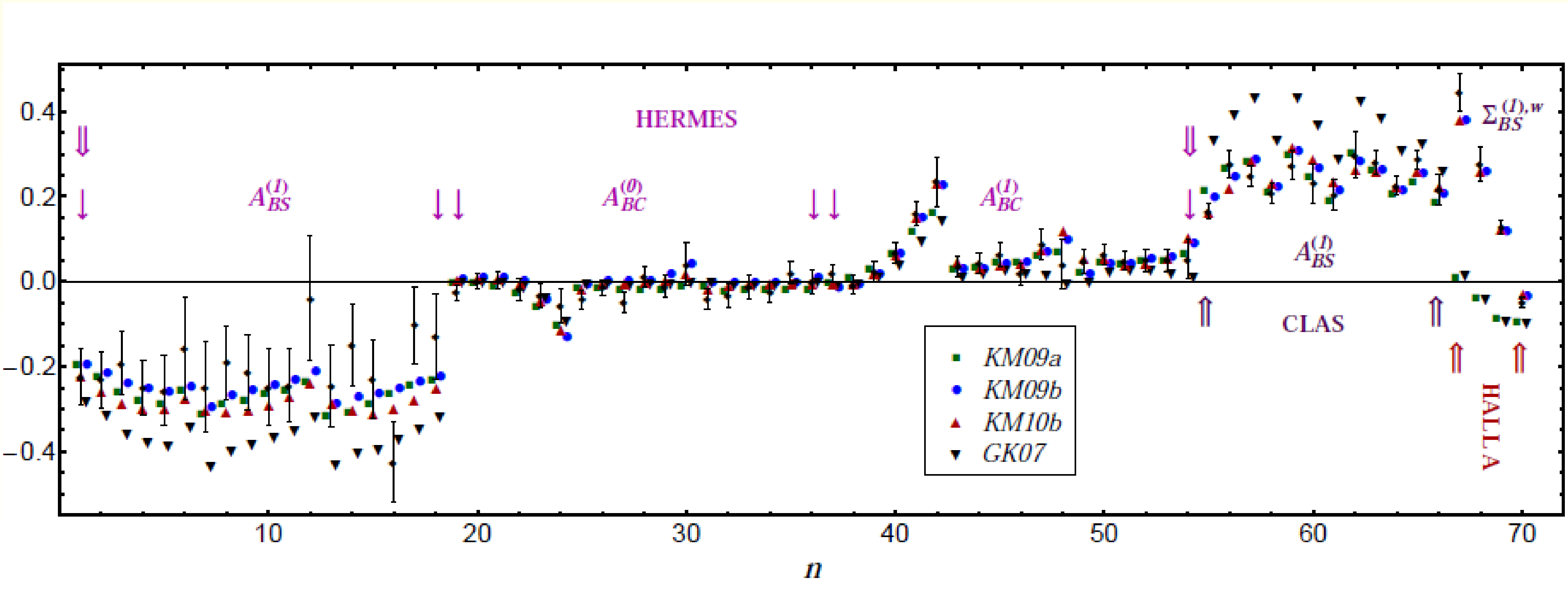
Fixed target DVCS data

- HERMES(02-12) 12x34 asymmetries $0.05 \leq <x> \leq 0.2$, $<|t|> \leq 0.4$ GeV^2
  [*sin(φ)*, ..., *cos(3 φ)*, $<<Q^2>> \approx 2.5$ GeV^2
  two kinds of electrons, all polarization options]

- HERMES(12) BSA with recoil detector
  (compatible with old data, different GPD interpretation)

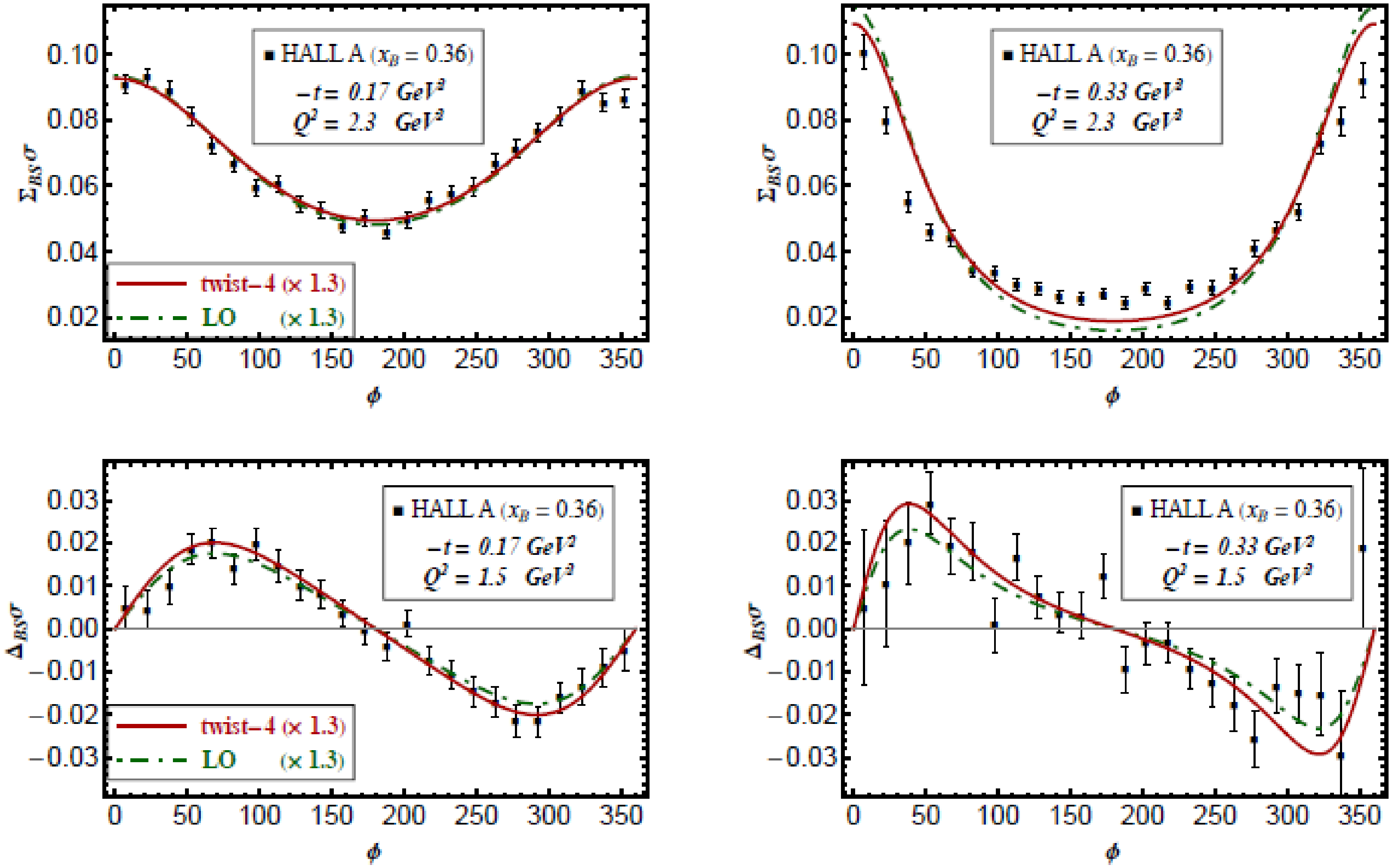
- CLAS(07) 12x12 [BSA(*φ*)] $0.14 \leq <x> \leq 0.35$, $<|t|> \leq 0.3$ GeV^2
  40x12 [BSA(*φ*)] (large |t| or bad sta.) $<<Q^2>> \approx 1.8$ GeV^2

- HALL A(06) 12x24 [*Δσ(φ)*] $<x> = 0.36$, $<|t|> \leq 0.33$ GeV^2
  3x24 [σ*(φ)*] $<<Q^2>> \approx 1.8$ GeV^2



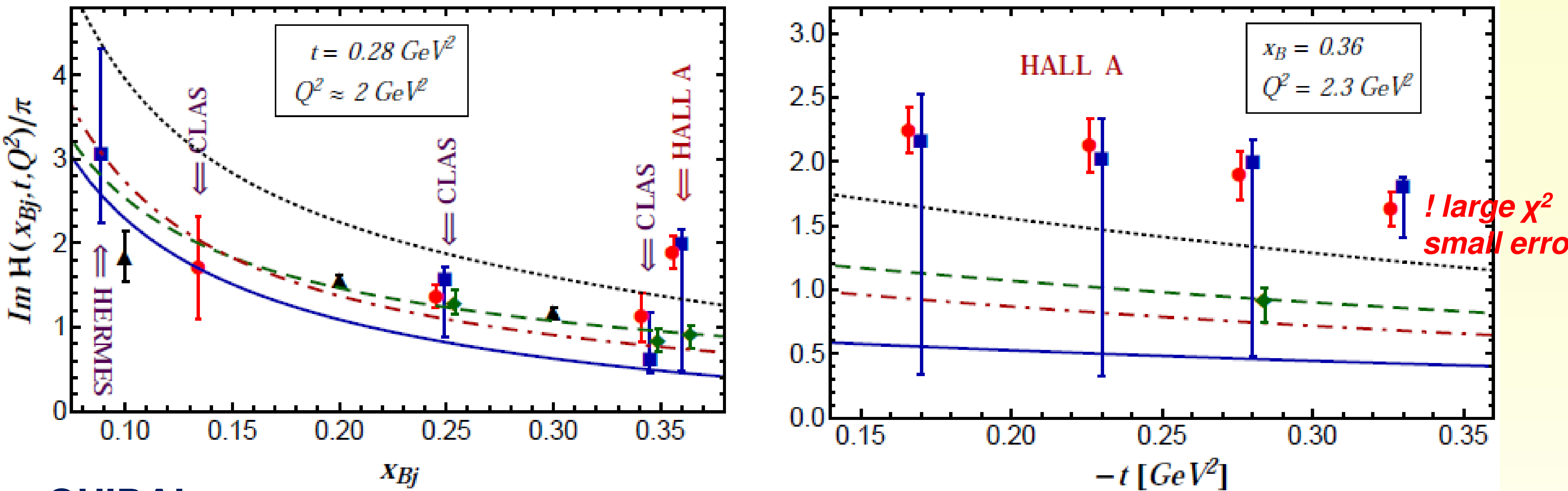
How to understand Hall A data?

[Braun, Manashov, Pirnay, DM]



- ➢ higher twist nor NLO corrections can explain data with standard models e.g., Guidal Polyakov Radyushkin Vanderhaeghen 04 model
- ➢ wrong understanding on CFF hierarchy?
- ➢ exclusivity issue in all other fixed target data?
- ➢ Is (QED) correction procedure understood?
- ➢ naive understanding of `power corrections' **[Vanderhaeghen et al. (99)]** is wrong

KM fits versus CFF fits & large-x “model” fit



GUIDAL twist-two dominance hypothesis
7 parameter fit to all harmonics of unpolarized cross section
propagated errors + “theoretical“ error estimate

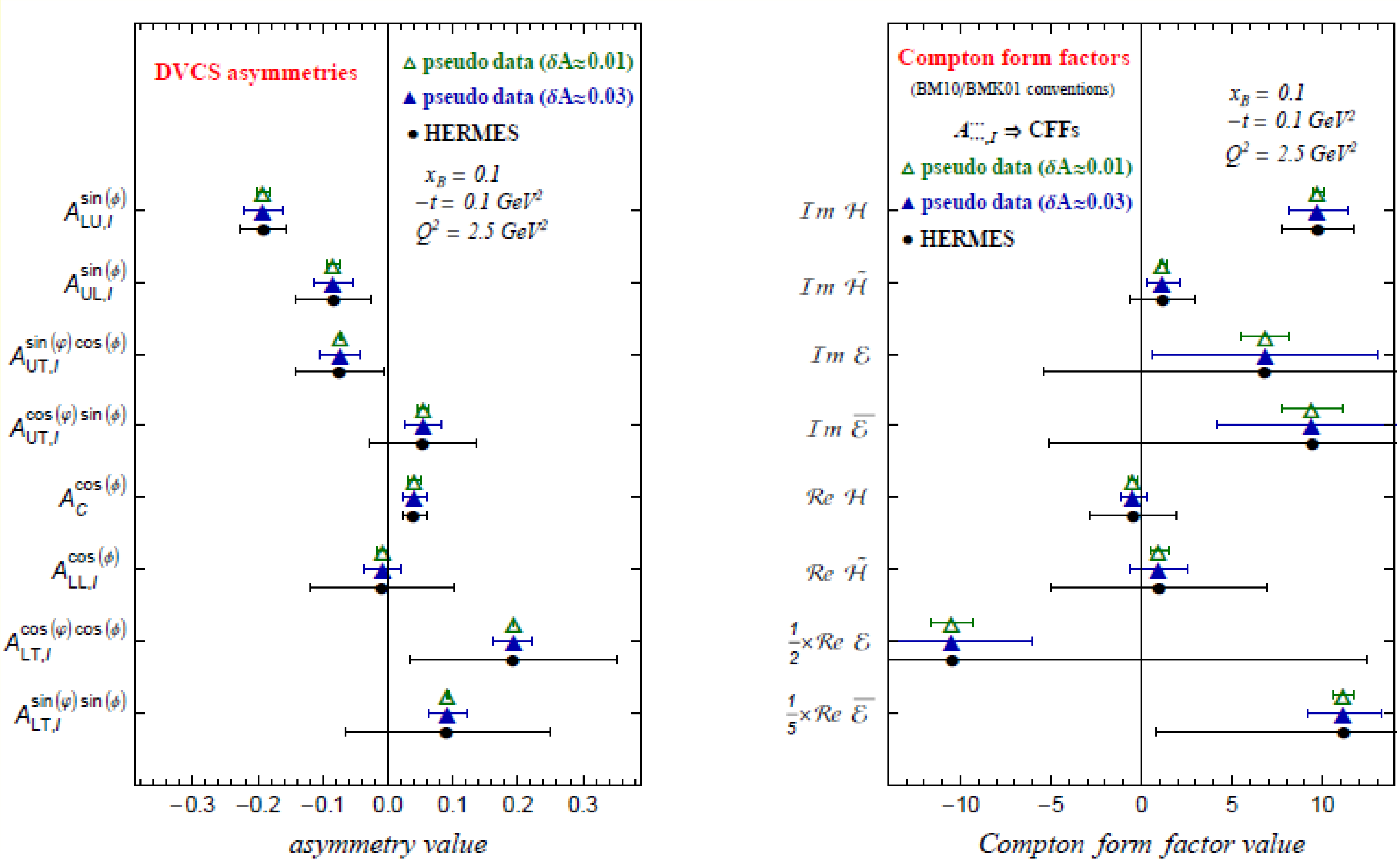
GUIDAL same + longitudinal TSA

Moutarde H dominance hypothesis within a smeared polynomial expansion
propagated errors + “theoretical“ error estimate

Kroll/Golokoskov (dashed curve) GPD model predictions from *vector meson* production

KM neural network within H dominance hypothesis
green (blue) curves (KM10) without (with) HALL A data (ratios)
red hybrid model fit with HALL A data

- a complete measurements allows in principle to pin down all CFFs **KK, DM, Murray (13)**
- missing information in incomplete measurements can be filled with noise

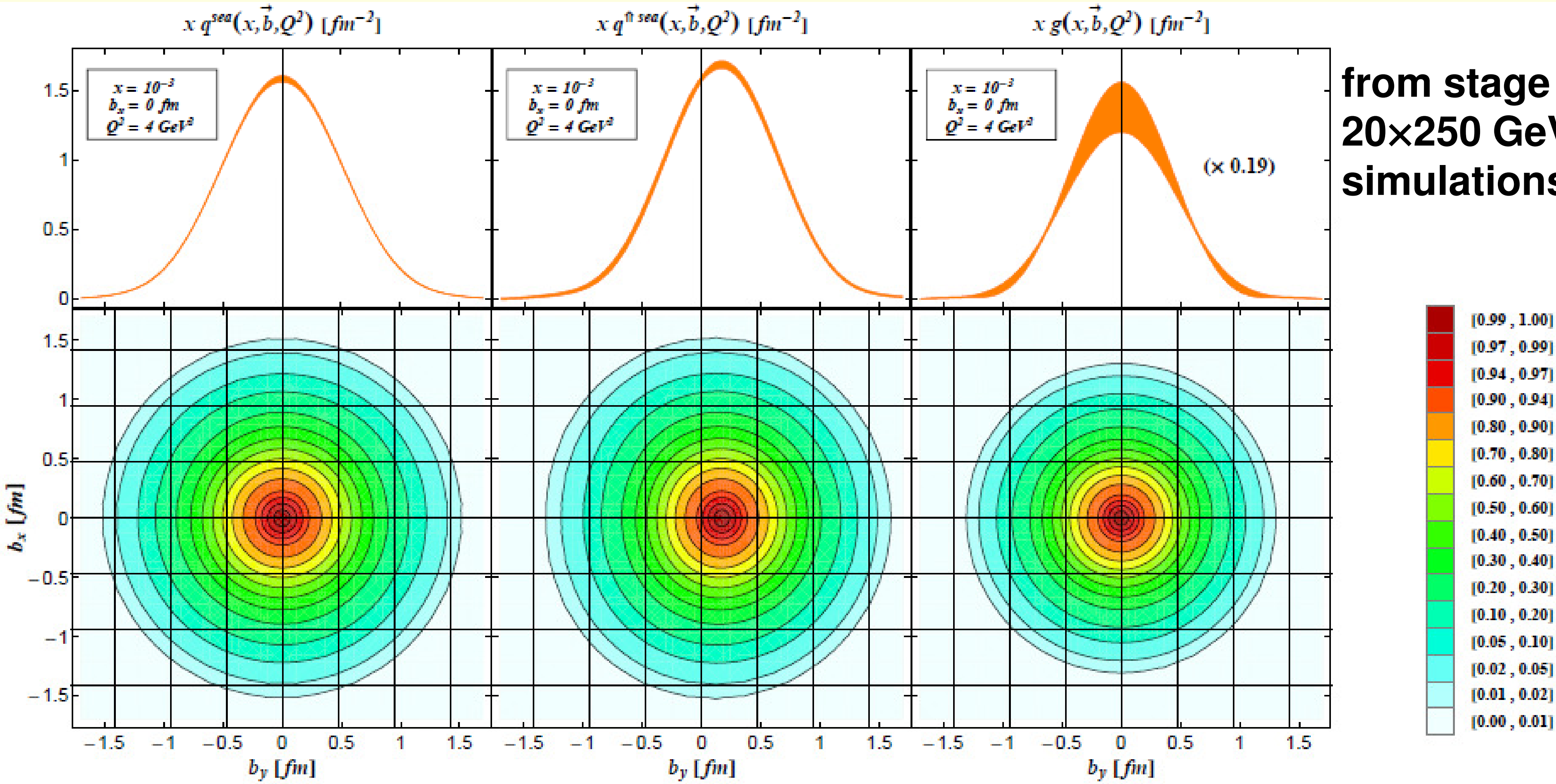


- KMM12 global fit to unpolarized + polarized data best we have χ^2/d.o.f. ≈ 1.6

The future

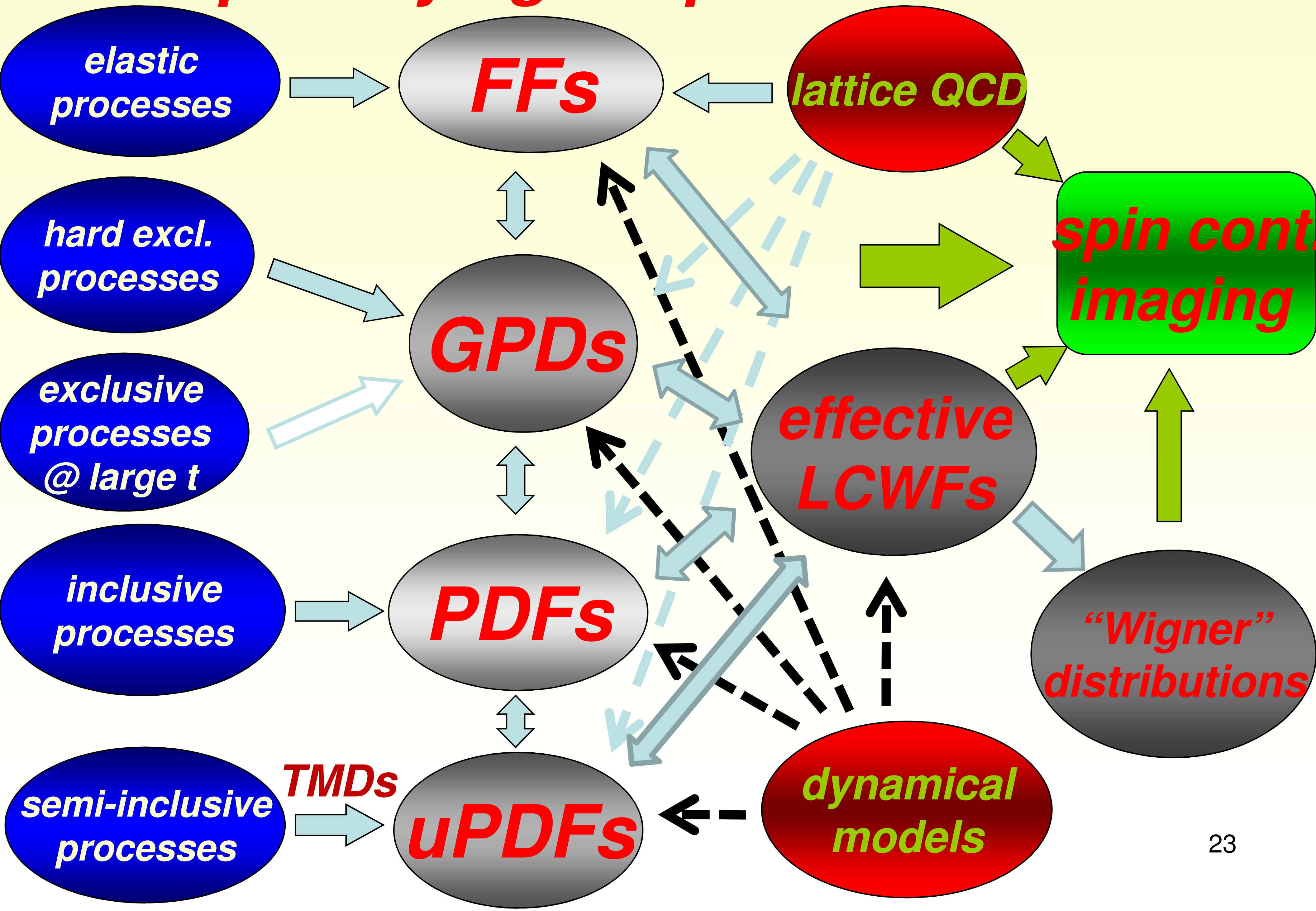
- ✓ Compass (kinematics)
- ✓ JLAB@12 GeV - high luminosity experiment
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB

Aschenauer, Firzo KK, DM (13)



from stage II 20×250 GeV^2 simulations

quantifying the partonic content



The concept of an effective LCWF

- *Is the LCWF approach well defined?*
- *certainly, infinite # of LCWFs are coupled by QCD dynamics*
- *a solution can not be expected in near future*
- *to stay with the LCWF concept, we introduce effective ones*

$$
\begin{aligned}
|P,S\rangle \;=\; & \sum_{h=-1/2}^{1/2} \int d\lambda^2 \int [dX\, d^2\mathbf{k}_\perp] \\
& \times \sqrt{\rho(\lambda^2)}\,\Psi_h^S(X_i,\mathbf{k}_{\perp i}|\lambda^2)\, \prod_{j=1}^{2} \frac{1}{\sqrt{X_j}}\, |\lambda^2, X_i P^+, X_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h\rangle
\end{aligned}
$$

Ψ_h describes the dynamics of the struck quark with spin h

λ^2 denotes some collective degrees of freedom

$\rho(\lambda^2)$ is a density

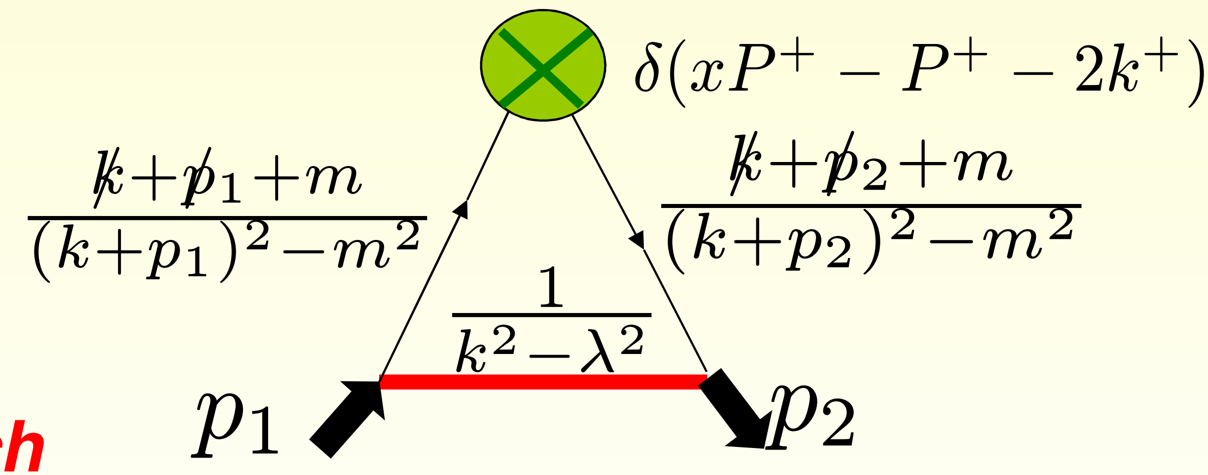
Scalar diquark model (Yukawa theory)

$$\mathcal{L} = \bar{\psi}\left(i\not{\partial} - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

struck spin-1/2 quark *collective scalar diquark spectator* *coupling knows about spin*

Diagrammatic approach:

via covariant time ordered perturbation theory



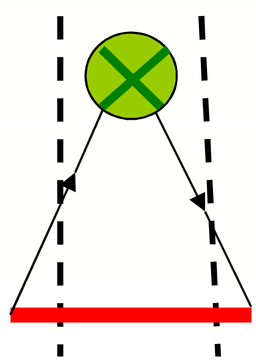
LC- Hamiltonian approach

$$k^\mu \to (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

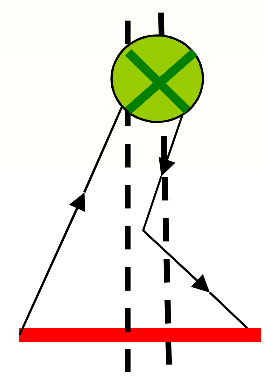
parton number conserved LCWF

(outer region)



parton number violating LCWF

(central region)



four parton number conserved LCWFs with angular momentum L

(! ΔL matters finally not L)

$$\psi^{\uparrow}_{+1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,,$$ *L=0*

$$\psi^{\downarrow}_{-1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,,$$ *L=0*

$$\psi^{\downarrow}_{+1/2}(X,\mathbf{k}_\perp) = \frac{k^1-ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,,$$ *L=-1*

$$\psi^{\uparrow}_{-1/2}(X,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,,$$ *L=+1*

in terms of one scalar LCWF (e.g., generalized Yukawa theory)

$$\varphi(X,\mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-X}}X^{-p}\left(M^2-\frac{\mathbf{k}_\perp^2+m^2}{X}-\frac{\mathbf{k}_\perp^2+\lambda^2}{1-X}\right)^{-p-1}\,,$$

- *Yukawa result follows with p=0*
- *p>0 serves as analytic regularization (model parameter)*

axial-vector LCWFs have L= {0,+1,-1}

Unintegrated PDFs (diquark model)

- *viewed as a phenomenological concept to describe data*

? *TMD counting, definition, factorization theorems, universality*

✓ *twist-2, -3 and -4 PDFs are expressed by one LCWF overlap*

L=0 ↓ *L=1* ↓ *overlap (one should wonder)*

$$f_1(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2+\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2), \qquad \mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \int d\lambda^2 \rho(\lambda^2)\,\frac{\left|\phi(x,\mathbf{k}_\perp|\lambda^2)\right|^2}{1-x},$$

$$g_1(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2-\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2), \qquad g_{1\mathrm{T}}^\perp(x,\mathbf{k}_\perp^2) = 2\frac{(m+xM)}{M}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2),$$

$$h_1(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2), \qquad h_{1\mathrm{T}}^\perp(x,\mathbf{k}_\perp^2) = -2\mathbf{\Phi}(x,\mathbf{k}_\perp^2),$$

$$h_{1\mathrm{L}}^\perp(x,\mathbf{k}_\perp^2) = -2\frac{(m+xM)}{M}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2), \qquad h_1^\perp(x,\mathbf{k}_\perp^2) = f_{1\mathrm{T}}^\perp(x,\mathbf{k}_\perp^2) = 0$$

✓ *elimination of LCWF overlap yields 8-1 constraints (symmetries)*

✓ *known results* ***[Jakob et al. (97)]*** *are recovered with*

$$\mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \frac{g^2}{M^2}\,\frac{(1-x)^{2p+1}}{\left[(1-x)\frac{m^2}{M^2}+x\frac{\lambda^2}{M^2}-x(1-x)+\frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

Classification of uPDF models

LCWF "spinors"
entries for n-parton contribution
are labeled by proton and struck quark spin

$$\boldsymbol{\psi}_{(n)}(X_i, \mathbf{k}_{\perp i}, s_i) = \begin{pmatrix} \psi^{\Rightarrow}_{\rightarrow,n} \\ \psi^{\Rightarrow}_{\leftarrow,n} \\ \psi^{\Leftarrow}_{\rightarrow,n} \\ \psi^{\Leftarrow}_{\leftarrow,n} \end{pmatrix} (X_i, \mathbf{k}_{\perp i}, s_i)$$

uPDF spin density matrix
(sum over all n-parton states)

$$\widetilde{\mathbf{\Phi}}(x, \mathbf{k}_\perp) = \sum_n \widetilde{\mathbf{\Phi}}_{(n)}(x, \mathbf{k}_\perp),$$

$$\widetilde{\mathbf{\Phi}}_{(n)}(x, \mathbf{k}_\perp) = \boldsymbol{\psi}^*_{(n)}(X_i, \mathbf{k}_{\perp i}, s_i) \overset{(n-1)}{\otimes} \boldsymbol{\psi}_{(n)}(X_i, \mathbf{k}_{\perp i}, s_i)$$

parameterized by uPDFs

$$\begin{pmatrix} \frac{f_1+g_1}{2} & \frac{|\mathbf{k}_\perp|e^{i\varphi}}{M}\frac{h_{1L}^\perp - i h_1^\perp}{2} & \frac{|\mathbf{k}_\perp|e^{-i\varphi}}{M}\frac{g_{1T}^\perp + i f_{1T}^\perp}{2} & h_1 \\ \frac{|\mathbf{k}_\perp|e^{-i\varphi}}{M}\frac{h_{1L}^\perp + i h_1^\perp}{2} & \frac{f_1-g_1}{2} & \frac{\mathbf{k}_\perp^2 e^{-i2\varphi}}{2M^2} h_{1T}^\perp & \frac{-|\mathbf{k}_\perp|e^{-i\varphi}}{M}\frac{g_{1T}^\perp - i f_{1T}^\perp}{2} \\ \frac{|\mathbf{k}_\perp|e^{i\varphi}}{M}\frac{g_{1T}^\perp - i f_{1T}^\perp}{2} & \frac{\mathbf{k}_\perp^2 e^{i2\varphi}}{2M^2} h_{1T}^\perp & \frac{f_1-g_1}{2} & \frac{-|\mathbf{k}_\perp|e^{i\varphi}}{M}\frac{h_{1L}^\perp + i h_1^\perp}{2} \\ h_1 & \frac{-|\mathbf{k}_\perp|e^{i\varphi}}{M}\frac{g_{1T}^\perp + i f_{1T}^\perp}{2} & \frac{-|\mathbf{k}_\perp|e^{-i\varphi}}{M}\frac{h_{1L}^\perp - i h_1^\perp}{2} & \frac{f_1+g_1}{2} \end{pmatrix}$$

degeneration yields classification scheme of models

Note: $\mathrm{Tr}\widetilde{\mathbf{\Phi}}(x, \mathbf{k}_\perp) = 2 f_1(x, \mathbf{k}_\perp)$

"spherical" models of rank-1 or rank-4

- *any model arising from one LCWF has rank-1 density matrix (three zero modes, called "spherical")*

$$\widetilde{\mathbf{\Phi}}^{\mathrm{rank}-1}(x,\mathbf{k}_\perp) = (\psi^{\Rightarrow}_{\rightarrow},\psi^{\Rightarrow}_{\leftarrow},\psi^{\Leftarrow}_{\rightarrow},\psi^{\Leftarrow}_{\leftarrow})^* \otimes \begin{pmatrix} \psi^{\Rightarrow}_{\rightarrow} \\ \psi^{\Rightarrow}_{\leftarrow} \\ \psi^{\Leftarrow}_{\rightarrow} \\ \psi^{\Leftarrow}_{\leftarrow} \end{pmatrix}(x,\mathbf{k}_\perp)$$

- *"spherical" model possesses three degenerated eigenvalues (eigenvalues $(f_1 + g_1 \mp 2h_1)/2$, rank-4 no zero modes)*

$$\widetilde{\mathbf{\Phi}}^{\text{"spheric"}}(x,\mathbf{k}_\perp) = \sum_{m=1}^{m_{\mathrm{min.}}} \widetilde{\mathbf{\Phi}}_{(m)}(x,\mathbf{k}_\perp)\,, \quad \text{with} \quad m_{\mathrm{min.}} \geq 4\,.$$

- *"spherical " symmetry relations*

$$g_{1\mathrm{T}}^{\perp} \pm h_{1\mathrm{L}}^{\perp} = 0\,, \quad f_{1\mathrm{T}}^{\perp} \mp h_{1}^{\perp} = 0\,, \quad g_1 \mp h_1 \mp \frac{\mathbf{k}_\perp^2}{2M^2} h_{1\mathrm{T}}^{\perp} = 0\,,$$

$$\left(h_{1\mathrm{L}}^{\perp}\right)^2 + \left(h_{1}^{\perp}\right)^2 + 2h_1 h_{1\mathrm{T}}^{\perp} = 0$$

- *rank-1 (zero mode) yields saturated uPDF Soffer bound*

$$h_1 = \pm(f_1 + g_1)/2$$

"spherical models" of rank-1:

- *scalar diquark model*
- *2u/3 –d/3 sector of a three quark LCWF model* **[Pasquini et al.]**

"spherical models" of rank-4, density matrix $\propto$ identity:

- *u/3 +4d/3 sector of a three quark LCWF model* **[Pasquini et al.]**

other "spherical models":

- *one version of an axial-vector diquark model* **[Jakob, Mulders, Rodrigues (97)]**
- *covariant parton model* **[Efremov,Schweitzer,Teryaev, Zavada (09)]**
- *bag model* **[Avakian, Efremov, Schweitzer, Yuan (10)]**
- *chiral quark soliton model* **[Lorce, Pasquini, Vanderhaeghen (11)]**
- *...*

representation of "spherical" uPDFs

from parity even LCWFs

to respect Lorentz symmetry g^{01} must be fixed

$$\psi^{\text{sca}} = \frac{1}{M}\begin{pmatrix} M \\ -g^{10}|\mathbf{k}_\perp|e^{i\varphi} \\ g^{10}|\mathbf{k}_\perp|e^{-i\varphi} \\ M \end{pmatrix}\phi^{\text{sca}}$$

f_1 - scalar overlap

$$\widetilde{\mathbf{\Phi}}^q \overset{\text{sph}}{=} \frac{1}{2}\mathbf{1}_{4\times 4}f_1^q + g_q^{\text{sca}}\left[\left(\psi^{*\ \text{sca}}\otimes\psi^{\text{sca}}\right) - \frac{1}{4}\mathbf{1}_{4\times 4}\text{Tr}\left(\psi^{*\ \text{sca}}\otimes\psi^{\text{sca}}\right)\right]$$

"axial-symmetric" models

two kinds of rank-2 models $$\tilde{\mathbf{\Phi}}^{\rm rank-2}(x,\mathbf{k}_\perp) = \sum_{m=1}^{2} \tilde{\mathbf{\Phi}}^{\rm rank-1}_{(m)}(x,\mathbf{k}_\perp)$$

first kind, e.g., **[Meissner et al. (07)]** *scalar diquark model + link factor*

$$g_{1\rm T}^{\perp} \pm h_{1\rm L}^{\perp} = 0,\ f_{1\rm T}^{\perp} \mp h_{1}^{\perp} = 0,\ g_1 \mp h_1 \mp \frac{\mathbf{k}_\perp^2}{2M^2} h_{1\rm T}^{\perp} = 0,\ h_1 = \pm\frac{1}{2}(f_1 + g_1)$$

second kind, e.g., transverse polarized axial-vector diquark **[Bacchetta et al. (08)]**

$$f_1^2 - g_1^2 - \frac{\mathbf{k}_\perp^2}{M^2}\left[\left(g_{1\rm T}^{\perp}\right)^2 + \left(h_{1\rm L}^{\perp}\right)^2 + \left(f_{1\rm T}^{\perp}\right)^2 + \left(h_{1}^{\perp}\right)^2 - \frac{\mathbf{k}_\perp^2}{4M^2}\left(h_{1\rm T}^{\perp}\right)^2 + 2h_1 h_{1\rm T}^{\perp}\right] = 0,$$

$$(f_1 - g_1)h_1 - \frac{\mathbf{k}_\perp^2}{2M^2}\left[(f_1 + g_1)h_{1\rm T}^{\perp} - 2(g_{1\rm T}^{\perp}h_{1\rm L}^{\perp} - f_{1\rm T}^{\perp}h_{1}^{\perp})\right] = 0$$

rank-3, e.g., axial-vector diquark model **[Bacchetta et al. (08)]**

$$(f_1 + g_1 + 2h_1)\left(f_1 - g_1 - \frac{\mathbf{k}_\perp^2}{M^2}h_{1\rm T}^{\perp}\right) - \frac{\mathbf{k}_\perp^2}{M^2}\left(g_{1\rm T}^{\perp} - h_{1\rm L}^{\perp}\right)^2 = 0$$

rank-4 models (analogous classification)

no example are known to us

Skewless GPD classification in impact space

Skewless GPD spin density matrix is parameterized by

$$\begin{pmatrix} \frac{H+\widetilde{H}}{2} & i\,e^{i\varphi}\,\frac{\overline{E}'_{\mathrm{T}}}{2} & -i\,e^{-i\varphi}\,\frac{E'}{2} & \overline{H}_{\mathrm{T}} \\ -i\,e^{-i\varphi}\,\frac{\overline{E}'_{\mathrm{T}}}{2} & \frac{H-\widetilde{H}}{2} & e^{-i2\varphi}\,\widetilde{H}''_{\mathrm{T}} & -i\,e^{-i\varphi}\,\frac{E'}{2} \\ i\,e^{i\varphi}\,\frac{E'}{2} & e^{i2\varphi}\,\widetilde{H}''_{\mathrm{T}} & \frac{H-\widetilde{H}}{2} & i\,e^{i\varphi}\,\frac{\overline{E}'_{\mathrm{T}}}{2} \\ \overline{H}_{\mathrm{T}} & i\,e^{i\varphi}\,\frac{E'}{2} & -i\,e^{-i\varphi}\,\frac{\overline{E}'_{\mathrm{T}}}{2} & \frac{H+\widetilde{H}}{2} \end{pmatrix}(x,b)$$

formal uPDF-GPD correspondences

$$H(x,b) \leftrightarrow f_1(x,\mathbf{k}_\perp)\,,\quad \widetilde{H}(x,b) \leftrightarrow g_1(x,\mathbf{k}_\perp)\,,\quad \overline{H}_{\mathrm{T}}(x,b) \leftrightarrow h_1(x,\mathbf{k}_\perp),$$

$$E'(x,b) \leftrightarrow -\frac{|\mathbf{k}_\perp|}{M} f^{\perp}_{1\mathrm{T}}(x,\mathbf{k}_\perp),\quad \overline{E}'_{\mathrm{T}}(x,b) \leftrightarrow -\frac{|\mathbf{k}_\perp|}{M} h^{\perp}_{1}(x,\mathbf{k}_\perp),$$

$$\widetilde{H}''_{\mathrm{T}}(x,b) \leftrightarrow \frac{\mathbf{k}_\perp^2}{2M^2} h^{\perp}_{1\mathrm{T}}(x,\mathbf{k}_\perp),\quad g^{\perp}_{1\mathrm{T}} = h^{\perp}_{1\mathrm{L}} = 0$$

analogous classification scheme as for uPDFs

*NOTE: uPDFs(not TMDs)-GPDs are tied by **3 (non)trivial** sum rules*
(all other claimed relations appear only in models, limited number of LCWFs)

Relations among integrated quantities

GPDs satisfy analog model relations as uPDFs, but

- *linear relations containing no k_T factors remain valid*
- *k_T- dependent linear relations turn into integral relations (use Lorentz symmetry)*
- *possible quadratic relations might be lost*

"spherical" GPD model of rank-1 relations:

$$H \overset{\mathrm{sph}^3}{=} \pm\left[\overline{H}_{\mathrm{T}}(x,\eta,t) - \frac{t}{4M^2}\widetilde{H}_{\mathrm{T}}(x,\eta,t) - \int_{-\infty}^{t}\frac{dt'}{4M^2}\,\widetilde{H}_{\mathrm{T}}(x,\eta,t') + \eta\widetilde{E}_{\mathrm{T}}(x,\eta,t)\right]$$

$$E \overset{\mathrm{sph}^3}{=} \pm\left[E_{\mathrm{T}}(x,\eta,t) + 2\widetilde{H}_{\mathrm{T}}(x,\eta,t) - \eta\widetilde{E}_{\mathrm{T}}(x,\eta,t)\right] \;\Rightarrow\; \overline{E}_T \overset{\mathrm{sph}}{\approx} E$$

consistent with lattice 4u/3- d/3

$$\widetilde{H} \overset{\mathrm{sph}^3}{=} \pm\left[\overline{H}_{\mathrm{T}}(x,\eta,t) + \frac{t}{4M^2}\widetilde{H}_{\mathrm{T}}(x,\eta,t) + \int_{-\infty}^{t}\frac{dt'}{4M^2}\,\widetilde{H}_{\mathrm{T}}(x,\eta,t')\right]$$

$$\widetilde{E} \overset{\mathrm{sph}^3}{=} \pm\left[E_{\mathrm{T}}(x,\eta,t) - \frac{1}{\eta}\widetilde{E}_{\mathrm{T}}(x,\eta,t)\right]$$

NOTE: 3 more relations arise in a scalar diquark model
(given in terms of double distributions)

Exploring the LCWF overlap

- ✓ *duality between s- and t-channel allows to find the central region*
- ➢ *folklore that central region always restores polynomiality*
  rather LCWFs have to respect hidden Lorentz covariance
- ✓ *positivity constraints should be automatically satisfied*
- ✓ *all twist-two and twist-three related quantities can be evaluated*
- ✓ *N, m, λ, and p can be used as model parameters*
- ✓ *concept: non-perturbative quantities in terms of LCWFs models*

missing ingredients and open questions:

- *Regge behavior as a collective phenomena is missing*
- *hard to understand from the naive point of view:*

t and $k_\perp$ are tied to each other
μ^2 evolution arises from large $k_\perp$ behavior

? *contradiction with power counting rules*

Converting $\Delta_\perp$ into t dependence

unintegrated scalar LCWF overlap:

$$\mathbf{\Phi}(x \geq \eta, \eta, \mathbf{\Delta}_\perp, \mathbf{k}_\perp) = \frac{2-\zeta}{2\sqrt{1-\zeta}\sqrt{1-X'}\sqrt{1-X}} \phi^*\left(X', (\mathbf{k}'_\perp - \mathbf{\Delta}'_\perp)^2\right) \phi\left(X, \mathbf{k}_\perp^2\right)$$

$$\frac{1-X}{1-\zeta}\mathbf{\Delta}_\perp$$

has to possesses a scalar DD representation

$$\mathbf{\Phi}(x, \eta, t) \; = \; \int_0^1 dy \int_{-1+y}^{1-y} dz \; \delta(x - y - z\eta) \, \hat{\mathbf{\Phi}}(y, z, t)$$

Mandelstam variable

in terms of an ***unintegrated double distribution***

$$\hat{\mathbf{\Phi}}(y, z, t) = \iint d\overline{\mathbf{k}}_\perp^2 \, \hat{\mathbf{\Phi}}\left(y, z, t, \overline{\mathbf{k}}_\perp^2\right) \,, \quad \overline{\mathbf{k}}_\perp = \mathbf{k}_\perp - (1 - y + z)\mathbf{\Delta}_\perp/2$$

a general ansatz (Laplace transform) solves the problem

$$\phi(X, \mathbf{k}_\perp) = \int_0^\infty d\alpha \; \varphi(X, \alpha) \exp\left\{-\alpha \frac{\mathbf{k}_\perp^2 - X(1-X)M^2}{(1-X)M^2}\right\}$$

Constrain for Laplace transformed LCWF

to restore polynomiality a (sufficient) constraint must be satisfied:

$$\frac{d}{d\eta}\left[\varphi^*\left(\frac{y-\eta(1-z)}{1-\eta}, A\frac{1-y+z}{2}\right)\varphi\left(\frac{y+\eta(1+z)}{1+\eta}, A\frac{1-y-z}{2}\right)\right]=0$$

a simple, however, non-trivial Laplace transformed LCWF:

$$\varphi(X,\alpha)=\varphi(\alpha)\exp\left\{-\alpha\frac{m^2}{M^2}-\alpha\frac{X}{1-X}\frac{\lambda^2}{M^2}\right\}$$

reduced LCWF is not fixed

yields the desired result:

$$\varphi^*(X',\alpha_2)\varphi(X,\alpha_1)$$

$$\to\varphi^*\left(A\frac{1-y+z}{2}\right)\varphi\left(A\frac{1-y-z}{2}\right)\exp\left\{-A(1-y)\frac{m^2}{M^2}-Ay\frac{\lambda^2}{M^2}\right\}$$

NOTE:

$\varphi\propto\delta(\alpha-\alpha_0)$ ➡ *exponential t-dependence* ***(disfavored @ large -t)***

$\varphi\propto\alpha^p$ ➡ *power-like t-dependence*

Spin structure of chiral even GPDs

specified spin coupling yields various GPD representations:

$$\left\{ \begin{array}{c} H \\ E \\ \widetilde{H} \\ \widetilde{E} \end{array} \right\} (x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-\eta z) \left\{ \begin{array}{c} h+(x-y)e \\ (1-x)e \\ \widetilde{h} \\ \widetilde{e}+(1-y-z/\eta)\, e \end{array} \right\} (y,z,t)$$

in terms of DDs:

$$\begin{aligned}
h &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) + \left[(1-y)^2 - z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
e &= 2\left(\frac{m}{M}+y\right)\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h} &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) - \left[(1-y)^2 - z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
\widetilde{e} &= 2\left((1-y)^2 - z^2\right)\hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

NOTE: $k_\perp$ *-integration can be replaced by t-integration*

three constraints among four GPDs

Spin structure of chiral odd GPDs

chiral odd GPDs arise from the interference of L=0 & L=1 LCWFs

$$F_{\mathrm{T}}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, f_{\mathrm{T}}(y,z,t) \text{ for } F_{\mathrm{T}} = \left\{ H_{\mathrm{T}}, E_{\mathrm{T}}, \widetilde{H}_{\mathrm{T}}, \widetilde{E}_{\mathrm{T}} \right\}$$

$$\begin{aligned}
h_{\mathrm{T}} &= \left[\left(\frac{m}{M}+y\right)^2 + \left((1-y)^2 - z^2\right)\frac{t}{4M^2}\right]\hat{\mathbf{\Phi}}(y,z,t) \\
e_{\mathrm{T}} &= 2\left[\left(\frac{m}{M}+y\right)(1-y) + (1-y)^2 - z^2\right]\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h}_{\mathrm{T}} &= -\left[(1-y)^2 - z^2\right]\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{e}_{\mathrm{T}} &= 2\left(y+\frac{m}{M}\right) z\,\hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

(common DD representation)

obviously, another set of constraints, e.g.,

$$\begin{aligned}
h_{\mathrm{T}}(y,z,t) &= \frac{1}{2}\left[h+\widetilde{h}\right](y,z,t) + \frac{t}{8M^2}\widetilde{e}(y,z,t), \quad \widetilde{h}_{\mathrm{T}}(y,z,t) = -\frac{1}{2}\widetilde{e}(y,z,t) \\
e_{\mathrm{T}}(y,z,t) &= (1-y)e(y,z,t) + \widetilde{e}(y,z,t)\,, \quad \widetilde{e}_{\mathrm{T}}(y,z,t) = z\,e(y,z,t)
\end{aligned}$$

Regge improved PDFs and GPDs

collective degrees of freedom:

- *g an effective coupling (normalization fixed by parton number)*
- *m struck quark mass (containing a bunch of partons)*
- *λ spectator diquark mass*

[Landshoff, Polkinghorne (71); Pobylitsa (03)]

naively, any transformation within these parameters is allowed

$$\rho_\alpha(\lambda, \lambda_c) = \theta(\lambda - \lambda_c)\frac{2^{-\alpha}\Gamma(2+2p)}{\Gamma(2+2p-\alpha)\Gamma(\alpha)} M^{-2\alpha}(\lambda^2 - \lambda_c^2)^{\alpha-1}$$

this simple Regge ansatz with intercept α yields PDFs and DDs:

$$\mathbf{\Phi}(x, \mathbf{k}_\perp^2) = \frac{g^2}{M^2} \frac{x^{-\alpha}(1-x)^{2p+1+\alpha}}{\left[(1-x)\frac{m^2}{M^2} + x\frac{\lambda^2}{M^2} - x(1-x) + \frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

$$\hat{\mathbf{\Phi}}(y, z, t) = N \frac{y^{-\alpha}\left((1-y)^2 - z^2\right)^{p+\alpha/2}}{\left[(1-y)\frac{m^2}{M^2} + y\frac{\lambda^2}{M^2} - y(1-y) - \left((1-y)^2 - z^2\right)\frac{t}{4M^2}\right]^{2p+1}}$$

NOTES:

- *a t-dependent α(t) might violate positivity constraints*
- *linear Regge trajectory has not been established from field theory*
- *t-dependence requires a more intricate integral transformation*

- *there is a mismatch of dimensional counting (P) and UV behavior*
- *Drell-Yan (perturbative behavior) and West (IR behavior) derived the same exclusive-inclusive (form factor-PDF) relation*

- *collinear gluon degrees of freedom are effectively absorbed*
- *twist-3 rel. unintegrated PDFs are affected by transverse gluons*

- *matching scale is ambiguous (evolution below <<1 GeV^2 ???)*
  ? not working GR(77) → GRV(98) (has steep gluons)

Building set for (u)PDFs & GPDs

- *choose two-body LCWF (respecting Lorentz symmetry)*
- *choose proton-quark spin coupling (respecting Lorentz-symmetry) (scalar, pseudo-scalar, minimal axial-vector and vector couplings)*
- *Regge improvement by convolution with a spectral mass density*
- *build (u)PDFs and GPDs and confront them with phenomenology*

e.g., minimal axial vector + scalar diquark coupling for f_1, H, E

(other quantities vanish, might be used for sea quarks)

$$f_1^{\rm sea}(x,\mathbf{k}_\perp) = \left[\left(\frac{m}{M}+x\right)^2 + \frac{\mathbf{k}_\perp^2}{M^2}\right]\mathbf{\Phi}(x,\mathbf{k}_\perp)$$

$$\begin{Bmatrix} H^{\rm sea} \\ E^{\rm sea} \end{Bmatrix}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-\eta z) \begin{Bmatrix} h^{\rm sea} + (x-y)e^{\rm sea} \\ (1-x)e^{\rm sea} \end{Bmatrix}(y,z,t)$$

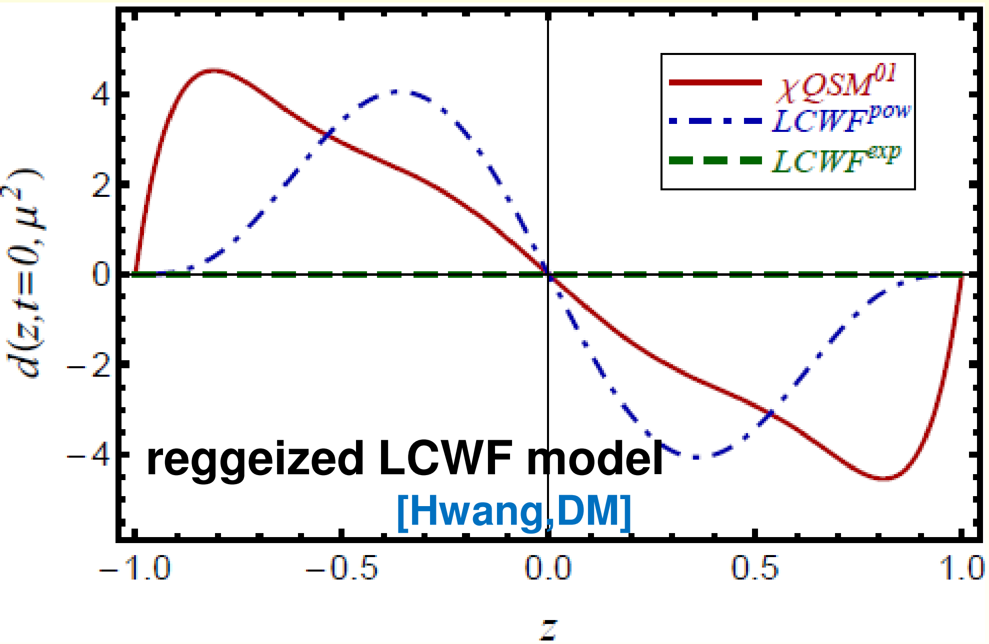
where double distributions are given in terms of off-diagonal LCWF overlap

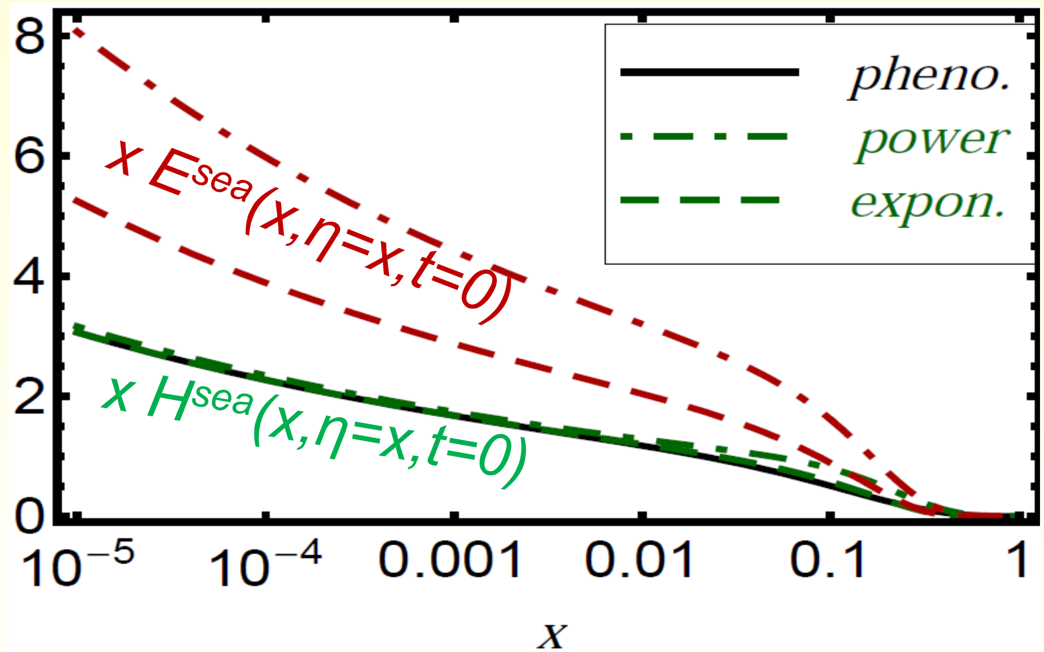
$$h = \left(\frac{m}{M}+y\right)^2 \hat{\mathbf{\Phi}}(y,z,t) + \left[(1-y)^2 - z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{4M^2}\hat{\mathbf{\Phi}}(y,z,t')\right]$$

$$e = 2\left(\frac{m}{M}+y\right)\hat{\mathbf{\Phi}}(y,z,t)$$

properties of the H/E model:

- *H and E are tied to each other (since of specific proton-quark spin coupling)*
- *effective "pomeron" trajectory is $\alpha \sim 1.1$ for $Q^2 \sim 4$ GeV^2*
- *polynomiality is completed (contains D-term contribution)*
- *large negative D-term (powerlike LCWF) contribution and large E^{sea} at small x*





- *essential for quark orbital angular momentum interpretation*

$$B^{\rm sea} \approx \begin{Bmatrix} 0.31 \\ 0.23 \end{Bmatrix} \quad \Rightarrow \quad J^{\rm sea} \approx \begin{Bmatrix} 0.23 \\ 0.19 \end{Bmatrix} \quad \text{for} \quad \begin{Bmatrix} \text{LCWF}^{\rm pow} \\ \text{LCWF}^{\rm exp} \end{Bmatrix}$$

- *E^{sea} will be clearly visible at small x_{Bj} via A_{UT} $ep \to ep\gamma$ asymmetry*
  (at a proposed Electron-Ion-Collider, E^{sea} can not be extracted from present data)

Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons
- to address the spin content of the nucleon
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (require efforts)

hard exclusive leptoproduction

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated in NLO and offers a new insight in QCD
- DVCS is widely considered as a theoretical clean process
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena

need :

tools/technology for global QCD fits (inclusive + exclusive)

Back Up

Models versus phenomenology

only for the scalar content of the proton, related to a ud-diquark
SU(6) symmetry states

$$|p\rangle = \frac{1}{\sqrt{2}}|u,(ud,0)^+\rangle + \frac{1}{\sqrt{6}}|u,(ud,0)^-\rangle - \frac{1}{\sqrt{3}}|d,(uu,1)^-\rangle$$

unintegrated PDFs *(as below + Regge improved LCWF overlap)*

GPDs:

$$H(x,\eta,t) = \frac{1}{3}\left[2H_{u_{\rm val}}(x,\eta,t) - H_{d_{\rm val}}(x,\eta,t)\right],$$

$$E(x,\eta,t) = \frac{1}{3}\left[2E_{u_{\rm val}}(x,\eta,t) - E_{d_{\rm val}}(x,\eta,t)\right],$$

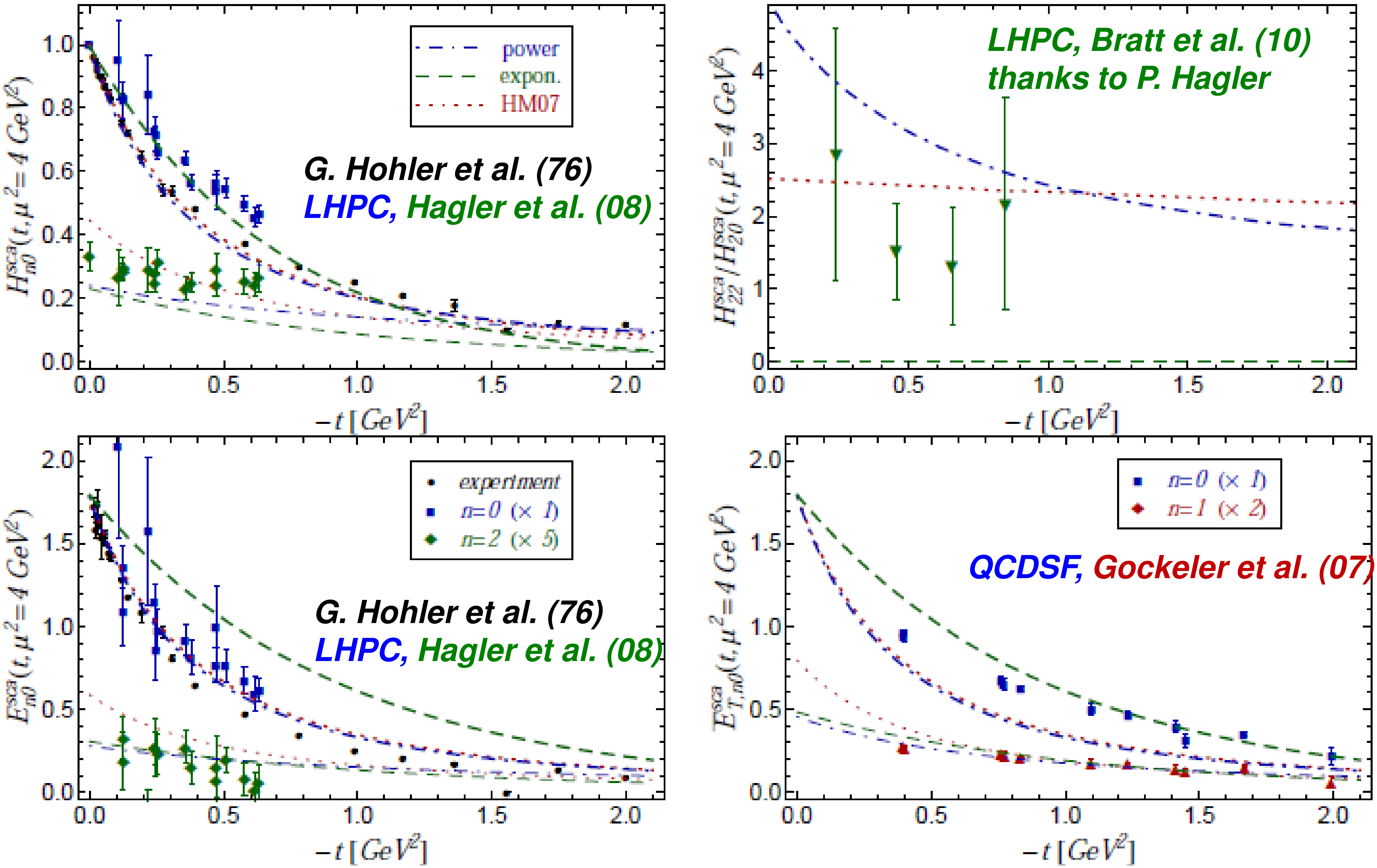
(generalized) form factors

$$F_1(t) = \int_{-\eta}^{1} dx H(x,\eta,t) \quad F_2(t) = \int_{-\eta}^{1} dx E(x,\eta,t)$$

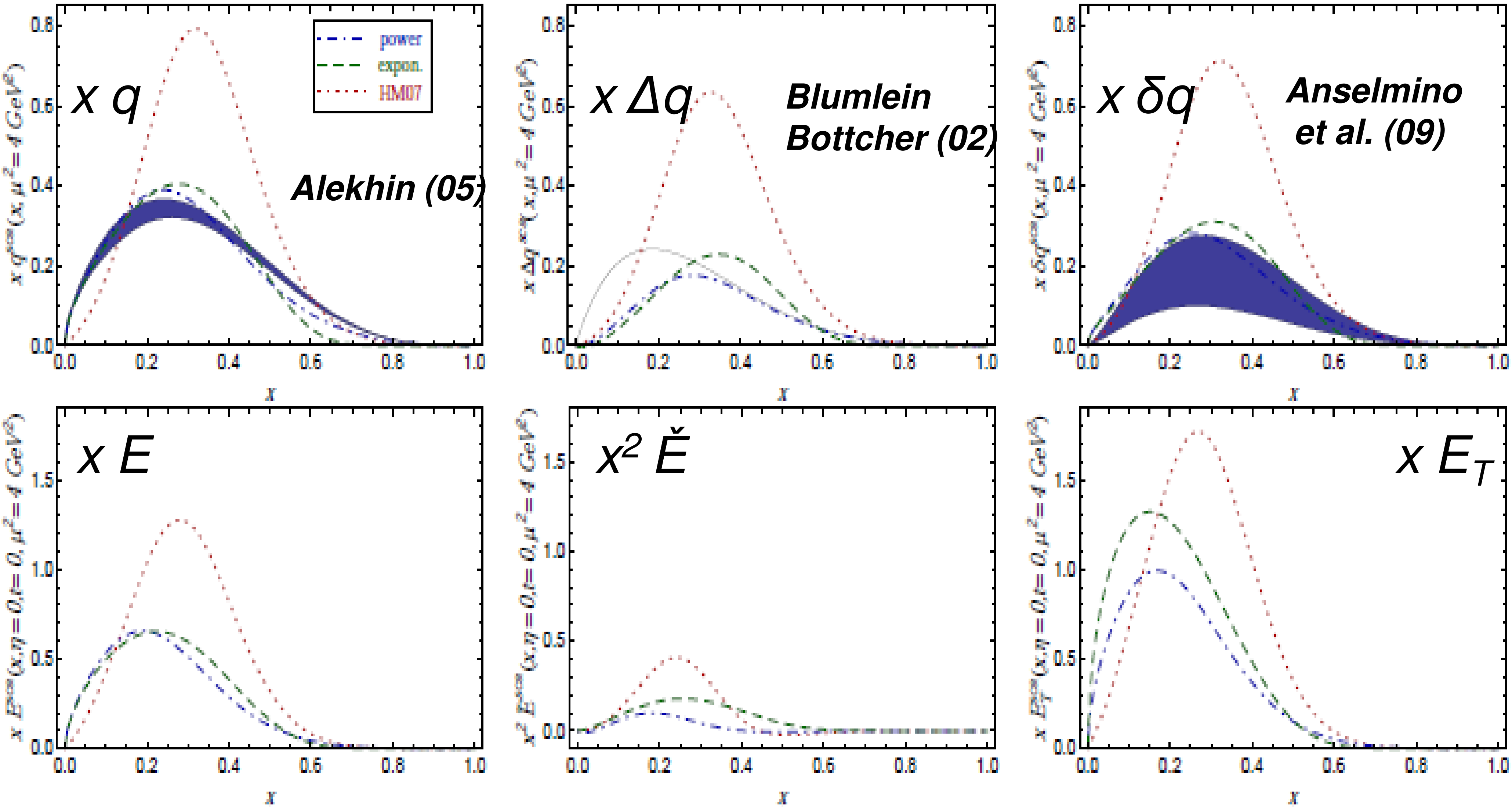
parton distribution functions

$$q(x) = H(x,\eta=0,t=0)$$

Comparison with Lattice data

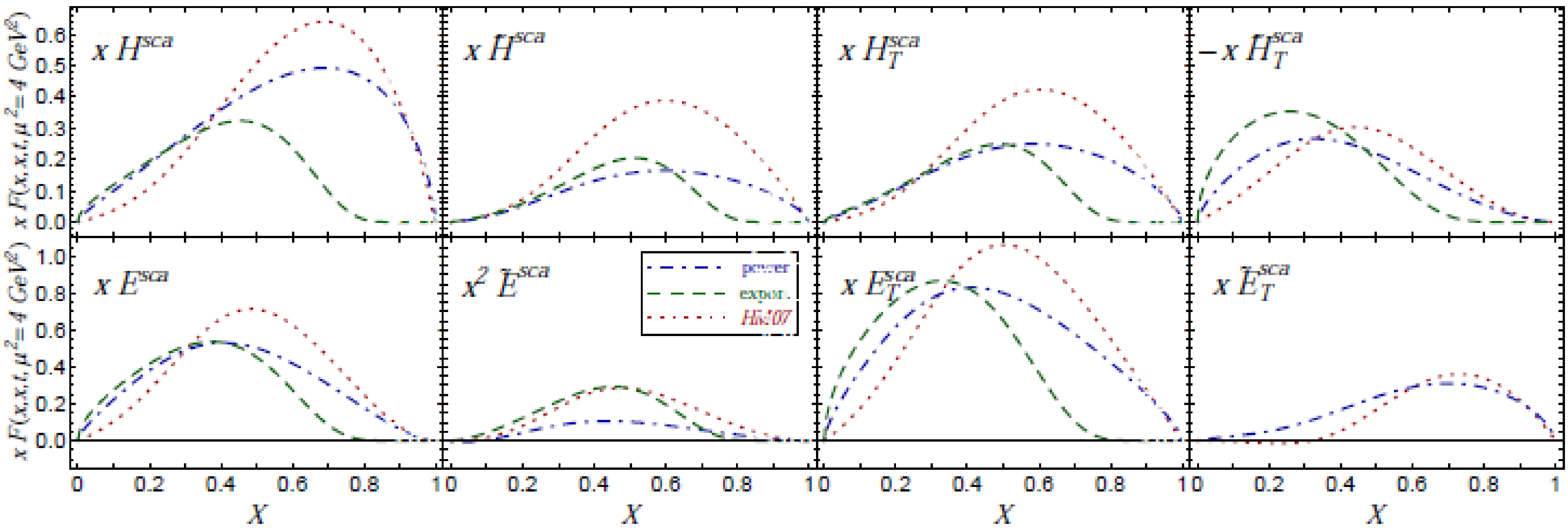


Comparison with PDFs and predictions for zero skewness GPDs



GPD predictions for η=x

GPD at η=x is accessible @ LO and fixed Q^2 + subtraction const.



- ✓ *1-x term "outside" of DD e completes polynomiality*
- ✓ *duality is respected, no ambiguous D-term*
- ✓ *projecting on the D-term* $D(x,t) = -\lim_{\eta\to\infty} E(x\eta,\eta,t)$
- ✓ *"pomeron" exchange α ≳ 1 we find a large negative D-term*

[Polyakov et al (01)]