Infra-red divergences in LFFT and Coherent State Formalism

Jai More

Department of Physics, University of Mumbai

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1. Introduction
   - Notation
   - Coherent State Formalism
2. Mass renormalization upto $O(e^2)$ in the fock state basis
3. Mass renormalization upto $O(e^2)$ in the coherent state basis
4. Mass renormalization upto $O(e^4)$ in the fock state basis

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$$x^\mu = (x^+, x^-, \mathbf{x}^\perp)$$

where

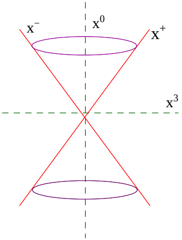
$$x^+ = \frac{(x^0 + x^3)}{\sqrt{2}}, x^- = \frac{(x^0 - x^3)}{\sqrt{2}}, \mathbf{x}^\perp = (x^1, x^2)$$

The metric tensor

$$g^{\mu\nu} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Another notation
$x^\pm = x^0 \pm x^3$.
We have used the notation of Mustaki et al.

Momentum is given by $p = (p^+, p^-, \mathbf{p}^\perp)$
Mass shell condition $p^- = \frac{\mathbf{p}_\perp^2 + m^2}{2p^+}$



[D. Mustaki, S. Pinsky, J. Shigemitsu and K.G. Wilson, Phys. Rev. D **43**, 3411 (1991).]

Method of Asymptotic Dynamics

The LSZ formalism is based on the assumption

$$H_{as} = \lim_{|t|\to\infty} H = H_0$$

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- $H_{as} \neq H_0$
  - there are long range interaction
  - incoming and outgoing states are bound states

In the limit $|t| \to \infty$, $H \longrightarrow H_{as}$

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Coherent states

$$|n \colon coh\rangle = \Omega^A_{\pm} |n\rangle \ ,$$

$|n\rangle$ is a Fock state, $\Omega^A_{\pm}$ are the asymptotic Möller operators

$$\Omega^A_{\pm} = T \ exp\left[-i \int_{\mp}^{0} V_{as}(t) dt \right]$$

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The transition matrix element calculated using these states is IR divergence free.

Nelson and Butler $\Longrightarrow$ Cancellation of IR div. in QCD (lowest order) D.R. Butler and C.A. Nelson, Phys. Rev. D **18**, 1196 (1978).
C.A. Nelson Nucl. Phys. **B181**, 141 (1981).
C.A. Nelson Nucl. Phys. **B186**, 187 (1981).

Greco etal. $\Longrightarrow$ M. Greco, F. Palumbo, G. Pancheri-Srivastava and Y. Srivastava, Phys. Lett. **B77**, 282 (1978).

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Harindranath and Vary $\Longrightarrow$ Relevance of coherent state formalism in LFFT. A. Harindranath and J. P. Vary, Phys. Rev. D **37**, 3010 (1988).
L. Martinovic and J. P. Vary, Phys. Lett. **B459**, 186 (1999).

Anuradha Misra $\Longrightarrow$ Coherent state formalism in LFFT cancell- -ation of IR divergences in 3-point vertex correction in QED and QCD at one loop level.
Anuradha Misra, Phys. Rev. D **50**, 4088 (1994).
Anuradha Misra, Phys. Rev. D **53**, 5874 (1996).
Anuradha Misra, Phys. Rev. D **62**, 125017 (2000).

IR Divergences in LFFT

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The coherent state method provides an alternative way of treating the true IR divergences.

Coherent state Formalism in LFFT

H_{as} is evaluated by taking the limit $x^+ \to \infty$ in $exp[-i(p_1^- + p_2^- + \cdots + p_n^-)x^+]$ of the interaction Hamiltonian H_{int}.

If $(p_1^- + p_2^- + \cdots + p_n^-) \to 0$ for some vertex, then the corresponding term in H_{int} does not vanish in large x^+ limit.

Use KF method to construct asymptotic Hamiltonian and coherent state basis.

Coherent state Formalism in LFQED

$$H_I(x^+) = V_1(x^+) + V_2(x^+) + V_3(x^+)$$

where

$$V_1(x^+) = e \sum_{i=1}^{4} \int d\nu_i^{(1)} [e^{-i\nu_i^{(1)} x^+} \tilde{h}_i^{(1)}(\nu_i^{(1)}) + e^{i\nu_i^{(1)} x^+} \tilde{h}_i^{(1)\dagger}(\nu_i^{(1)})]$$

$\tilde{h}_i^{(1)}(\nu_i^{(1)})$ and $\nu_i^{(1)}$ are three point QED interaction vertices and the light-front energy transferred at the vertex $\tilde{h}^{(1)}$ respectively.

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For example,

$$\tilde{h}_1^{(1)} = \sum_{s,s',\lambda} b^\dagger(\overline{p}, s')b(p,s)a(k,\lambda)\overline{u}(\overline{p}, s')\gamma^\mu u(p,s)\epsilon_\mu^\lambda ,$$

$\nu_i^{(1)} = p^- - k^- - (p-k)^-$ and

$$\int d\nu^{(1)} = \frac{1}{(2\pi)^{3/2}}\int \frac{[dp][dk]}{\sqrt{2\overline{p}^+}} ,$$

The 3-point asymptotic Hamiltonian is defined by the following expression

$$V_{1as}(x^+) = e \sum_{i=1,4} \int d\nu_i^{(1)} \Theta_\Delta(k)[e^{-i\nu_i^{(1)}x^+} \tilde{h}_i^{(1)}(\nu_i^{(1)}) + e^{i\nu_i^{(1)}x^+} \tilde{h}_i^{\dagger}(\nu_i^{(1)})]$$

where

$$\Theta_\Delta(k) = \begin{cases} 1 & \text{in the asymptotic region} \\ 0 & \text{elsewhere.} \end{cases}$$

Asymptotic region can be considered to be consisting of all points $(k^+, \mathbf{k}_\perp)$ satisfying:

$$\mathbf{k}_\perp^2 < \frac{k^+\Delta}{p^+}, \quad k^+ < \frac{p^+\Delta}{m^2} \ .$$

leading to

$$\Theta_\Delta(k) = \theta\left(\frac{k^+\Delta}{p^+} - \mathbf{k}_\perp^2\right)\theta\left(\frac{p^+\Delta}{m^2} - k^+\right)$$

Asymptotic states:

$$\Omega^{A}_{\pm}|n\colon p_i\rangle = exp\Big[-e\int dp^{+}d^{2}\mathbf{p}_{\perp}\sum_{\lambda=1,2}[d^{3}k][f(k,\lambda : p)a^{\dagger}(k,\lambda)$$

$$- f^{*}(k,\lambda : p)a(k,\lambda)] + e^{2}\int dp^{+}d^{2}\mathbf{p}_{\perp}\sum_{\lambda_1,\lambda_2=1,2}[d^{3}k_1][d^{3}k_2]$$

$$[g_1(k_1,k_2,\lambda_1,\lambda_2\colon p)a^{\dagger}(k_2,\lambda_2)a(k_1,\lambda_1)$$

$$- g_2(k_1,k_2,\lambda_1,\lambda_2\colon p)a(k_2,\lambda_2)a^{\dagger}(k_1,\lambda_1)]\rho(p)\Big]|n\colon p_i\rangle$$

[Jai D. More and Anuradha Misra, Phys. Rev. D **86**, 065037 (2012)]
Here

$$[d^{3}k] = \int\frac{d^{2}\mathbf{k}_{\perp}}{(2\pi)^{3/2}}\int\frac{dk^{+}}{\sqrt{2k^{+}}}$$

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$$[d^3k] = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^{3/2}}\int \frac{dk^+}{\sqrt{2k^+}}$$

$$f(k,\lambda\colon p) = \frac{p_\mu \epsilon^\mu_\lambda(k)}{p\cdot k}\theta\left(\frac{k^+\Delta}{p^+}-\mathbf{k}^2_\perp\right)\theta\left(\frac{p^+\Delta}{m^2}-k^+\right),$$
$$f(k,\lambda\colon p) = f^*(k,\lambda\colon p),$$

One fermion coherent state:

$$|p,\sigma : f(p)\rangle = exp\Big[-e\sum_{\lambda=1,2}[d^3k][f(k,\lambda : p)a^\dagger(k,\lambda) - f^*(k,\lambda : p)a(k,\lambda)]$$

$$+e^2\sum_{\lambda_1,\lambda_2=1,2}[d^3k_1][d^3k_2][g_1(k_1,k_2,\lambda_1,\lambda_2 : p)a^\dagger(k_2,\lambda_2)a(k_1,\lambda_1)$$

$$-g_2(k_1,k_2,\lambda_1,\lambda_2 : p)a(k_2,\lambda_2)a^\dagger(k_1,\lambda_1)\Big]|p,\sigma\rangle$$

Light-Front QED in LF gauge

Light-front QED Hamiltonian in the light-front gauge ($A^+ = 0$)

$$P^- = H \equiv H_0 + V_1 + V_2 + V_3 \ ,$$

Here

$$H_0 = \int d^2\mathbf{x}_\perp dx^- \{ \frac{i}{2}\bar{\xi}\gamma^- \overleftrightarrow{\partial}_- \xi + \frac{1}{2}(F_{12})^2 - \frac{1}{2} a_+ \partial_- \partial_k a_k \}$$

$$V_1 = e \int d^2\mathbf{x}_\perp dx^- \bar{\xi}\gamma^\mu \xi a_\mu$$

$$V_2 = -\frac{i}{4} e^2 \int d^2\mathbf{x}_\perp dx^- dy^- \epsilon(x^- - y^-)(\bar{\xi} a_k \gamma^k)(x)\gamma^+(a_j \gamma^j \xi)(y)$$

$$V_3 = -\frac{e^2}{4} \int d^2\mathbf{x}_\perp dx^- dy^- (\bar{\xi}\gamma^+ \xi)(x)|x^- - y^-|(\bar{\xi}\gamma^+ \xi)(y)$$

$\xi(x)$ and $a_\mu(x)$ can be expanded in terms of creation and annihilation operators as

$$\xi(x) = \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^{3/2}} \int \frac{dp^+}{\sqrt{2p^+}} \sum_{s=\pm\frac{1}{2}} [u(p,s)e^{-i(p^+x^- - \mathbf{p}_\perp x_\perp)} b(p,s,x^+) + v(p,s)e^{i(p^+x^- - \mathbf{p}_\perp x_\perp)} d^\dagger(p,s,x^+)],$$

$$a_\mu(x) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^{3/2}} \int \frac{dq^+}{\sqrt{2q^+}} \sum_{\lambda=1,2} \epsilon_\mu^\lambda(q)[e^{-i(q^+x^- - \mathbf{q}_\perp x_\perp)} a(q,\lambda,x^+) + e^{i(q^+x^- - \mathbf{q}_\perp x_\perp)} a^\dagger(q,\lambda,x^+)],$$

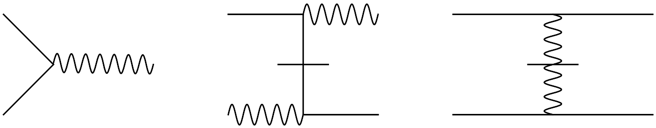
operators satisfy

$$\{b(p,s), b^\dagger(p',s')\} = \delta(p^+ - p'^+)\delta^2(\mathbf{p}_\perp - \mathbf{p}'_\perp)\delta_{ss'} = \{d(p,s), d^\dagger(p',s')\},$$

$$\left[a(q,\lambda), a^\dagger(q',\lambda')\right] = \delta(q^+ q'^+)\delta^2(\mathbf{q}_\perp - \mathbf{q}'_\perp)\delta_{\lambda\lambda'}.$$

QED interaction vertices

QED



Mass renormalization in LFQED

The transition matrix is given by

$$T = V + V\frac{1}{p^- - H_0}V + \cdots$$

The electron mass shift is obtained by calculating T_{pp}

$$\boxed{\delta m^2 = p^+ \sum_s T_{pp}}$$

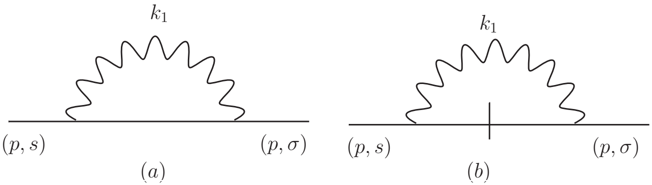
and

$$T_{pp} = \langle p, s|T|p, s\rangle$$

We expand T_{pp} in powers of e^2 as

$$T_{pp} = T^{(1)} + T^{(2)} + \cdots$$

$$T^{(1)}_{pp} \equiv T^{(1)}(p,p) = \langle p, s|V_1\frac{1}{p^- - H_0}V_1|p, s\rangle + \langle p, s|V_2|p, s\rangle$$



Diagrams for $O(e^2)$ self energy correction in fock basis corresponding to T_1

$$T^{(1)}_{pp} \equiv T^{(1)}(p,p) = \langle p, s| V_1 \frac{1}{p^- - H_0} V_1 |p, s\rangle$$

In the limit $k_1^+ \to 0$, $\mathbf{k}_{1\perp} \to 0$,

$$\boxed{(\delta m^2_{1a})^{IR} = -\frac{e^2}{(2\pi)^3} \int d^2\mathbf{k}_{1\perp} \int \frac{dk_1^+}{k_1^+} \frac{(p \cdot \epsilon(k_1))^2}{(p \cdot k_1)}}$$

Mass renormalization in LFQED: Feynman gauge

QED Lagrangian in Feynman gauge with additional PV fields

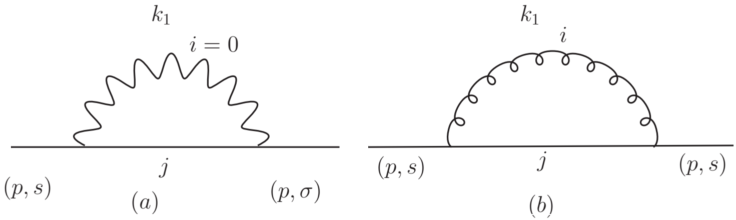
$$\mathcal{L} = \sum_{i=0}^{2}(-1)^i\left[-\frac{1}{4}F_i^{\mu\nu}F_{i,\mu\nu} + \frac{1}{2}\mu_i^2 A_i^\mu A_{i\mu} - \frac{1}{2}\left(\partial^\mu A_{i\mu}\right)^2\right] + \sum_{i=0}^{2}(-1)^i\bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i - e\bar{\psi}\gamma^\mu\psi A_\mu.$$

Here

$$\psi = \sum_{i=0}^{2}\sqrt{\beta_i}\psi_i,\quad A_\mu = \sum_{i=0}^{2}\sqrt{\xi_i}A_{i\mu},\quad F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu},$$

$i = 0$ corresponds to a physical field, and $i = 1$ and 2, to PV fields. The photon fields have mass m_i, $m_0 \to 0$ for the physical photon. β_i and ξ_i are coupling coefficients.

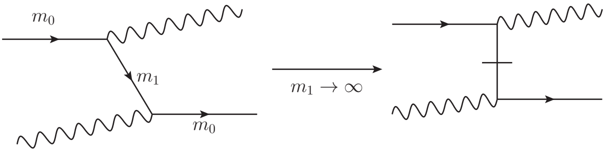
[S. S. Chabysheva and J. R. Hiller, Phys. Rev. D **84**, 034001 (2011).]



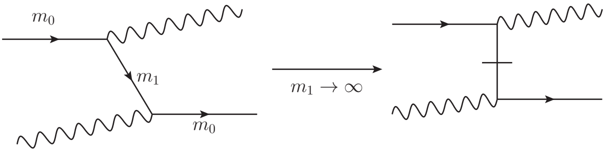
Diagrams for $O(e^2)$ self energy correction in fock basis corresponding to T_{1a} and T_{1b}. In (a), wavy line corresponds to physical photon ($i = 0$) and in (b) curly line corresponds to PV photon ($i = 1, 2$).

- The four point instantaneous terms do not appear in LF Hamiltonian

- In the infinite PV mass limit one can obtain the instantaneous four point interaction terms



In the infinite PV mass limit the PV fermion line reduces to an instantaneous four point interaction term denoted by a dash on the fermion line. [S. S. Chabysheva and J. R. Hiller, Phys. Rev. D **84**, 034001 (2011).]



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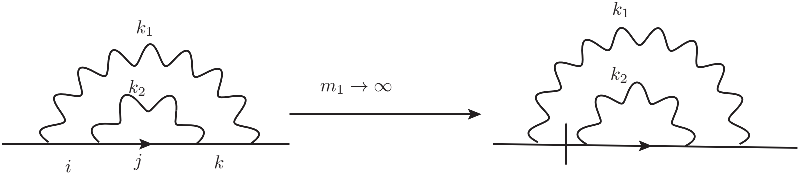
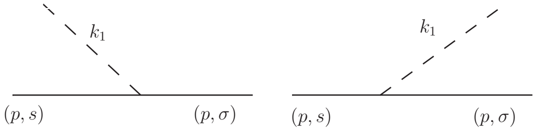


$3(a)$ $4(a)$

In the infinite PV mass limit the diagram on the left reduces to a diagram involving instantaneous interaction. Here $i = 1$ while $j = k = 0$.

Mass renormalization in the coherent state basis



Additional diagrams in coherent state basis for $O(e^2)$ self energy correction corresponding to T_2.

$$T'(p, p) = \langle p, s : f(p)|V_1|p, s : f(p)\rangle$$

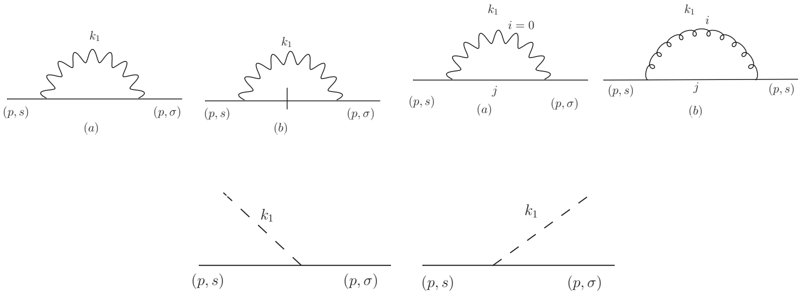
$$T'(p,p) = \frac{e^2}{(2\pi)^3}\int \frac{d^2\mathbf{k}_{1\perp}}{2p^+}\int \frac{dk_1^+}{2k_1^+}\overline{u}(\overline{p},s')\not\epsilon^{\lambda}(k_1)u(p,s)f(k_1,\lambda:p)$$

where

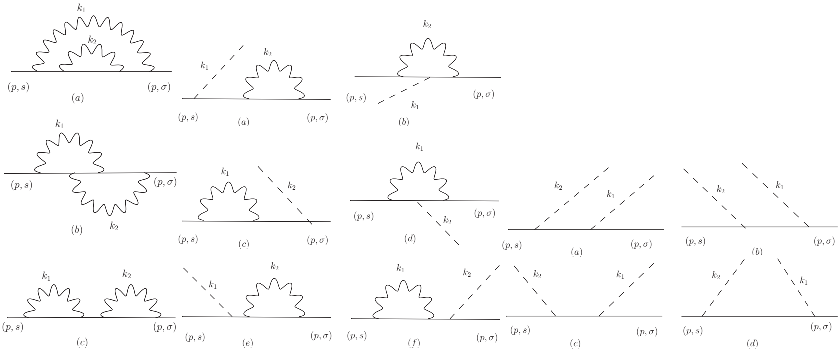
$$f(k,\lambda\colon p) = \frac{p_\mu \epsilon_\lambda^\mu(k)}{p\cdot k}\theta\left(\frac{k^+\Delta}{p^+} - \mathbf{k}_\perp^2\right)\theta\left(\frac{p^+\Delta}{m^2} - k^+\right),$$

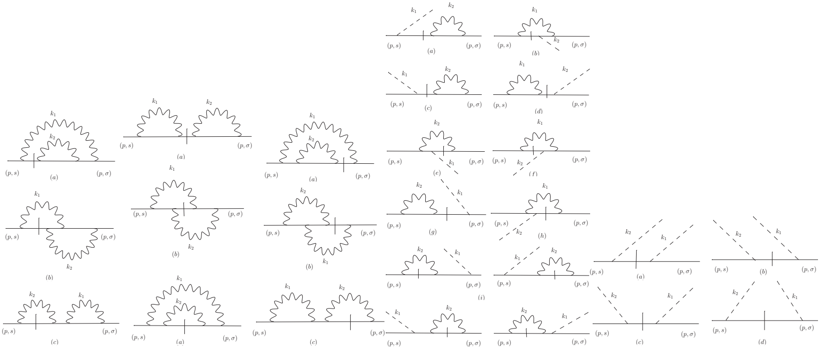
$$\boxed{(\delta m^2)' = \frac{e^2}{(2\pi)^3}\int d^2\mathbf{k}_{1\perp}\int \frac{dk_1^+}{k_1^+}\frac{(p\cdot\epsilon(k_1))^2\Theta_\Delta(k_1)}{p\cdot k_1}}$$

Mass Renormalization up to $O(e^2)$ is IR finite

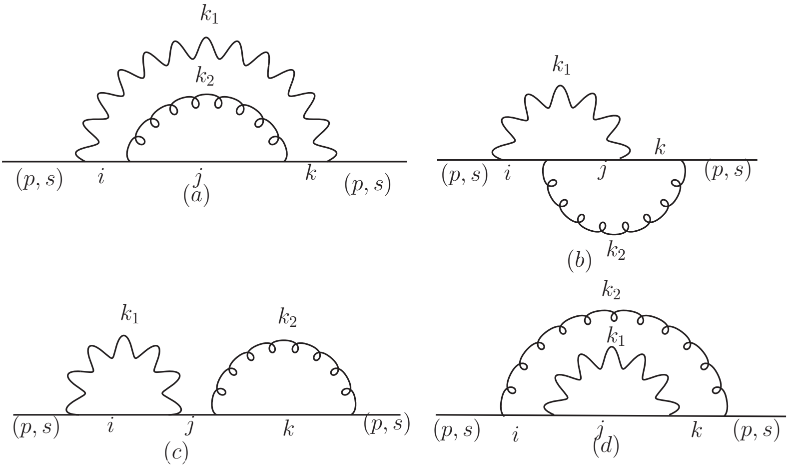


Diagrams for $O(e^4)$ self energy correction in LF gauge

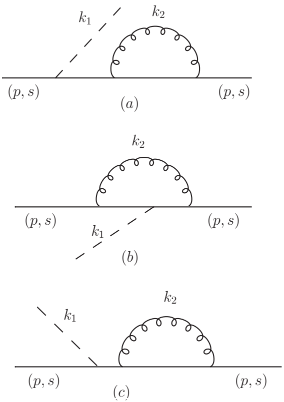




[Jai D. More and Anuradha Misra, Phys. Rev. D **86**, 065037 (2012)]

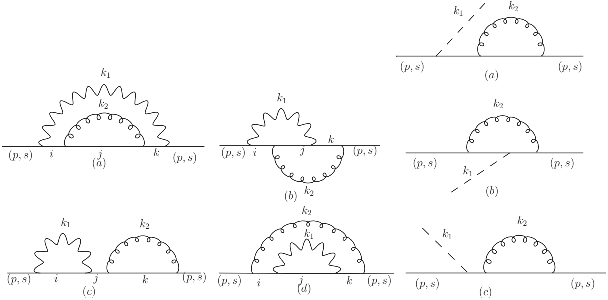


Diagrams for $O(e^4)$ self energy correction in fock basis using Feynman gauge.



Diagrams for $O(e^4)$ self energy correction in coherent state basis using Feynman gauge.

Mass Renormalization up to O(e^4) in Feynman gauge is IR finite



[Jai D. More and Anuradha Misra, Phys. Rev. D **87**, 085052 (2013)]

Improved Method of Asymptotic Dynamics

- In equal time formulation of QED, IR divergences cancel to all orders. In QCD such a proof does not exists
- In QCD asymptotic state are bound state. So the asymptotic Hamiltonian obtained by KF method is not sufficient for the cancellation of IR divergences
- An *'improved' method of asymptotic dynamics takes into account the separation of particles also*

[R. Horan, M. Lavelle, and D. McMullan, Pramana **51**, 317 (1998).

R. Horan, M. Lavelle, and D. McMullan, Report No. PLY-MS-99-9, hep-th/9909044, (1999).

R. Horan, M. Lavelle, and D. McMullan, hep-th /0002206 (2000).]

The key observation is that it is more appropriate in QFT to work at the level of matrix elements than at the level of operators.

$$\langle \psi_{out} | H_{int} | \psi_{in} \rangle$$

is a time dependent complex number and, therefore, to investigate its asymptotic limit, one can use the method of stationary phase.
Thus, if the above matrix element is given by

$$\langle \psi_{out} | H_{int} | \psi_{in} \rangle = \int d\nu_i f(p_1) g(p_2) \cdots exp[-i\nu_i x^+]$$

then, according to the method of stationary phase, this integral approaches zero as $|x^+| \to \infty$ provided there is no point in the region of integration at which all first order partial derivatives of ν_i vanish.
The second criteria is to take into account the binding of particle at asymptotic limit.

[Anuradha Misra, Few-Body Systems 36, 201-204 (2005).]

Criteria in the Method of Asymptotic Dynamics

In LFQED,

$$\nu_i = p^- - k^- - (p-k)^-$$

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Condition to obtain the asymptotic region

KF approach : $\nu_i = 0$

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Improved Method:

1. $\frac{\partial \nu_i}{\partial p_\perp} = \frac{\partial \nu_i}{\partial p^+} = \frac{\partial \nu_i}{\partial k_\perp} = \frac{\partial \nu_i}{\partial k^+} = 0$
2. Separation of particles at large distances

Work in progress:

- For QED, both methods give the same asymptotic conditions
- In case of theories with 4-point coupling the asymptotic regions obtained by KF method and first criteria of improve method do not match.
- Developing the improved method of asymptotic dynamics in LFFT for simple model like Yukawa theory, ϕ^4 theory
- Extending this method to QCD to analyze the nature of IR divergences
- To construct an artificial potential that is needed for bound state calculation in LFQCD.

Summary

- The *true* IR divergences get cancelled when coherent state basis is used to calculate the matrix elements in lepton self energy correction in light-front QED up to $O(e^4)$ in LF gauge as well as in Feynman gauge.
- The proof can be generalized to general covariant gauge (in preparation)
- In LFQCD, the second criteria of **Improved** Method of Asymptotic dynamics must take into account the binding between the quarks and gluons.

Future Plans

- The cancellation of IR divergences between real and virtual processes is known to hold in equal time QED to all orders. It would be interesting to verify this all order cancellation in LFQED.
- It is well known that IR divergences do not cancel in QCD in higher orders. The reason may lie in the wrong choice of asymptotic states.
- Connection between asymptotic dynamics and IR divergences can possibly be exploited to construct an artificial potential that may be used in bound state calculation in LFQCD

Thank you