Chiral corrections to nucleon GPDs

Alena M.Moiseeva, Alexey A.Vladimirov

Institut für theoretische Physik II
Bochum

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Definition of GPD

- Generalized parton distributions (GPDs) are phenomenological functions accumulating information about long-distance hadron dynamics.

Nucleon GPD parameterizes the matrix element of light-cone non-local operator:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p'|\bar{q}(\frac{\lambda n}{2})\, \not{n}\, \tau^A q(-\frac{\lambda n}{2})|p\rangle = \frac{1}{P_+}\bar{u}(p')\left[\not{n}\, H(x,\xi,\Delta^2) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E(x,\xi,\Delta^2)\right]\tau^A u(p),$$

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p'|\bar{q}(\frac{\lambda n}{2})\, \not{n}\, \gamma_5 \tau^A q(-\frac{\lambda n}{2})|p\rangle = \frac{1}{P_+}\bar{u}(p')\left[\not{n}\, \gamma_5 \widetilde{H}(x,\xi,\Delta^2) + \gamma_5 \frac{(n\Delta)}{2M} \widetilde{E}(x,\xi,\Delta^2)\right]\tau^A u(p)$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p, \quad n^2 = 0.$$

- λ is the light-cone distance
- x – hadron momentum fraction carried by a quark, $\xi = -\frac{\Delta_+}{2P_+}$ – skewedness

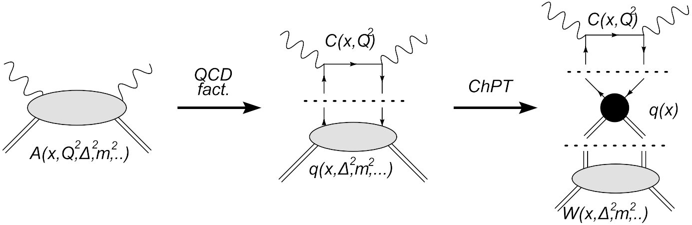
- GPD can not be calculated within QCD, since it defines long-distance dynamics (low momenta).
- Dependence of GPDs on low-scale parameters (m_π, Δ^2) can be extracted within framework of Effective Field Theories (EFT).

$$q(x, \Delta^2) = \mathring{q}(x) + a_\chi q^{(1)}(x, \mathring{q}) + a_\chi^2 q^{(2)}(x, \mathring{q}) + ...,$$

Chiral expansion parameter $a_\chi^2 \sim \frac{m_\pi^2}{(4\pi F_\pi)^2} \sim \frac{\Delta^2}{(4\pi F_\pi)^2} \sim 10^{-2}$.

- $\mathring{q}(x)$ is PDF in the chiral limit ($m \to 0$, $\Delta \to 0$), and $q^{(i)}(x, \mathring{q})$ perturbativly calculable in EFT functions.

Chiral expansion $\Leftrightarrow$ QCD factorization (factorization "head over heels"):



- The lowest chiral order Lagrangian:

$$\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} = \bar{N}\left[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - M + \frac{g_a}{2}u_\mu\gamma^\mu\gamma^5\right]N + \frac{F_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U\partial^\mu U^\dagger + \chi^\dagger U + \chi U^\dagger\right]$$

$M \simeq 940$ MeV – the nucleon mass and $F_\pi \simeq 93$ MeV – the pion decay constant.

$$u^2 = U = e^{\frac{i\pi^a\tau^a}{F_\pi}}, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right], \quad u_\mu = iu^\dagger\partial_\mu U u^\dagger, \quad \chi \sim m_\pi^2$$

⊙ Not typical for ChPT scale presents: $\frac{M^2}{(4\pi F_\pi)^2} \sim 1$ vs $\frac{m_\pi^2}{(4\pi F_\pi)^2} \sim 10^{-2}$.

Violation of the chiral counting rules!

There are several methods to overcome the hierarchy problem of baryon ChPT, e.g:

- Heavy baryon approach to baryon ChPT, [Jenkins, Manohar,90]
- Infrared regularization scheme, [Becher, Letwyler,01]
- Extended on-mass shell regularization scheme, [Gegelia et., 2003]

However the first and the second are not applied to investigation of non-local operators: non-local operators are very sensitive to the analytic properties of theory

Matching operators

We stay in the Lorentz-invariant ChPT and need to construct GPD non-local operators.

- Quantum numbers of EFT operator and QCD operator should coincide
- Operators in EFT are made of $U(x), u(x), N(x)$ and ∂ (various chiral orders are possible).

The lowest chiral order operators (zero derivatives): [Kivel, Polyakov, 04]

$$\bar{q}(\frac{\lambda n}{2})\, \not{n}\, \tau^A q(-\frac{\lambda n}{2}) \quad \Rightarrow \quad \frac{1}{2}\int d\beta d\alpha \bar{N}(x)\left[F_1(\beta,\alpha)\, \not{n}\, t_+^A(x,y) + F_2(\beta,\alpha)\, \not{n}\, \gamma^5 t_-^A(x,y)\right] N(y)$$

$$\bar{q}(\frac{\lambda n}{2})\, \not{n}\, \gamma_5 \tau^A q(-\frac{\lambda n}{2}) \quad \Rightarrow \quad \frac{1}{2}\int d\beta d\alpha \bar{N}(x)\left[F_2(\beta,\alpha)\, \not{n}\, \gamma^5 t_+^A(x,y) + F_1(\beta,\alpha)\, \not{n}\, t_-^A(x,y)\right] N(y)$$

$$t_\pm^A(x,y) = u^\dagger(x)\tau^A u(y) \pm u(x)\tau^A u^\dagger(y), \qquad x = \frac{(\alpha+\beta)\lambda n}{2}, y = \frac{(\alpha-\beta)\lambda n}{2}$$

$F(\beta,\alpha)$ are generating function for low energy constants (LECs).
The operator with the same quantum numbers also can be constructed from pion fields only:

$$\bar{q}(\frac{\lambda n}{2})\, \not{n}\, \tau^A q(-\frac{\lambda n}{2}) \Rightarrow \frac{-i\mathcal{F}_\pi^2}{4}\int d\beta d\alpha\; F(\beta,\alpha)\mathrm{Tr}\left[\tau^A\left(U(x) \overleftrightarrow{\partial}_+ U^\dagger(y) + U^\dagger(x) \overleftrightarrow{\partial}_+ U(y)\right)\right]$$

Tree level matrix element of effective operator represents GPD at the chiral limit:

$$\int_{-1}^{1} d\beta \int_{-1+\beta}^{1-\beta} d\alpha\ F_{1(2)}(\beta,\alpha)\delta(x-\alpha\xi-\beta) = \mathring{H}\left(\mathring{\tilde{H}}\right)(x,\xi)$$

- F_i, $\mathcal{F}$ are Double Distributions in the chiral limit, [Radyushkin,97].
- Generating functions at the forward limit ($\xi = 0$) are PDFs:

$$\int_{-1+\beta}^{1-\beta} d\alpha F_1(\beta,\alpha) = \mathring{q}(\beta), \qquad \int_{-1+\beta}^{1-\beta} d\alpha F_2(\beta,\alpha) = \Delta\mathring{q}(\beta)$$

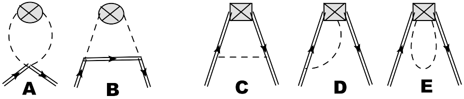
- Parton distributions are normalized from the form factor operators:

$$\int_{-1}^{1} d\beta \mathring{q}(\beta) = 1, \qquad \int_{-1}^{1} d\beta \Delta\mathring{q}(\beta) = g_a.$$

g_a is axial coupling constant.

Next-to-Leading chiral order GPDs.

- Loop expansion provides us chiral corrections to the tree order GPDs, $\mathring{H}(x,\xi) \sim a_\chi^0$.
- One-loop graphs gives next-to-leading chiral corrections $\sim a_\chi^2$



- Diagrams A and B describes the contribution of the pion cloud to the nucleon. Diagram D mixes up the vector and axial PDFs.
- The calculation has been done in the extended on mass-shell normalization scheme (EOMS)
- The heavy baryon and similar schemes gives "mostly wrong" results, (often divergent)

Leading order PDF.

- We present parton distributions $q(x,\Delta^2)$, $\Delta q(x,\Delta^2)$ and $E(x,\Delta^2)$ in one-loop order. They correspond to GPDs: $H(x,\xi=0,\Delta^2)$, $\widetilde{H}(x,\xi=0,\Delta^2)$, $E(x,\xi=0,\Delta^2)$, $\widetilde{E}$.

$$q^I(x,\Delta^2) = \mathring{q}(x) + \frac{1}{(4\pi F_\pi)^2}\int_{-1}^{1}\frac{d\beta}{\beta}\theta\left(0<\frac{x}{\beta}<1\right) \times\left[\mathring{q}(\beta)C^I\left(\frac{x}{\beta},\Delta^2\right)+\Delta\mathring{q}(\beta)\Delta C^I\left(\frac{x}{\beta}\right)+\mathring{Q}(\beta)C^I_\pi\left(\frac{x}{\beta},\Delta^2\right)\right]$$

-

$$C^0\left(y,\Delta^2\right) = -\frac{3g_a^2}{2}\delta(1-y)m^2\ln\frac{m^2}{\mu^2}-3g_a^2\int_0^{\bar{y}}d\eta\frac{ym^2-\Delta^2\eta\bar{y}}{\bar{y}^2+\alpha^2y-\frac{\Delta^2}{M^2}\eta(\bar{\eta}-y)}$$

$$\Delta C^0(y) = -\frac{3g_a}{2}\delta(1-y)m^2\ln\frac{m^2}{\mu^2}-3g_aM^2\bar{y}\ln\left(1+\frac{y\alpha^2}{\bar{y}^2}\right)$$

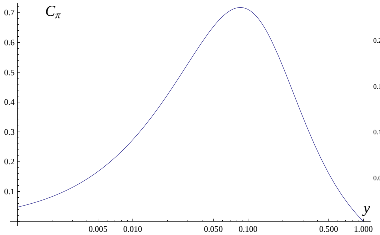
$$C^0_\pi(y,\Delta^2) = \frac{3g_a^2}{2}\delta(y)\int_0^1 d\eta R(\eta)\ln\frac{R(\eta)}{\mu^2} +6g_a^2M^2y\ln\left(1+\frac{\alpha^2\bar{y}}{y^2}\right)+6g_a^2y\int_0^{\bar{y}}d\eta\frac{m^2-\eta\Delta^2}{y^2+\frac{R(\eta)}{M^2}-y\frac{m^2-\eta\Delta^2}{M^2}}$$

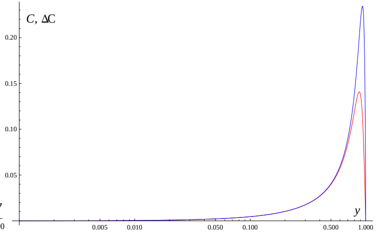
Leading order PDF.

$$C^0\left(y,\Delta^2\right) = -\frac{3g_a^2}{2}\delta(1-y)m^2\ln\frac{m^2}{\mu^2} - 3g_a^2\int_0^{\bar y}d\eta\frac{ym^2-\Delta^2\eta\bar y}{\bar y^2+\alpha^2 y-\frac{\Delta^2}{M^2}\eta(\bar\eta-y)}$$

$$\Delta C^0(y) = -\frac{3g_a}{2}\delta(1-y)m^2\ln\frac{m^2}{\mu^2} - 3g_a M^2\bar y\ln\left(1+\frac{y\alpha^2}{\bar y^2}\right)$$

$$C_\pi^0(y,\Delta^2) = \frac{3g_a^2}{2}\delta(y)\int_0^1 d\eta R(\eta)\ln\frac{R(\eta)}{\mu^2}$$
$$+6g_a^2M^2y\ln\left(1+\frac{\alpha^2\bar y}{y^2}\right)+6g_a^2y\int_0^{\bar y}d\eta\frac{m^2-\eta\Delta^2}{y^2+\frac{R(\eta)}{M^2}-y\frac{m^2-\eta\Delta^2}{M^2}}$$





Leading order PDF.

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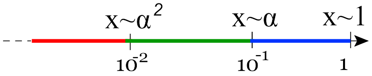
$$C_\pi^0(y,\Delta^2) = \frac{3g_a^2}{2}\delta(y)\int_0^1 d\eta R(\eta)\ln\frac{R(\eta)}{\mu^2}$$

$$+6g_a^2M^2y\ln\left(1+\frac{\alpha^2\bar y}{y^2}\right) + 6g_a^2 y\int_0^{\bar y} d\eta\frac{m^2-\eta\Delta^2}{y^2+\frac{R(\eta)}{M^2}-y\frac{m^2-\eta\Delta^2}{M^2}}$$



- Red terms are responsible for small-x region $x\sim\alpha^2$. They describe the pion cloud at large impact parameters $b_\perp$.
- Green terms become significant in the intermediate region $x\sim\alpha$.
- Blue terms are responsible for the region $x\sim 1$.

⊙ The consideration of the Mellin moments gives $\Delta C = 0$ and $C\sim\delta(1-y)$.

Why so different?

- $$M_N = \int_{-\infty}^{\infty} \mathrm{d}x x^N q(x,\Delta^2) \quad \Leftrightarrow \quad q(x,\Delta^2) = \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}N}{2\pi i} x^{-N-1} M_N$$
- The perturbative calculation leads to $\bar{M}_N = M_N^{(0)} + a_\chi^2 M_N^{(1)}$, i.e. $M_N - \bar{M}_N = \mathcal{O}(a_\chi^3)$.
- The restored from $\bar{M}_N$ PDF $\bar{q}(x)$
- $$q(x) - \bar{q}(x) = \mathcal{O}(a_\chi^3)??$$

⊙ The restored from the Mellin moments PDF can be of absolutely different order. It depends on how the perturbative expansion effects the analytic properties of PDF.

- For QCD, where α_s is dimensionless and presumably does not change the analytic properties the difference would appear only for very small x.
- For EFT, where a_χ is of kinematic origin the (non-accurate) perturbative expansion changes the analytic properties of PDF (shifts positions of poles, etc).
- These effects are well known for pion PDF (see [Kivel,Polyakov,AV]), and should be taken into account for nucleon case.

Mellin moments

$$O^A_{\pi N}(\lambda) = \int \mathrm{d}\beta \mathrm{d}\alpha \bar{N}(x) \left[F_1(\beta,\alpha)\frac{\not{n}}{2} t^A_+(x,y) + F_2(\beta,\alpha)\frac{\not{n}\gamma^5}{2} t^A_-(x,y)\right] N(y)$$

The corresponded Mellin moment operator reads (here we extract the large component of nucleon field $N_v(x) = e^{iM(vx)}N(x)$)

$$O_N = \int \mathrm{d}x x^N \int_{-\infty}^{\infty} d\lambda \, e^{-i\lambda x P_+} O_{\pi N}(\lambda) = \frac{\mathring{M}_N}{P_+^{N+1}} \bar{N}_v(0) \left[\frac{i \overleftrightarrow{\partial}_+}{2} - Mv_+\right]^N N_v(0) + (pions)$$

- Since λ is a dimensional parameter the standard counting rules of gradient expansion should be modified, in the regions of large λ.
- xP_+ and λ are "canonical conjugated" variables (Fourier integral demands $xP_+\lambda \sim 1$). Therefore, x "remembers" about different regions of integration over λ.
- Thus, restoration of PDF in different regions of x requires different counting rules for Mellin moment.

Region $x \sim 1$

- Let us consider standard chiral counting rules.

$$\begin{matrix} x\lambda(P\cdot n)\sim 1 \\ M\lambda(v\cdot n)\sim 1 \end{matrix} \quad \Bigg|\Bigg| \quad \begin{matrix} n\sim 1 \\ x\sim 1 \end{matrix} \quad \Rightarrow \quad \partial_+\sim q, \quad \lambda\sim\frac{1}{M}, \quad v_+\sim 1$$

- Small light-cone distances justify the expansion over λ (gives local MM-operators).
- Under such chiral counting rules one can forget about derivatives (at least for higher moments). MM-operators correspond to local operator for GPD(no parton dynamics!):

$$\frac{\mathring{M}_N}{P_+^{N+1}}\bar{N}_v(0)\Big[\underbrace{\frac{i\overleftrightarrow{\partial}_+}{2}}_{soft}-\underbrace{Mv_+}_{hard}\Big]^N N_v(0) \;\Rightarrow\; O_{\pi N}(\lambda)=\int d\beta\mathring{q}(\beta)e^{iMv_+\beta\lambda}\bar{N}_v(0)\frac{v_+}{2}N_v(0)$$

- Implies the multiplicative approximation for PDF: $q(x,\Delta^2)=q(x)F(\Delta^2)$, $F(\Delta^2)$–corresponding Form Factor.
- This regime was considered in all previous calculations of nucleon PDF and GPDs [Belitsky,Ji,02],[Diehl,et al,06],[Ando,Chen,Kao,06],[Dorati,et al,08]

Region $x \sim \frac{m}{M} \sim 10^{-1}$

- Counting rules for the intermediate region of x:

$$\begin{array}{c} x\lambda(P\cdot n)\sim 1 \\ M\lambda(v\cdot n)\sim 1 \end{array} \quad \Bigg\| \quad \begin{array}{c} n\sim 1/q \\ x\sim q/M \end{array} \quad \Rightarrow \quad \partial_+\sim 1, \quad \lambda\sim 1, \quad v_+\sim\frac{1}{M}$$

- Under such chiral counting rules MM-operators correspond to the truly non-local operator:

$$\frac{\mathring{M}_N}{P_+^{N+1}}\bar{N}_v(0)\Bigg[\underbrace{\frac{i\overleftrightarrow{\partial}_+}{2}-Mv_+}_{same\quad order}\Bigg]^N N_v(0) \;\Rightarrow\; \int d\beta\,\mathring{q}(\beta)e^{iMv_++\beta\lambda}\bar{N}_v\left(\frac{\beta\lambda n}{2}\right)\frac{v_+}{2}N_v\left(-\frac{\beta\lambda n}{2}\right)$$

⊙ Consideration of this region requires the all order summation of particular diagrams in Mellin moment representations, but automatically covered by the consideration of the non-local operator.

Bright representative of this region is parity mixing term ΔC

$$\Delta C^0(y) = -\frac{3g_a}{2}\delta(1-y)m^2\ln\frac{m^2}{\mu^2} - 3g_a M^2\bar{y}\ln\left(1+\frac{y\alpha^2}{\bar{y}^2}\right)$$

$$\Delta M_N = 0+\mathcal{O}\left(\frac{m^4}{M^4}\right)$$

$x \sim \frac{m^2}{M^2} \sim 10^{-2}$

-

$$\begin{matrix} x\lambda(P \cdot n) \sim 1 \\ M\lambda(v \cdot n) \sim 1 \end{matrix} \quad \Bigg\| \quad \begin{matrix} n \sim M/q \\ x \sim q^2/M^2 \end{matrix} \quad \Rightarrow \quad \partial_+ \sim M, \quad \lambda \sim \frac{M}{q^2}, \quad v_+ \sim q^2/M^2$$

- Under such chiral counting rules one can forget about Mv_+ term. The non-local operator loses its exponential factor:

$$\frac{\mathring{M}_N}{P_+^{N+1}} \bar{N}_v(0) \Big[\underbrace{\frac{i \overset{\leftrightarrow}{\partial}_+}{2}}_{hard} - \underbrace{M v_+}_{soft} \Big]^N N_v(0) \;\Rightarrow\; O_{\pi N}(\lambda) = \int \mathrm{d}\beta \mathring{q}(\beta) \bar{N}_v \left(\frac{\beta\lambda n}{2} \right) \frac{v_+}{2} N_v \left(-\frac{\beta\lambda n}{2} \right)$$

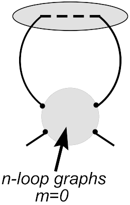
⊙ In this region one need to sum up all orders diagrams in order.
⊙ This region is of the special importance for the pion PDF, and for the pion cloud contribution to the nucleon PDF.

$$C^0_\pi(y,\Delta^2) = \frac{3g_a^2}{2}\delta(y)\int_0^1 d\eta R(\eta)\ln\frac{R(\eta)}{\mu^2}$$
$$+6g_a^2 M^2 y\ln\left(1+\frac{\alpha^2\bar{y}}{y^2}\right)+6g_a^2 y\int_0^{\bar{y}} d\eta\frac{m^2-\eta\Delta^2}{y^2+\frac{R(\eta)}{M^2}-y\frac{m^2-\eta\Delta^2}{M^2}}$$

⊙ This δ-function is only the first term for the series of δ-functions

$$C_\pi(y,0) = (\text{regular terms}) + c_1\delta(y)a_\chi\ln a_\chi + c_2\delta'(y)a_\chi^2\ln^2 a_\chi + c_3\delta''(y)a_\chi^3\ln^3 a_\chi + \ldots$$
$$= (\text{regular terms}) + f\left(\frac{x}{a_\chi\ln a_\chi}\right)$$

- The coefficients c_n are proportional to the leading logarithm coefficients of $\pi\pi$-scattering [Kivel, Polyakov, 0707.2208].
- Can be obtained via renormalization group recursive equations [Kivel,Polyakov,Vladimirov,0809.3236]
- Or alternatively can be obtained via the analytic properties of the amplitude [Koschinski,Polyakov,Vladimirov,1004.2197]



Conclusion

⊙ We have performed the detailed analysis for nucleon GPD operators in EFT [AM,Vladimirov,1208.1714].

- Reconstruction of GPDs in different regions of x requires different counting rules for Mellin moment.
- For non-local operators we do not need special counting rules.
- At low $x \sim \frac{m_\pi^2}{M_N^2} \sim 10^{-2}$ a resummation of the singular terms is required.

⊙ We have calculated GPD functions at the leading order of chiral expansion in the limits $\xi = 0$ and $\xi \neq 0$.

- A parity mixing contribution appears at a_χ^2 chiral order. It was believed to be strongly suppressed ($\sim a_\chi^4$) from MM calculations.
- Calculations with non-local GPD operators explore a rich structure of chiral corrections in x-space.

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Thank you for your attention.