Pion-photon transition form factor in light-cone sum rules: BaBar faces Belle

based on 1202.1781[hep-ph], will appear in PRD

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Experiments to $e^+e^- \longrightarrow e^+e^-\pi^0$

One of the **most accurate** results on exclusive reactions is data on transition FF $F^{\gamma^*\gamma^*\pi^0}(-Q^2 = q_1^2, q_2^2)$ provided by the experiments $e^+e^- \to e^+e^-\pi^0$, $q_2^2 \approx 0$.

CELLO (1991) $Q^2 : 0.7 - 2.2$ GeV^2 ,

CLEO (1998) $Q^2 : 1.6 - 8.0$ GeV^2 ,

followed to collinear QCD.

BaBar (2009) $Q^2 : 4 - 40$ GeV^2

Q^2**·FF certainly growth with** Q^2**,**

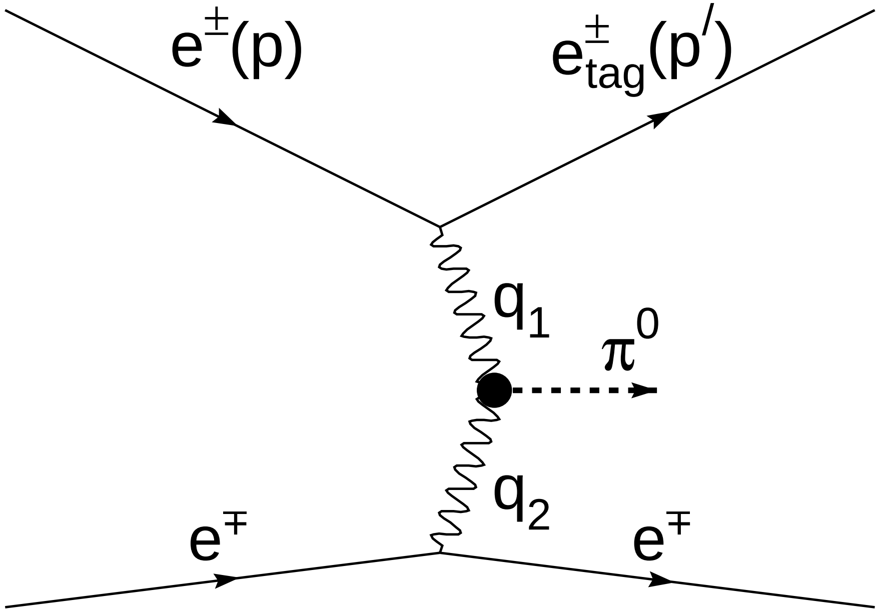
creating the “BaBar puzzle”,

Belle (2012) $Q^2 : 4 - 40$ GeV^2

return to collinear QCD?

BESIII (????) $Q^2 \leq 5$ GeV^2,

promises very precise data



Outline:

1. **Experimental background on Pion-photon transition form factor (TFF)**

2. **Pion-photon TFF in collinear QCD, its components**

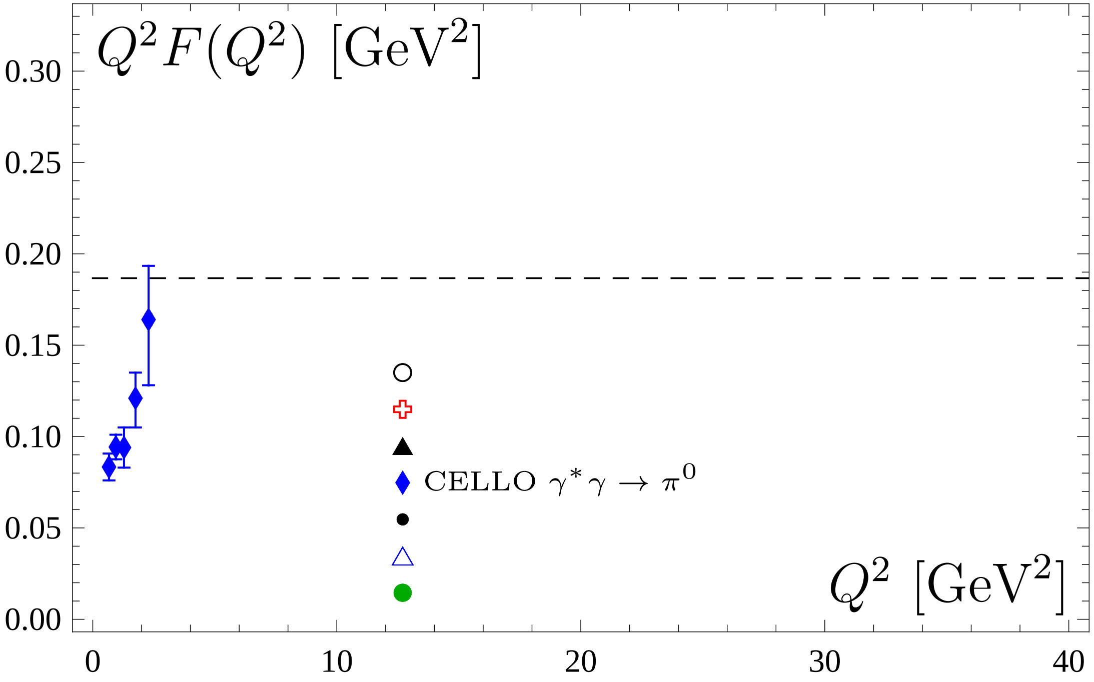
3. **Light Cone Sum Rules for Transition FF, there ingredients**

4. **TFF and experiments: predictions and determination of Pion distribution amplitude in 2- and 3-harmonic analysis**

5. **Conclusions**

Experimental background on Pion TFF

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**



Data	Collab.
◆	**CELLO (1991)**

dashed line = $\sqrt{2}\, f_\pi$

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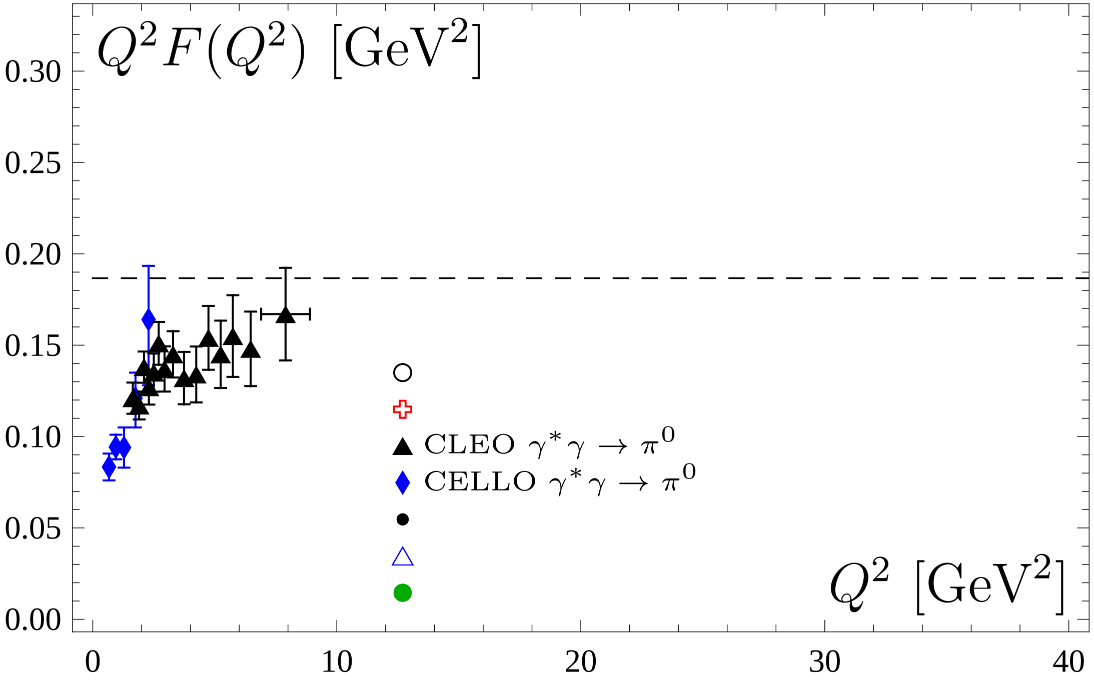


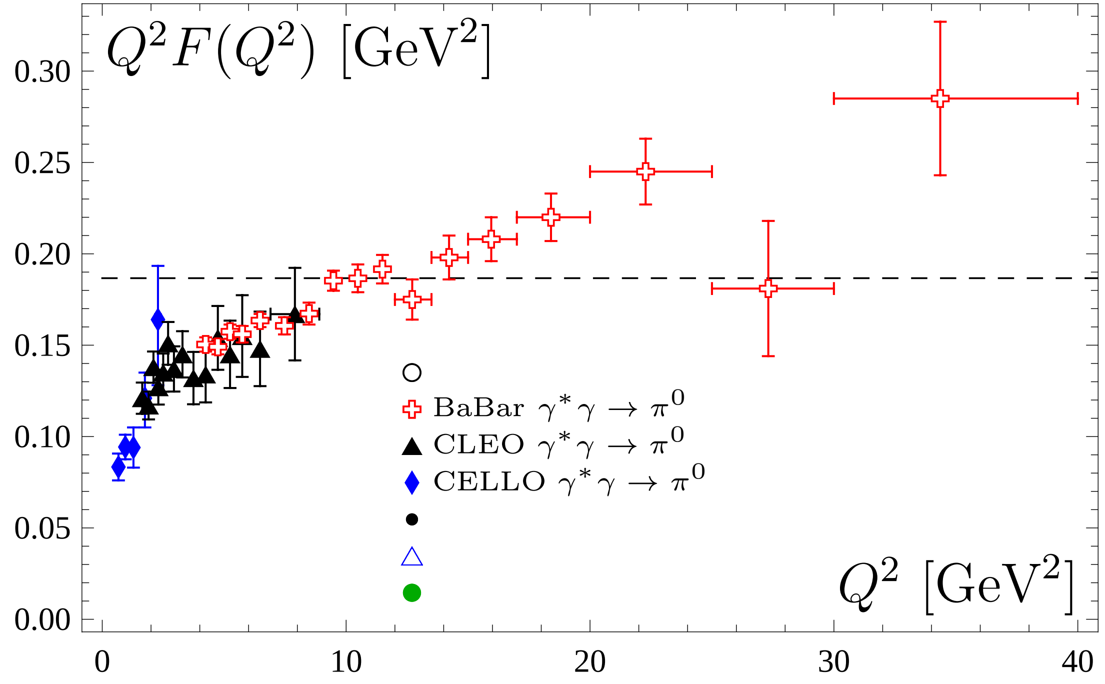
Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)

dashed line = $\sqrt{2}\, f_\pi$

CELLO and CLEO data good agree with QCD collinear factorization, [BMS2003-06] within NLO QCD$\oplus$twist-4$\oplus$[end-point suppressed DA]

Experimental background on Pion TFF

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**



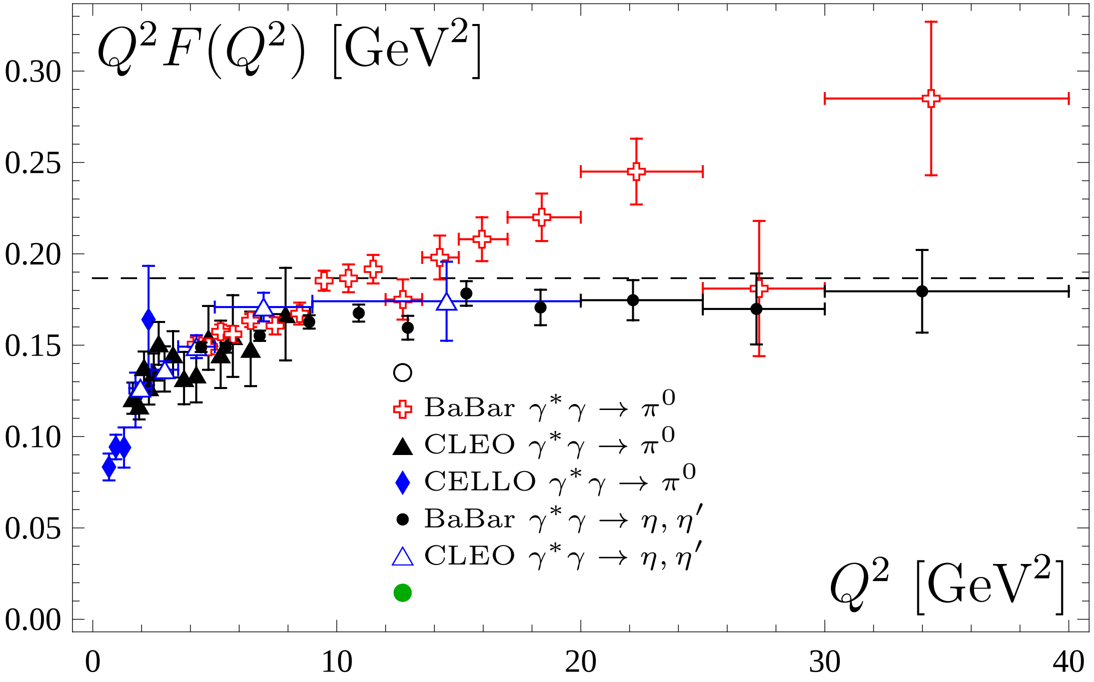
Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✚	**BaBar (2009)**

dashed line = $\sqrt{2}\, f_\pi$

[arXiv:1101.1142]: "If the experiment is correct, many theoretical predictions should be revised. . . "

Experimental background on Pion TFF

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**



Data	Collab.
◆	**CELLO (1991)**
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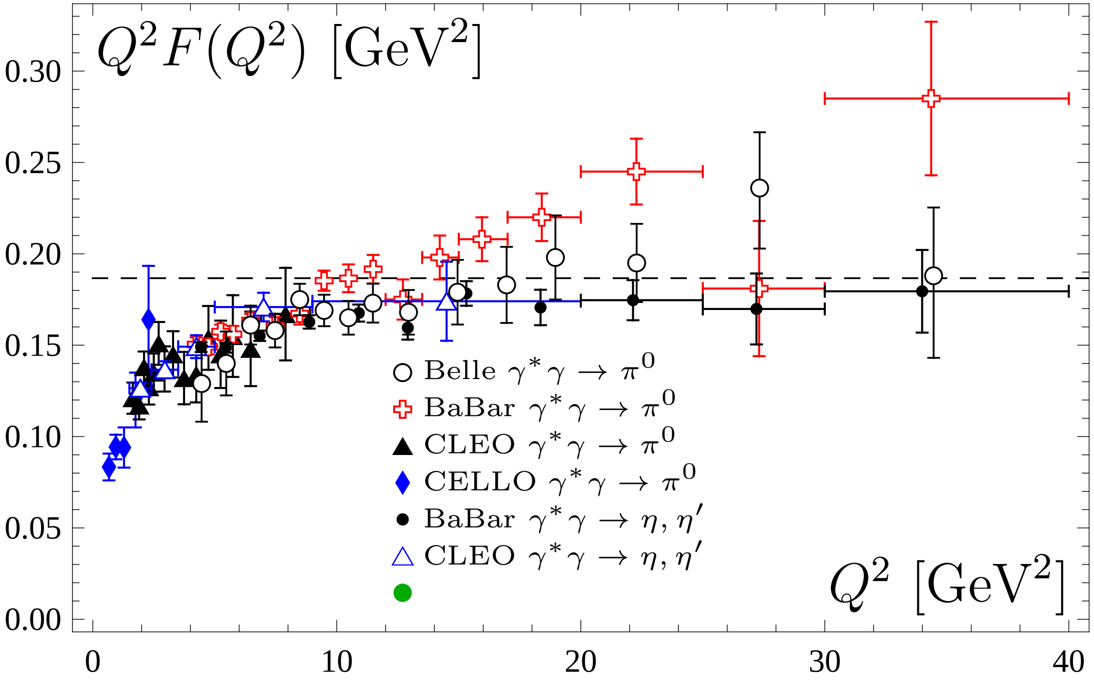
✚	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$, (2011)**

dashed line = $\sqrt{2}\, f_\pi$

[NPBSub,234,2013]: "It comes out as a surprising result that the Q^2 dependence of the non-strange TFF is in strong disagreement with the π^0 TFF."

Experimental background on Pion TFF

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**



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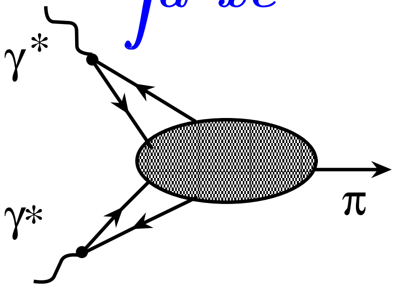
❍	**Belle** (2012)
dashed line = $\sqrt{2}\, f_\pi$	

[NPBSub,234,2013]: “Recent of Belle data is in conflict with BaBar.”

They do not confirm auxetic form factor behavior above 10 GeV^2.

Factorization $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD



$$\int d^4x e^{-iq_1\cdot z}\langle\pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$

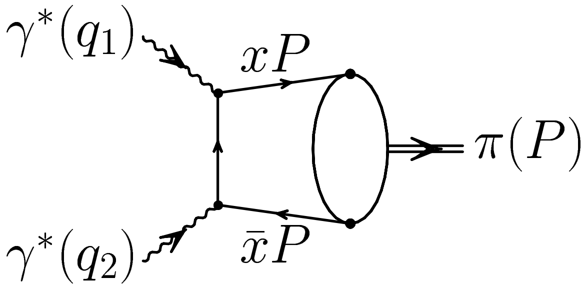
where $-q_1^2 = Q^2 > 0, \quad -q_2^2 = q^2 \geq 0$

Collinear factorization at $Q^2, q^2 \gg$ **(hadron scale $\sim m_\rho$)**^{2} for the leading twist

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = T(Q^2,q^2,\mu_F^2;x)\otimes\varphi_\pi(x;\mu_F^2) + O(\frac{1}{Q^4})\,,$$

μ_F^2 – boundary between large scale Q^2 and hadronic one. At the parton level

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}}{3}f_\pi\int_0^1 dx\frac{1}{Q^2x+q^2\bar{x}}\varphi_\pi(x).$$



$$Q^2F^{\gamma^*\gamma\pi}(Q^2,q^2\to 0) = \frac{\sqrt{2}}{3}f_\pi\int_0^1\frac{dx}{x}\varphi_\pi(x) \equiv \frac{\sqrt{2}}{3}f_\pi\langle x^{-1}\rangle_\pi$$

Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

- Pion DA parameterizes this matrix element:

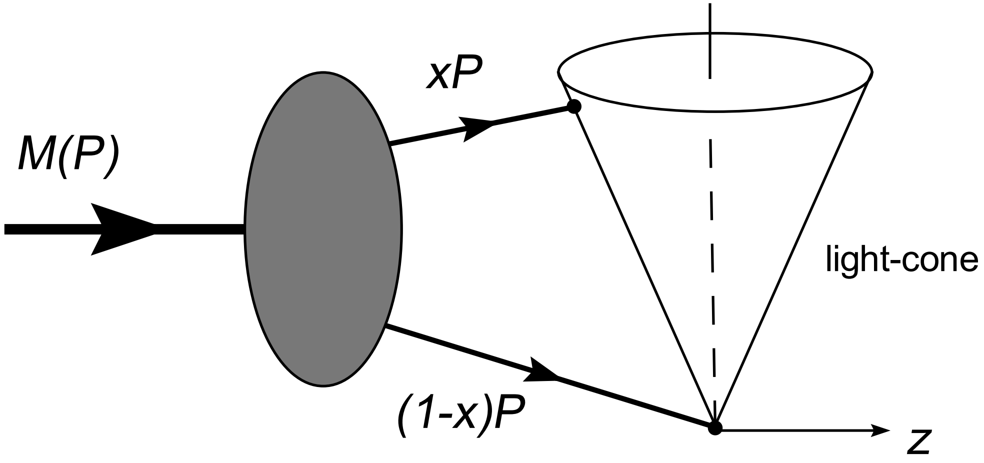
$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\; e^{ix(zP)}\varphi_\pi(x,\mu^2)\,.$$

where path-ordered exponential

$$[z,0] = \mathcal{P}\, exp\left[ig\int\limits_0^z t^a A_\mu^a(y)dy^\mu\right],$$

i.e., light-like gauge link, ensures gauge invariance.

- Pion DA describes transition of physical pion into two valence quarks, separated at light cone.



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Distribution amplitudes are **nonperturbative** quantities to be derived from

- QCD SR **[CZ 1984]**,
  NLC QCD SR **[M&Radyushkin 1986-91, Bakulev&M&S 1998,2001–04,13]**
- instanton-vacuum approaches, e.g.
  [Polyakov *et al.* 1998, 2009; Dorokhov *et al.* 2000,07]
- Light-front quark model **[Choi&Ji 2007]**
- Lattice QCD, **[Braun *et al.* 2006; Donnellan *et al.* 2007; Arthur *et al.* 2011]**
- from experimental data **[Schmedding&Yakovlev 2000, Khodjamirian *et al.* 2000,2002, BM(P)S 2003–12]**

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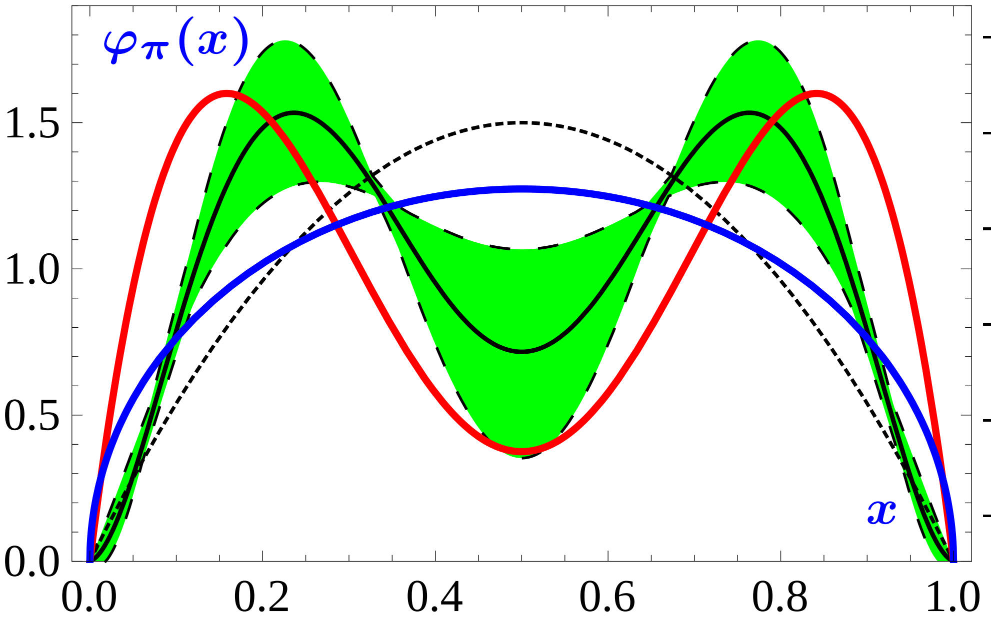
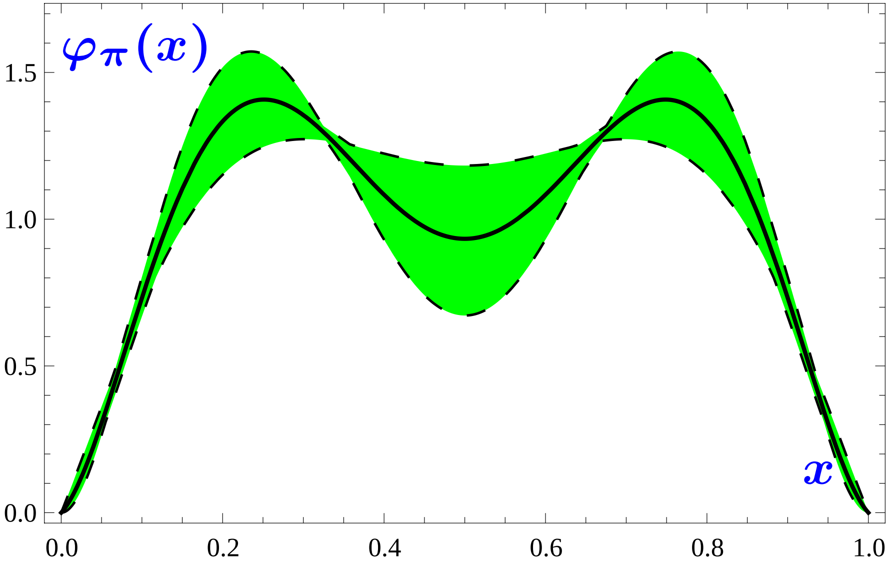


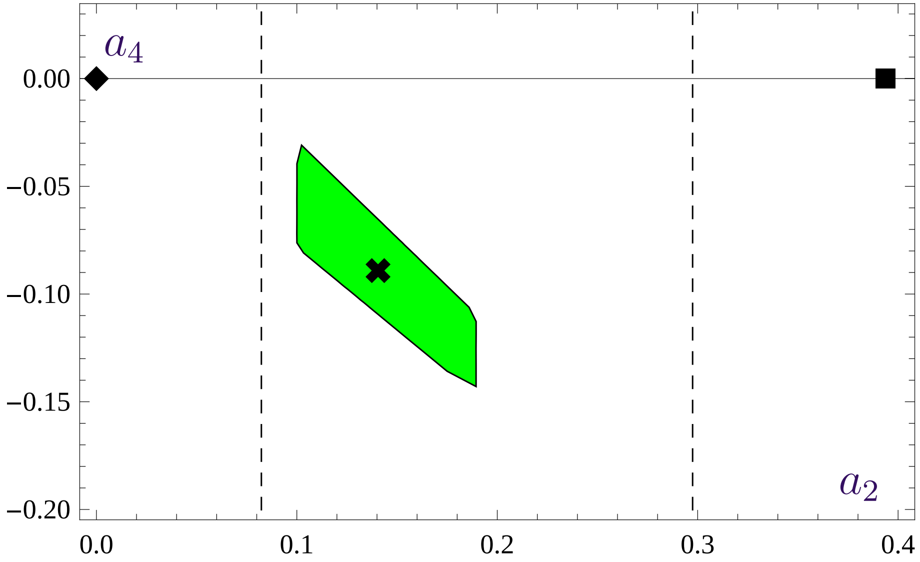
Curve	Approach
- - - - -	Asymptotic
🟩	BMS DA, NLC QCD SR
🟥	CZ from QCD SR
🟦	AdS/QCD result

- DA evolution with μ^2, according to ERBL equation **[79-80]**.
- Gegenbauer expansion of pion DA: $\varphi_\pi(x,\mu^2) \Leftrightarrow a_2(\mu^2), a_4(\mu^2), \ldots, a_n$

$$\varphi_\pi(x,\mu^2) = 6x\bar{x}(1 + a_2 C_2^{3/2}(x-\bar{x}) + a_4 C_4^{3/2}(x-\bar{x}) + a_6 C_6^{3/2}(x-\bar{x}) + \ldots)$$

Pion DA from QCD SR with NLC

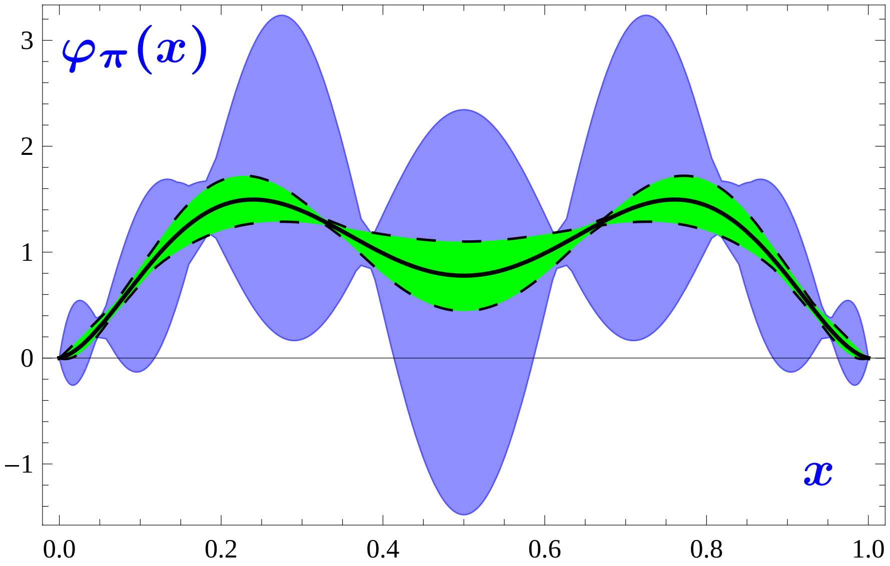


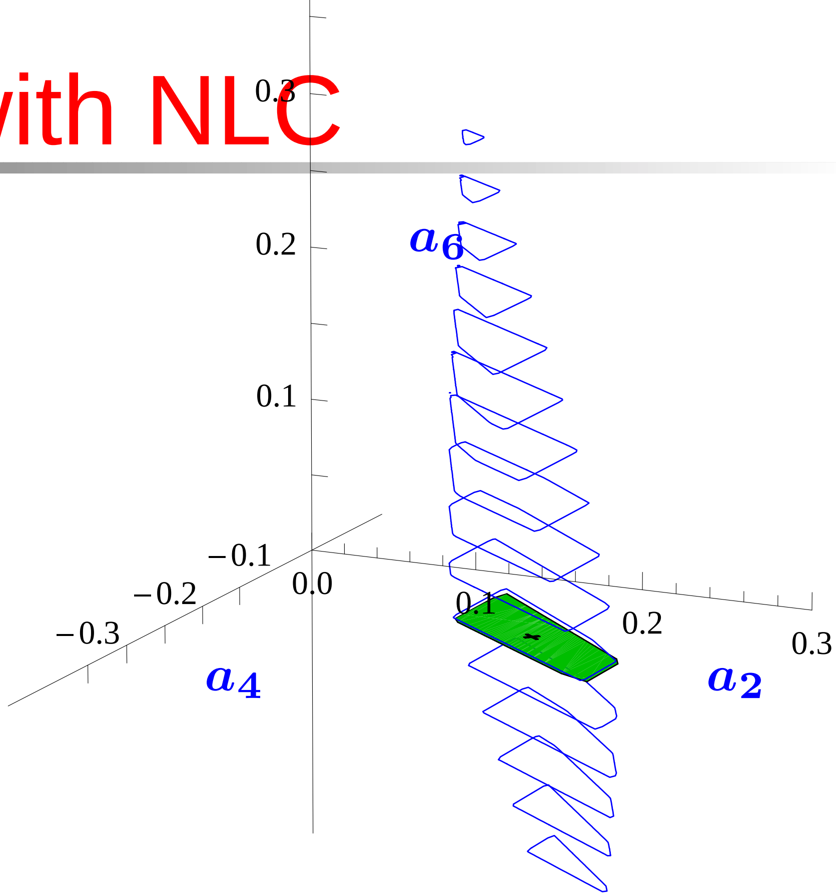


BMS DA model and bunch were obtained using Gaussian condensate model with single nonlocality parameter $\lambda_q^2 = 0.4\,\text{GeV}^2$.

- higher Gegenbauer coefficients $a_{n\geq 6} = 0$ be equal to 0, that is supported by QCD SR.
- QCD SR with NLC provides end-poind suppressed pion DA with slope $\varphi'_\pi(0) \approx 6$ that depends on behavior of quark-condensate at large distances.

Pion DA from QCD SR with NLC





BMS DA model and bunch were obtained using Gaussian condensate model with single nonlocality parameter $\lambda_q^2 = 0.4\,\text{GeV}^2$.

- large uncertainties at $a_6 \approx 0$ are taken into account; higher Gegenbauer coefficients $a_{n\geq 8} = 0$ that is supported by QCD SR.

NLO and NNLO amplitudes.

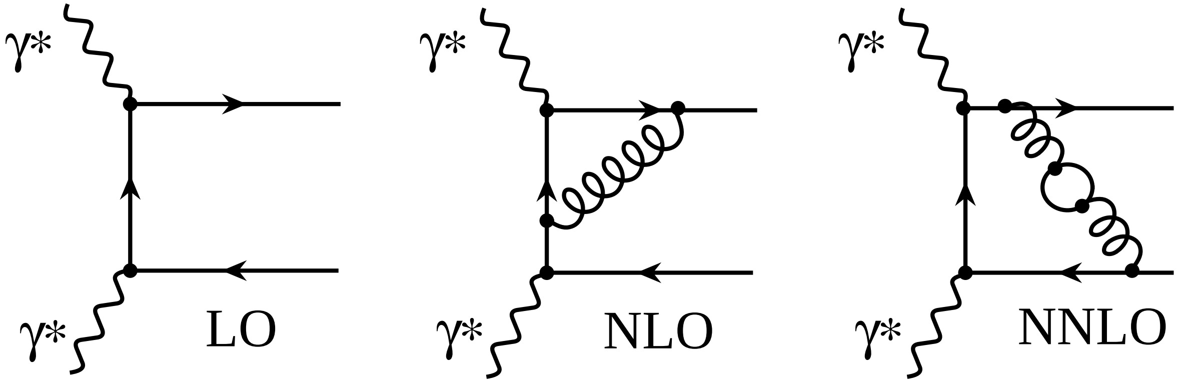
Collinear factorization **[Efremov&Radyushkin 1978]**

$$F^{\gamma^*\gamma^*\pi} \sim \Big(T_0(Q^2,q^2;x) + \ a_s^1\, T_1(Q^2,q^2;\mu_F^2;x)$$
$$+ \ a_s^2\, T_2(Q^2,q^2;\mu_{\rm F}^2;\mu_{\rm R}^2;x) + \ldots\Big) \otimes \ \varphi_\pi^{(2)}(x;\mu_{\rm F}^2)$$
$$- \ \delta_{tw4}^2(\mu_F^2)\cdot T_0^2(Q^2,q^2;x) \otimes \ \varphi_\pi^{(4)}(x)$$

Hard amplitudes T_i — calculable in pQCD, coupling constant — $a_s = \alpha_s(\mu_{\rm R}^2)/(4\pi)$.

Usual setting $\mu_{\rm R}^2 = \mu_F^2 = \langle Q^2\rangle$ to simplify and minimize rad. corrections.

Twist-4 scale parameter – $\delta_{tw4}^2 = (0.19 \pm 0.02)$ GeV^2.

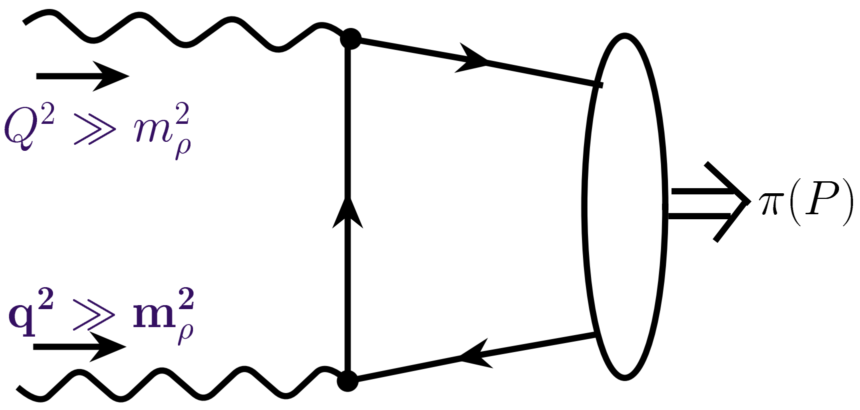


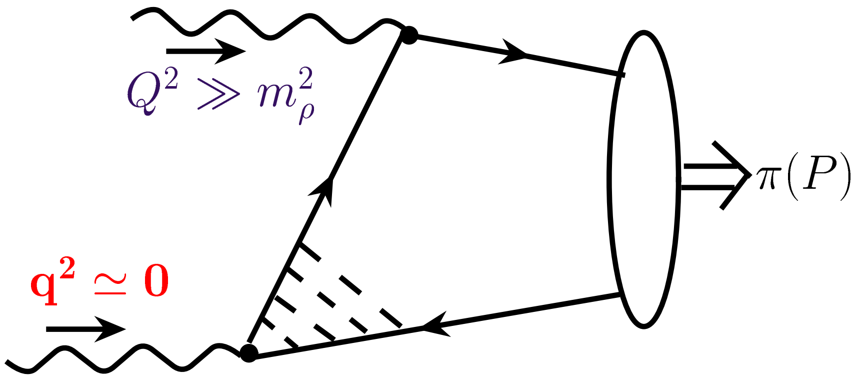
LO hard amplitude: $$T_0(Q^2,q^2;x) = \frac{1}{x\,Q^2 + \bar{x}\,q^2}$$

$\gamma^*\gamma \to \pi$: Why Light-Cone Sum Rules?

- For $Q^2 \gg m_\rho^2,\ \ q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are important **[Radyushkin–Ruskov, NPB (1996)]**.

- Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$





pQCD is OK **LCSRs should be applied**

$\gamma^*\gamma \longrightarrow \pi$: Light-Cone Sum Rules

LCSR effectively takes into account long-distances effects of real photon using **quark-hadron duality** in vector channel and **dispersion relation** in q^2 (Balitsky et. al.-**[NPB (1989)]**, Khodjamirian **[EJPC (1999)]**)

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) \;=\; \int_0^{s_0} \frac{\rho^{\text{PT}}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \int_{s_0}^{\infty} \frac{\rho^{\text{PT}}(Q^2, s)}{s + q^2} ds\,,$$

where $s_0 \simeq 1.5$ **GeV^2** – effective threshold in vector channel, M^2 – Borel parameter ($0.5 - 0.9$ GeV^2). Limit to real-photon $q^2 \to 0$ can be done.

Spectral density was calculated in QCD:

$$\rho^{\text{PT}}(Q^2, s) = \text{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, -s - \imath\varepsilon) = \text{Tw-2} + \text{Tw-4} + \text{“Tw-6”} + \ldots\,,$$

twists contributions are given in a form of convolution with pion DA:

$$\text{Tw-2} \sim \left(T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}_{\beta_0}} + \ldots\right) \otimes \varphi_\pi^{\text{Tw2}}(x, \mu)\,.$$

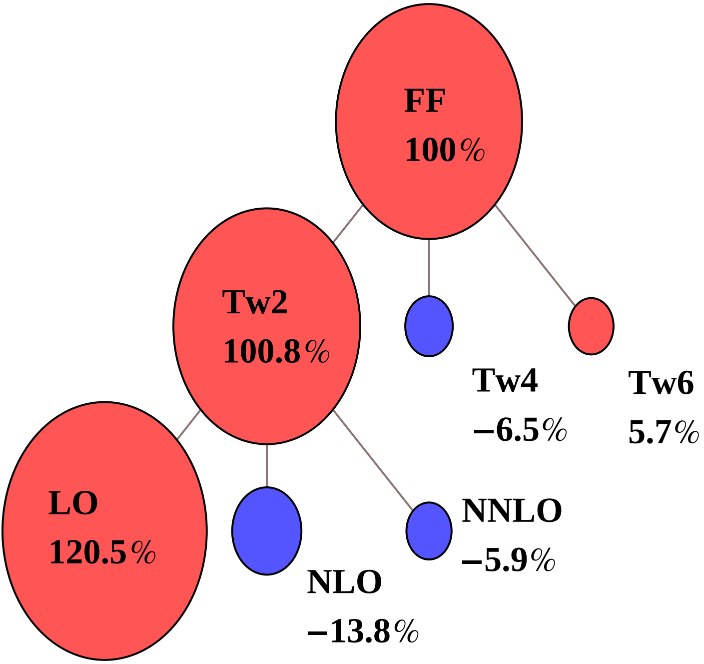
Main Ingredients of Spectral Density

- **LO** Spectral Density, **Tw-4** term **[Khodjamirian(1999)]**
- **NLO** Spectral Density, **[M&Stefanis(2009)]**,corr. in **[Agaev et.al.(2011)]**
- **NNLO**$_{\beta_0}$ Spectral Density, **[M&Stefanis(2009)]**
- **“Tw-6”** contribution, **[Agaev et.al.(ABOP 2011)]**

- **NLO** evolution of pion DA **[Kadantseva&M&R, S.J.NP.(1986)]**

Terms of Pion-Photon FF at $Q^2 = 8$ GeV^2

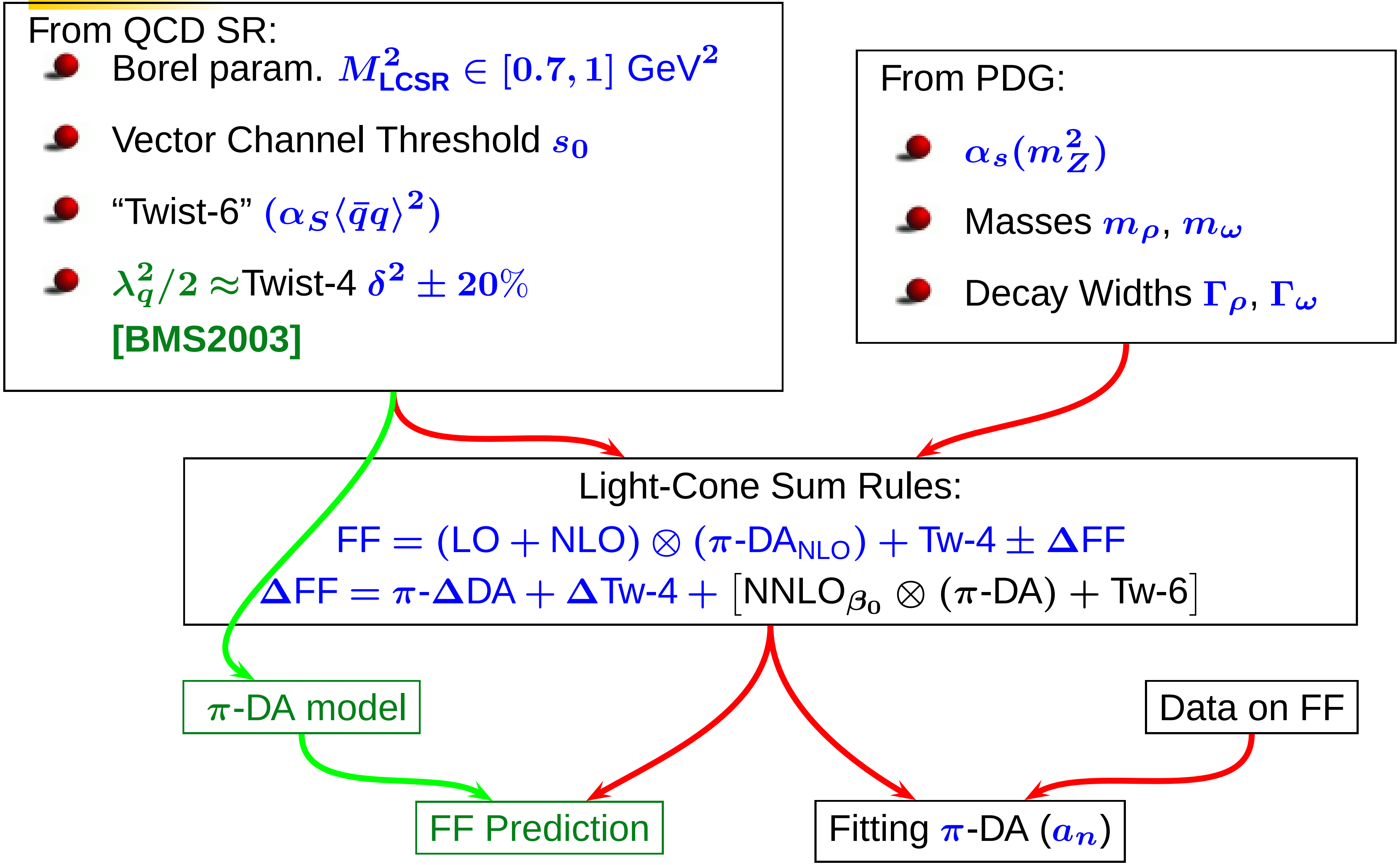
- Result is dominated by the Twist-2 LO and NLO.
- **“Twist-6”** contribution is taken into account together with **NNLO**$_{\beta_0}$ one — they has close absolute values and opposite signs.



Blue - negative terms

Red - positive terms

Parameters of LC SR



From QCD SR:

- Borel param. $M^2_{\text{LCSR}} \in [0.7, 1]$ GeV^2
- Vector Channel Threshold s_0
- “Twist-6” $(\alpha_S\langle\bar{q}q\rangle^2)$
- $\lambda_q^2/2 \approx$Twist-4 $\delta^2 \pm 20\%$ **[BMS2003]**

From PDG:

- $\alpha_s(m_Z^2)$
- Masses m_ρ, m_ω
- Decay Widths Γ_ρ, Γ_ω

Light-Cone Sum Rules:

$$\text{FF} = (\text{LO} + \text{NLO}) \otimes (\pi\text{-DA}_{\text{NLO}}) + \text{Tw-4} \pm \Delta\text{FF}$$

$$\Delta\text{FF} = \pi\text{-}\Delta\text{DA} + \Delta\text{Tw-4} + \left[\text{NNLO}_{\beta_0} \otimes (\pi\text{-DA}) + \text{Tw-6}\right]$$

π-DA model

Data on FF

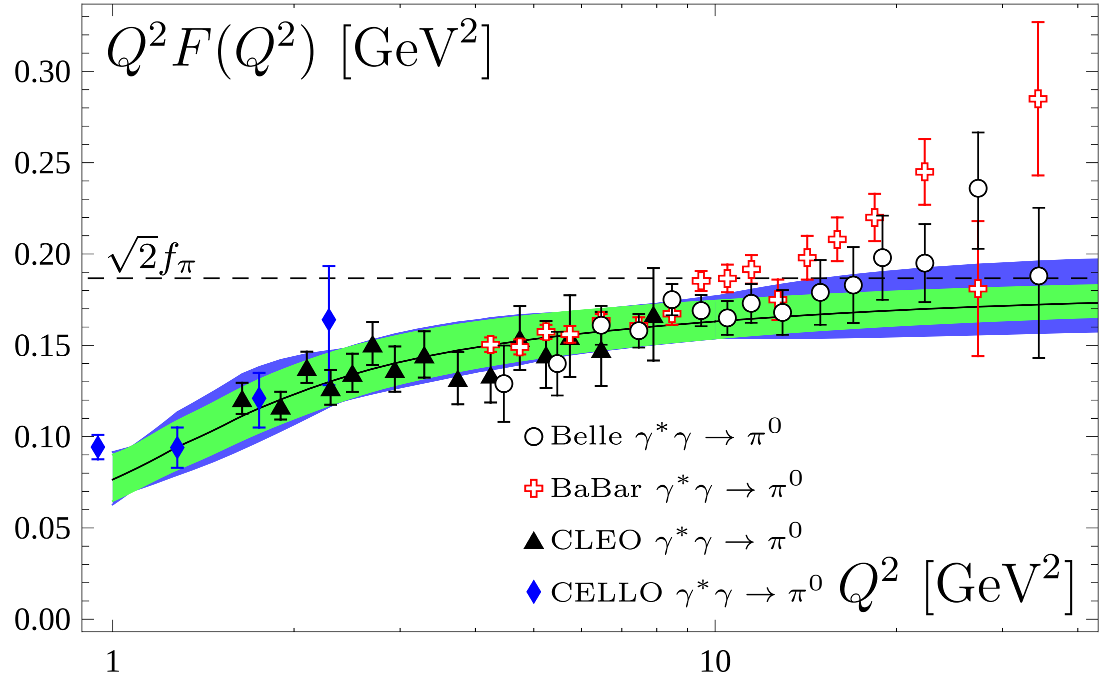
FF Prediction

Fitting π-DA (a_n)

Pion-gamma transition FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**

[PRD86,092007(2012)]

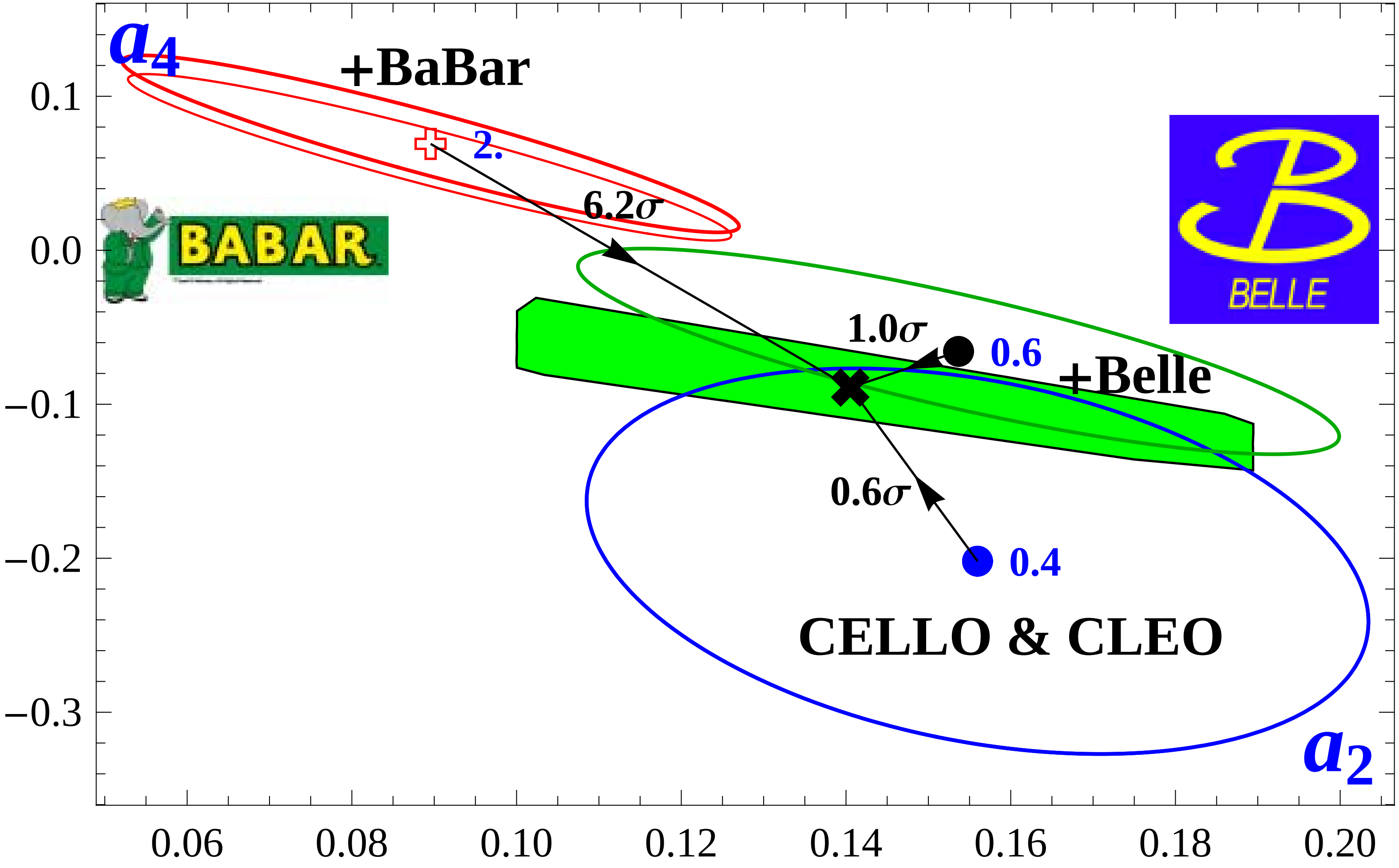


Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✙	**BaBar (2009)**
❍	**Belle (2012)**
🟩	our result **BMPS**

Belle data do not confirm auxetic form factor behavior above 10 GeV^2 (except outlier at $Q^2 = 27.33$ **GeV^2**).

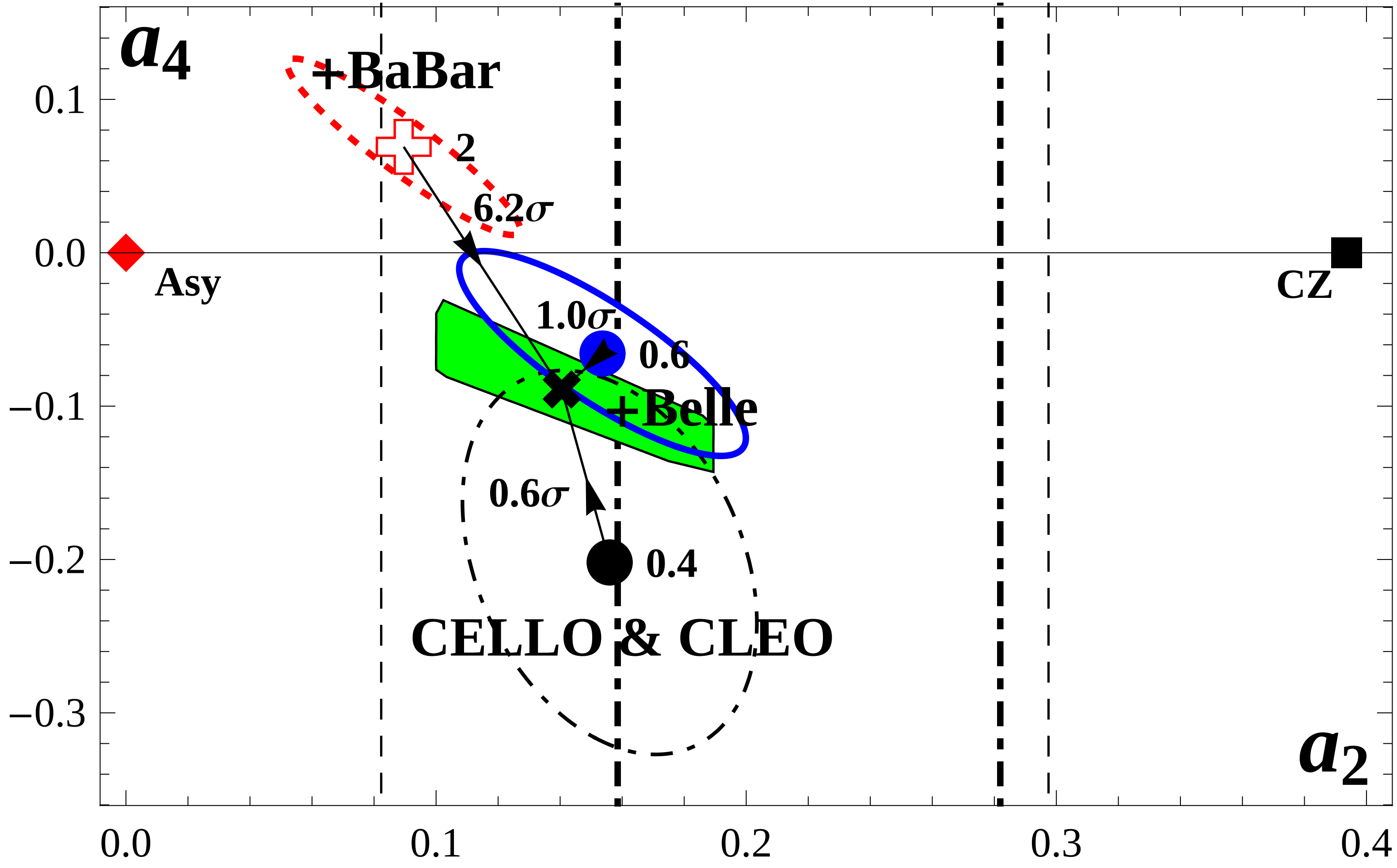
Predicted FF agrees well with CELLO, CLEO, BaBar$_{Q^2<9\ \mathrm{GeV}^2}$ (2009), BaBar$^{\eta}_{\eta'}$ (2011), and most **Belle** (2012).

Confidential regions in 2D (a_2, a_4)



- In vertexes of a triangle – χ^2/ndf, all estimates at $\mu_{\text{SY}} = 2.4$ GeV.
- On sides of triangle: discrepancy in terms of stand. deviation ($1\sigma \approx 68\%$)

Confidential regions in 2D (a_2, a_4)

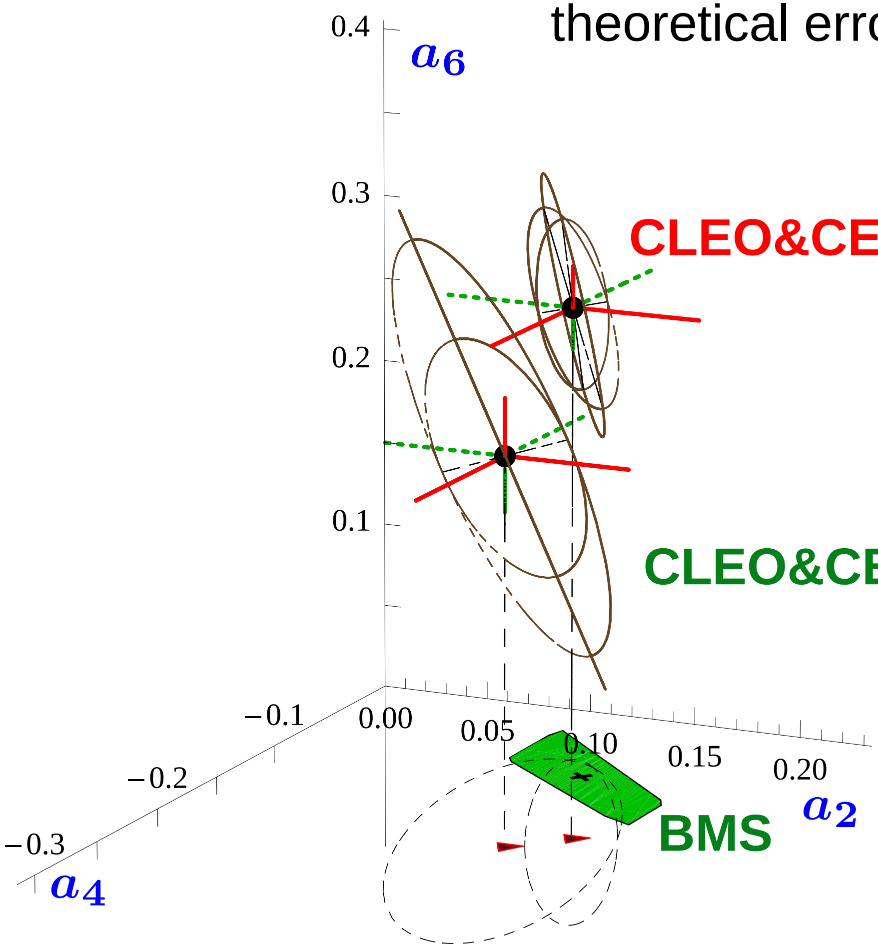


- **BMS DA (✖ &green bunch)** from QCD SR with nonlocal condensates: $\lesssim 1\sigma$.
- ◆ **Asy**mptotic DA, CZ DA: $> 6\sigma$.
- Vertic. lines – lattice constraints: wider – **Braun et al.[2006]**, thinner – **[2011]**.

NLC SR Results vs 3D Constraints (a_2, a_4, a_6)

3D 1σ-error ellipsoid for $(a_2,\ a_4,\ a_6)$ at $\mu_{\text{SY}} = 2.4$ GeV scale
with theoretical $\mp\Delta\delta^2_{\text{tw4}}$-error shown by **green($-$)** and **red($+$)** length.

theoretical errors are correlated for different data sets.



CLEO&CELLO&BaBar Data, $\chi^2_{\text{ndf}} \approx 1.0$

CLEO&CELLO&Belle Data, $\chi^2_{\text{ndf}} \approx 0.4$

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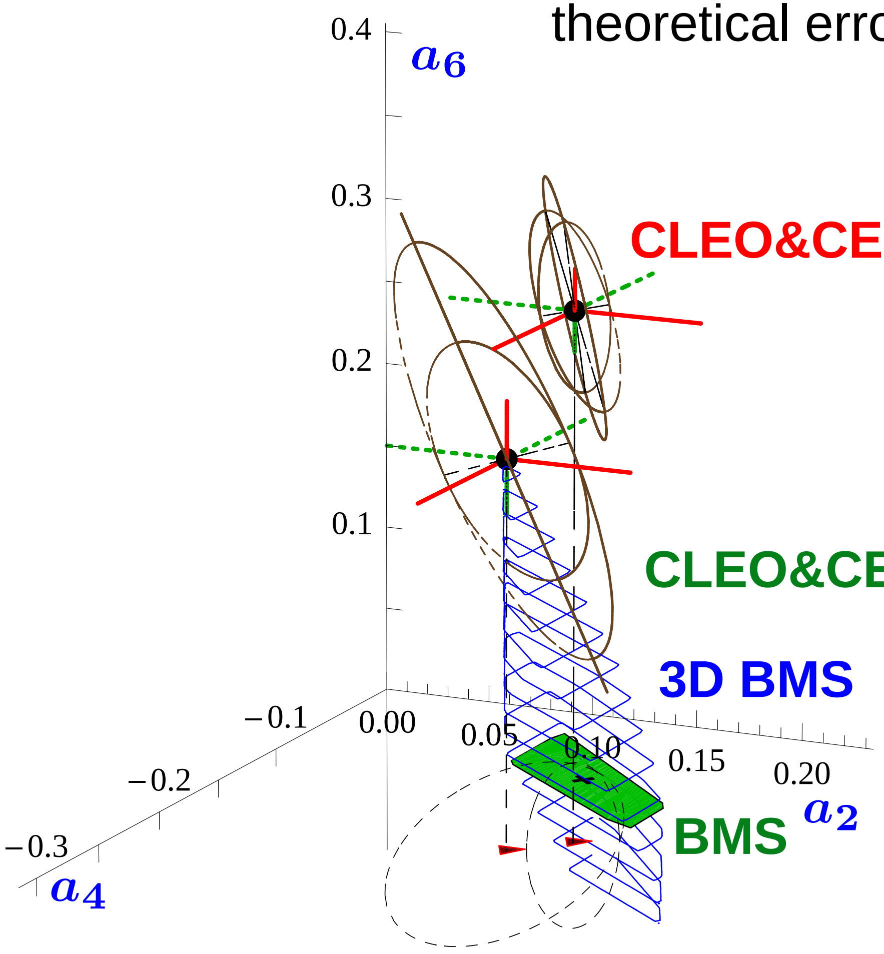

- QCD SR with Nonlocal Condensate result: $a_6 \approx 0 \pm 0.2$.
- $a_{n\geq 6} \rightarrow 0$ assumption gives **2D BMS** pion DA bunch.
- $a_{n\geq 8} \rightarrow 0$ assumption leads to **3D BMS** pion DA bunch.

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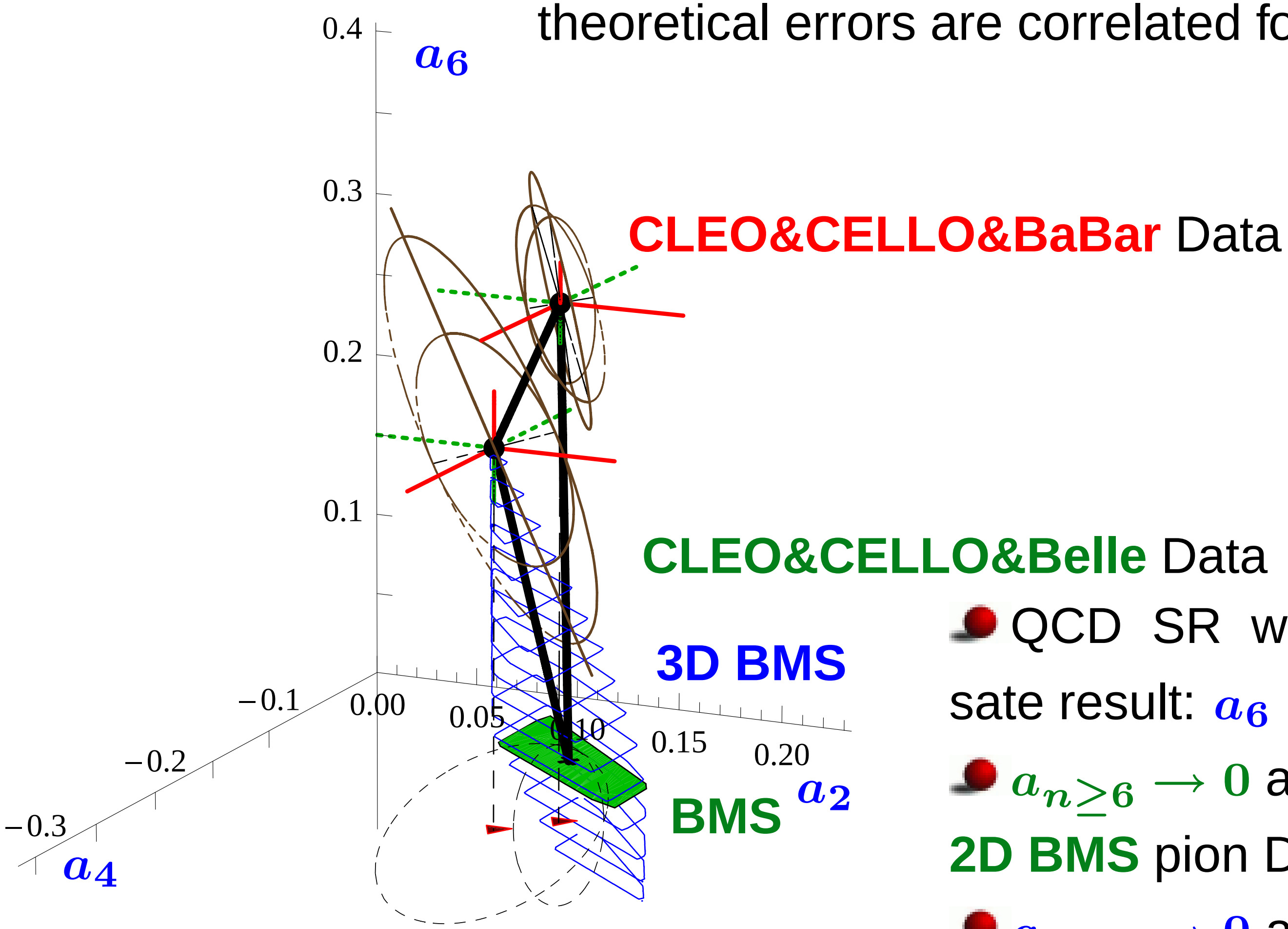

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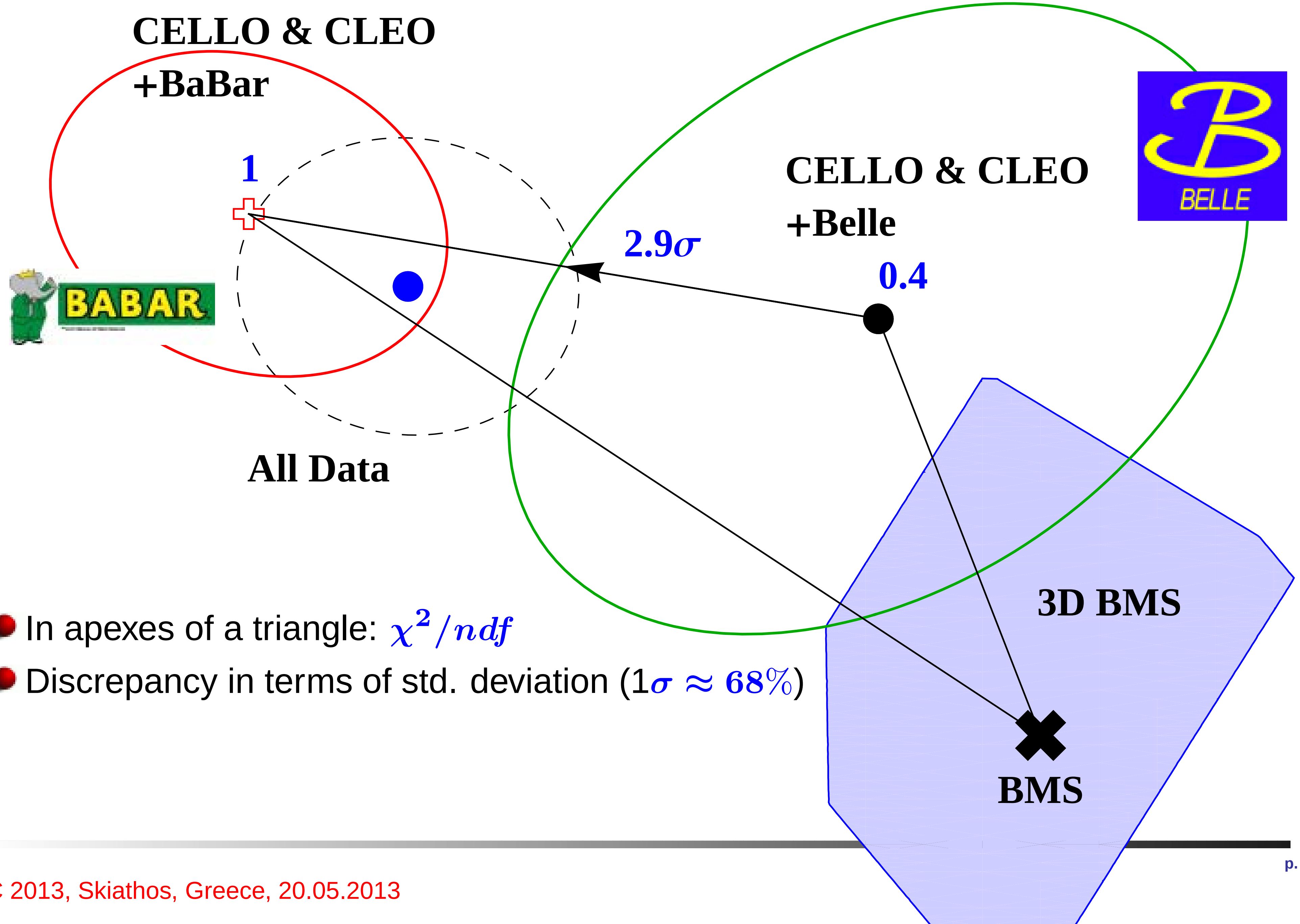
2D cut of 3D confidence regions ((a_2, a_4, a_6))



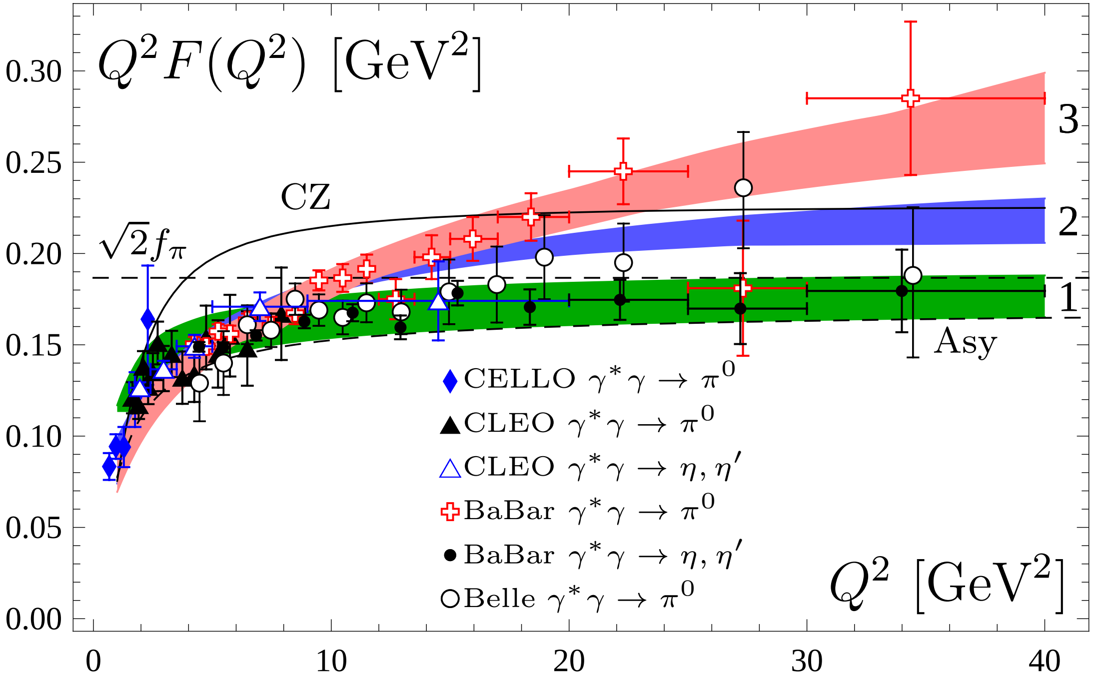
- In apexes of a triangle: χ^2/ndf
- Discrepancy in terms of std. deviation (1$\sigma \approx 68\%$)

Conclusions

1. Performed **2D** and **3D** analysis of **CELLO**, **CLEO**, **BaBar**, **Belle** data using LCSRs at NLO and Tw-4 term and taking [NNLO$_\beta$ + “twist6"] as uncertainties.
2. We showed that the data from **CELLO**, **CLEO**, **BaBar**, and **Belle** at $\boldsymbol{Q^2 = 1 - 9}$ **GeV^2** in 2D analysis favor a pion DA with endpoint suppression, like **BMS bunch**.
3. Beyond $\boldsymbol{Q^2 = 9}$ **GeV^2**, the best fit to data including BaBar on $\boldsymbol{F_{\gamma^*\gamma\to\pi}}$ requires a sizeable coefficient $\boldsymbol{a_6}$.
4. **2D, 3D** analysis of CLEO-CELLO-**Belle** data **agrees** with **BMS bunch** and **does not agree** with CLEO-CELLO-**BaBar** one.
5. **3D** analysis of CLEO-CELLO-**Belle** certainly **does not agree** with CLEO-CELLO-**BaBar** one.
6. The promising **fine accuracy** of future **BESSIII** experiment can clarify choice between **BaBar** and **Belle** results.

Store

Pion TFF Data and Models

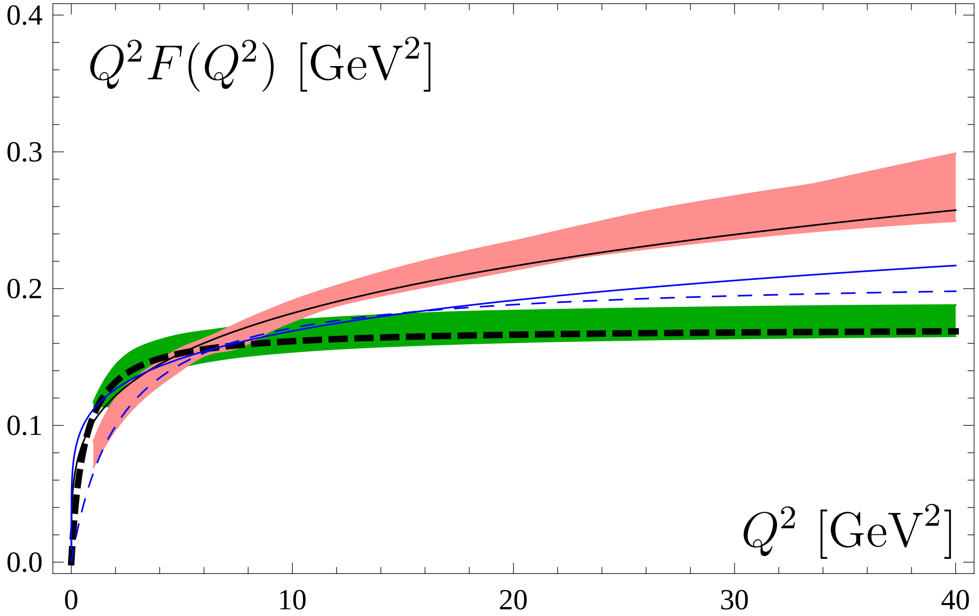


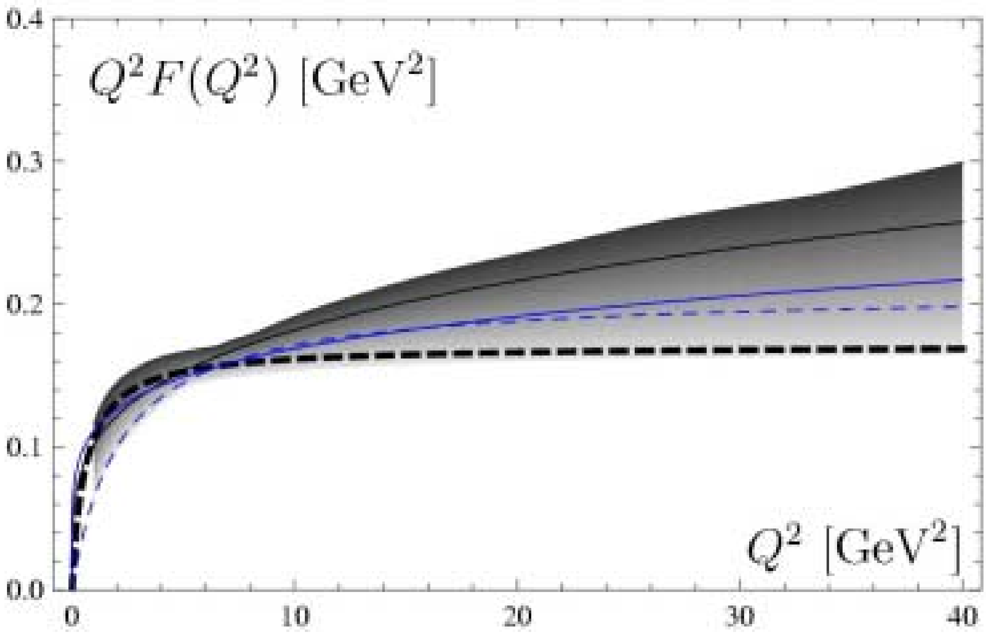
Data	Collab.
🟥🟥	**BaBar, 8 models**
🟦🟦	**Intermediate, 5 models**
🟩🟩	**BMPS & Holography, 4 models**
	New **[ABOP 2012] model** lie on

- Most data points either inside **green "Belle" strip** (**scaling**)
  or within **red "BaBar" strip** (**auxesis**).
- BaBar η, η' data are **within green strip**
- **Blue strip** mostly theoretical.

Alternatives for Pion-Gamma FF Analysis

Alternative: To consider data as forming **two independent data strips** (left) or **one single data strip** (right) **[1205.3770]**





We suggest to explore the first **Alternative**:

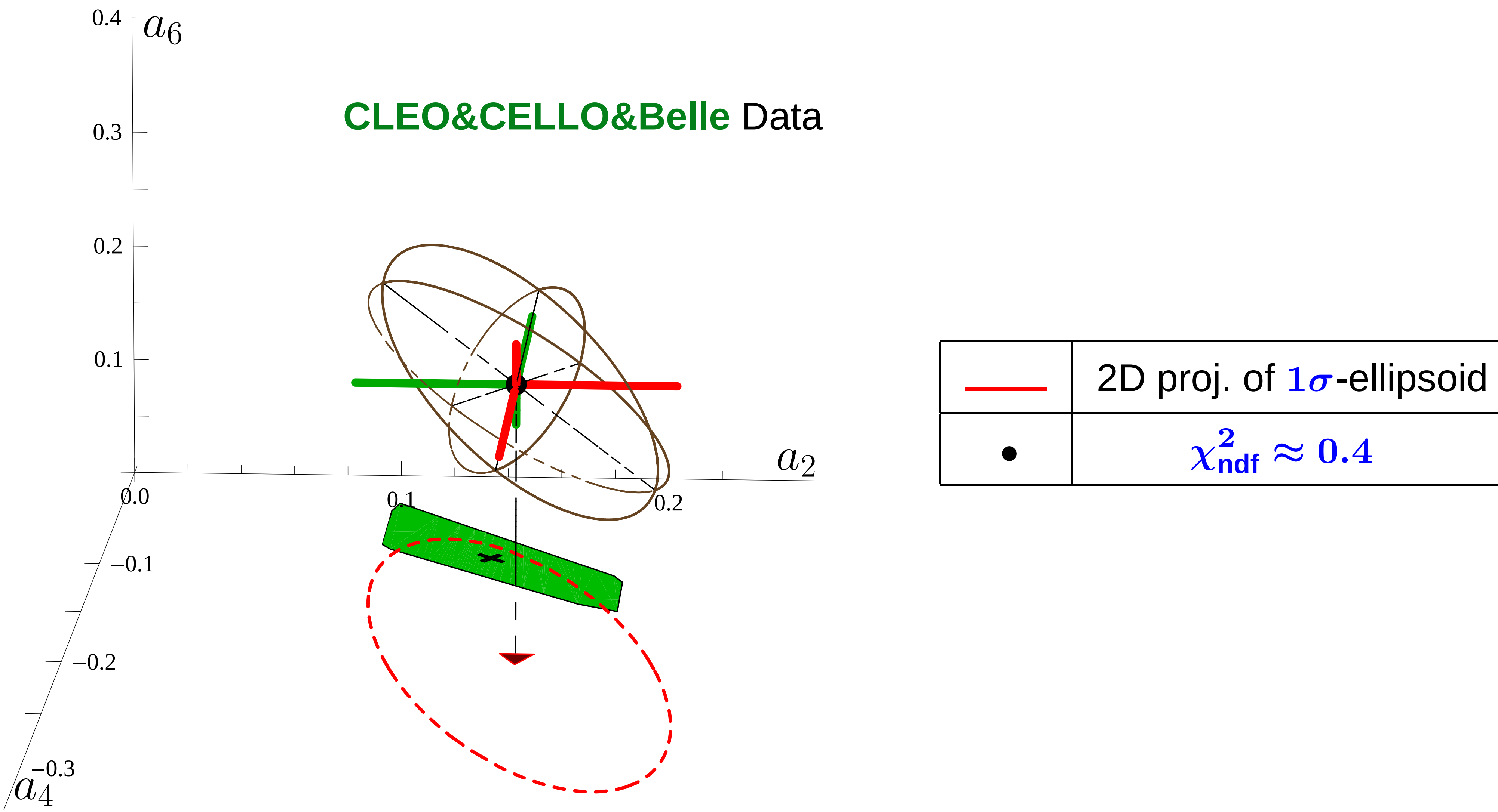
To consider all data as forming

two independent data strips,

namely, **CELLO&CLEO&Belle** and **CELLO&CLEO&BaBar**

NLC SR Results vs 3D Constraints

3D $\mathbf{1\sigma}$-error ellipsoid for $(\boldsymbol{a_2},\ \boldsymbol{a_4},\ \boldsymbol{a_6})$ at $\mu_{\text{SY}} = \mathbf{2.4}$ GeV scale with theoretical $\mp\boldsymbol{\Delta\delta^2_{\text{tw4}}}$-error shown by **green($-$)** and **red($+$)** length.



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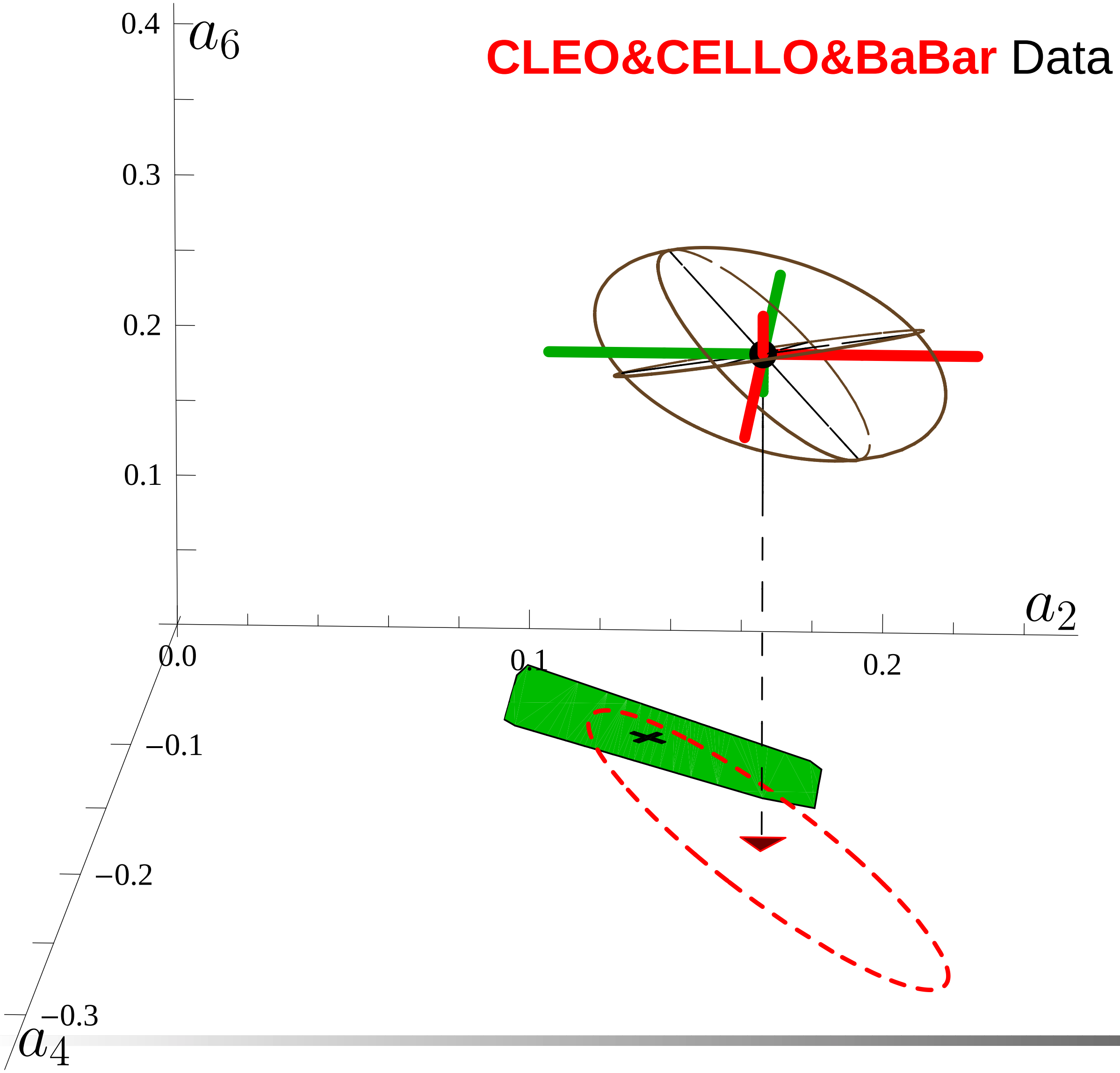

—	2D proj. of $\boldsymbol{1\sigma}$-ellipsoid
●	$\chi^{\mathbf{2}}_{\textbf{ndf}} \approx\ \mathbf{1.0}$

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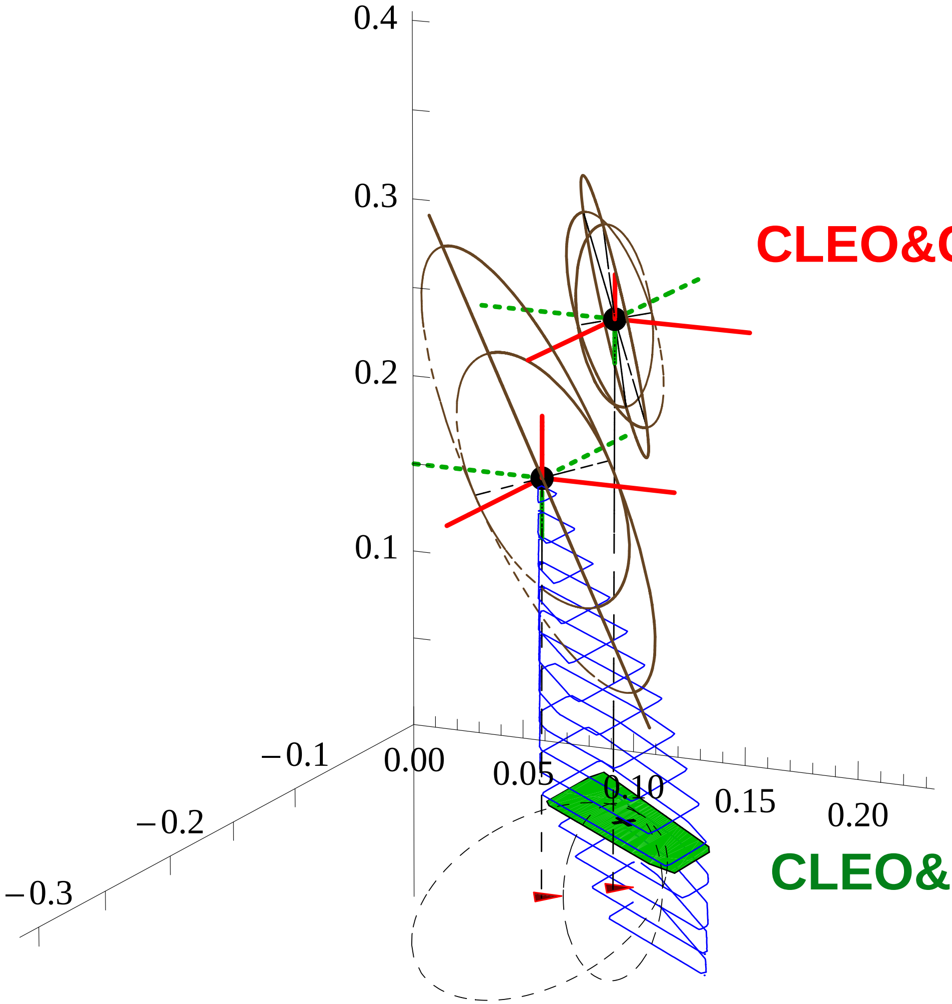


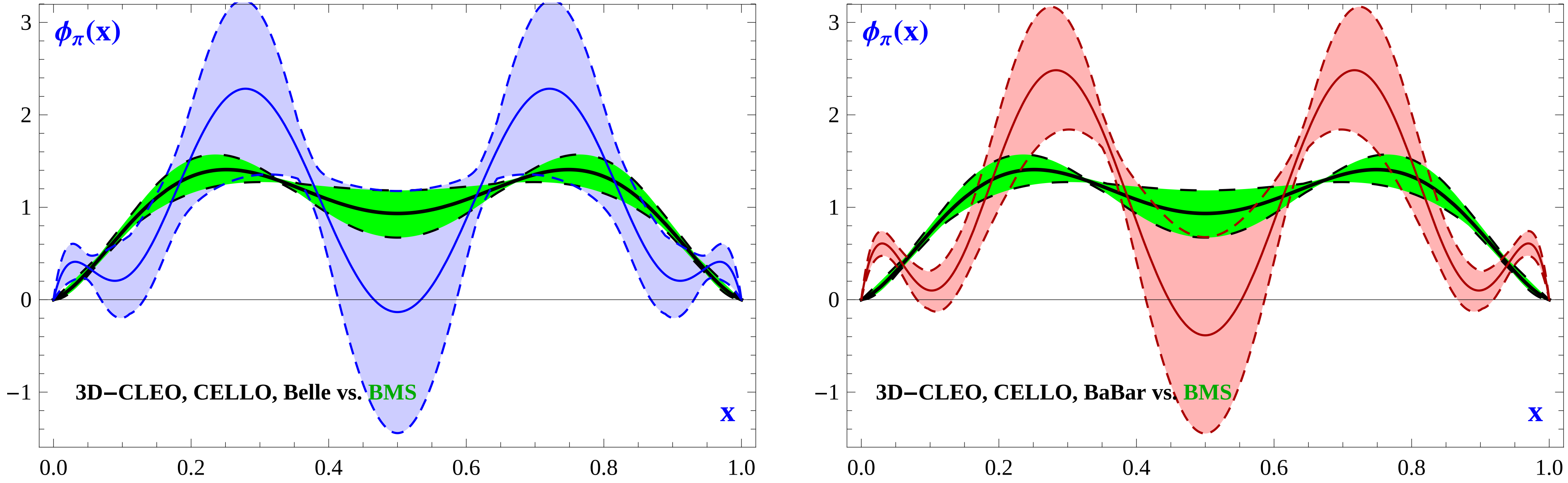
CLEO&CELLO&BaBar Data

CLEO&CELLO&Belle Data

—	2D proj. of $\mathbf{1\sigma}$-ellipsoid
●	$\chi^2_{\text{ndf}} \approx \mathbf{1.0}$

Data fit of pion DA vs **NLC SR profiles**

🟦 $\longrightarrow$ Belle, 🟩 $\longrightarrow$ **BMS**, $\mathbf{1-40}$ GeV^2 at $\mu_{\rm SY}=\mathbf{2.4}$ GeV, 🟥 $\longrightarrow$ **BaBar**



average incline: $\mathbf{17.2 \pm 8.5}$; $\mathbf{25.6 \pm 5.25}$

- The main difference – **a sharper behaviour** of **BaBar** near endpoints.
- **CELLO, CLEO, Belle** data agrees with **BMS bunch** based on NLC QCD SR.
- **BaBar** data above 10 GeV^2 does not support BMS bunch.