Pion loops in effective field theory: *pseudovector vs. pseudoscalar theories*

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Greek Australia

- ~ 700,000 Greeks in Australia
  (beginning with 7 Greek pirate convicts in 1829)

- Melbourne is third largest Greek city in the world, after Athens & Thessaloniki
  (also ~150,000 Australian citizens in Greece)

- Staple Australian dish:
  γύρος (yiros)

souvlaki bros
Charcoal Yiros

Con the fruiterer

(and his 6 daughters: Roula, Toula, Soula, Voula, Foula & Agape)

Outline

- *Motivation*: can one understand flavor asymmetries in the nucleon ($e.g.\ \bar{d} - \bar{u}$) from QCD?
  - ⟶ origin of 5-quark Fock components $|qqq\,\bar{q}q\rangle$ of nucleon

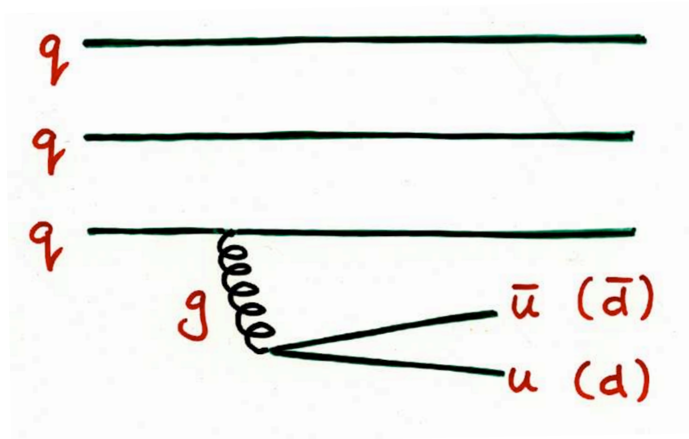
- Effective pion-nucleon interactions
  - ⟶ pseudovector *vs*. pseudoscalar coupling

- *Example*: self-energy of nucleon dressed by pions
  - ⟶ equivalence of equal-time and light-front formulations

- Vertex corrections
  - ⟶ light-cone momentum distributions
  - ⟶ PDF moments: χPT *vs*. "Sullivan" formulations

Flavor asymmetry

- Antiquarks in the proton "sea" produced predominantly by gluon radiation into quark-antiquark pairs, $g \to q\bar{q}$



→ since u and d quark masses are similar, expect flavor-symmetric sea, $\bar{d} \approx \bar{u}$

- Experimentally, one finds *large excess* of $\bar{d}$ over $\bar{u}$

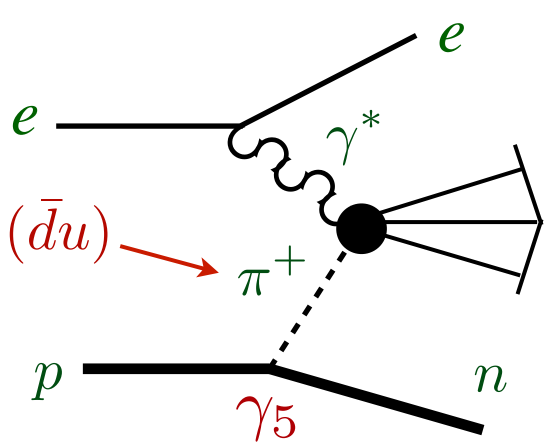
$$\int_0^1 dx\ (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

*E866 (Fermilab), PRD **64**, 052002 (2001)*

Flavor asymmetry

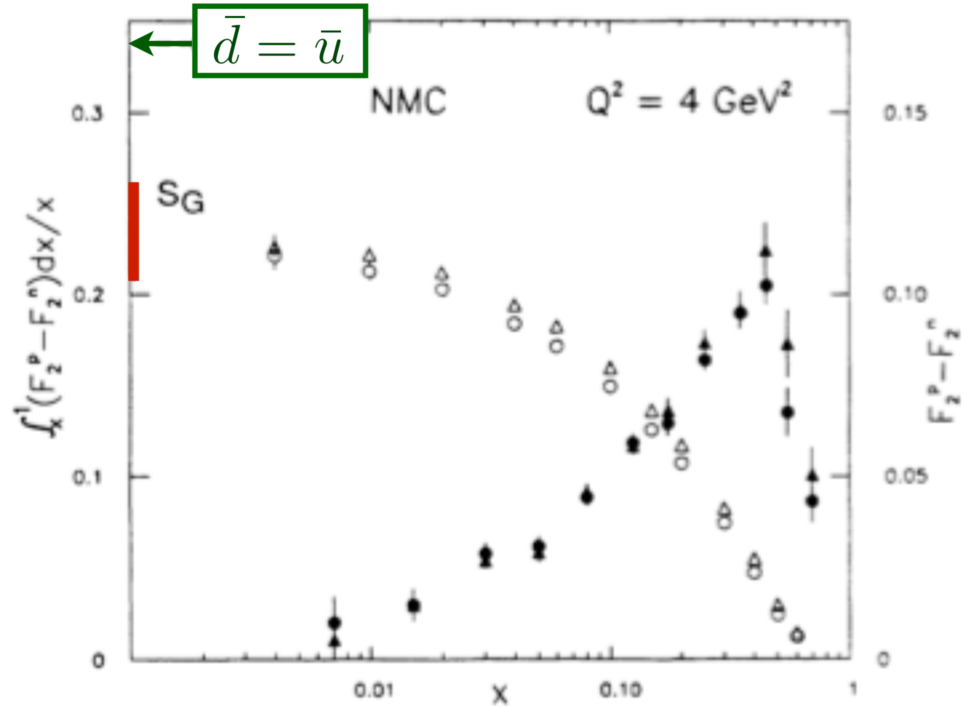
- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*

$$\bar{d} > \bar{u}$$



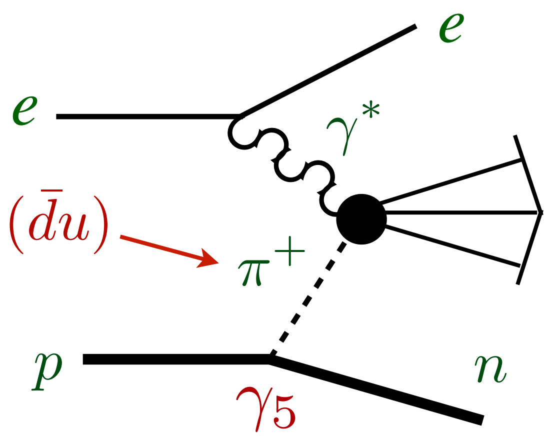
$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

New Muon Collaboration, PRD ***50****, 1 (1994)*

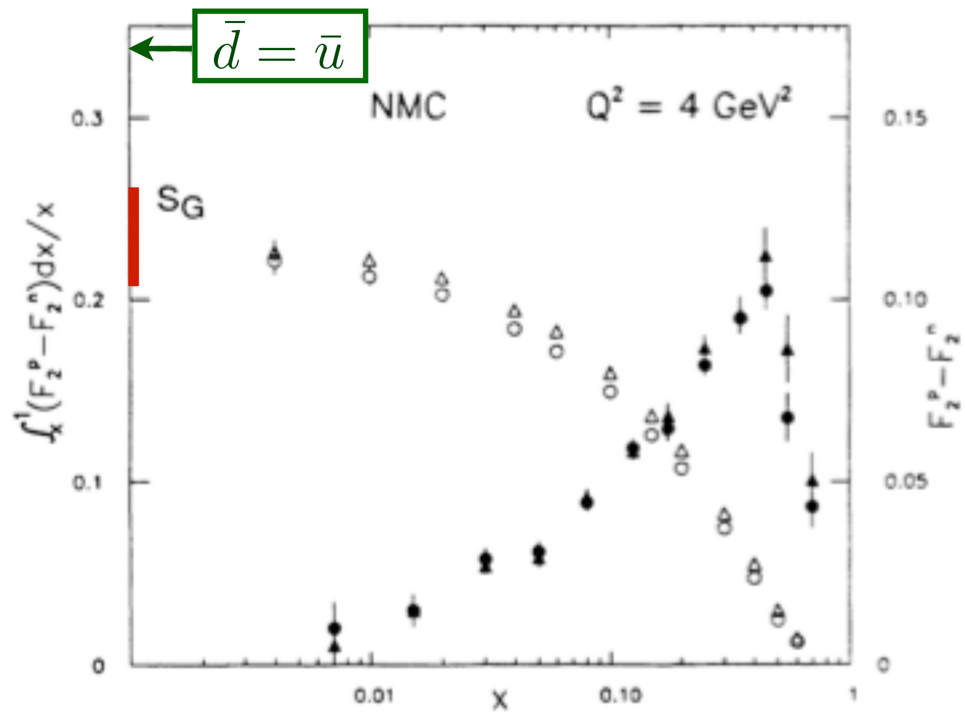
Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} \, f_\pi(y) \; \bar{q}^\pi(x/y)$$

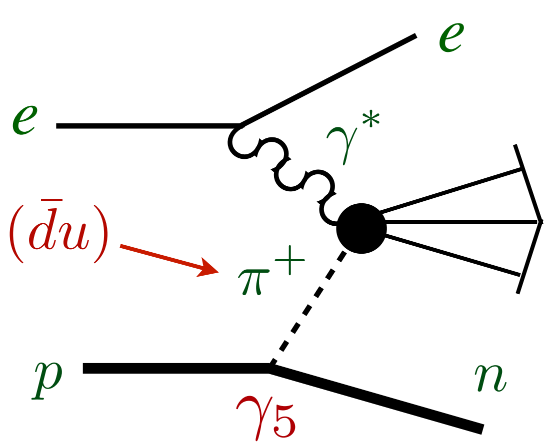
pion light-cone momentum distribution in nucleon

$$f_\pi(y) = \frac{3 g_{\pi NN}^2}{16\pi^2} \, y \int dt \, \frac{-t \; \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}$$

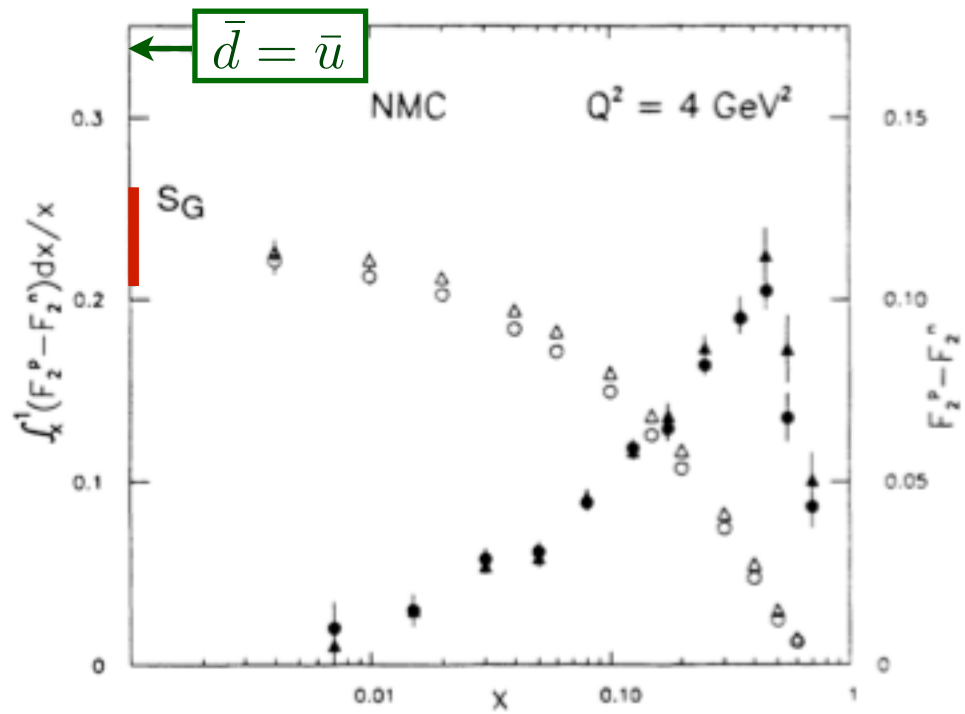
Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y)\ \bar{q}^\pi(x/y)$$

$$f_\pi(y) = \frac{3g_{\pi NN}^2}{16\pi^2}\, y \int dt \frac{-t\ \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}$$

connection to QCD?

Connection with QCD?

- Chiral expansion of moments of $f_\pi(y)$

→ *model-independent* leading nonanalytic (LNA) behavior

$$\langle x^0 \rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u})$$

$$= \frac{2}{3}\int_0^1 dy\, f_\pi(y)$$

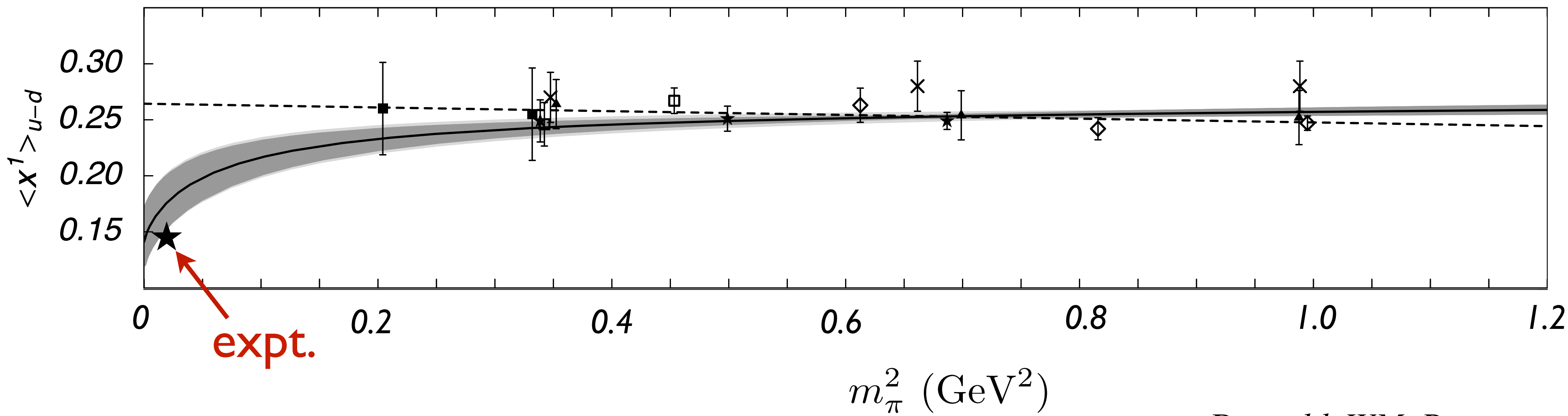
$$= \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \;+\; \text{analytic terms}$$

Thomas, WM, Steffens
PRL ***85****, 2892 (2000)*

→ can *only* be generated by pion cloud!

Connection with QCD?

- Nonanalytic behavior vital for chiral extrapolation of lattice data



*Detmold, WM, Renner et al. PRL **87**, 172001 (2001)*

→ allows lattice QCD calculations to be reconciled with experiment

Connection with QCD?

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

cf. $4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$

Chen, X. Ji, PLB ***523****, 107 (2001)*
Arndt, Savage, NPA ***692****, 429 (2002)*

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

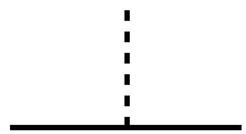
→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

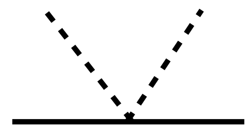
→ consider simple test case: nucleon self-energy

πN Lagrangian

Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar\psi_N \gamma^\mu \gamma_5\, \vec\tau \cdot \partial_\mu \vec\pi\, \psi_N - \frac{1}{(2f_\pi)^2}\,\bar\psi_N \gamma^\mu\, \vec\tau \cdot (\vec\pi \times \partial_\mu \vec\pi)\, \psi_N$$





$g_A = 1.267$

$f_\pi = 93$ MeV

$\longrightarrow$ lowest order approximation of chiral perturbation theory Lagrangian

Pseudoscalar Lagrangian

$$\mathcal{L}^{\mathrm{PS}}_{\pi N} = -g_{\pi N N}\,\bar\psi_N\, i\gamma_5 \vec\tau \cdot \vec\pi\, \psi_N$$

Self-energy

- From lowest order PV Lagrangian

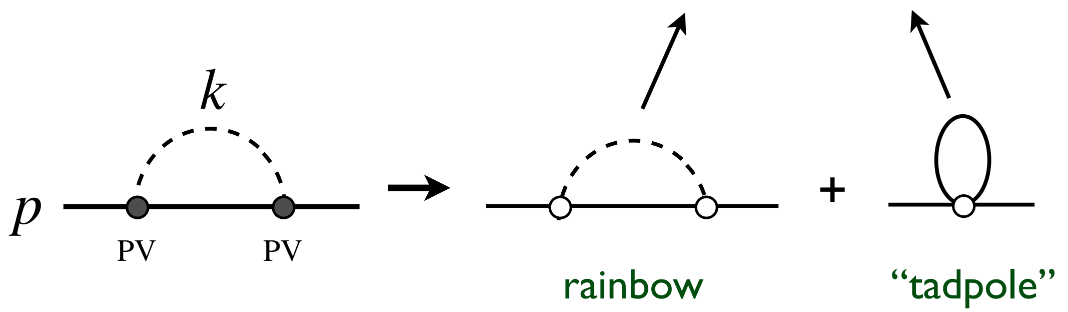
$$\Sigma = i\left(\frac{g_{\pi NN}}{2M}\right)^2 \overline{u}(p)\int\frac{d^4k}{(2\pi)^4}\,(\not{k}\gamma_5\vec{\tau})\,\frac{i\,(\not{p}-\not{k}+M)}{D_N}\,(\gamma_5\not{k}\vec{\tau})\,\frac{i}{D_\pi^2}\,u(p)$$

Goldberger-Treiman $\quad \dfrac{g_A}{f_\pi} = \dfrac{g_{\pi NN}}{M}$

$$D_\pi \equiv k^2 - m_\pi^2 + i\varepsilon$$
$$D_N \equiv (p-k)^2 - M^2 + i\varepsilon$$

$\longrightarrow$ rearrange in more transparent "reduced" form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{2M}\left[4M^2\left(\frac{m_\pi^2}{D_\pi D_N}+\frac{1}{D_N}\right)+\frac{2p\cdot k}{D_\pi}\right]$$



*C.-R. Ji, WM, Thomas, PRD **80**, 054018 (2009)*

Self-energy

- Covariant (dimensional regularization)

$$\int d^{4-2\varepsilon}k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \int_0^1 dx \log \frac{(1-x)^2 M^2 + x m_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon) \right)$$

$$\int d^{4-2\varepsilon}k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon) \right)$$

→ gives well-known m_π^3 LNA behavior (from $1/D_\pi D_N$ term)

$$\Sigma_{\rm cov}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- <u>Equal time</u> (rest frame)

$$\int d^4k \frac{1}{D_\pi D_N} = \int d^3k \int_{-\infty}^{\infty} dk_0 \frac{1}{(-2)(\omega_k - i\varepsilon)} \left(\frac{1}{k_0 - \omega_k + i\varepsilon} - \frac{1}{k_0 + \omega_k - i\varepsilon} \right)$$
$$\times \frac{1}{2(E' - i\varepsilon)} \left(\frac{1}{k_0 - E + E' - i\varepsilon} - \frac{1}{k_0 - E - E' + i\varepsilon} \right)$$

$$\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}\ , \quad E' = \sqrt{\mathbf{k}^2 + M^2}$$

→ four time-orderings

$$\Sigma_{\rm ET}^{(+-)},\quad \Sigma_{\rm ET}^{(-+)},\quad \Sigma_{\rm ET}^{(++)},\quad \Sigma_{\rm ET}^{(--)}$$

positive energy → $\Sigma_{\rm ET}^{(+-)}$

"Z-graph" → $\Sigma_{\rm ET}^{(-+)}$

= 0 → $\Sigma_{\rm ET}^{(++)}$, $\Sigma_{\rm ET}^{(--)}$

$$\Sigma_{\rm ET}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{3}{4\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

$$\Sigma_{\rm ET}^{(-+)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(- \frac{1}{4\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- Equal time (infinite momentum frame)

$$\Sigma_{\rm IMF}^{(+-)} = -\frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E-E'-\omega_k)}$$

$$= \frac{3g_A^2 M}{32\pi^2 f_\pi^2}\int_0^1 dy \int_0^{\Lambda^2} dk_\perp^2 \frac{m_\pi^2}{k_\perp^2 + M^2(1-y)^2 + m_\pi^2 y}$$

$$p_z \equiv P \to \infty$$

$$y = p_z'/p_z$$

$$\Sigma_{\rm IMF}^{(-+)} = \frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E+E'+\omega_k)} = \mathcal{O}(1/P^2)$$

$\longrightarrow$ nonanalytic behavior same as in rest frame

$$\Sigma_{\rm IMF}^{\rm LNA} = \Sigma_{\rm IMF}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- Light-front

$$\int dk^+ dk^- d^2k_\perp \frac{1}{D_\pi D_N} = \frac{1}{p^+}\int_{-\infty}^{\infty} \frac{dx}{x(x-1)} d^2k_\perp \int dk^- \left(k^- - \frac{k_\perp^2 + m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+}\right)^{-1} \times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2 + M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+}\right)^{-1}$$

$$= 2\pi^2 i \int_0^1 dx\ dk_\perp^2\ \frac{1}{k_\perp^2 + (1-x)m_\pi^2 + x^2 M^2}$$

$$x = k^+/p^+$$

$\longrightarrow$ identical nonanalytic results as covariant & instant form

$$\Sigma_{\rm LF}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

C.-R. Ji, WM, Thomas, PRD ***80***, *054018 (2009)*

Self-energy

- **Light-front**

→ $1/D_N$ "tadpole" term has k^- pole that depends on k^+ and moves to infinity as $k^+ \to 0$
("treacherous" in LF dynamics)

→ use LF cylindrical coordinates $k^+ = r\cos\phi,\ k^- = r\sin\phi$

$$\int d^4k \frac{1}{D_N} = \frac{1}{2}\int d^2k_\perp \int \frac{dk^+}{k^+} \int dk^- \left(k^- - \frac{k_\perp^2 + M^2}{k^+} + \frac{i\varepsilon}{k^+}\right)^{-1}$$

$$= -2\pi \int d^2k_\perp \left[\int_0^{r_0} dr \frac{r}{\sqrt{r_0^4 - r^4}} + i \lim_{R\to\infty} \int_{r_0}^{R} dr \frac{r}{\sqrt{r^4 - r_0^4}}\right]$$

$$= \frac{1}{2}\int d^2k_\perp \lim_{R\to\infty}\left(-\pi^2 + 2\pi i\ \log\frac{r_0^2}{R^2} + \mathcal{O}(1/R^4)\right)$$

relevant also for $1/D_\pi$ term

$$r_0 = \sqrt{2(k_\perp^2 + M^2)}$$

contains $\log(k_\perp^2 + M^2)$ term as required

Self-energy

- Pseudoscalar interaction

$$\Sigma^{\rm PS} = i g_{\pi NN}^2\ \overline{u}(p) \int \frac{d^4k}{(2\pi)^4}\, (\gamma_5 \vec{\tau})\, \frac{i\,(\not{p} - \not{k} + M)}{D_N}\, (\gamma_5 \vec{\tau})\, \frac{i}{D_\pi^2}\, u(p)$$

$$= -\frac{3 i g_A^2 M}{2 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]$$

→ contains additional ("treacherous") pion "tadpole" term

→ similar evaluation as for $1/D_N$ term

$$\Sigma^{\rm PS}_{\rm LNA} = \frac{3 g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Self-energy

- Alberg & Miller claim on light-front $\Sigma^{\rm PS} = \Sigma^{\rm PV}$
  – "form factor removes $k^+ = 0$ contribution" *PRL* **108**, 172001 (2012)

- In practice, AM drop "treacherous" $k^+ = 0$ (end-point) term

$$\Sigma^{\rm PS} = \Sigma^{\rm PV} + \Sigma^{\rm PS}_{\rm end-pt}$$

  after which PS result happens to coincide with PV

→ but, *even with* form factors, end-point term is non-zero

$$\Sigma^{\rm PS}_{\rm end-pt} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2} \int_0^\infty dt \frac{\sqrt{t}\, F^2(m_\pi^2, -t)}{\sqrt{t + m_\pi^2}} \quad \xrightarrow{\rm LNA} \quad \frac{3g_A^2}{32\pi f_\pi^2} \frac{M}{\pi} m_\pi^2 \log m_\pi^2$$

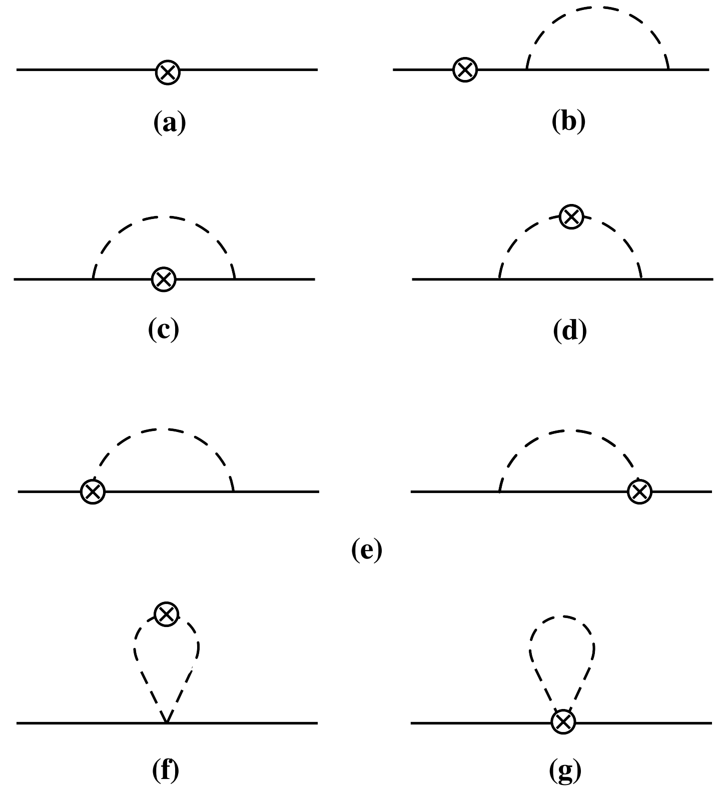
Ji, WM, Thomas, PRL ***110***, *179191 (2013)*

→ *ansatz* does not work for other quantities
  e.g. vertex renormalization

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$

→ $e.g.$ for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial \hat{\Sigma}}{\partial p^\mu}$$

Vertex corrections

- Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

for isovector $(p-n)$ distribution

where

$$f_\pi(y) = 4f^{(\text{on})}(y) + 4f^{(\delta)}(y)$$
$$f_N(y) = -f^{(\text{on})}(y) - f^{(\text{off})}(y) + f^{(\delta)}(y)$$
$$f_{\text{KR}}(y) = 4f^{(\text{off})}(y) - 8f^{(\delta)}(y)$$
$$f_{\pi(\text{tad})}(y) = -f_{N(\text{tad})}(y) \;=\; 2f^{(\text{tad})}(y)$$

with components

$$f^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \, \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$

$$f^{(\text{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \, \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$

$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

$$f^{(\text{tad})}(y) = -\frac{1}{(4\pi f_\pi)^2} \int dk_\perp^2 \, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

Burkardt, Hendricks, Ji, WM, Thomas, PRD ***87****, 056009 (2013)*

- Pion distribution $f_\pi(y)$ contains *on-shell* contribution $f^{(\mathrm{on})}(y)$ equivalent to PS ("Sullivan") result

- Nucleon distribution $f_N(y)$ contains in addition new *off-shell* contribution $f^{(\mathrm{off})}(y)$

- Both contain singular $\delta(y)$ components $f^{(\delta)}(y)$, which are present only in PV theory

- Kroll-Ruderman term $f_{\mathrm{KR}}(y)$ needed for gauge invariance

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{KR}})$$

- Nucleon and pion tadpole terms equal & opposite

$$(1 - Z_1^{\pi\,(\mathrm{tad})}) + (1 - Z_1^{N\,(\mathrm{tad})}) = 0$$

Nonanalytic behavior of vertex renormalization factors

$$1 - Z_1^N \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \pi \frac{m_\pi^3}{M} - \frac{2m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^\pi \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \frac{5\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{\text{KR}} \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ + \frac{2\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{N\,(\text{tad})} \xrightarrow{\text{NA}} -\frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$1 - Z_1^{\pi\,(\text{tad})} \xrightarrow{\text{NA}} \frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

→ cancellation of $m_\pi^2 \log m_\pi^2$ terms in KR contribution

→ demonstration of gauge invariance condition
(in fact, to *all* orders!)

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1-Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1-Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1-Z_1^{\rm KR}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1-Z_1^{N\,{\rm tad}}$	0	0	0	$-1/2$	$-1/2$	0
$1-Z_1^{\pi\,{\rm tad}}$	0	0	0	$1/2$	$1/2$	0

* also in PS

in units of $\dfrac{1}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$

→ origin of ChPT *vs*. Sullivan process difference clear!

$$\left(1 - Z_1^{N\,(\rm PV)}\right)_{\rm LNA} = \frac{3}{4}\left(1 - Z_1^{N\,(\rm PS)}\right)_{\rm LNA}$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N|\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n}|N\rangle \;=\; 2\,\langle x^{n-1}\rangle_q\; p^{\{\mu_1}\cdots p^{\mu_n\}}$$

→ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1}\rangle_q \;=\; \int_0^1 dx\, x^{n-1}\,(q(x)+(-1)^n\bar{q}(x))$$

→ operator is $\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n} \;=\; \bar{\psi}\gamma^{\{\mu_1}\, iD^{\mu_2}\cdots iD^{\mu_n\}}\psi \;-\; \text{traces}$

- Lowest (n=1) moment $\langle x^0\rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

Moments of PDFs

- For couplings involving nucleons

$$\mathcal{M}_N^{(p)} = Z_2 + (1 - Z_1^N) + (1 - Z_1^{N\,(\mathrm{tad})})$$
$$\mathcal{M}_N^{(n)} = 2(1 - Z_1^N) - (1 - Z_1^{N\,(\mathrm{tad})})$$

$\longrightarrow$ wave function renormalization

$$1 - Z_2 = (1 - Z_1^p) + (1 - Z_1^n) \equiv 3(1 - Z_1^N)$$

- For couplings involving only pions

$$\mathcal{M}_\pi^{(p)} = 2(1 - Z_1^\pi) + 2(1 - Z_1^{\mathrm{KR}}) + (1 - Z_1^{\pi\,(\mathrm{tad})})$$
$$\mathcal{M}_\pi^{(n)} = -2(1 - Z_1^\pi) - 2(1 - Z_1^{\mathrm{KR}}) - (1 - Z_1^{\pi\,(\mathrm{tad})})$$

Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

→ no pion corrections to isosclar moments

→ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

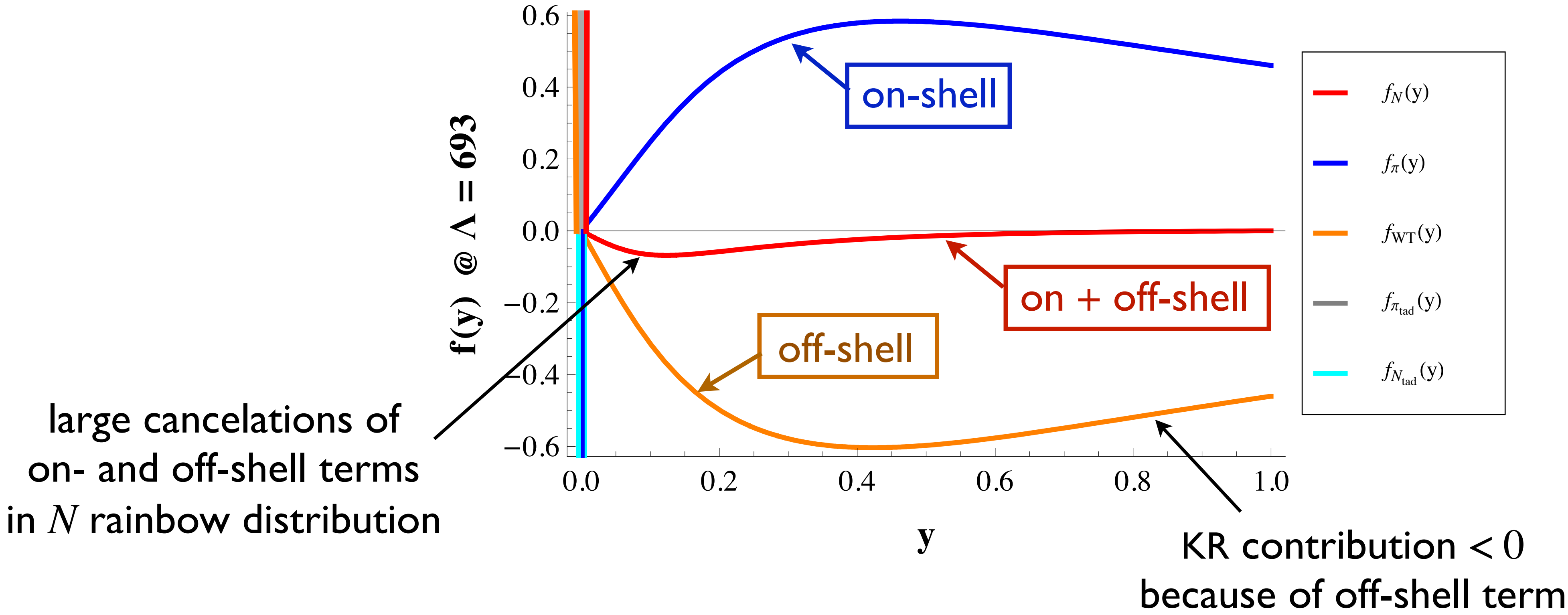
PS (“on-shell”) contribution

δ-function contribution

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ for transverse momentum cut-off $F(k_\perp) = \Theta(k_\perp^2 - \Lambda^2)$

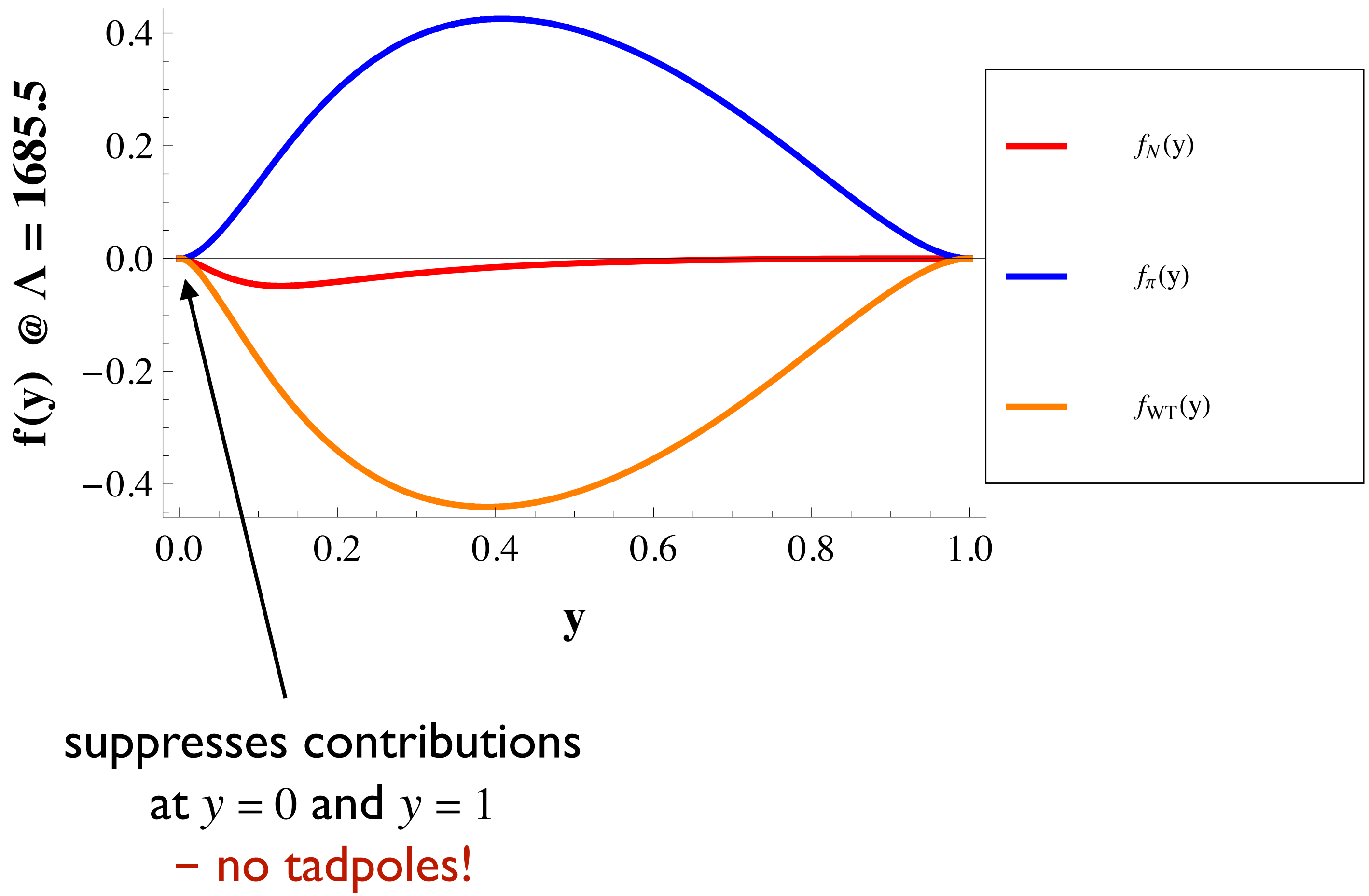


Hendricks, Ji, WM, Thomas (2013)

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ s-dependent (dipole) form factor $s_{\pi N} = \frac{k_\perp^2 + m_\pi^2}{y} + \frac{k_\perp^2 + M^2}{1-y}$



Hendricks, Ji, WM, Thomas (2013)

Summary

- Equivalence demonstrated between self-energy in equal-time, covariant, and light-front formalisms

$$\Sigma_{\rm cov}^{\rm LNA} = \Sigma_{\rm ET}^{(+-)\rm LNA} + \Sigma_{\rm ET}^{(-+)\rm LNA} = \Sigma_{\rm IMF}^{(+-)\rm LNA} = \Sigma_{\rm LF}^{\rm LNA}$$

  → non-trivial due to end-point singularities

  → PV and PS results clearly differ

- Gauge invariance relations for vertex corrections verified to all orders in m_π

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\rm KR})$$

  → difference between PDF moments in ChPT (PV) & "Sullivan" process (PS)

  → model-independent constraints on LC distributions $f_i(y)$

  → impact on $\bar{d} - \bar{u}$ data analysis in progress

Ευχαριστώ!