Systematic regularization (and renormalization) scheme in non-perturbative calculations within covariant light-front dynamics

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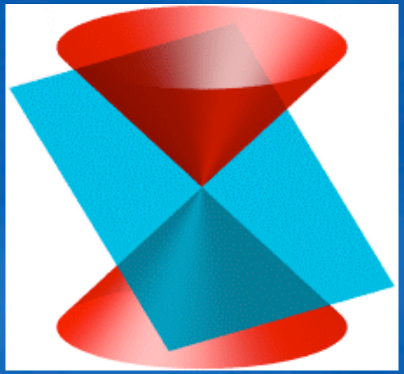
PLAN

- Covariant light-front dynamics
- Non-perturbative calculations
- Renormalization conditions
- Systematic regularization scheme
- Conclusions

Covariant light-front dynamics

- **Control of rotational invariance**



➢ arbitrary position ω of the light-front

$$\tau = \omega . x \quad \text{with} \quad \omega^2 = 0$$

➢ control of rotational invariance at each order of perturbation theory or at each order of any approximative scheme (ex. Fock space truncation)

➢ Either calculated physical observables are rotational invariant, or we can extract the physical, ω -independent contribution

ex. renormalization condition

V. Karmanov 1976, J. Carbonell et al. Phys. Rep. 300 (1998) 215

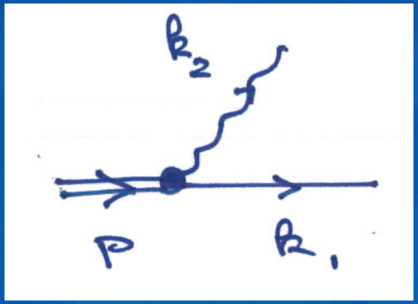
❑ Sources of violation of rotational invariance

Two very different origins in an ab-initio calculation

- ➢ Regularization scheme
  - ➥ ex. cut-off in transverse momentum space
  - ➥ use of rotational invariant regularization scheme

- ➢ Truncation of the Fock space (or any other approximative scheme which does not preserve rotational invariance)
  - ➥ use of appropriate counterterms to restore rotational invariance if needed

❑ Off-shell energy/momentum



$$p^2 = M^2 \qquad k_1^2 = m^2 \qquad k_2^2 = \mu^2$$

$$p = k_1 + k_2 - \omega\tau$$

$$\tau = \frac{s - M^2}{2\ \omega . p} \quad \textbf{with} \quad s = (k_1 + k_2)^2$$

➢ τ **fixed from the kinematics of the two-body state**

➢ **systematic calculations of energy/momentum conservation**

Non-perturbative calculations

- **Poincaré group equations**

$$P^2\phi(p) = M^2\phi(p)$$

$\phi(p)$ **: state vector**

- **Fock space expansion**

$$\Phi = \sum_i \phi_i(k_1 \dots k_i)$$

➢ **Truncation of the Fock space (to order N) for obvious technical reasons**

➢ **System of coupled equations which sums to all orders a given class of irreducible contributions with up to N particles in the intermediate states**

- **Non-perturbative quantity** ϕ_i **or more conveniently** Γ_i

  ➢ $$\phi_i = \frac{\Gamma_i}{s_i - M^2}$$ Γ_i **: vertex function**

  $$s_i = (k_1 + \ldots k_i)^2$$

  ➢ **ex.** $\Gamma_2(x, k_\perp^2)$ **with** $\Gamma_2(x, k_\perp^2) \to cte$ **for** $k_\perp^2 \to \infty$

- **Covariant decomposition of** Γ_i

  ➢ $$\Gamma_2 = a_2 + b_2 \frac{M \not{\omega}}{\omega\cdot p}$$ **for the Yukawa model**

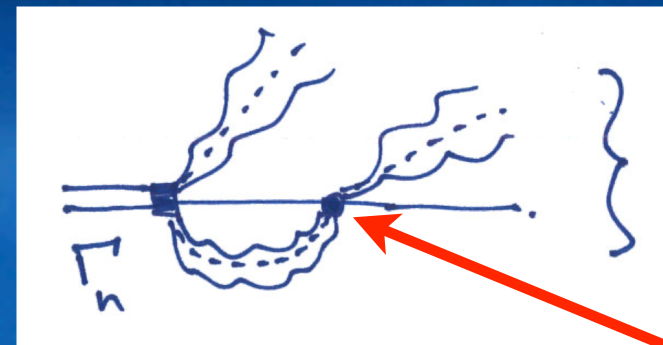
  $$\Gamma_3 = a_2 + b_2 \frac{M \not{\omega}}{\omega\cdot p} + C_{ps}\left(c_2 + d_2 \frac{M \not{\omega}}{\omega\cdot p}\right)\gamma_5$$

  $$C_{ps} = i\frac{1}{M^2 \omega\cdot p}\epsilon_{\mu\nu\rho\sigma}\ k_2^\mu\ k_3^\nu\ p^\sigma\ \omega^\rho$$

  ➢ **in general** $\Gamma_n = \sum_i a_i\ O_i$ ← **vertex operators**

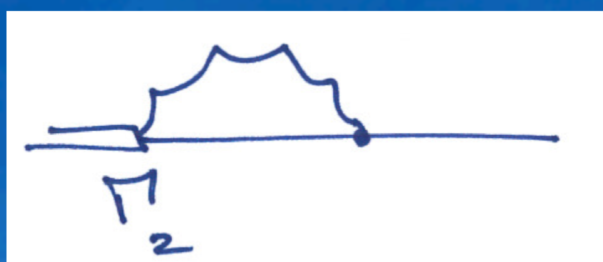
General contribution

- general covariant decomposition



off-shell momentum k

H_{int}



- **simplest non-perturbative contribution** in the Yukawa model

$$A(k^2) + B(k^2)\ \frac{\not{k}}{M} + C(k^2)\frac{M\ \not{\omega}}{\omega{\cdot}k} + D(k^2)\frac{\not{\omega}\ \not{k} - \not{k}\ \not{\omega}}{\omega{\cdot}k}$$

$$A(k^2) \sim \int dx\ dk_\perp^2 \frac{a_2(x,k_\perp^2)}{k_\perp^2 + M^2 x + \mu^2(1-x) - k^2 x(1-x)}$$

up to a regularization scheme

Renormalization scheme

- **Physical mass**

physical mass

$$P^2\phi(p) = M^2\phi(p)$$

➢ Solution of the homogeneous coupled equations defines the appropriate (non-perturbative) mass counterterm

- **Physical coupling constant**

➢ using the standard procedure

physical coupling constant

$$\Gamma_2(s_2 = M^2) = g\sqrt{Z_f}$$

(in the quenched approximation)

➢ rem. $\Gamma_2(s_2 = M^2)$ should be independent of x (cf V. Karmanov)

➢ $\Gamma_2(s_2 = M^2)$ should be also independent of the position of the light front

➢ necessary condition to define the PHYSICAL coupling constant g

- one **should extract** (or remove with an appropriate counterterm) the ω - independent contribution to $\Gamma_2(s_2 = M^2)$

- this is **only possible** in the **covariant formulation** of light-front dynamics (unless the calculation is exact!)

$$\Gamma_2(s_2 - M^2) = a_2(s_2 - M^2) + b_2(s_2 - M^2)\, \frac{M\, \not{\omega}}{\omega\cdot p}$$

$a_2 \leftarrow g\sqrt{Z_f}$

- defines the **minimal number** of counterterms to be introduced

Systematic regularization scheme

- **The key role of non-perturbative regularization scheme**
  - first approach with Pauli-Villars boson and fermion fields
    - no contact interactions
    - infinite mass limit for both PV fermion and boson
    - delicate numerical cancellations
  - need for a more systematic approach which preserves rotational invariance and which does not involve infinitely large energy/momentum scales
  - Taylor-Lagrange regularization scheme

❑ Reminder of the Taylor-Lagrange regularization scheme

P. Grangé, E. Werner (2006)

➢ Fields defined as **distributions** acting on (super-regular) test functions of **space-time extension a**

$$\varphi_a(x) = \int \frac{d^3p}{(2\pi)^3} \frac{f_a(\epsilon_p^2, \mathbf{p}^2)}{2\epsilon_p} \left[a_p^\dagger e^{i\mathbf{p}.\mathbf{x}} + a_p e^{-i\mathbf{p}.\mathbf{x}}\right]$$

➢ Use of the **Lagrange formula** (in one dimension X for a simple logarithmic divergence)

$$f_a(\lambda X) = -\int_\lambda^\infty dt\ \partial_t\ f_a(Xt)$$

λ : intrinsic scale

➢ Continuum limit $a \to 0$ or in momentum space cut-off $H \to \infty$

ultra-soft cut-off

$$H \to H(X) \equiv \eta^2\ X^\alpha\ +\ cte \qquad \eta^2 > 1 \qquad 0 \leq \alpha < 1$$

$H \to \infty$ for $\alpha \to 1$

➢ Arbitrary scale η plays a very similar rôle to the unit of mass μ in dimensional regularization: completely arbitrary and physical observables should not depend on it

➢ for an amplitude of the form $\mathcal{A}_a = \int dX\ T(X)\ f_a(X)$

and a singular operator $T(X) = \frac{1}{X+\lambda}$

➢ integration by part and change of variable $Y = \frac{X}{\lambda} t$

$$\mathcal{A} = \int dY \int_\lambda^{\eta^2} \partial_t \left[\frac{1}{t}\ T\left(\frac{Y}{t} \right) \right]\ f(Y)$$

in the continuum limit $a \to 0$

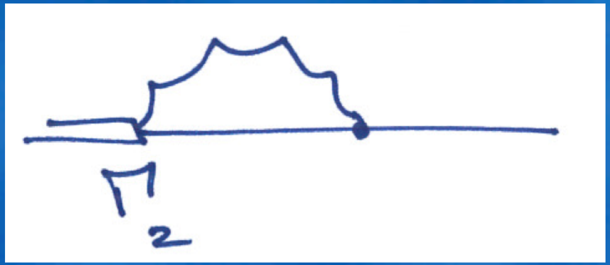
$$\mathcal{A} = \int dY\ f(Y) \left[\frac{1}{Y+\eta^2} - \frac{1}{Y+\lambda} \right]$$

❑ Application to non-perturbative calculations in light-front dynamics

➢ **very simple implementation** of the Taylor-Lagrange regularization scheme in covariant light-front dynamics

$$\Gamma_n(k_1 \ldots k_n) \to \bar{\Gamma}_n(k_1 \ldots k_n) = \Gamma_n(k_1 \ldots k_n) f(k_1^2) \ldots (k_n^2)$$

➢ the **properties of the test functions** are now embedded in $\bar{\Gamma}_n$ and in its covariant components



➢ **simplest non-perturbative contribution** in the Yukawa model

$$A(k^2) \sim \int dx\ dk_\perp^2 \frac{a_2(x,k_\perp^2) f_a \left[\left(\frac{k_\perp^2}{2xM\Lambda} \right)^2 \right]}{k_\perp^2 + M^2 x + \mu^2(1-x) - k^2 x(1-x)}$$

$\bar{a}_2$

Λ : arbitrary momentum scale

➢ with the identification $X = \frac{k_\perp^2}{2xM\Lambda}$

➢ and the intrinsic scale $\lambda(x) = \frac{M^2 x + \mu^2(1-x) - k^2 x(1-x)}{M^2}$

➢ one gets

$$A(k^2) \sim \int dx \; dk_\perp^2 \left[\frac{1}{k_\perp^2 + M^2 \; \eta^2 \; \lambda(x)} - \frac{1}{k_\perp^2 + M^2 \; \lambda(x)^2} \right] a_2(x, k_\perp^2)$$

➢ with $a_2 = 1$ one recovers the perturbative result in terms of the arbitrary scale η^2

$$A(k^2) \sim \int dx \;\; \left(Log[\eta^2] - Log[\lambda(x)] \right)$$

Conclusions

- The general framework is now settled

- First non-perturbative calculations in the Yukawa model
  - show that it is indeed applicable and successfull in the Yukawa model
  - extension to N > 3 to be done (see also V. Karmanov)

- Main difficulty : application to gauge theories
  - expansion of the vertex functions in covariant components may become rather tedious
  - is it absolutely necessary? (see also V. Karmanov and J. Vary)