Solvable models in the conventional and light-front field theory

Recent progress

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ABSTRACT: We discuss a few new results in the field of exactly solvable models in the conventional field theory as well as the light-front field theory. The models include the Thirring, Rothe-Stamatescu, the massive and massleess Federbush and the Thirring-Wess model. We work within the hamiltonian framework and pay a careful attention to the correct definition of interacting currents, to the right choice of field variables and the ccorresponding Hamiltonian including its true physical ground state.

INTRODUCTION

Soluble models: simple relativistic field theories in two-dimenional space-time in which explicit and non-approximative solutions of the field equations can be found on the quantum level

massless and massive model with derivative coupling, Rothe-Stamatescu, Thirring, Federbush, Thirring-Wess, Schwinger

DISADVANTGES: not realistic, ”toy” models, mostly not gauge theories, 2-dimensional - extrapolation of the results to higher dimensions not guaranteed

ADVANTAGES: non-perturbative studies possible, explicit solutions of the Heisenberg field equations, complete information on the NP dynamics, insights into the structure of QFT, vacuum properties (often neglected, Fock vacuum taken as the physical ground state), comparison between the conventional (SL) and light-front (LF) forms of the theory

SL and LF Hamiltonians in many models have different structure:

for example, g^2 and gm terms in the LF Yukawa Hamiltonian, only $O(g)$ terms in the SL counterpart + different status of vacuum states $\Rightarrow$ **do they agree in the physical predictions?**

difficult to answer in realistic theories, solvable models very helpful

studied over decades in the usual SL form

critical reexaminations revealed certain inconsistencies/mistakes

Here we would like to discuss these ”weak points”, to clarify the situation and to suggest more adequate formulations

IMPORTANT NEW INGREDIENT: choice of the right field variables - solutions of the field equations automatically expressed in terms of free fields $\Rightarrow$ the Hamiltonian (Lagrangian) has to be reexpressed in terms of the free fields also

for a few solvable models, this step removed discrepancies between the SL and LF forms (Hamiltonians)

another element - a consistent definition of quantum currents:

EXAMPLE: free fermionic vector current

defined as normal ordered product

$$j^\mu(x) =: \psi^\dagger(x)\gamma^0\gamma^\mu\psi(x) : \qquad (1)$$

Alternatively regularized by the point splitting, $x \pm \frac{\epsilon}{2}$

Observation: singular vacuum parts cancel - normal-ordered result obtained automatically, if one does the point splitting in a hermitean way, defining

$$j^{\mu}(x) = \lim_{\epsilon \to 0} \frac{1}{2}\left[\psi^{\dagger}(x+\frac{\epsilon}{2})\gamma^{0}\gamma^{\mu}\psi(x-\frac{\epsilon}{2}) + \psi^{\dagger}(x-\frac{\epsilon}{2})\gamma^{0}\gamma^{\mu}\psi(x+\frac{\epsilon}{2})\right]. \qquad (2)$$

Cancelation of the singular parts works also for the interacting currents - the Rothe-Stamatescu model for a simple illustration

ROTHE-STAMATESCU MODEL

Lagrangian density (Ann. Phys. 95 (1975))

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^{\mu} \overleftrightarrow{\partial}_{\mu} \Psi - m\overline{\Psi}\Psi + \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}\mu^{2}\phi^{2} - g\partial_{\mu}\phi J_{5}^{\mu},$$
$$J_{5}^{\mu} = \overline{\Psi}\gamma^{\mu}\gamma^{5}\Psi. \qquad (3)$$

Field equations:

$$i\gamma^\mu\partial_\mu\Psi = m\Psi + g\partial_\mu\phi\gamma^\mu\gamma^5\Psi,$$
$$\partial_\mu\partial^\mu\phi + \mu^2\phi^2 = g\partial_\mu J_5^\mu = 2img\overline{\Psi}\gamma^5\Psi. \tag{4}$$

Scalar field is not free as it was the case for the vector-current interaction (DCM). Dirac eq. seems to have an operator solution (Belvedere and Rodrigues, Ann. Phys. 321 (2006)) similar to the one from the DCM:

$$\Psi(x) = e^{-ig\gamma^5\phi(x)}\psi(x). \tag{5}$$

Check:

$$i\gamma^\mu\partial_\mu\Psi(x) = i\gamma^\mu\big[-ig\gamma^5\partial_\mu\phi(x)\Psi(x) + e^{-ig\gamma^5\phi(x)}\partial_\mu\psi(x)\big] =$$

$$= g\partial_\mu \phi(x)\gamma^\mu\gamma^5\Psi(x) + e^{+ig\gamma^5\phi(x)} i\gamma^\mu\partial_\mu\psi(x), \qquad (6)$$

where $i\gamma^\mu\partial_\mu\psi = m\psi$. The sign in the last exponential is opposite due to $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$.

Thus, the massive RS model is not exactly solvable. The original massless RS model (Rothe and Stamatescu, Annals of Physics 1977): The massless axial current is conserved, hence scalar field is free and the Dirac eq. is exactly solved by (5).

Vector and axial-vector anomaly found - based on the definition of the vector current:

$$j^\mu_\epsilon(x) = \overline{\Psi}(x+\epsilon)\gamma^\mu\Psi(x)\exp\left(ig\int\limits_x^{x+\epsilon} dy_\lambda \epsilon^{\lambda\nu}\partial_\nu\phi(y)\right) - VEV. \qquad (7)$$

GAUGE INVARIANCE?

analogy with the Schwinger model invoked – the point-splitting regularization of the product of operators at the same point used and gauge invariance maintained by introducing the gauge exponential: In the RS model no symmetry present $\Rightarrow$ **construction not correct**

Careful regularized treatment based on the known solution

In quantum theory, singular operator products (Lagrangian) have to be regularized

We do not define quantum solution $\Psi(x)$ (5) as a normal-ordered exponential, but simply regularize it by the point-splitting of the positive and negative-frequency part of the scalar field in the exponential and by applying the BCH operator identity $e^A e^B = e^{\frac{1}{2}[A,B]} e^{A+B}$ valid, if A and B

commute with $[A, B]$:

$$\Psi(x) = Z^{1/2}(\epsilon) e^{-ig\gamma^5 \phi^{(-)}(x)} e^{-ig\gamma^5 \phi^{(+)}(x)} \psi(x), \tag{8}$$

where $Z^{1/2}(\epsilon) = \exp\big\{g^2[\phi^{(+)}(x - \frac{\epsilon}{2}), \phi^{(-)}(x + \frac{\epsilon}{2})]\big\} = \exp\big\{ - ig^2 D^{(+)}(\epsilon)\big\}$ and $D^{(+)}(x - y)$ is the corresponding two-point function.

Difference: we keep the regularized (infinite) constant $Z(\epsilon)$ – no need to define renormalized solution, the regularized factors automatically cancel in the point-split interacting currents:

$$J^\mu(x) = s \lim_{\epsilon \to 0} \frac{1}{2}\big\{Z(\epsilon)\overline{\psi}(x + \frac{\epsilon}{2}) e^{ig\gamma^5 \phi^{(-)}(x+\frac{\epsilon}{2})} e^{ig\gamma^5 \phi^{(+)}(x+\frac{\epsilon}{2})}$$

$$\times \gamma^\mu e^{-ig\gamma^5 \phi^{(-)}(x-\frac{\epsilon}{2})} e^{-ig\gamma^5 \phi^{(+)}(x-\frac{\epsilon}{2})} \psi(x - \frac{\epsilon}{2}) + H.c.\big\} =$$

$$=: \overline{\psi}(x)\gamma^\mu \psi(x) : + \frac{g}{2\pi} \epsilon^{\mu\nu} \partial_\nu \phi(x). \tag{9}$$

$s \lim$ designates the symmetric limit, the free-field relation $\overline{\psi}(x+\epsilon/2)\gamma^\mu\psi(x-\epsilon/2) =: \overline{\psi}(x)\gamma^\mu\psi(x) : -\frac{i}{\pi}\frac{\epsilon^\mu}{\epsilon^2}$ used

all singular terms have been automatically cancelled in (9) due to the manifestly hermitean definition of the current – no vacuum subtractions are needed. The constant $Z(\epsilon)$ got cancelled by the factor $Z^{-1}(\epsilon)$ coming from normal-ordering of the two exponentials sandwiching γ^μ in (9).

$J^\mu(x)$ conserved due to $\epsilon^{\mu\nu}$ in the quantum correction

The axial vector current obtained analogously

$$J_5^\mu(x) =: \overline{\psi}(x)\gamma^5\gamma^\mu\psi(x) : +\frac{g}{2\pi}\partial^\mu\phi(x) \qquad (10)$$

NOT conserved, $\partial_\mu J^\mu(x) = \frac{g}{2\pi}\partial_\mu\partial^\mu\phi(x)$.

the only effect of the anomaly is to renormalize the scalar field mass,

$$\partial_\mu \partial^\mu \phi + \tilde{\mu}^2 \phi = 0, \quad \tilde{\mu}^2 = \frac{\mu^2}{1 - \frac{g^2}{2\pi}}. \tag{11}$$

The conjugate momenta $\Pi_\phi = \partial_0 \phi(x) - g J_5^0, \ \Pi_\Psi = \frac{i}{2}\Psi^\dagger, \ \Pi_{\Psi^\dagger} = -\frac{i}{2}\Psi$ lead from the Lagrangian (3) to the Hamiltonian $H = H_{0B} + H^{'}$. H_{0B} corresponds to the free massive scalar field and

$$H^{'} = \int_{-\infty}^{+\infty} \mathrm{d}x^1 \big[-i\Psi^\dagger \alpha^1 \partial_1 \Psi + g \partial_1 \phi J_5^1 \big]. \tag{12}$$

However, re-expressing the Lagrangian in terms of true dynamical variables $\psi(x)$ and $\phi(x)$ leads to the sum of free fermion and boson

Hamiltonians – spectrum: free massless fermions and massive bosons (mass renormalized)

Correlation functions composed from the free fermion and boson correlation two-point functions, but depend on the coupling constant

the momentum operator contains interacting piece if the knowledge of the operator solution not taken into account

THE THIRRING MODEL

one of the prototype quantum field theories

operator solution studied by Klaiber (Boulder lectures 1967)

systematic hamiltonian treatment based on the model's solvability, explicit construction of the physical ground state (L. Martinovic, LC Dallas, Few Body Systems 52)

HERE: generalization – truly interacting current included (L. Martinovic and P. Grange, accepted in Phys. Lett. B)

The Lagrangian density of the massless Thirring model and the corresponding field equations read

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^{\mu} \overleftrightarrow{\partial_{\mu}} \Psi - \frac{1}{2} g J_{\mu} J^{\mu}, \quad (13)$$

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = gJ^{\mu}(x)\gamma_{\mu}\Psi(x), \tag{14}$$

solution of the Dirac equation (14) (Klaiber)

$$\Psi(x) = e^{i(g/\sqrt{\pi})\left(\alpha j(x) - \beta\gamma^{5}\tilde{j}(x)\right)}\psi(x),$$
$$\gamma^{\mu}\partial_{\mu}\psi(x) = 0. \tag{15}$$

The coefficients α and β satisfy $\alpha + \beta = 1$. The "potentials" $j(x)$ and $\tilde{j}(x)$ are connected to the free current $j^{\mu}(x)$ (taken as normal-ordered product of free fermion fields) according to $\partial_{\mu}j(x) = -\sqrt{\pi}j_{\mu}(x)$ and $\partial_{\mu}\tilde{j}(x) = \sqrt{\pi}\epsilon_{\mu\nu}j^{\nu}(x)$. This corresponds to replacing $J^{\mu}(x)$ by $j^{\mu}(x)$ in the field equation (14) – rather restrictive assumption

MORE GENERAL TREATMENT: $\beta = 0$ for simplicity, consider the solution

$$\Psi(x) = e^{i(g/\sqrt{\pi})J(x)}\psi(x), \tag{16}$$

with the unknown potential $J(x)$ of the interacting current $J^\mu(x)$, i.e. defining $\partial_\mu J(x) = -\sqrt{\pi} J_\mu(x)$. Compute the interacting current from the solution (16) using the point-splitting regularization as in Eq.(9):

$$J^\mu(x) =: \overline{\psi}(x)\gamma^\mu\psi(x) : +\frac{g}{2\pi}J^\mu(x). \quad (17)$$

$\Rightarrow$ the interacting current is simply the renormalized free current:

$$J^\mu(x) = G(g)j^\mu(x), \;\; G(g) = (1-\frac{g}{2\pi})^{-1}. \quad (18)$$

The Klaiber's solution is qualitatively correct, the factor $G(g)$ missed. This may have consequences for some aspects of bosonization of the massive Thirring model (Coleman, Phys. Rev. D 11 (1975)).

The rest of the study as before: bosonization of the free vector current plus a Bogoliubov transformation to diagonalize the Hamiltonian and to find the lowest-energy eigenstate

expansion of the massless spinor field, ($\hat{p}.x = |p^1|t - p^1x^1$)

$$\psi(x) = \int \frac{dp^1}{\sqrt{2\pi}}\{b(p^1)u(p^1)e^{-i\hat{p}.x} + d^\dagger(p^1)v(p^1)e^{i\hat{p}.x}\}, \tag{19}$$

$$\{b(p^1), b^\dagger(q^1)\} = \{d(p^1), d^\dagger(q^1)\} = \delta(p^1 - q^1),$$

$$b(k^1)|0\rangle = d(k^1)|0\rangle = 0, \;\; u^\dagger = (\theta(-p^1), \theta(p^1)), \;\; v^\dagger = (-\theta(-p^1), \theta(p^1)).$$

After the Fourier transformation, the current $j^\mu(x)$ expressed in terms of boson operators $c(k^1)$:

$$j^\mu(x) = \frac{-i}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dk^1 k^\mu}{\sqrt{2|k^1|}}\{c(k^1)e^{-i\hat{k}.x} - c^\dagger(k^1)e^{i\hat{k}.x}\},$$

$$c(k^1) = \frac{i}{\sqrt{|k^1|}} \int\limits_{-\infty}^{+\infty} dp^1 \{ \theta\big(p^1 k^1\big) \big[b^\dagger(p^1) b(p^1 + k^1) -$$

$$-(b \rightarrow d) + \epsilon(p^1)\theta\big(p^1(k^1 - p^1)\big) d(k^1 - p^1) b(p^1) \big\}, \tag{20}$$

The composite operators $c, c^\dagger$ obey the canonical commutation relation $\big[c(p^1), c^\dagger(q^1)\big] = \delta(p^1 - q^1), \;\; c(k^1)|0\rangle = 0$.

The Hamiltonian derived from the Lagrangian (13) after inserting the solution (15) into it. The contribution of the term $(i/2)\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi$ reverses the sign of the interacting term,

$$H = \int\limits_{-\infty}^{+\infty} \mathrm{d}x^1 \Big[- i\psi^\dagger \alpha^1 \partial_1 \psi - \frac{1}{2} g\big(J^0 J^0 - J^1 J^1\big) \Big]. \tag{21}$$

In Fock representation, $H = H_0 + H_g$ has the form

$$H_0 = \int_{-\infty}^{+\infty} \mathrm{d}p^1 |p^1| \Big[b^\dagger(p^1) b(p^1) + d^\dagger(p^1) d(p^1) \Big], \tag{22}$$

$$H_g = G^2(g) \frac{g}{\pi} \int_{-\infty}^{+\infty} dk^1 |k^1| \Big[c^\dagger(k^1) c^\dagger(-k^1) + c(k^1) c(-k^1) \Big].$$

H_g is not diagonal and thus $|0\rangle$ is not an eigenstate of the full Hamiltonian. To diagonalize H, form the new Hamiltonians $\hat{H}_0 = H_0 - T$, $\hat{H}_g = H_g + T$ where $T = \int_{-\infty}^{+\infty} \mathrm{d}k^1 |k^1| c^\dagger(k^1) c(k^1)$, and implement a Bogoliubov transformation by the unitary operator $U = e^{iS}$,

$$iS = \frac{1}{2} \int\limits_{-\infty}^{+\infty} \mathrm{d}k^1 \gamma(k^1) \Big[c^\dagger(k^1) c^\dagger(-k^1) - c(k^1) c(-k^1) \Big],$$

with an unknown function $\gamma(k^1)$. $\hat{H}_0$ is invariant with respect to U. The operators $c(k^1)$ transform as

$$c(k^1) \to c(k^1) \cosh \gamma(k^1) - c^\dagger(-k^1) \sinh \gamma(k^1). \tag{23}$$

The new interaction Hamiltonian $e^{iS} \hat{H}_g e^{-iS}$ will be diagonal,

$$\hat{H}_g^d = \frac{1}{\cosh 2\gamma_d} \int\limits_{-\infty}^{+\infty} \mathrm{d}k^1 |k^1| c^\dagger(k^1) c(k^1), \tag{24}$$

if $\gamma(k^1) = \gamma_d = \frac{1}{2}\mathrm{artanh}\, 2G(g)\frac{g}{\pi}$. Then

$$e^{iS}\big(\hat{H}_0 + \hat{H}_g\big)e^{-iS}|0\rangle = 0 \tag{25}$$

and $|\Omega\rangle = e^{-iS}|0\rangle$ is the new vacuum state,

$$|\Omega\rangle = N \exp\Big[-\kappa \int\limits_{-\infty}^{+\infty} \mathrm{d}p^1 c^\dagger(p^1) c^\dagger(-p^1)\Big]|0\rangle, \tag{26}$$

$\kappa = \frac{1}{2}\tanh\gamma_d$. $|\Omega\rangle$ is a coherent state of pairs of composite bosons with zero total momentum, $P^1|\Omega\rangle = 0$. The vacuum $|\Omega\rangle$ is invariant under axial $U(1)$ transformations

$$V(\beta)|\Omega\rangle = |\Omega\rangle, \quad V(\beta) = e^{i\beta Q_5},$$

$$Q_5 = \int_{-\infty}^{+\infty} \mathrm{d}k^1 \epsilon(k^1) \left[b^\dagger(k^1) b(k^1) - d^\dagger(k^1) d(k^1) \right]. \tag{27}$$

Thus, no chiral symmetry breaking occurs (contrary to some claims in literature).

Correlation functions have to be calculated using the vacuum $|\Omega\rangle$ and the solution (16)

$$J(x) = \frac{G(g)}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}q^1 \frac{c(q^1)}{\sqrt{2|q^1|}} \theta\left(|q^1| - \eta\right) e^{-i\hat{q}.x} + H.c. \tag{28}$$

η is the conventional infrared cutoff.

THE FEDERBUSH MODEL

a very brief description of the main steps of the solution in the hamiltonian form

demonstration that our modified canonical procedure removes the discrepancy between the structure of the SL and LF Hamiltonians of the Federbush model

clear advantages of the light-front formalism

very suitable for a detail comparison between the SL and LF forms of QFT

Lagrangian

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi - m\overline{\Psi}\Psi + \frac{i}{2}\overline{\Phi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Phi - \mu\overline{\Phi}\Phi -$$

$$-g\epsilon_{\mu\nu}J^\mu H^\nu, \tag{29}$$

two species of coupled fermion fields with masses m and μ. Both currents $J^\mu = \overline{\Psi}\gamma^\mu\Psi, H^\mu = \overline{\Phi}\gamma^\mu\Phi$ are conserved. The coupled field equations

$$\begin{aligned} i\gamma^\mu\partial_\mu\Psi(x) &= m\Psi(x) + g\epsilon_{\mu\nu}\gamma^\mu H^\nu(x)\Psi(x), \\ i\gamma^\mu\partial_\mu\Phi(x) &= \mu\Phi(x) - g\epsilon_{\mu\nu}\gamma^\mu J^\nu(x)\Phi(x) \end{aligned} \tag{30}$$

exactly solvable even for non-zero masses:

$$\begin{aligned} \Psi(x) &= e^{-i(g/\sqrt{\pi})h(x)}\psi(x),\ i\gamma^\mu\partial_\mu\psi(x) = m\psi(x), \\ \Phi(x) &= e^{i(g/\sqrt{\pi})j(x)}\varphi(x),\ i\gamma^\mu\partial_\mu\varphi(x) = \mu\varphi(x). \end{aligned} \tag{31}$$

In quantum theory, the above exponentials regularized by the ”triple-dot ordering” (Wightman, Schroer et al.) The potentials $j(x)$ and $h(x)$ defined as $\partial_\mu j(x) = \sqrt{\pi}\epsilon_{\mu\nu} j^\nu(x)$, $\partial_\mu h(x) = \sqrt{\pi}\epsilon_{\mu\nu} h^\nu(x)$ enter into the solutions (31) in an "off-diagonal" way. After inserting the solutions into the Lagrangian (29), the interaction term changes its sign yielding the Hamiltonian (H_0 is the sum of two free Hamiltonians)

$$H = H_0 + g \int_{-\infty}^{+\infty} \mathrm{d}x^1 \big(j^0 h^1 - j^1 h^0\big). \tag{32}$$

The LF field equations are also solved by (31) with the free LF fields $\psi(x)$, $\varphi(x)$; $j(x), h(x)$ are given by $2\partial_- j(x) = \sqrt{\pi} j^+(x)$, $2\partial_- h(x) = \sqrt{\pi} h^+(x)$. Usual LF treatment: one inserts the solution of the fermionic constraint into $\mathcal{L}$. $\Rightarrow$ free LF Hamiltonian! Only when inserting the full solution like in the

SL case the four-fermion interaction term also in the LF case:

$$P_g^- = \frac{1}{2} g \int_{-\infty}^{+\infty} \frac{\mathrm{d}x^-}{2} \left(j^+ h^- - j^- h^+ \right). \tag{33}$$

The interacting SL Hamitonian (32) contains terms composed solely from creation or annihilation operators, so the Fock vacuum is not its eigenstate. The diagonalization can be performed by a Bogoliubov transformation using a *massive* current bosonization – considerably more complicated than the massless case. The massive analog (up to the kinematical factors) of the boson operator $c(k^1)$ (20) is

$$A(k^1, t) = i \int_{-\infty}^{+\infty} \frac{dp^1}{\sqrt{E(k^1)}} \Big\{ \big[b^\dagger(p^1) b(k^1 + p^1) - (b \rightarrow d) \big]$$

$$
\begin{aligned}
&\times \tilde{f}_1(p^1, p^1 + k^1) e^{i\left(E(p^1) - E(k^1 + p^1)\right)t} \theta(k^1 p^1) \\
&+ \frac{1}{2}\left[b^\dagger(-p^1) b(k^1 - p^1) - (b \to d)\right] \theta(p^1(k^1 - p^1)) \\
&\times \tilde{f}_1(-p^1, k^1 - p^1) e^{i\left(E(p^1) - E(k^1 - p^1)\right)t} \\
&+ d(p^1) b(k^1 - p^1) \epsilon(p^1) \theta(p^1(k^1 - p^1)) \\
&\times \tilde{f}_2(p^1, k^1 - p^1) e^{-i\left(E(p^1) + E(k^1 - p^1)\right)t} \\
&+ d(p^1 + k^1) b(-p^1) \theta(p^1 k^1) \\
&\times \tilde{f}_2(-p^1, p^1 + k^1) e^{-i\left(E(p^1) + E(k^1 + p^1)\right)t} \\
&- b(p^1) d(-(p^1 - k^1)) \theta(k^1(p^1 - k^1)) \\
&\times \tilde{f}_2(p^1, -(p^1 - k^1)) e^{-i\left(E(p^1) + E(k^1 - p^1)\right)t} \Big\}. \qquad (34)
\end{aligned}
$$

The quantities

$$\tilde{f}_i(p^1, q^1) = \frac{f_i(p^1, q^1)}{\sqrt{2E(p^1)}\sqrt{2E(q^1)}}, \; i = 1, 2 \tag{35}$$

with $f_1(p^1, q^1) = \sqrt{p^+q^+} + \sqrt{p^-q^-}, f_2(p^1, q^1) = \sqrt{p^+q^+} - \sqrt{p^-q^-}$ are two coefficient functions appearing in four spinor products of the form $u^\dagger(p^1)\gamma^0\gamma^\mu u(q^1)$ etc., which arise when one calculates the free vector current in the Fock representation from the expansion of the free massive fermion field:

$$j^0(x) = \int\limits_{-\infty}^{+\infty} d\tilde{p}^1 \int\limits_{-\infty}^{+\infty} d\tilde{q}^1 \Big\{ \big[b^\dagger(p^1) b(q^1) - (b \rightarrow d) \big] \times$$

$$\times e^{i(\hat{p}-\hat{q})\dot{x}} f_1(p^1, q^1) + \big[b^\dagger(p^1) d^\dagger(q^1) e^{i(\hat{p}+\hat{q})\dot{x}} +$$

$$+d(q^1)b(p^1)e^{-i(\hat{p}+\hat{q}\dot{x}}\big]f_2(p^1,q^1)\Big\}. \tag{36}$$

For the component $j^1(x)$, the functions f_1 and f_2 are interchanged. $A(t,x^1)$ and $A^\dagger(t,k^1)$ (34) obtained by an inverse Fourier transformation from the assumed form of the current density

$$j^0(x) = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} \mathrm{d}k^1\big[\tilde{A}(t,k^1)e^{ik^1x^1} + \tilde{A}^\dagger(t,k^1)e^{-ik^1x^1}\big] \tag{37}$$

after inserting the fermion representation (36) for $j^0(x)$. They reduce to Klaiber's $c(k^1)$ for $m = 0$. These operators are complicated (no common $\hat{k}^\mu$ factor, separate time dependence of terms), but a useful concept – their algebraic properties are simple at equal times and the Hamiltonian of the model becomes quadratic when expressed in terms of them.

The corresponding massive charge density in the bosonized form is then written as

$$j^0(x) = \frac{-i}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dk^1 E(k^1)}{\sqrt{2E(k^1)}} A(k^1,t) e^{ik^1x^1} + H.c. \tag{38}$$

The analogous LF operators $\hat{A}, \hat{A}^\dagger$ are much simpler and have a structure similar to the massless SL case (20):

$$\hat{A}(k^+,x^+) = i \int_0^{+\infty} \frac{dp^+}{\sqrt{k^+}} \Big\{ \big[\hat{b}^\dagger(p^+)\hat{b}(k^+ + p^+) - (\hat{b} \to d) \big]$$
$$\times e^{\frac{i}{2}\frac{m^2 k^+ x^+}{p^+(k^+ + p^+)}} + \hat{d}(p^+)\hat{b}(k^+ - p^+) e^{-\frac{i}{2}\frac{m^2 k^+ x^+}{p^+(k^+ - p^+)}} \Big\}, \tag{39}$$

where

$$j^+(x) = \frac{-i}{2\pi} \int\limits_0^\infty \frac{dk^+}{\sqrt{k^+}}\, k^+ \hat{A}(k^+, x^+) e^{-\frac{i}{2}k^+x^-} + H.c. \tag{40}$$

In deriving $\hat{A}(k^+, x^+)$, the Fock expansion

$$\psi_2(x) = \frac{1}{\sqrt{4\pi}} \int\limits_0^{+\infty} \mathrm{d}p^+ \big[\hat{b}(p^+) e^{-i\hat{p}.x} + \hat{d}^\dagger(p^+) e^{i\hat{p}.x}\big],$$

$$\{\hat{b}(p^+), \hat{b}^\dagger(q^+)\} = \{\hat{d}(p^+), \hat{d}^\dagger(q^+)\} = \delta(p^+ - q^+). \tag{41}$$

The field $\varphi_2(x)$ expanded analogously. Due to $[\hat{A}(k^+), \hat{A}^\dagger(l^+)] = \delta(k^+ - l^+)$, valid at $x^+ = y^+$, the LF form of the solution (31) can be regularized by point-splitting,

$$\Phi(x) = Z(\epsilon) \exp\left\{i\frac{g}{\sqrt{\pi}}\hat{A}^\dagger(x)\right\} \exp\left\{i\frac{g}{\sqrt{\pi}}\hat{A}(x)\right\}\varphi(x),$$

$$\hat{A}(x) = \frac{1}{\sqrt{4\pi}}\int_0^\infty \frac{dk^+}{\sqrt{k^+}} A(k^+, x^+) e^{-\frac{i}{2}k^+x^-}. \quad (42)$$

Similar formulae hold for the solution $\Psi(x)$ built from the operators $\hat{B}(k^+, x^+), \hat{B}^\dagger(k^+, x^+)$ which are constructed from $h^+(x)$. The j^- and h^- currents contain the boson operators $\hat{C}(k^+, x^+), \hat{D}(k^+, x^+)$ and their conjugates, related to $\hat{A}, \hat{A}^\dagger, \hat{B}, \hat{B}^\dagger$ via the current conservation. In contrast to its SL analog, **the interacting LF Hamiltonian is diagonal** and therefore

$|0\rangle$ is its lowest-energy eigenstate:

$$P_g^- = \frac{g}{8\pi} \int\limits_0^{+\infty} \mathrm{d}k^+ k^+ \Big[\hat{A}^\dagger(k^+)\hat{D}(k^+) + \hat{D}^\dagger(k^+)\hat{A}(k^+) - \hat{B}^\dagger(k^+)\hat{C}(k^+) - \hat{C}^\dagger(k^+)\hat{B}(k^+) \Big]. \tag{43}$$

The next step – compute the correlation functions in both schemes

not simple since one needs to know the commutators of the composite boson operators at unequal times. This is the place where complexities of the usual triple-dot ordering technique enter into our bosonization approach. Irrespectively of this, **the LF calculation will be much simpler:** it works with Fock vacuum and simple operator structures while the

SL formalism requires nontrivial coherent-state vacuum and complicated operator terms.

MASSLESS VERSION MUCH SIMPLER

massless limit of the LF correlation functions

truly interacting currents, treatment like in the Thirring model

$$\Psi(x) = e^{-i\frac{g}{\sqrt{\pi}}\left(\alpha H(x) - \beta\gamma^5 \tilde{H}(x)\right)},$$
$$\Phi(x) = e^{i\frac{g}{\sqrt{\pi}}\left(\alpha J(x) - \beta\gamma^5 \tilde{J}(x)\right)}, \quad (44)$$

where

$$\partial_\mu \tilde{J} = -\sqrt{\pi} J_\mu(x), \;\; \partial_\mu J = \sqrt{\pi}\epsilon_{\mu\nu} J^\nu,$$

$$\partial_\mu \tilde{H} = -\sqrt{\pi} H_\mu(x), \quad \partial_\mu H = \sqrt{\pi} \epsilon_{\mu\nu} H^\nu, \tag{45}$$

The interacting currents are

$$\begin{aligned} J^\mu(x) &= \big(1 + \frac{g^2}{4\pi^2}\big)^{-1} \big[j^\mu(x) + \frac{g}{2\pi}(\alpha - \beta)\epsilon^{\mu\nu} h_\nu(x) \big], \\ H^\mu(x) &= \big(1 + \frac{g^2}{4\pi^2}\big)^{-1} \big[h^\mu(x) - \frac{g}{2\pi}(\alpha - \beta)\epsilon^{\mu\nu} j_\nu(x) \big]. \end{aligned} \tag{46}$$

SUMMARY AND OUTLOOK

- solvable models – good laboratory for studying subtleties of QFT, the vacuum problem and comparison between the SL and LF forms of the relativistic theory

- importance of the correct choice of the field variables and form of the Hamiltonians

- importance of the correctly defined currents - regularization by the point-splitting

- the Rothe-Stamatescu model reformulated in a ”minimal way”, quantum corrections to the currents found, improved regularization of the operator solution

- generalization of the Klaiber's Thirring-model solution, truly interacting currents, Hamiltonian diagonalization by a Bogoliubov transformation, true vacuum state derived as a coherent state - a very simple example of the complicated ground state in the SL field theory

- hamiltonian approach to the Federbush model sketched, bosonization of the massive current, LF treatment much simpler, study of its massless limit suggested along with the independent analysis of the massless SL model – non-trivial (cf. DCM) non-perturbative comparison between the SL and LF versions of the model

- quantum currents from the regularized operator solution of the Thirring-Wess model – axial anomaly usually found in the Schwinger model (cf. talk in Krakow); next step – derive the Hamiltonian and diagonalize it by a BT

- axial anomaly in a covariant-gauge Schwinger model based on the operator solution in terms of the fields present in the starting fermionic Lagrangian (no Ansaetze), subtleties of the residual gauge invariance and truly gauge-invariant current definition – anomaly only in the zero-mode sector, reformulation of the mass generation of the Schwinger boson

  original formulation of Lowenstein and Swieca incomplete - Hamiltonian in terms of their ”building-block fields” would be non-diagonal

- the biggest challenge: complete solution of the Schwinger model in the new formulation (new form of the operator solution in the finite-volume treatment) – role of large gauge transformations, vacuum degeneracy, chiral symmetry... IN PROGRESS