Baryon Structure in AdS/QCD

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Outline

- Introduction
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  - Classification of AdS/QCD models
- Baryons in soft-wall model
  - Basic blocks of effective action
  - EOM for the bulk profiles: mass spectra and wave functions
  - Inclusion of high Fock states
  - Inclusion photons
- Applications
  - Electromagnetic form factors
  - Generalized parton distributions
  - Roper resonance $N(1440)$

Introduction

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors
- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al) Extra-Dimensional (ED) theories including gravity are holographically equivalent to gauge theories living on boundary of ED space
- Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

Introduction

- AdS metric $ds^2 = \frac{R^2}{z^2}\Big(dx_\mu dx^\mu - dz^2\Big)$ Poincaré form

  z is extra dimensional (holographic) coordinate; $z = 0$ is UV boundary
  Light-Front Holography (Brodsky-Teramond) $z \to \zeta$ with $\zeta^2 = \mathbf{b}_\perp^2 x(1-x)$

- Fields in AdS$_5$ classified by representation $D(\Delta, J_1, J_2)$ of $SO(4,2)$
  Δ – conformal dimension, J_1 and J_2 – spins related to $SU(2)$

- Example: Scalar fields $\Phi(x,z)$ according $D(3,0,0)$ with $\Delta = 3$ and $J_1 = J_2 = 0$
  Holographic analogue of scalar mesons with $J = 0$

- Action: $S = \frac{1}{2}\int d^4x dz \sqrt{g}\Big[\partial_M \Phi(x,z)\partial^M \Phi(x,z) - m^2\,\Phi^2(x,z)\Big]$
  where m – 5D mass, $m^2 R^2 = \Delta(\Delta - 4)$.

- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum\limits_n \phi_n(x)\,\Phi_n(z)$

- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons

- Bulk profiles $\Phi_n(z)$ — dual to hadronic wafe functions.

- Constrained dynamics: breaking conformal symmeetry and introducing confinement due to background fields (dilaton) or infrared cut of z-coordinate

Introduction

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- Bound state problem:
  Derive Schrödinger-type EOM for $\Phi_n(z)$, solve it on mass-shell $p^2 = M_H^2$.
  Solutions: hadronic w.f. and mass spectra.

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- Scattering problem for AdS field (without KK expansion) gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $V(Q, z)$ [Fourier-trasform of AdS field $V(x, z)$].

  On UV boundary $V(Q, z = 0) = 1$.

- Hadron properties

  $$F_n(Q^2) = \int\limits_0^\infty dz V(Q, z)\Phi_n^2(z)$$

  $F_n(0) \equiv 1$ due to $V(0, z) = 1$ and $\int\limits_0^\infty dz\, \Phi_n^2(z) = 1$

- Nonlocality of strong interactions implemented by the 5-th (holographic) coordinate and hidden in the bulk profiles $\Phi_n(z)$ and bulk propagators $V(Q, z)$

Introduction

- Top-down approaches Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)

- Bottom-up approaches More phenomenological use the features of QCD to construct 5D dual theory including gravity on AdS space

- Towards to QCD:
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) $\leftrightarrow$ Hadron wave functions
  – Spectrum of Kaluza-Klein modes $\leftrightarrow$ Hadrons spectrum

- Hard-wall:
  AdS geometry is cutted by two branes UV $(z = \epsilon \longrightarrow 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence on $J(L)$ of hadron masses

- Soft-wall:
  Soft cuttoff of AdS space by dilaton field $e^{-\varphi(z)}$
  Analytical solution of EOM, Regge behavior $M^2 \sim J(L)$

Baryons in soft-wall model

- Baryons in soft-wall model: Forkel–Beyer–Frederico, Brodsky–Teramond, Abidin–Carlson, Gutsche–Lyubovitskij–Schmidt–Vega, . . .
- Talks at LC2013: Stan Brodsky, Alfredo Vega
- SW holographic approach for baryons with inclusion of high Fock states dual to bulk fermion fields of higher dimension.
- Objective: Application to nucleon form factors, GPDs, nucleon resonances (Roper)

Baryons in soft-wall model

- Bulk fermion fields
  $\Psi_+(x,z)$ and $\Psi_-(x,z)$ dual to $\mathcal{O}_R = (p_R, n_R)$ and $\mathcal{O}_L = (p_L, n_L)$
- Bulk fermion mass $\pm m = \pm(\Delta - 3/2)$, where Δ - scaling dimension
- Scaling dimension $\equiv$ Twist-dimension $\tau = N + L$,
  N - number of partons, $L = \max|L_z|$
- Action for the fermion field of twist τ

$$S_\tau = \int d^4x dz \sqrt{g}\, e^{-\varphi(z)} \sum_{i=+,-} \bar{\Psi}_{i,\tau}(x,z)\, \hat{\mathcal{D}}_i(z)\, \Psi_{i,\tau}(x,z)\,,$$

$$\hat{\mathcal{D}}_\pm(z) = \frac{i}{2}\Gamma^M \overset{\leftrightarrow}{\partial}_M \mp \frac{m + \varphi(z)}{R}$$

- dilaton $\varphi(z) = \kappa^2 z^2$ (Regge behavior of hadron masses)
- metric $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}, \quad g = |\mathrm{det} g_{MN}|$
- vielbein $\epsilon^a_M(z) = e^{A(z)}\delta^a_M, \quad A(z) = \log(R/z)$ (conformal)
- interval $ds^2 = g_{MN}dx^M dx^N = e^{2A(z)}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$

Baryons in soft-wall model

- P-transformations

$$U_P^{-1}\Psi_{\tau,\pm}(t,\vec{x},z)U_P = \pm\gamma^0\gamma^5\,\Psi_{\tau,\mp}(t,-\vec{x},z)$$

$$U_P^{-1}\bar\Psi_{\tau,\pm}(t,\vec{x},z)U_P = \pm\bar\Psi_{\tau,\mp}(t,-\vec{x},z)\,\gamma^0\gamma^5$$

$$\pm U_P^{-1}\,\bar\Psi_{\pm,\tau}(t,\vec{x},z)\,\Psi_{\pm,\tau}(t,\vec{x},z)\,U_P = \mp\bar\Psi_{\mp,\tau}(t,-\vec{x},z)\,\Psi_{\mp,\tau}(t,\vec{x},z)\,,$$

$$U_P^{-1}\,S_\tau^{\pm}\,U_P = S_\tau^{\mp}$$

- C-transformations

$$U_C^{-1}\Psi_{\pm}(x,z)U_C = \mp\,C\,\gamma^5\,\bar\Psi_{\mp}^T(x,z)$$

$$U_C^{-1}\bar\Psi_{\pm}(x,z)U_C \;=\; \pm\Psi_{\mp}^T(x,z)\gamma^5 C$$

$$\pm\,U_C^{-1}\bar\Psi_{\pm}(x,z)\Psi_{\pm}(x,z)U_C = \mp\,\bar\Psi_{\mp}(x,z)\Psi_{\mp}(x,z)$$

$$U_C^{-1}\,S_\tau^{\pm}\,U_C = S_\tau^{\mp}$$

Baryons in soft-wall model

- Redefinition $\Psi_{i,\tau}(x,z) = e^{\varphi(z)/2-2A(z)}\psi_{i,\tau}(x,z)$
- Expansion on left- and right-chirality components (eigenstates of γ^5)
  $\psi_{i,\tau}(x,z) = \psi^L_{i,\tau}(x,z) + \psi^R_{i,\tau}(x,z)$
- Kaluza-Klein expansion

$$\psi^{L/R}_{i,\tau}(x,z) = \frac{1}{\sqrt{2}} \sum_n \psi^{L/R}_n(x)\, f^{L/R}_{i,\tau,n}(z)\,,$$

- Relations between bulk profiles

$$\begin{aligned} f^R_{\tau,n}(z) &\equiv f^R_{+,\tau,n}(z) = -f^L_{-,\tau,n}(z)\,, \\ f^L_{\tau,n}(z) &\equiv f^L_{+,\tau,n}(z) = f^R_{-,\tau,n}(z)\,. \end{aligned}$$

- EOM

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(m \mp \frac{1}{2}\right) + \frac{m(m\pm 1)}{z^2}\right] f^{L/R}_{\tau,n}(z) = M^2_{n\tau}\, f^{L/R}_{\tau,n}(z),$$

Baryons in soft-wall model

- Solutions

$$
\begin{aligned}
f^L_{\tau,n}(z) &= \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}}\,\kappa^\tau\ z^{\tau-1/2}\ e^{-\kappa^2 z^2/2}\ L_n^{\tau-1}(\kappa^2 z^2)\,, \\
f^R_{\tau,n}(z) &= \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\,\kappa^{\tau-1}\ z^{\tau-3/2}\ e^{-\kappa^2 z^2/2}\ L_n^{\tau-2}(\kappa^2 z^2)
\end{aligned}
$$

and

$$
M^2_{n\tau} = 4\kappa^2\Big(n+\tau-1\Big)
$$

with

$$
\int\limits_0^\infty dz\ f^{L/R}_{\tau,n_1}(z)\, f^{L/R}_{\tau,n_2}(z) \;=\; \delta_{n_1 n_2}
$$

Baryons in soft-wall model

- Inclusion of high Fock states

$$S = \sum_\tau c_\tau \, S_\tau$$

c_τ - set of free parameters

- Integration over z using normalizazion condition for $f^{L/R}$

$$S \ = \ \int d^4x \, \bar\psi_n(x) \Big[\underbrace{\sum_\tau c_\tau}_{=1} \, i \, \not\partial - \underbrace{\sum_\tau c_\tau M_{n\tau}}_{=M_n} \Big] \psi_n(x) \, .$$

Correct normalization of kinetic term of 4D spinor field

$$\sum_\tau c_\tau = 1 \, , \qquad \sum_\tau c_\tau M_{n\tau} = M_n \quad \text{(baryon mass)}$$

Electromagnetic stucture of nucleons

- Abidin-Carlson: First application of SW model (3q configurations)
- Coupling of bulk vector and fermion fields

$$\mathcal{L}_{\rm int}(x,z) = \sum_{i=+,-} \sum_{\tau} c_\tau \, \bar{\Psi}_{i,\tau}(x,z) \, \hat{\mathcal{V}}_i(x,z) \, \Psi_{i,\tau}(x,z)$$

$$\hat{\mathcal{V}}_\pm(x,z) = \underbrace{Q_N \, \Gamma^M V_M(x,z)}_{\text{min. coupling}} \pm \underbrace{\frac{i}{4} \eta_V \, [\Gamma^M, \Gamma^N] \, V_{MN}(x,z) \pm g_V \, \tau_3 \, \Gamma^M \, i\Gamma^z \, V_M(x,z)}_{\text{nonmin. coupling}}$$

$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p') \left[\gamma^\mu F_1^N(t) + \frac{i}{2m_N} \sigma^{\mu\nu} \, q_\nu F_2^N(t) \right] u(p)$$

$$\begin{aligned}
F_1^p(Q^2) &= C_1(Q^2) + g_V C_2(Q^2) + \eta_V^p C_3(Q^2) \\
F_2^p(Q^2) &= \eta_V^p C_4(Q^2) \\
F_1^n(Q^2) &= -g_V C_2(Q^2) + \eta_V^n C_3(Q^2) \\
F_2^n(Q^2) &= \eta_V^n C_4(Q^2)
\end{aligned}$$

Electromagnetic stucture of nucleons

- $V(Q,z)$ – propagator of trans. massless vector field (analogue of EM field)

$$V(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$

$$V(0,z) = 1\,, \qquad V(Q,0) = 1\,, \qquad V(Q,\infty) = 0\,.$$

Electromagnetic stucture of nucleons

Matveev-Muradyan-Tavkhelidze-Brodsky-Farrar quark-counting rules at large Q^2

- $$C_1(Q^2) = \frac{1}{2}\int_0^\infty dz\, V(Q,z)\,\sum_\tau c_\tau \left([f^L_\tau(z)]^2 + [f^R_\tau(z)]^2\right) = \sum_\tau c_\tau\, B(a+1,\tau)\,\left(\tau + \frac{a}{2}\right) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_2(Q^2) = \frac{1}{2}\int_0^\infty dz\, V(Q,z)\,\sum_\tau c_\tau \left([f^R_\tau(z)]^2 - [f^L_\tau(z)]^2\right) = \frac{a}{2}\sum_\tau c_\tau\, B(a+1,\tau) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_3(Q^2) = \frac{1}{2}\int_0^\infty dz z\, \partial_z V(Q,z)\,\sum_\tau c_\tau \left([f^L_\tau(z)]^2 - [f^R_\tau(z)]^2\right) = a\sum_\tau c_\tau\, B(a+1,\tau+1)\,\frac{a(\tau-1)-1}{\tau} \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_4(Q^2) = 2m_N\int_0^\infty dz z\, V(Q,z)\,\sum_\tau c_\tau\, f^L_\tau(z) f^R_\tau(z) = \frac{2m_N}{\kappa}\sum_\tau c_\tau\,(a+1+\tau)\,B(a+1,\tau+1)\,\sqrt{\tau-1} \sim \sum_\tau \frac{c_\tau}{a^\tau}$$

- $a = \frac{Q^2}{4\kappa^2}$, $\quad B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ is the beta-function.

Electromagnetic stucture of nucleons

Choice of free parameters

- $\kappa = 383\ \mathrm{MeV}\,, \quad c_3 = 1.25\,, \quad c_4 = 0.16\,, \quad g_V = 0.3$
- c_5 is expressed through c_3 and c_4

$$c_5 = 1 - c_3 - c_4 = -0.41$$

- c_3, c_4 are constrained by the nucleon mass
- κ is fixed by the nucleon mass and nucleon electromagnetic radii
- g_V is fixed by fine tuning of the neutron electromagnetic radii
- Nonminimal couplings $\eta_V^{p,n}$ from nucleon magnetic moments

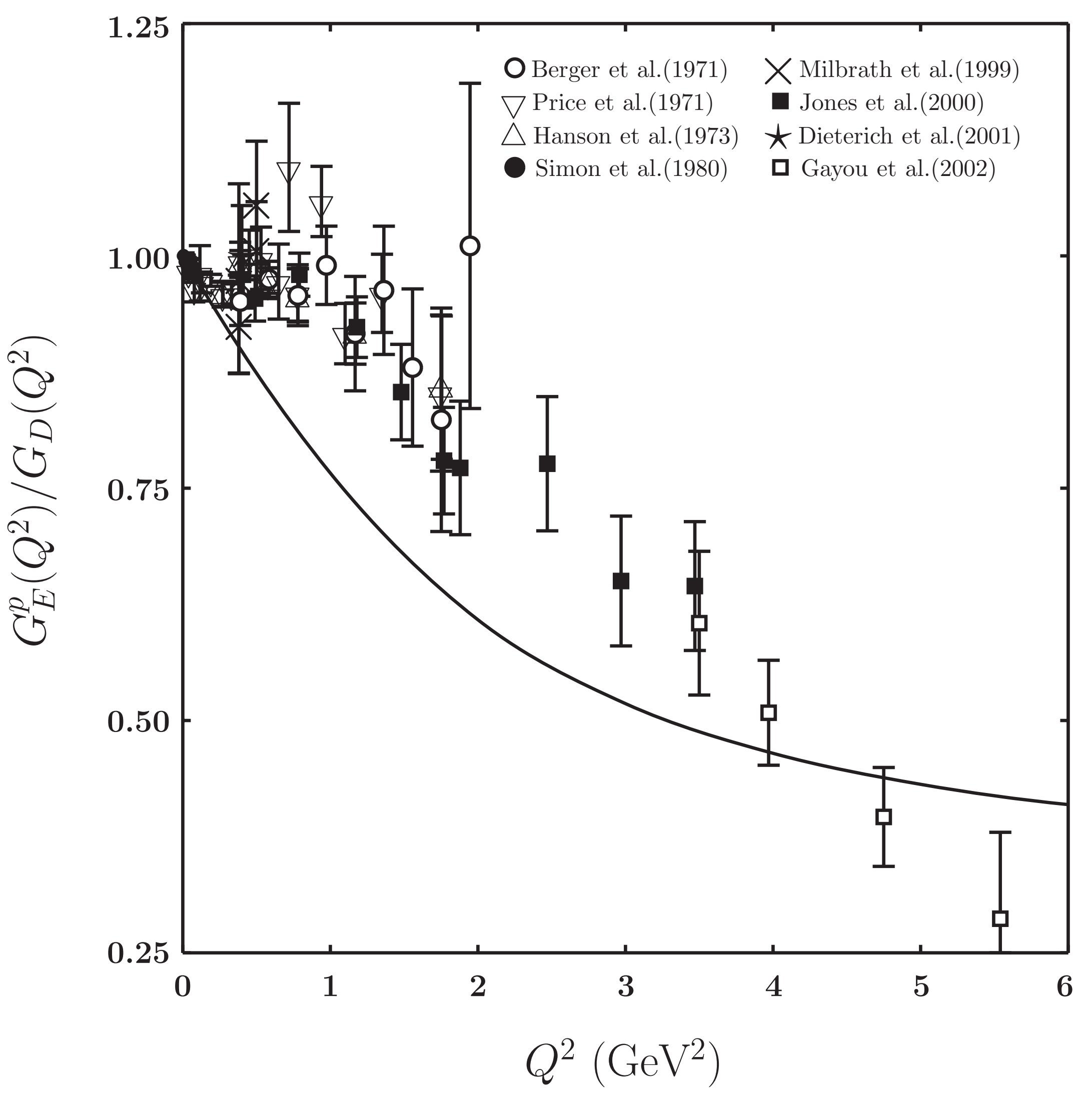
$$\eta_V^p = \frac{\kappa\,(\mu_p - 1)}{2m_N\,C_0} = 0.30\,, \quad \eta_V^n = \frac{\kappa\,\mu_n}{2m_N\,C_0} = -0.32\,, \quad C_0 = \sqrt{2}\,c_3 + \sqrt{3}\,c_4 + 2c_5$$

Electromagnetic stucture of nucleons

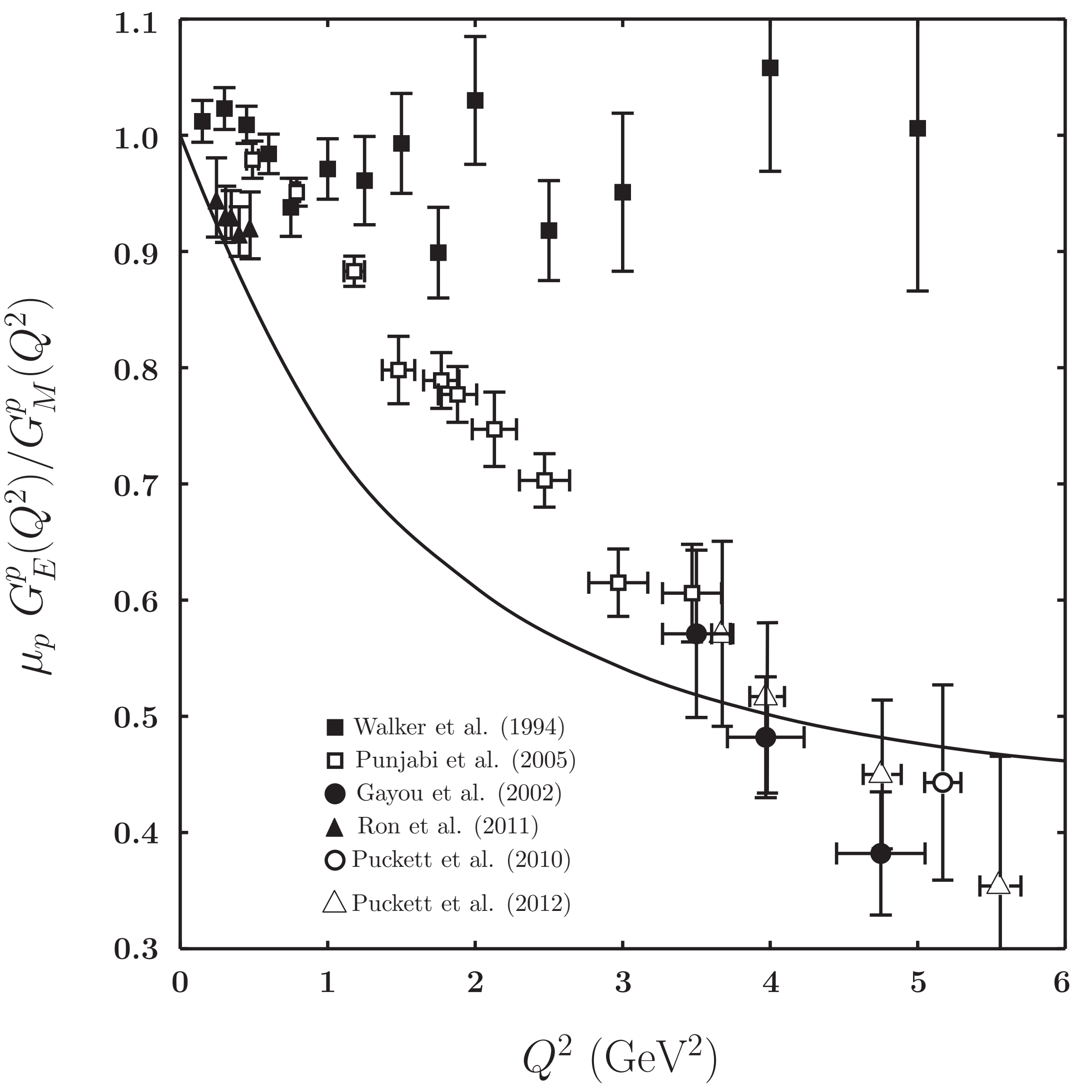
Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	0.862$^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

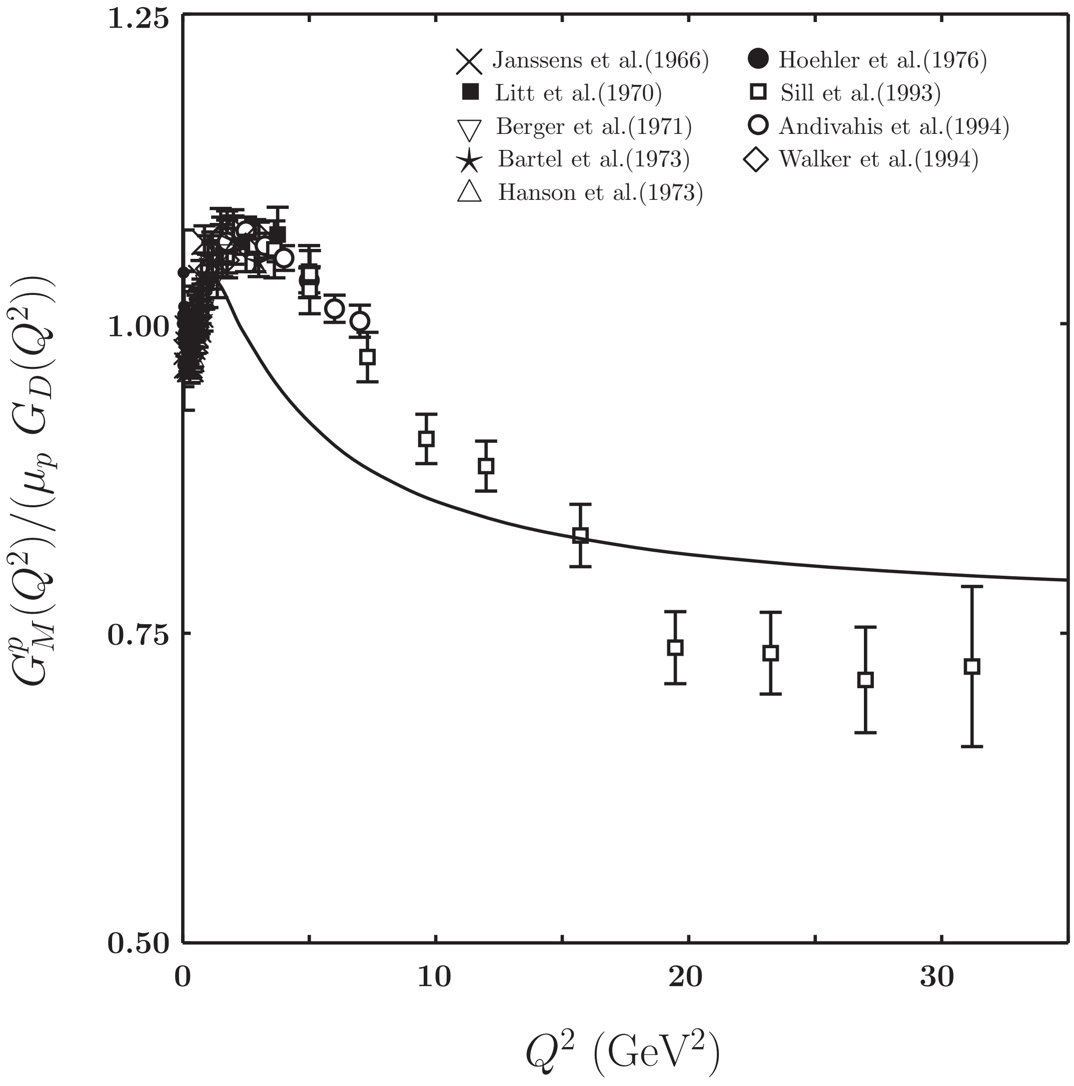
Electromagnetic stucture of nucleons



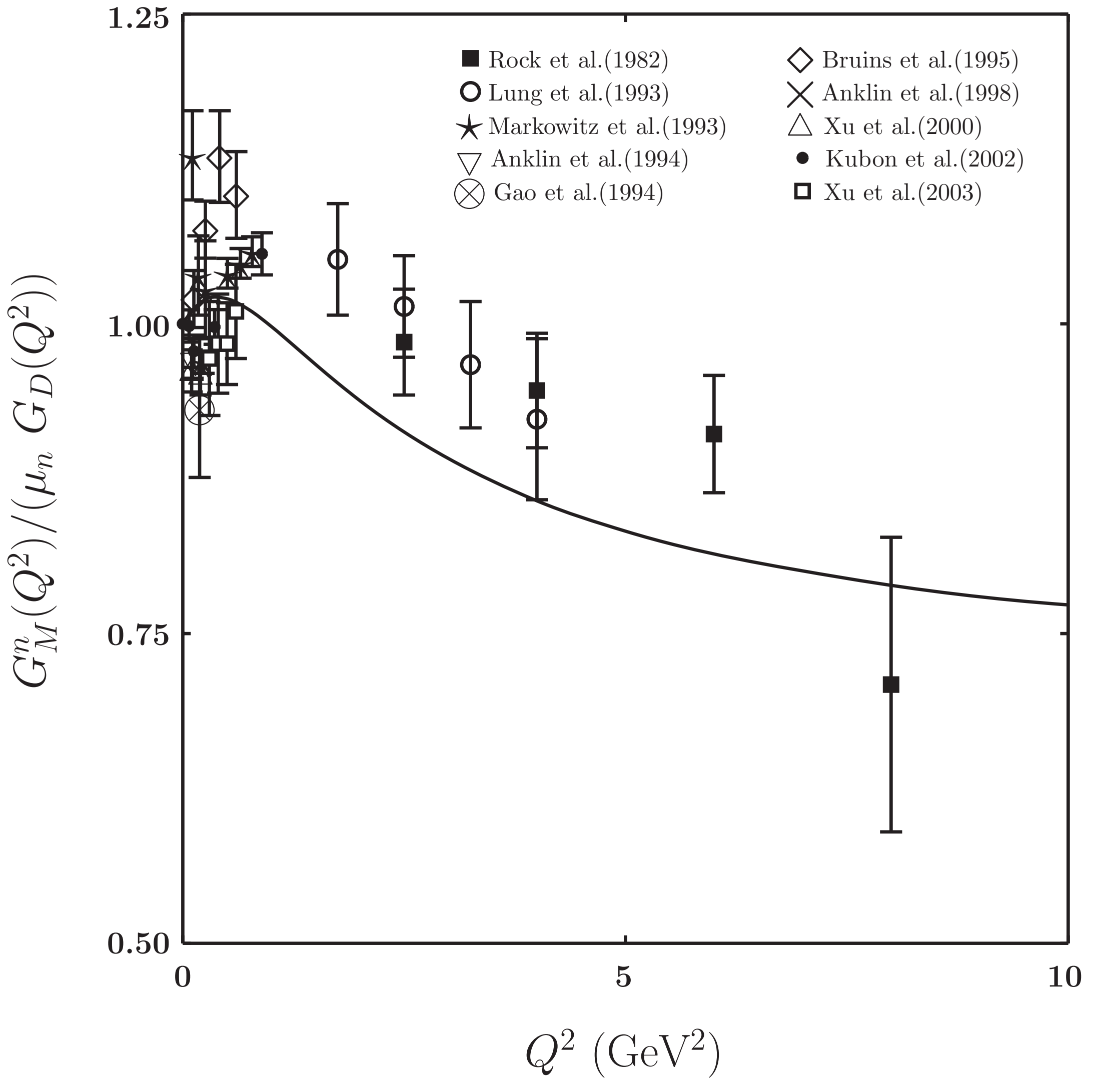
Electromagnetic stucture of nucleons



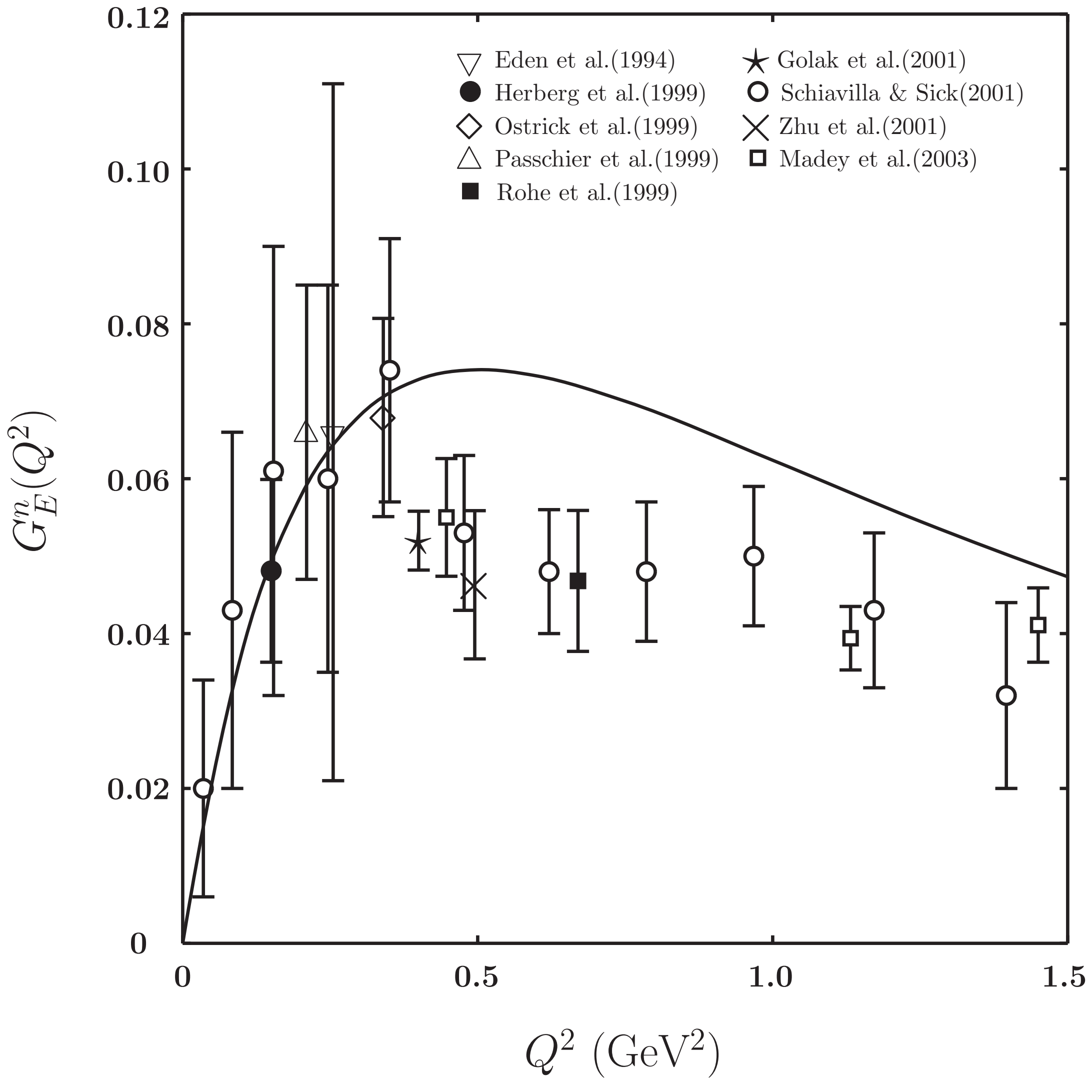
Electromagnetic stucture of nucleons



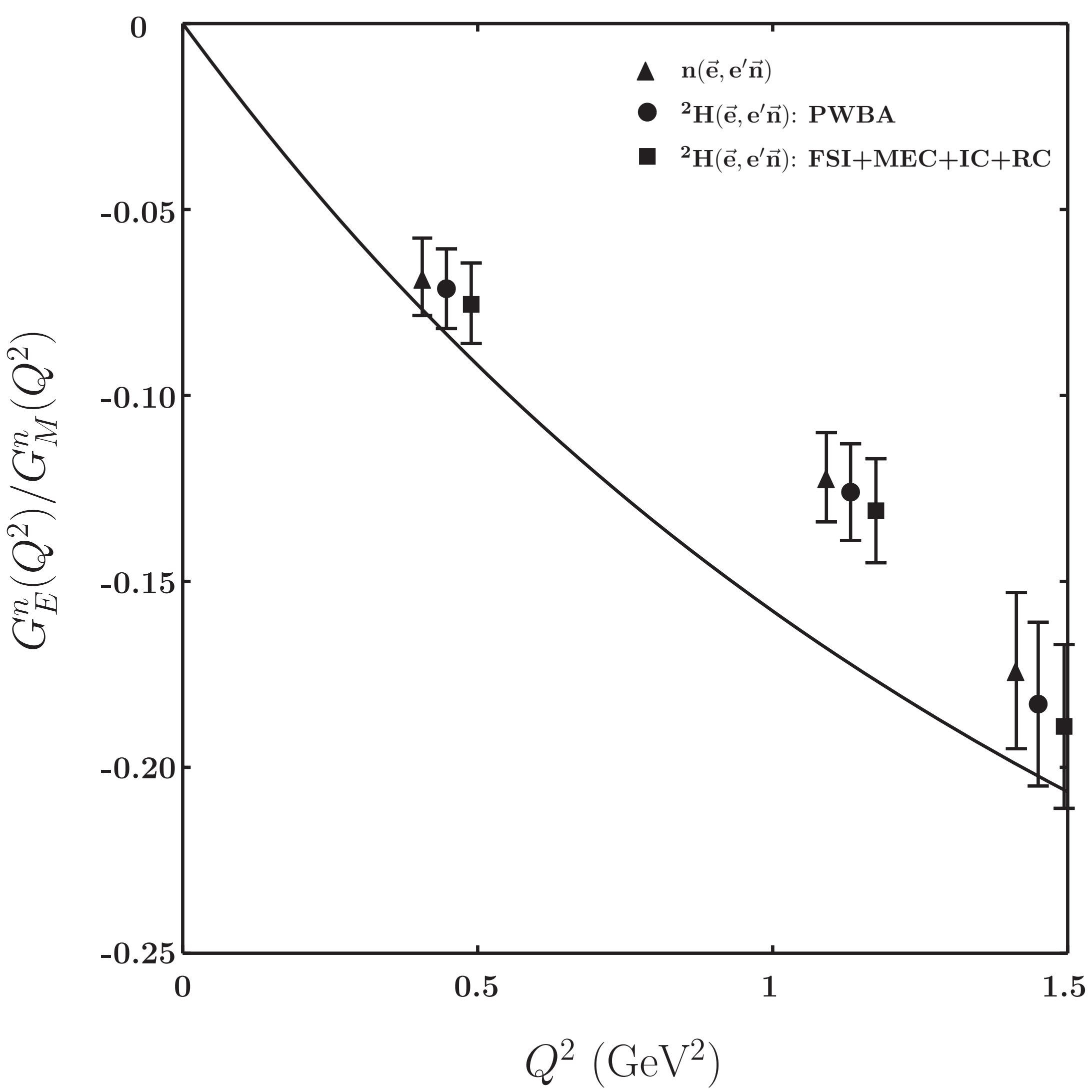
Electromagnetic stucture of nucleons



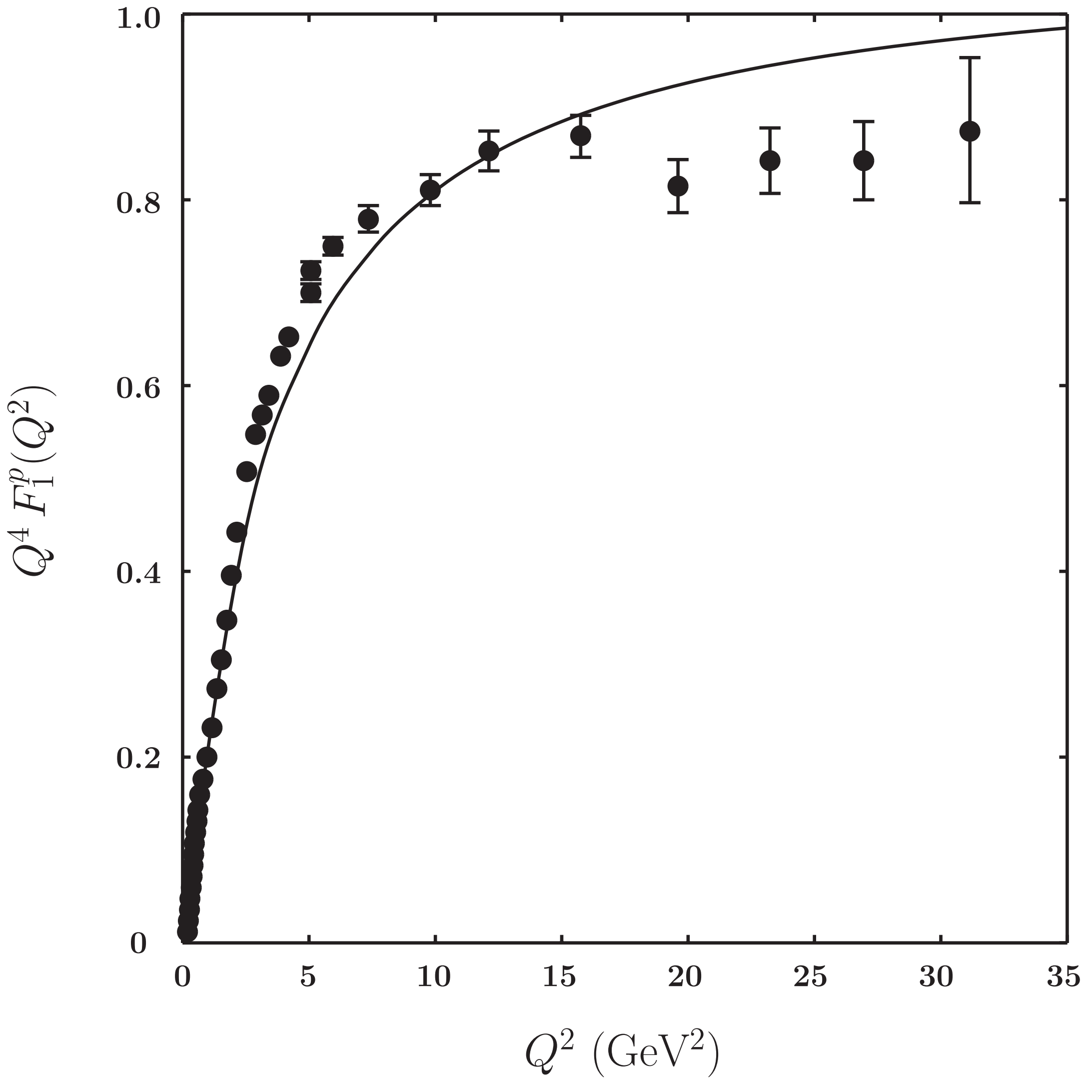
Electromagnetic stucture of nucleons



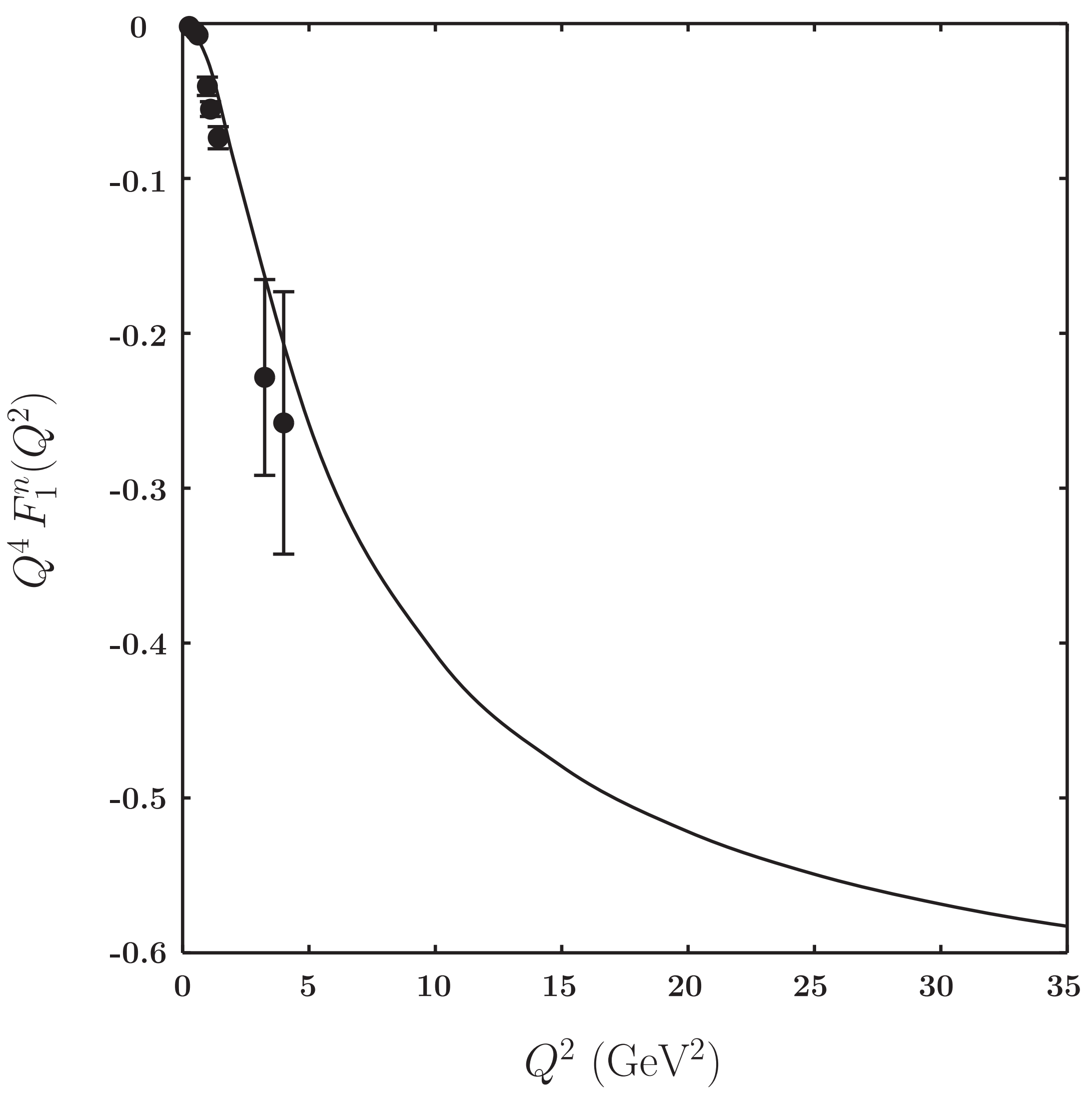
Electromagnetic stucture of nucleons



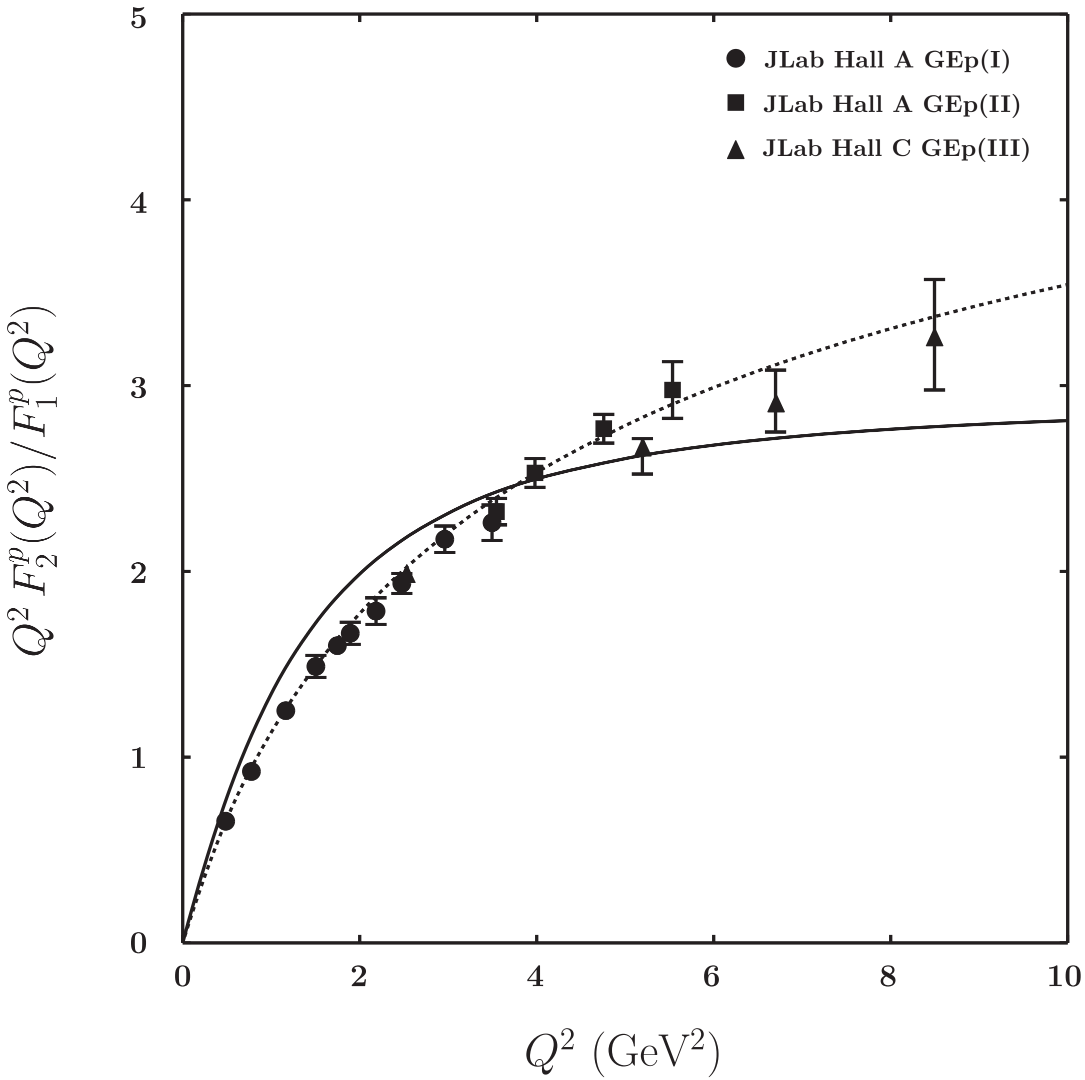
Electromagnetic stucture of nucleons



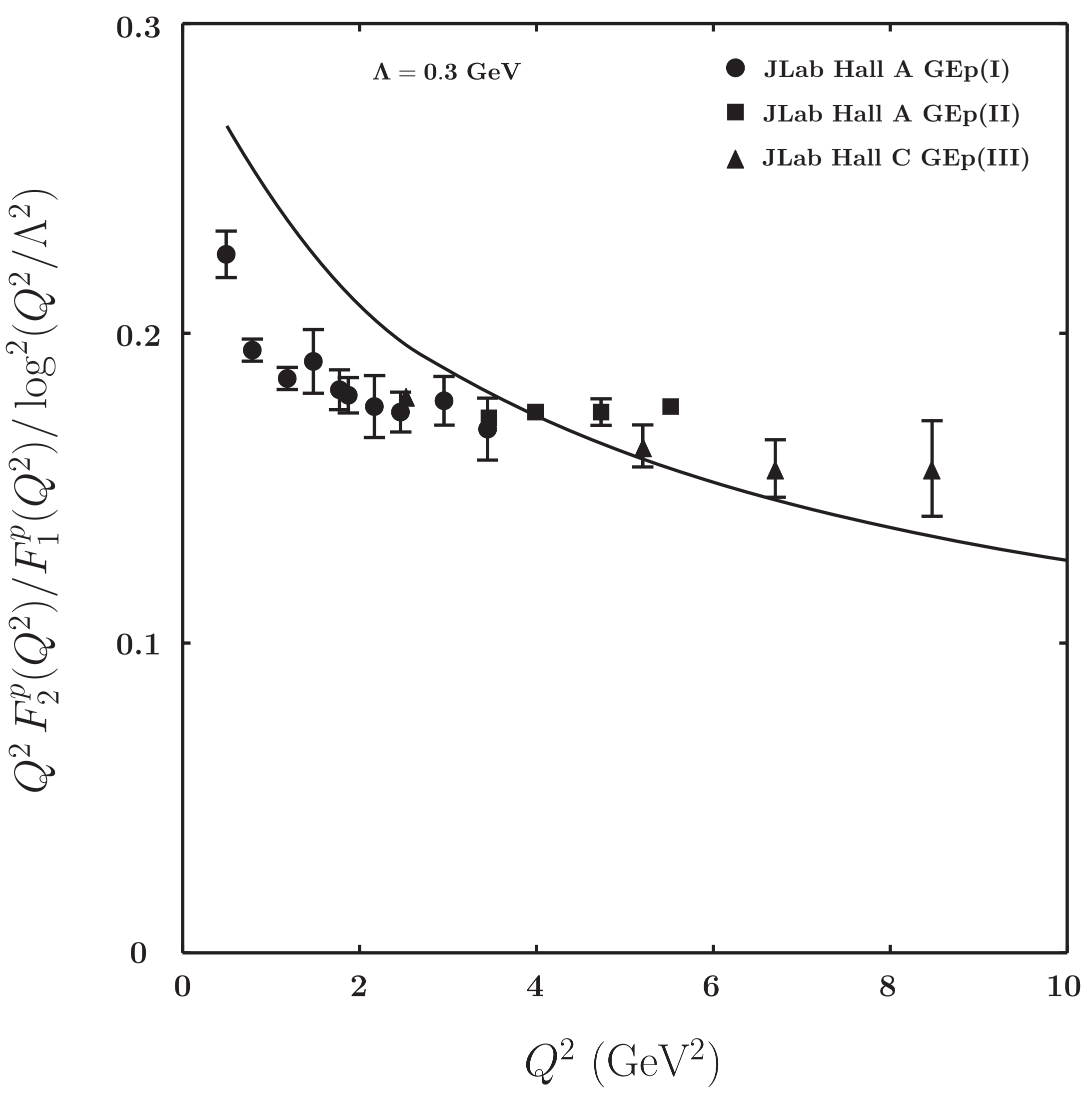
Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



GPDs

Substitute the identity

$$1 \quad = \quad -\int_0^1 d\Big((1-x)^2\, e^{-k^2 z^2 x/(1-x)}\Big) = \int_0^1 dx\,(1-x)\, e^{-k^2 z^2 x/(1-x)}\,\Big(2+\frac{k^2 z^2}{1-x}\Big)$$

in the normalization condition for the twist-τ hadronic w.f.

$$\begin{aligned} 1 &= \int_0^\infty dz \varphi_\tau^2(z) \\ &= (\tau+1)\int_0^1 dx\,(1-x)^\tau = \int_0^1 dx\,\rho_\tau(x)\,, \end{aligned}$$

where $\varphi_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\,\kappa^{\tau-1}\, z^{\tau-3/2}\, e^{-\kappa^2 z^2}$

$\rho_\tau(x) = (\tau+1)\,(1-x)^\tau$ is the PDF at $x \to 1$

GPDs

Hadronic form factor is given by

$$F_\tau(Q^2) = \int_0^\infty dz\, \varphi_\tau^2(z)\, V(Q^2, z^2) = \int_0^1 dx\, \rho_\tau(x)\, f_\tau(x, Q^2)$$

$$f_\tau(x, Q^2) = \frac{1}{(\tau+1)\,\Gamma(\tau-1)} \int_0^\infty dt t^{\tau-2}\, e^{-t}(2+t)\, V(Q^2, t(1-x))$$

$$V(Q^2, t(1-x)) \to V(Q^2, 0) \equiv 1$$

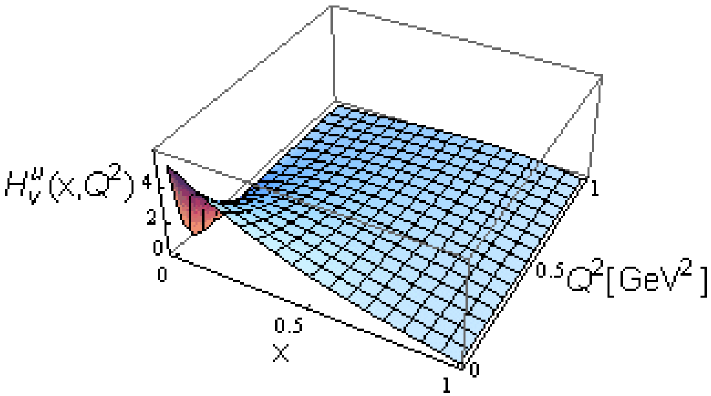
as required by model-independent result and $f_\tau(x, Q^2) \to 1$
The GPD $H_\tau(x, Q^2)$ is defined as

$$\mathcal{H}_\tau(x, Q^2) = \rho_\tau(x)\, f_\tau(x, Q^2)$$

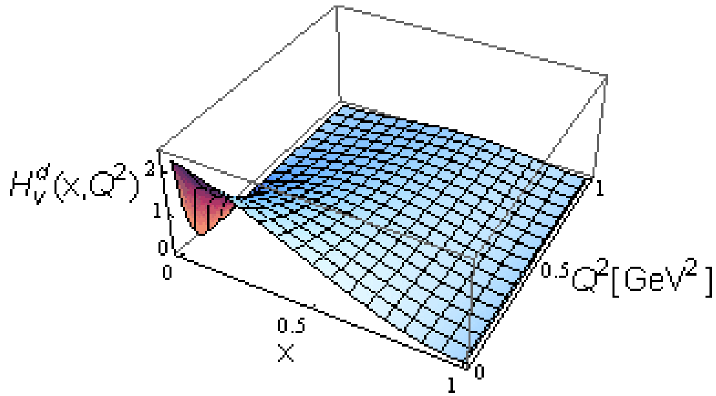
with correct behavior at $x \to 1$

$$\mathcal{H}_\tau(x, Q^2) \to \rho_\tau(x) = (\tau+1)(1-x)^\tau$$

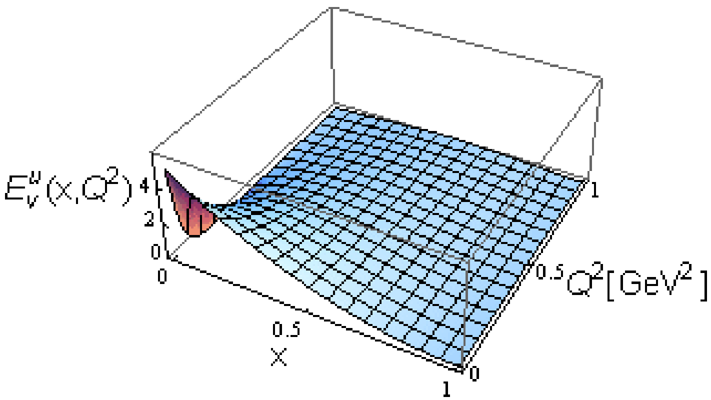
Nucleon GPDs



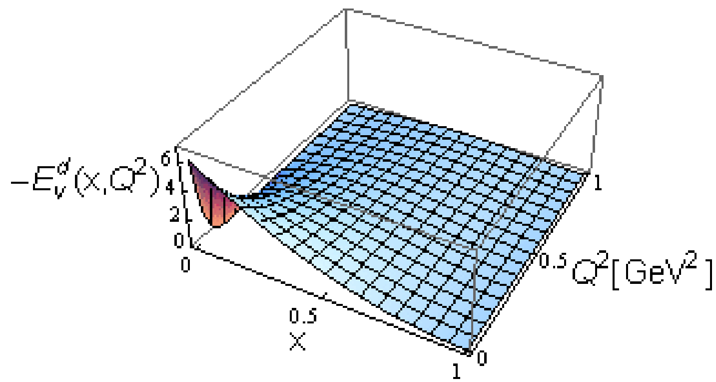
$H_v^u(x,Q^2)$



$H_v^d(x,Q^2)$

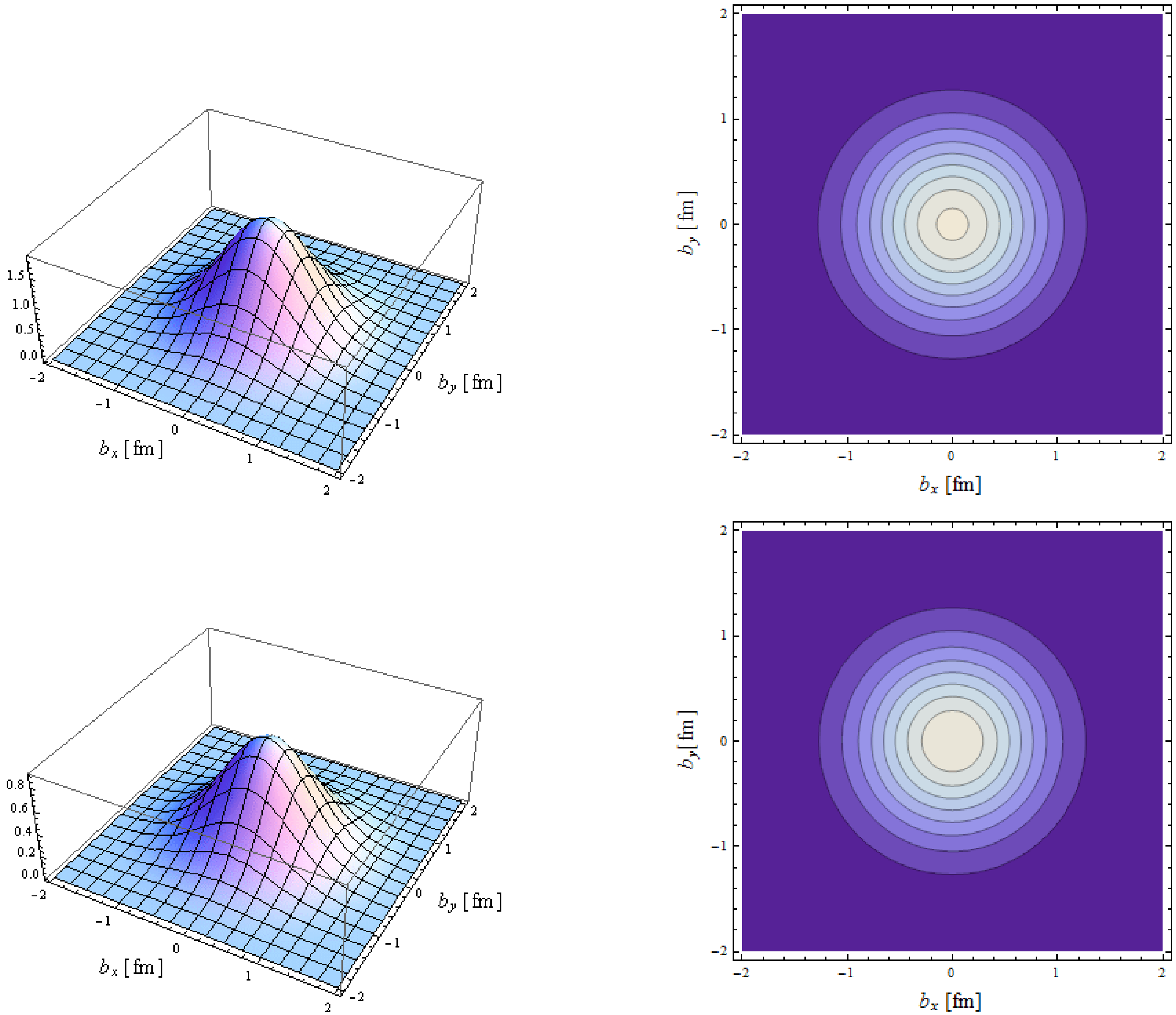


$E_v^u(x,Q^2)$



$E_v^d(x,Q^2)$

Nucleon GPDs



Plots for $q(x, \mathbf{b}_\perp)$ for $x = 0.1$: $u(x, \mathbf{b}_\perp)$ - upper pannels, $d(x, \mathbf{b}_\perp)$ - lower pannels

Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^\mu = \bar{u}_{\mathcal{R}} \left[\gamma_\perp^\mu \, F_1(q^2) \, + \, i\sigma^{\mu\nu} \frac{q_\nu}{M_{\mathcal{R}}} \, F_2(q^2) \right] u_N \, , \quad \gamma_\perp^\mu = \gamma^\mu - q^\mu \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm\frac{1}{2}0} \;\; = \;\; \sqrt{\frac{Q_-}{Q^2}} \left(F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right)$$

$$H_{\pm\frac{1}{2}\pm 1} \;\; = \;\; -\sqrt{2Q_-} \left(F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} \;\; = \;\; -b \, H_{\frac{1}{2}1} \, , \quad S_{1/2} \, = \, b \, \frac{|\mathbf{p}|}{\sqrt{Q^2}} \, H_{\frac{1}{2}0} \, ,$$

$$Q_\pm \;\; = \;\; M_\pm^2 + Q^2 \, , \quad M_\pm = M_{\mathcal{R}} \pm M_N \, , \quad b \, = \, \sqrt{\frac{\pi\alpha}{2 E M_{\mathcal{R}} M_N}}$$

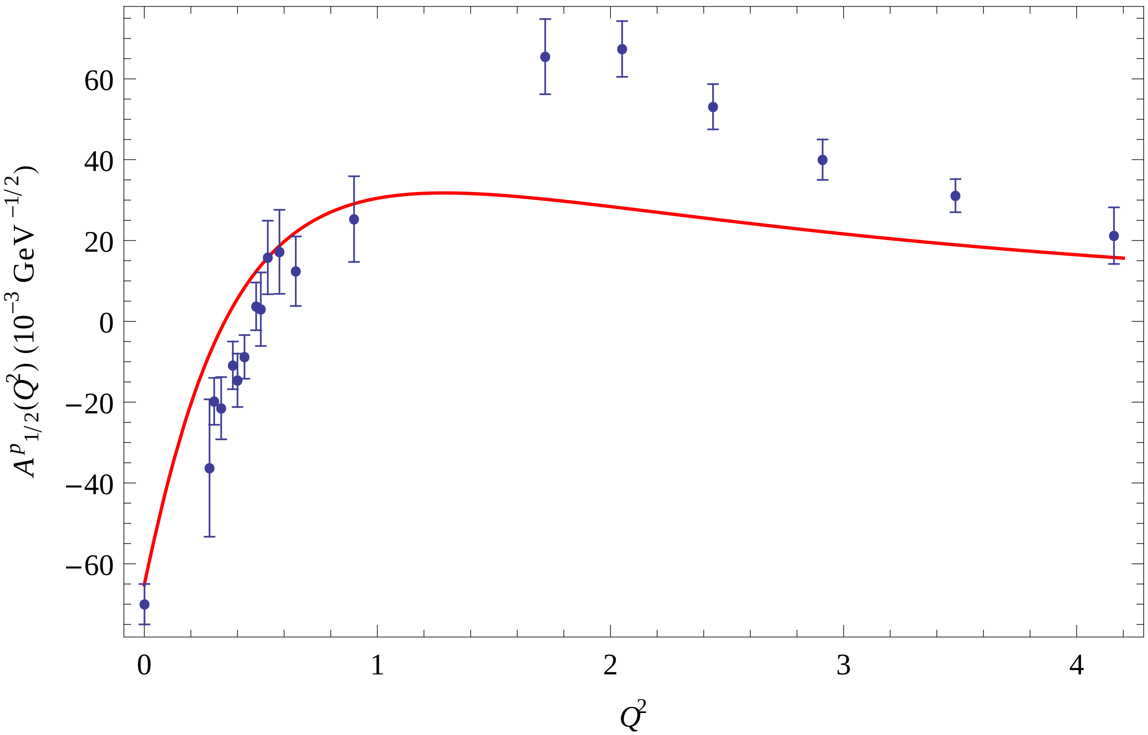
Roper resonance $N(1440)$

Helicity amplitudes $A^N_{1/2}(0)\,,\ S^N_{1/2}(0)$

Quantity	Our results	Data
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.040	

Roper resonance $N(1440)$

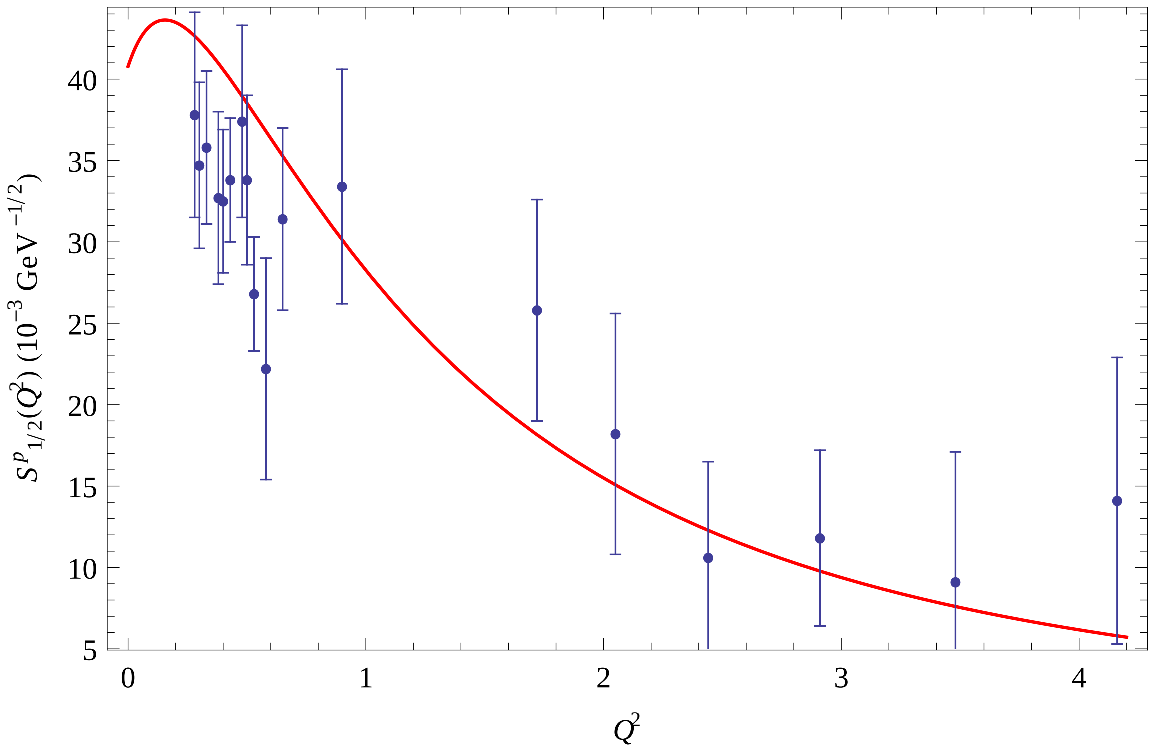
Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

Roper resonance $N(1440)$

Helicity amplitude $S^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states
- Future work: nucleon TMDs, DVCS