Nuclei on the Lattice: Chiral Effective Field Theory Approach

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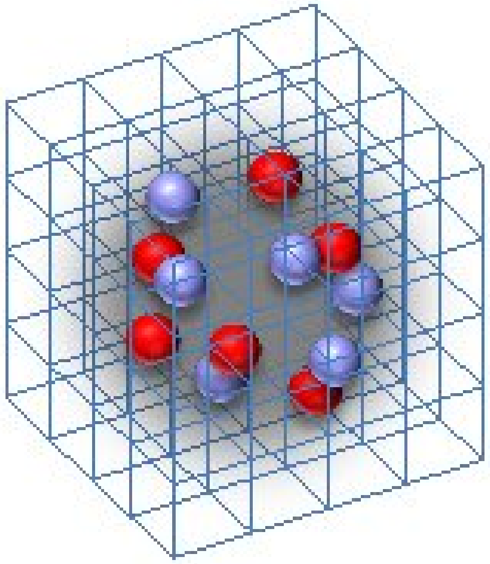
Satellite meeting in LC2013
Bethe-Salpeter equation and relations with Euclidean- and Minkowski-based approaches

May 25, 2013, Skiathos, Greece

- **N**uclear **L**attice **E**ffective **F**ield **T**heory collaboration



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Ulf-G. Meißner (Bonn/Jülich)

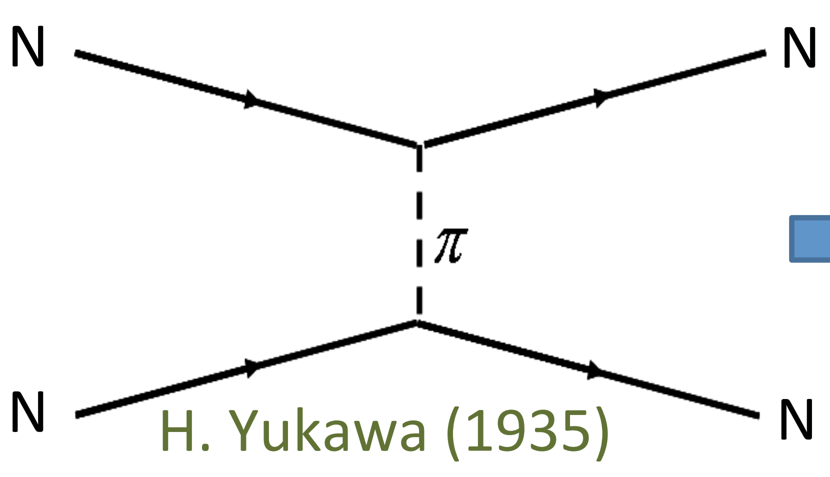
Gautam Rupak (Mississippi State)

Outline

- ChPT and low energy QCD
- Nuclear forces in chiral EFT
- Lattice EFT as an access to many-body physics
- Nucleon-Nucleon scattering within Lattice EFT
- Application to light nuclei
- Summary & Outlook

Nucleon-Nucleon forces

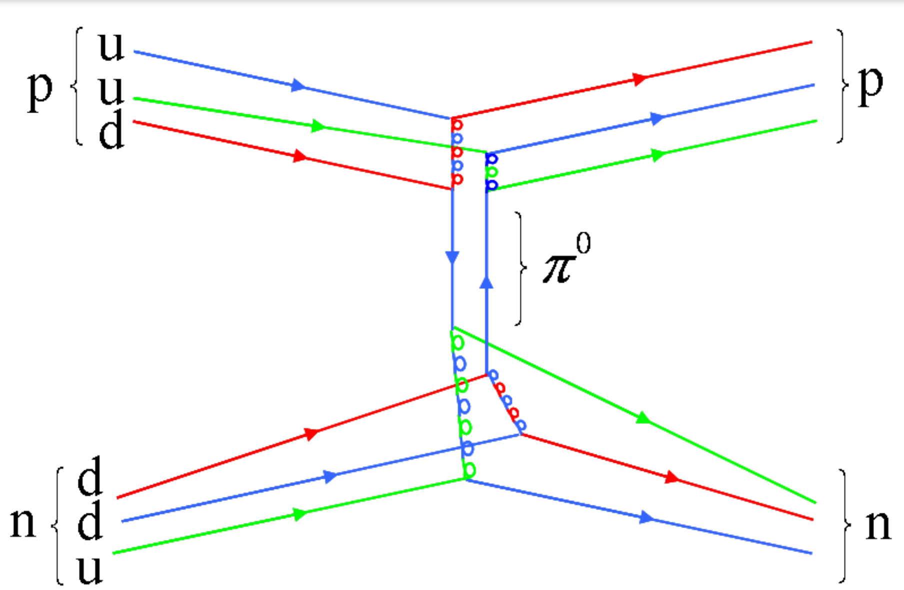
Phenomenological description by Meson-exchange



- Boson-Exchange Models as basis for NN-force
- Highly sophisticated phen. NN potentials
- Excellent description of many experimental data
- Connection to QCD is unclear

QCD Interpretation of NN forces

- NN force as residual strong interaction between hadrons



Chiral EFT Interpretation of NN forces

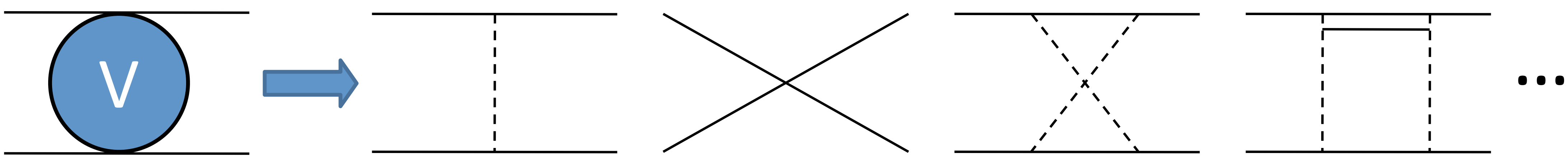
- Model independent treatment
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials
- Underlying QCD symmetries implemented by construction

Weinberg's scheme for NN

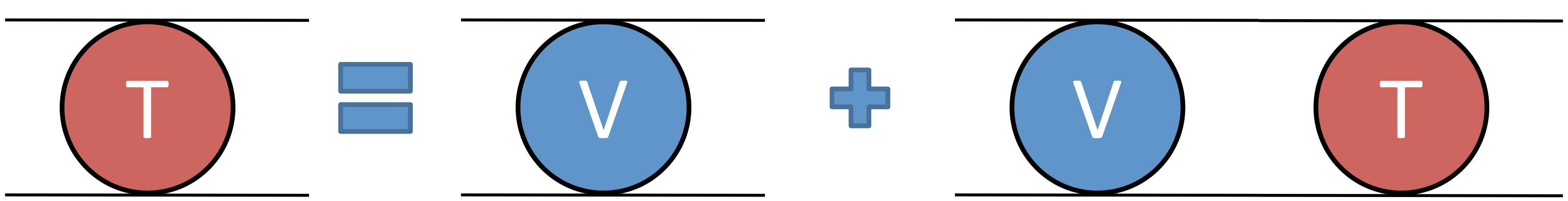
Weinberg, Nucl. Phys. B 363: 3 (1991)

- No perturbative description for bound states

- Construct effective potential perturbatively



- Solve Lippmann-Schwinger equation nonperturbatively



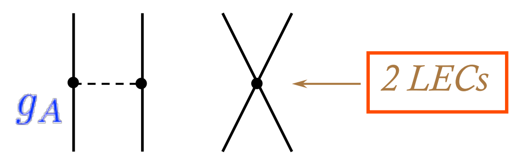
Nucleon-nucleon force up to N^{3}LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; ...

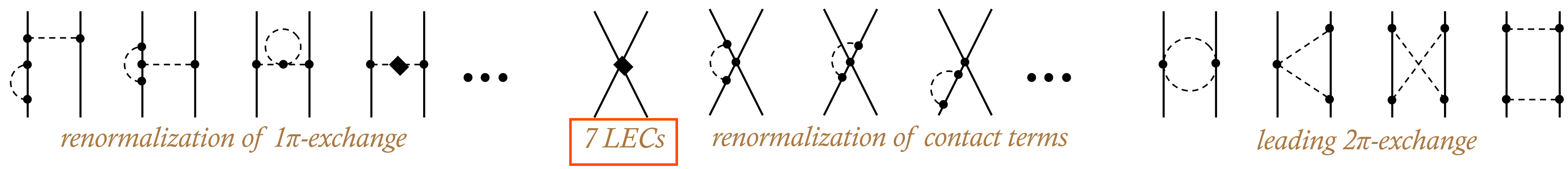
Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \dots$$

- **LO:**

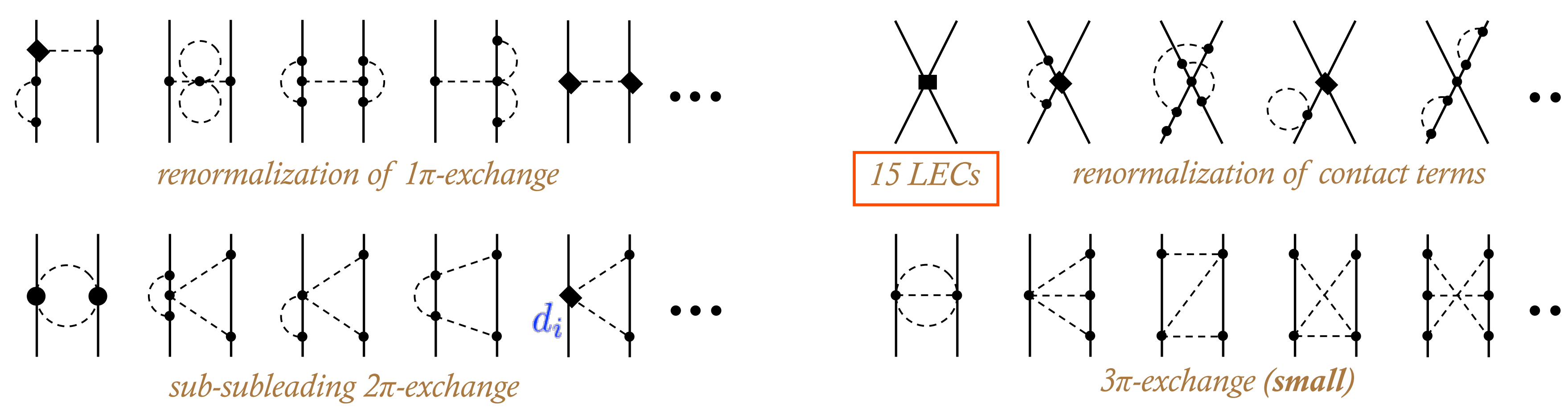


- **NLO:**



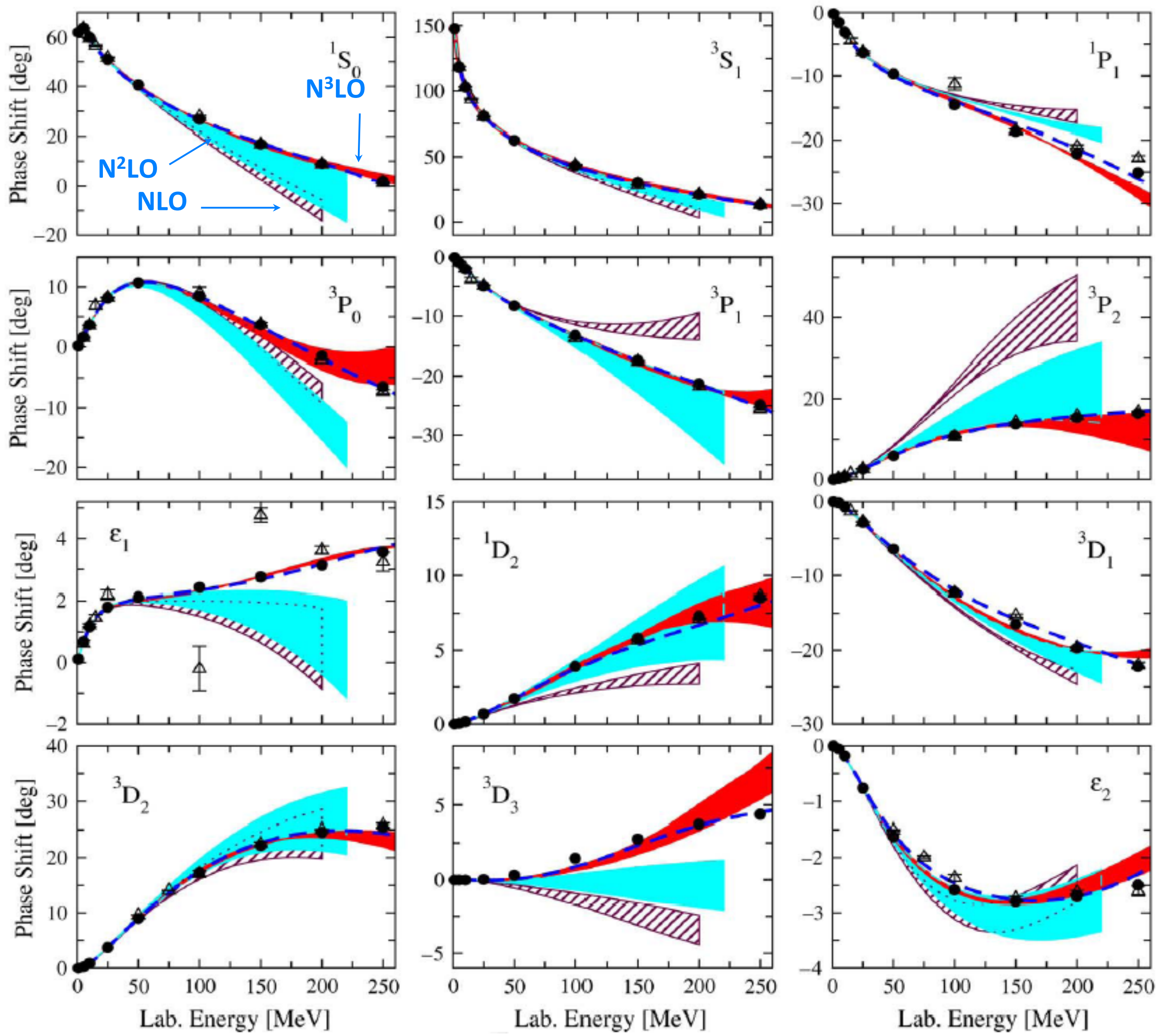
- **N^{2}LO:**

- **N^{3}LO:**

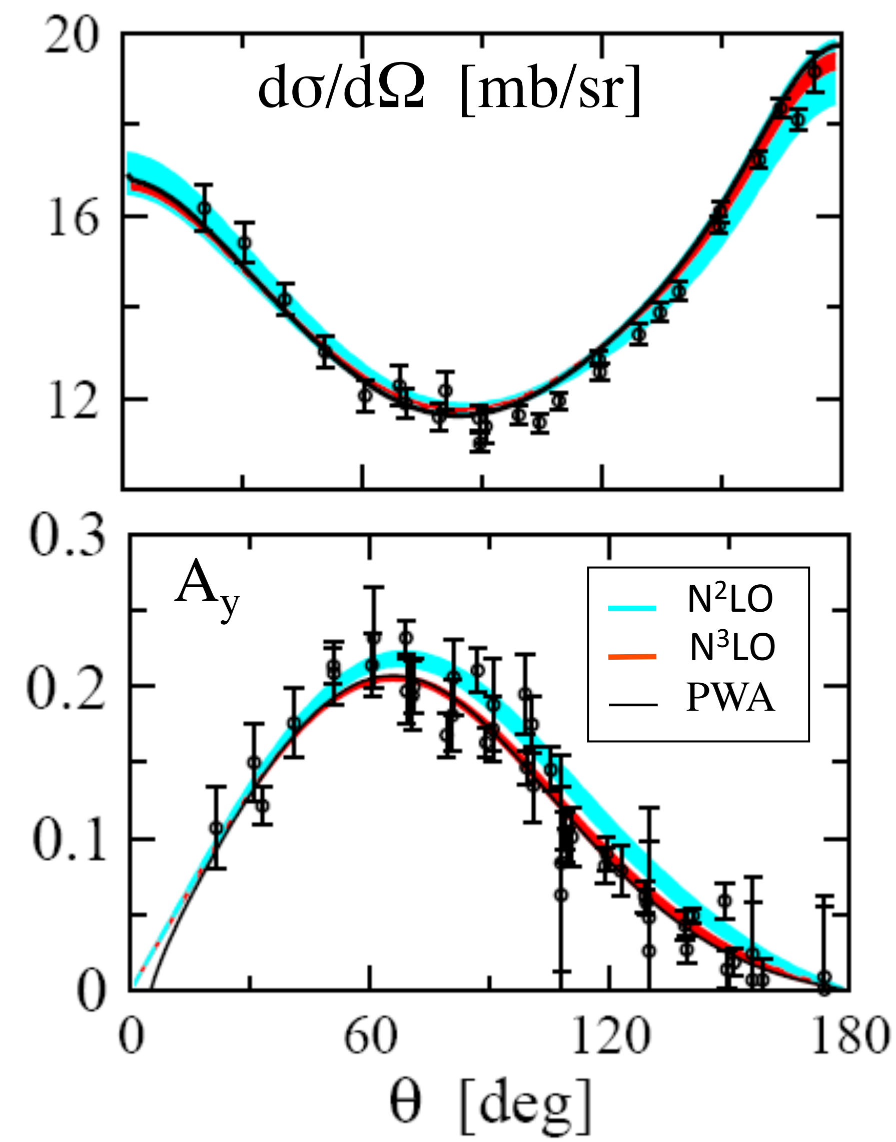


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N^3LO



np scattering at 50 MeV



Deuteron binding energy & asymptotic normalizations A_s and η_d

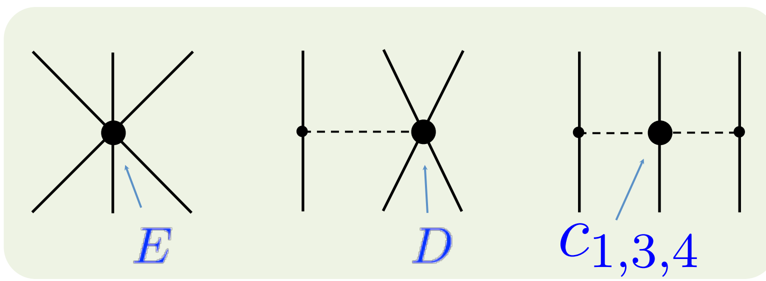
	NLO	N^2LO	N^3LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [$\text{fm}^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

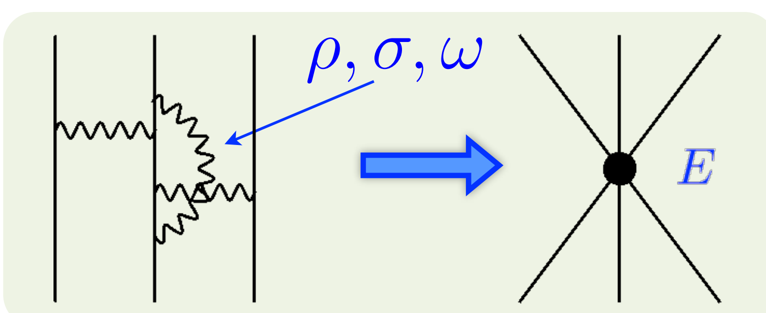
  (U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)



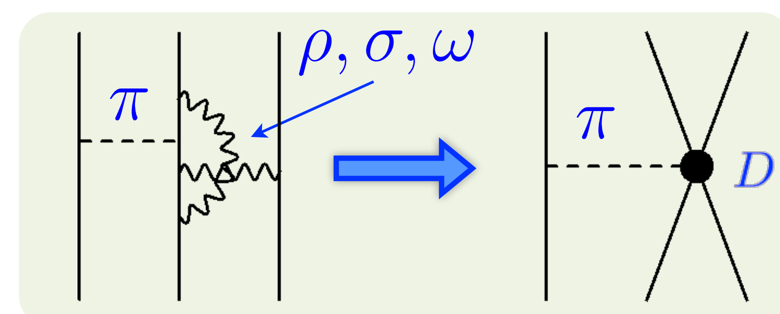
$c_{1,3,4}$ from the fit to πN-scattering data

D, E from ${}^3\mathrm{H}, {}^4\mathrm{He}, {}^{10}\mathrm{B}$ binding energy + coherent nd - scattering length

- LECs D and E incorporate short-range contr.

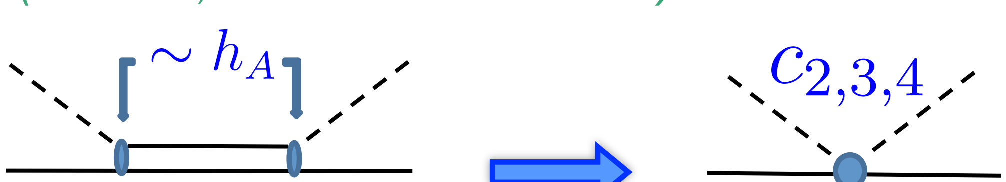


Resonance saturation interpretation of LECs



- Delta contributions encoded in LECs

  (Bernard, Kaiser & Meißner '97)

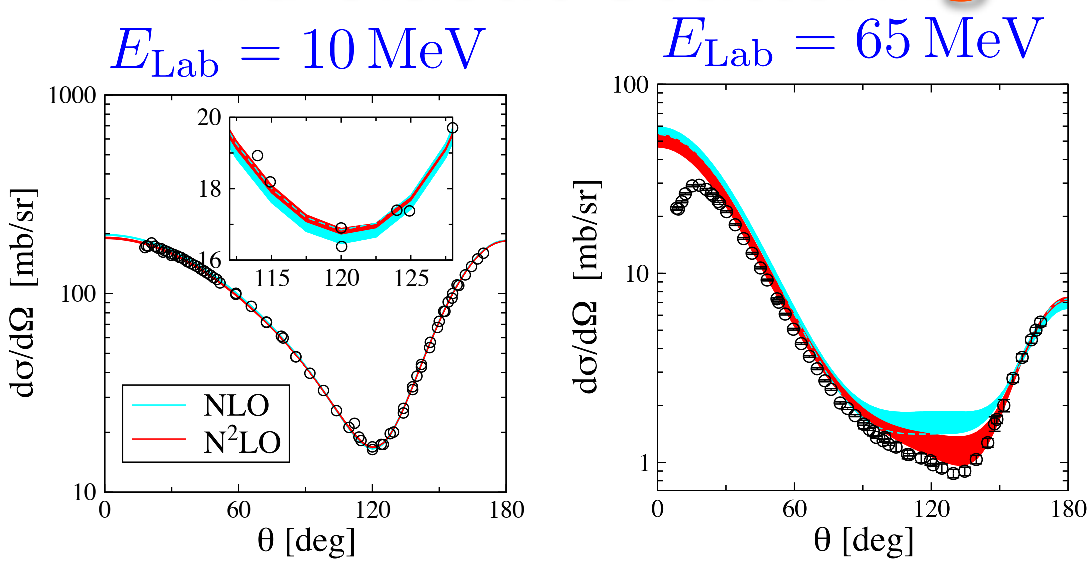


Delta-resonance saturation

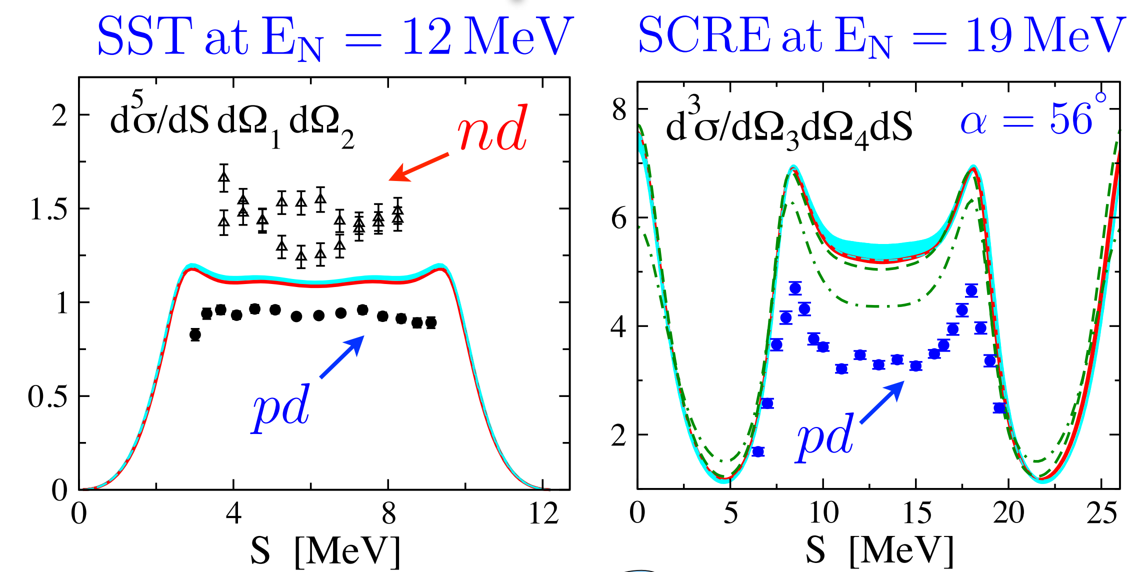
$$c_3 = -2c_4 = c_3(\not\Delta) - \frac{4h_A^2}{9\Delta}$$

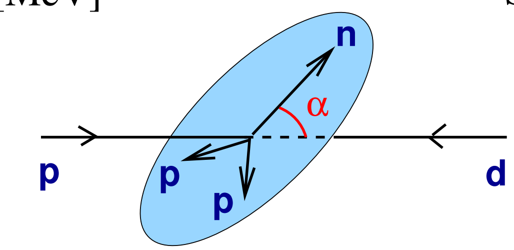
Enlargement due to Delta contribution

nd elastic scattering

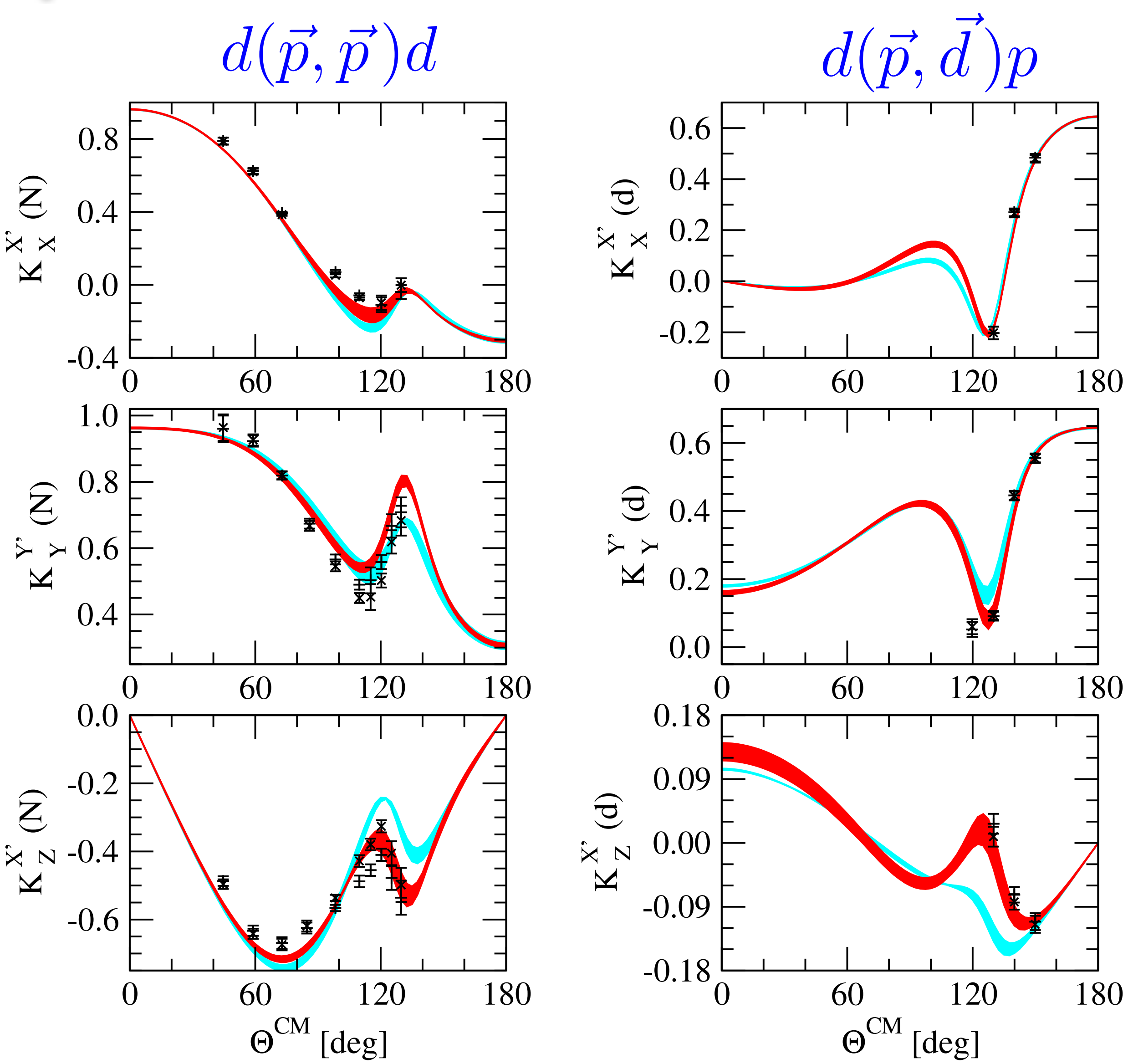


nd break-up $[\mathrm{mb\,MeV^{-1}sr^{-2}}]$





polarization transfer: $E_p^{\mathrm{Lab}} = 22.7\,\mathrm{MeV}$



For references see recent reviews:

Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773

Entem, Machleidt, Phys. Rept. 503 (11) 1

Epelbaum, Meißner, arXiv:1201.2136, to appear in Ann. Rev. Nucl. Part. Sci.

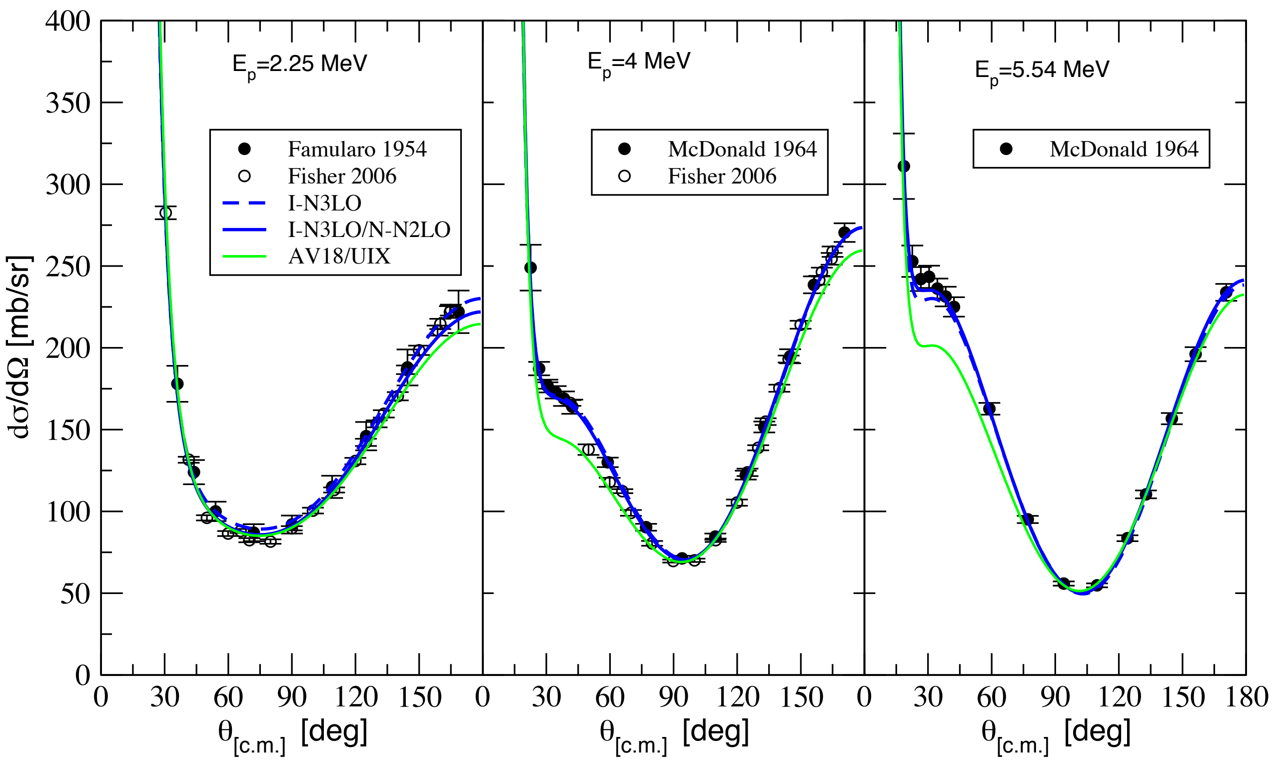
Kalantar et al. Rep. Prog. Phys. 75 (12) 016301

- Generally good description of data. But some discrepancies arise. E.g. break-up observables for SCRE/SST configuration at low energy
- Hope for improvement at N³LO

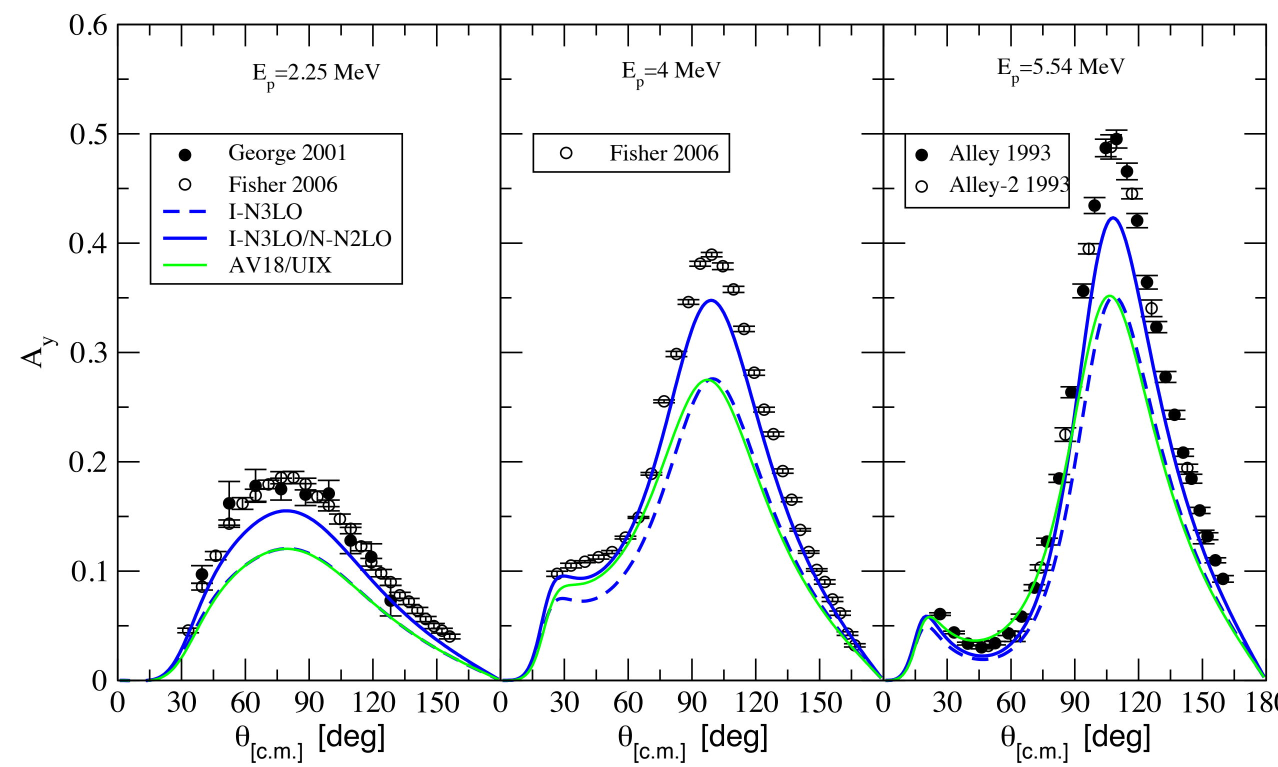
Proton-^{3}He elastic scattering

Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306

p-^{3}He differential cross section at low energies



proton vector analyzing power A_y-puzzle



As in n-d scattering case N^{2}LO 3NF's are not enough to resolve underprediction of A_y

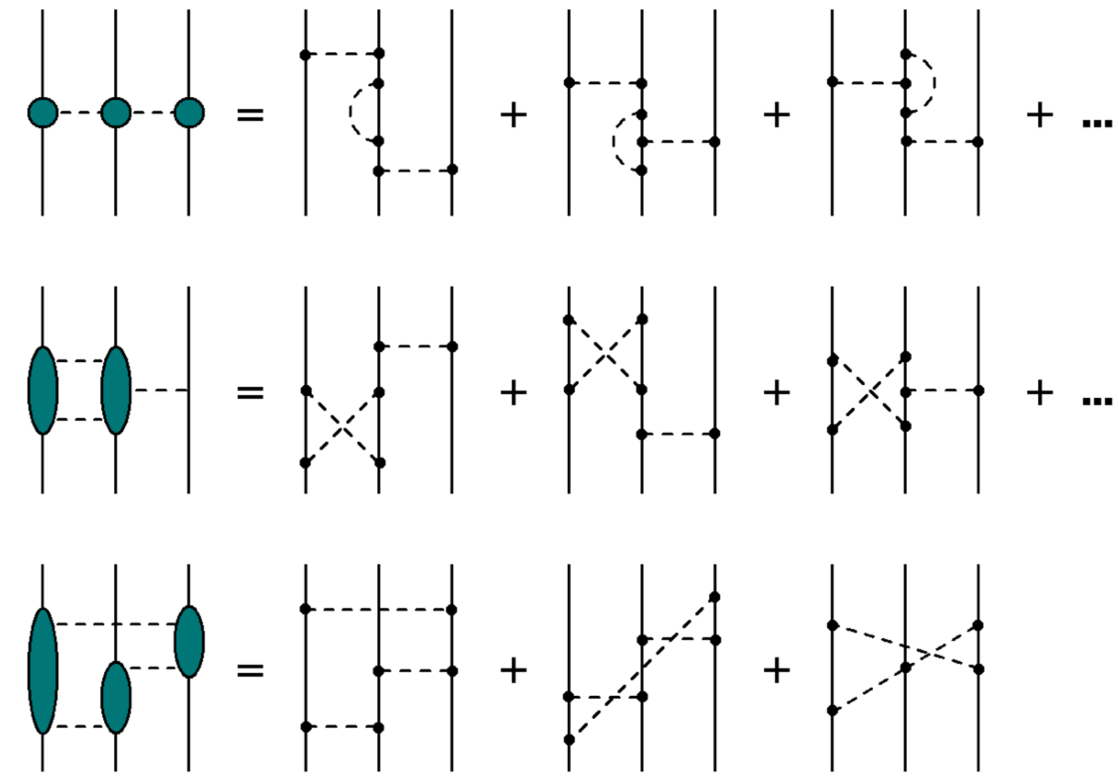
→ Hope for improvement at higher orders

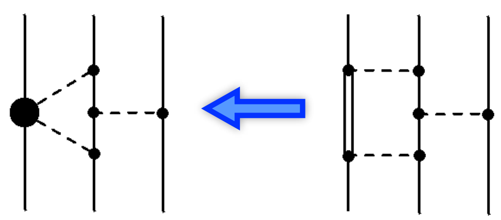
Three-nucleon forces

- Three-nucleon forces at N^{3}LO

Long range contributions *Bernard, Epelbaum, H.K., Meißner ´08; Ishikawa, Robilotta ´07*

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important

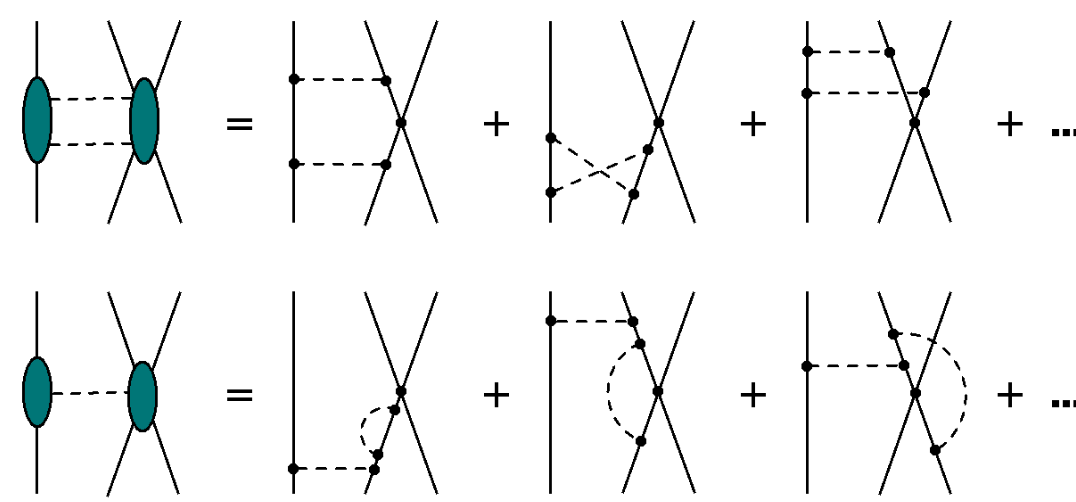


Large c_i $\sim$ 

Shorter range contributions *Bernard, Epelbaum, H.K., Meißner ´11*

- LECs needed for shorter range contr.

  g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF



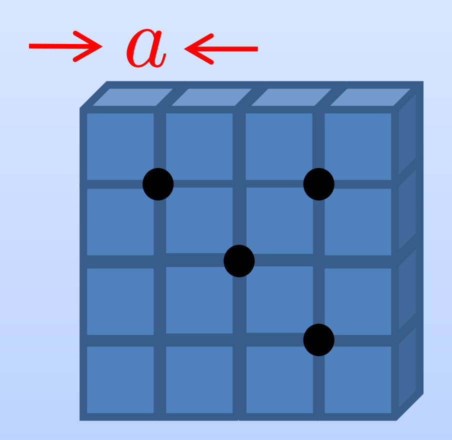
Nuclear lattice simulations

Borasoy, H.K., Lee, Meißner, Nucl.Phys. A768 (2006) 179

- Low-energy constants fixed from experimental data with $A \leq 3$
- All nuclear spectra for $A \geq 4$ pure prediction
- Explicit methods for $A \geq 4$ too complicated ⇨ use EFT on the Lattice

- Nucleons are represented as point-like Grassmann fields
- Point-like instantaneous pions to reproduce effective potential
- Typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\mathrm{MeV}(a \simeq 2\,\mathrm{fm})$$
$$L \simeq 20\,\mathrm{fm}$$



- Strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry
- Method: hybrid Monte Carlo & transfer matrix (similar to LQCD)

Transfer matrix method

- Correlator function for A nucleons $\quad Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

Slater Determinants for A free Nucleons

- Ground state energy by time derivative of the correlator

$$E(t) = -\frac{d}{dt} \ln Z_A(t)$$

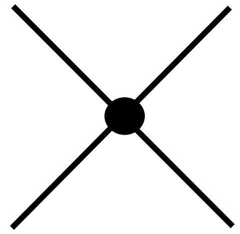
At large time only ground states survive $\Rightarrow$ $E_A^0 = \lim_{t\to\infty} E_A(t)$

- Expectation value of normal ordered operator $\mathcal{O}$:

$$Z_A^{\mathcal{O}}(t) = \langle \Psi_A | \exp(-tH/2) \mathcal{O} \exp(-tH/2) | \Psi_A \rangle$$

$$\lim_{t\to\infty} \frac{Z_A^{\mathcal{O}}(t)}{Z_A(t)} = \langle \Psi_A^0 | \mathcal{O} | \Psi_A^0 \rangle$$

Monte Carlo with auxiliary fields

- Contact interactions  can be represented by auxiliary fields

$$\exp(\rho^2/2) \sim \int_{-\infty}^{\infty} ds\, \exp(-s^2/2 - s\,\rho)$$

Hubbard-Stratonovich field

- Correlation function as path-integral over pions and auxiliary fields

$$Z_A(t) \propto \int_{-\infty}^{\infty} \mathcal{D}s \prod_{I=1,2,3} \mathcal{D}s_I \mathcal{D}\pi_I \exp(-S_{\pi\pi} - S_{ss}) \det \mathcal{M}(\pi_I, s, s_I)$$

Slater-determinant of single nucleon matrix elements

- Single-nucleon matrix elements

L_t -th temporal lattice step

$$\mathcal{M}_{i,j}(\pi_I, s, s_I) = \langle \psi_{i,X} | M_X^{(L_t-1)} \cdots M_X^{(0)} | \psi_{j,X} \rangle$$

Free nucleons and pions

- Positive definite free action: $\alpha_t = a_t/a$

$$S_{ss}(s, s_I) = \frac{1}{2}\sum_{\vec{n}} s(\vec{n})^2 + \frac{1}{2}\sum_{I=1,2,3}\sum_{\vec{n}} s_I(\vec{n})^2$$

Auxiliary field contributions

$$S_{\pi\pi}(\pi_I) = \frac{\alpha_t}{2}\sum_{I=1,2,3}\sum_{\vec{n}} \pi_I(\vec{n})(-\Delta + M_\pi^2)\pi_I(\vec{n})$$

Free instantaneous pions

- $O(a^4)$-improved free nucleon lattice Hamiltonian $\quad f_{0,1,2,3} = \frac{49}{2}, -\frac{3}{4}, \frac{3}{40}, -\frac{1}{180}$

$$H_{\rm free} = \frac{1}{m}\sum_{k=0}^{3}\sum_{\vec{n}_s, l_s, i, j} f_k \left[a^\dagger_{i,j}(\vec{n}_s)\left[a_{i,j}(\vec{n}_s + k\,\hat{l}_s) + a_{i,j}(\vec{n}_s - k\,\hat{l}_s)\right]\right]$$

- Nucleon density operators with different spin-isospin polarizations

$$\rho^{a^\dagger, a}(\vec{n}_s) = \sum_{i,j} a^\dagger_{i,j}(\vec{n}_s)a_{i,j}(\vec{n}_s) \qquad \rho_I^{a^\dagger, a}(\vec{n}_s) = \sum_{i,j,j'} a^\dagger_{i,j}(\vec{n}_s)[\tau_I]_{j,j'}a_{i,j'}(\vec{n}_s)$$

$$\rho_{S,I}^{a^\dagger, a}(\vec{n}_s) = \sum_{i,i',j,j'} a^\dagger_{i,j}(\vec{n}_s)[\sigma_S]_{i,i'}[\tau_I]_{j,j'}a_{i',j'}(\vec{n}_s)$$

Leading order interactions

- Transfer matrix from n_t - th step in temporal direction $C < 0, C_I > 0$

Small sign oscillation from pion-nucleon vertex

$$M^{(n_t)} = :\exp\left[-H_{\rm free}\alpha_t - \frac{g_A\alpha_t}{2F_\pi}\sum_{S,I}\nabla_S\pi_I(\vec{n}_s,n_t)\rho^{a^\dagger,a}_{S,I}(\vec{n}_s) \right.$$
$$\left. +\sqrt{-C\alpha_t}\sum_{\vec{n}_s}s(\vec{n}_s,n_t)\rho^{a^\dagger,a}(\vec{n}_s) + i\sqrt{C_I\alpha_t}\sum_{I}\sum_{\vec{n}_s}s_I(\vec{n}_s,n_t)\rho^{a^\dagger,a}_I(\vec{n}_s)\right]:$$

No sign oscillation for contact interactions if the number of protons and neutrons are equal

- Real determinant in the pion-less case $\tau_2\mathcal{M}\tau_2 = \mathcal{M}^* \Longrightarrow \det\mathcal{M}^* = \det\mathcal{M}$

Lee, Phys. Rev. C70: 064002 (2004)

Antisymmetry of τ_2 $\Longrightarrow$ real eigenvalues of $\mathcal{M}$ are doubly degenerate

$$\det\mathcal{M} \geq 0$$

Approximate SU(4) symmetry

- Wigner spin-isospin SU(4) symmetry transformation:

Wigner, Phys. Rev. 51 (1937) 106

$$\delta N = \alpha_{\mu\nu}\sigma^{\mu}\tau^{\nu} N \quad \text{with} \quad \sigma^{\mu} = (1, \vec{\sigma}), \tau^{\mu} = (1, \vec{\tau})$$

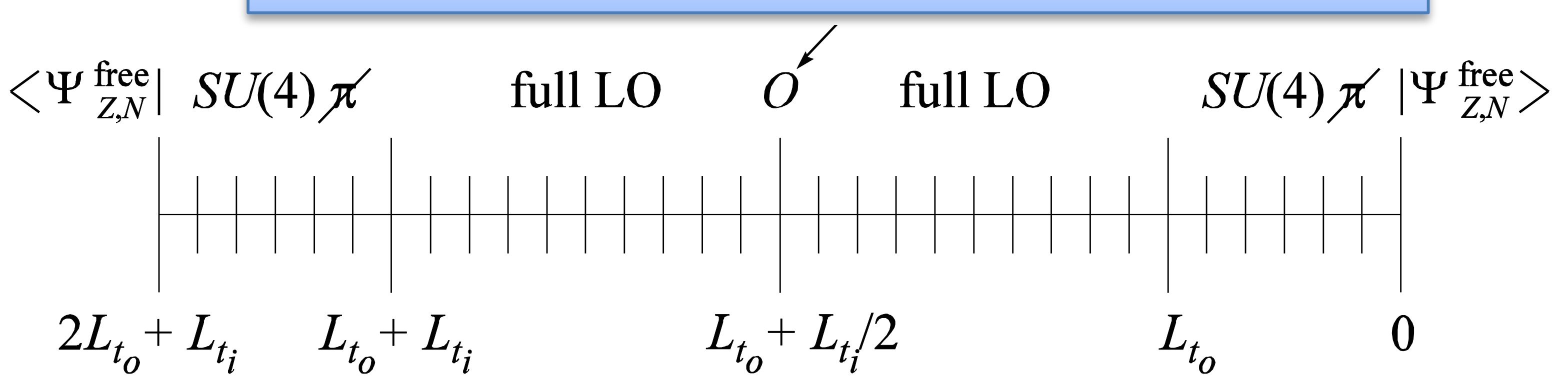
SU(4) invariance in the limit of infinite $a(^1S_0)$ and $a(^3S_1)$ scattering length
SU(4)-breaking terms $\sim 1/a(^3S_1) - 1/a(^1S_0), q/\Lambda_\chi$

Mehen, Stewart, Wise, Phys. Rev. Lett. 83 (1999) 931

- Large NN scattering length ⟹ approximate SU(4) symmetry

$$M_{\mathrm{SU(4)}}^{(n_t)} = : \exp\left[-H_{\mathrm{free}}\alpha_t + \sqrt{-C\alpha_t}\sum_{\vec{n}_s}[s(\vec{n}_s, n_t)\rho^{a^\dagger,a}(\vec{n}_s)\right] :$$

Operator insertion for expectation value



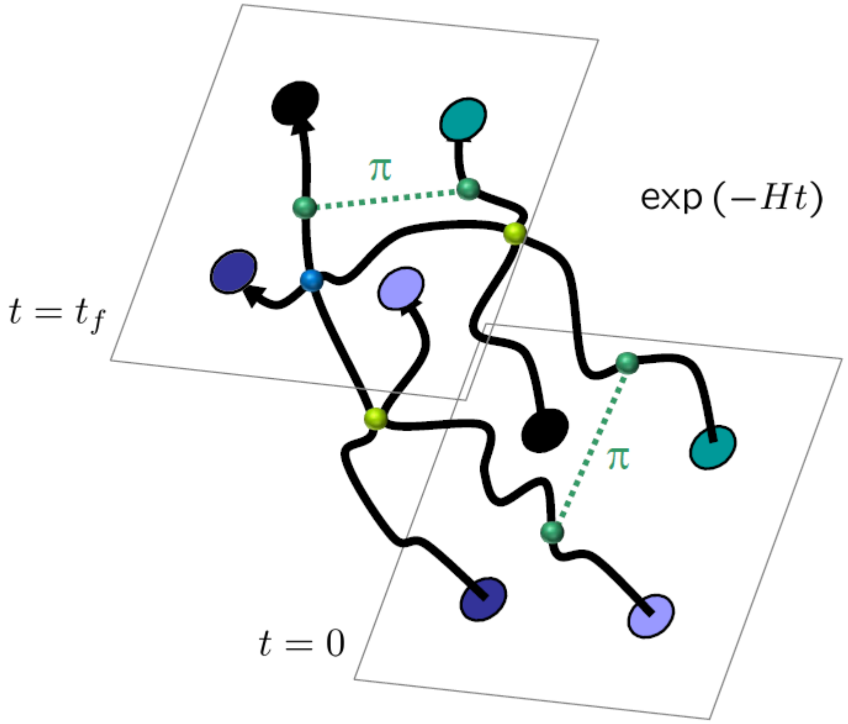
- Pionless SU(4)-symmetric simulations are cheaper
- Decrease computational effort by using SU(4)-filter

Hybrid Monte Carlo

- Introduce conjugate fields p_{π_I}, p_s, p_{s_I}

$$H_{\rm HMC} = \frac{1}{2}\sum_{I,\vec{n}}(p^2_{\pi_I}(\vec{n}) + p^2_s(\vec{n}) + p^2_{s_I}(\vec{n})) + V(\pi_I, s, s_I)$$

$$V(\pi_I, s, s_I) = S_{\pi\pi} + S_{ss} - \log\{|\det \mathcal{M}|\}$$



Repeat the steps many times

Generate new configurations for $p_{\pi_I}, p_s, p_{s_I}, \pi_I, s, s_I$ by molecular dynamics trajectories

Apply Metropolis accept or reject step for the new configuration according to probability $\exp(-H_{\rm HMC})$

Scattering from finite volume

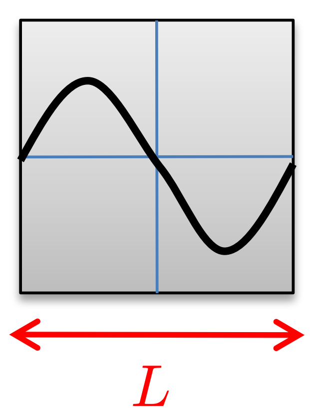
- Rotation group $SO(3) \longrightarrow SO(3,Z)$

Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0,1,3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0,2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1,2,3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$

Every irreducible repr. includes definite $J \bmod 4$ quantum numbers

$$Z_{0,0}(s,q^2) = \sqrt{1/4\pi} \sum_{n \in Z^3} \frac{1}{(n^2-q^2)^s}$$

- Scattering from finite volume



Large L

$$\exp(2\,i\,\delta_0) = \frac{Z_{0,0}(1;q^2) + i\,\pi^{3/2} q}{Z_{0,0}(1;q^2) - i\,\pi^{3/2} q}$$

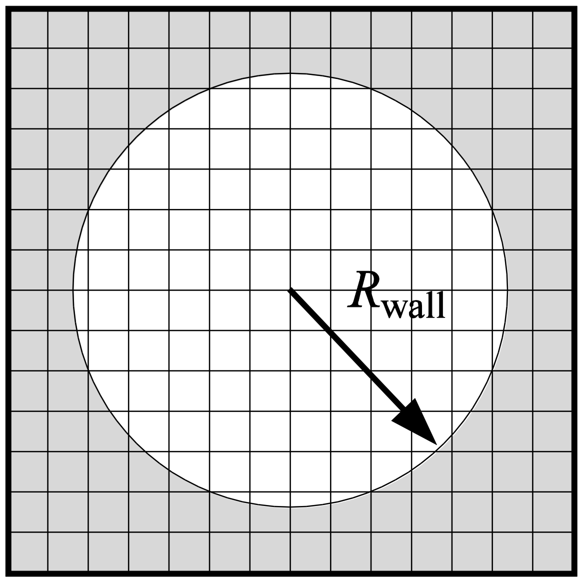
Lüscher formula for phase-shifts

- Note: No extension to mixing angles available

Spherical wall method

Borasoy, Epelbaum, H.K.,Lee, Meißner, Eur.Phys.J.A34 (2007) 185

- Spherical wall imposed in the center-of-mass frame



$$\Psi(\vec{r}) = [\cos\delta_L j_L(kr) - \sin\delta_L y_L(kr)] Y_{L,m}(\theta,\phi)$$

$$\Psi(\vec{R}_{\text{Wall}}) = 0$$

$$\tan\delta_L = \frac{j_L(kR_{\text{Wall}})}{y_L(kR_{\text{Wall}})}$$

Similar for Spin-triplet case

- Spherical wall removes copies of interactions due to periodic boundaries
- Energy spectrum by solving Schrödinger Eq. on the lattice (Lanczos method)
- Illustration for a toy-model: $C = -2\,\text{MeV}, R_0 = 2\times 10^{-2}\,\text{MeV}^{-1}$

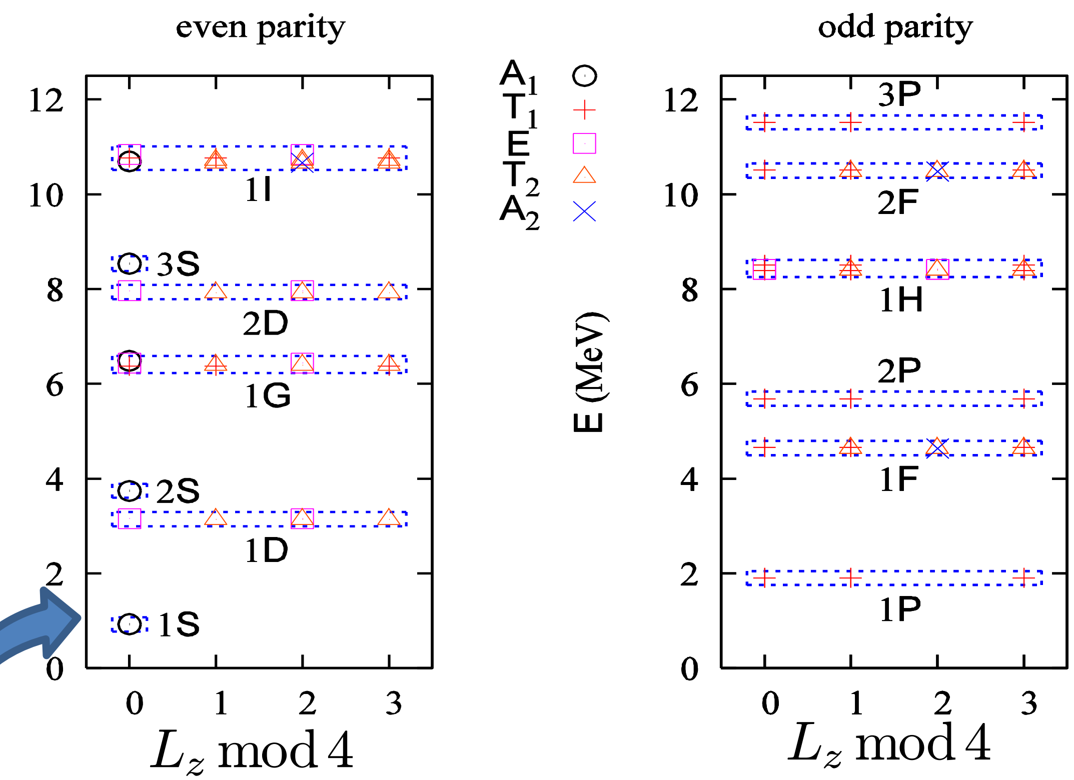
$$V(\vec{r}) = C\left\{1 + \frac{r^2}{R_0^2}[3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2]\right\}\exp\left(-\frac{1}{2}\frac{r^2}{R_0^2}\right)$$

Very shallow bound state in the ${}^3S(D)_1$ channel with energy $-0.155\,\text{MeV}$

Illustration for the toy model

Borasoy, Epelbaum, H.K.,Lee, Meißner, Eur.Phys.J.A34 (2007) 185

Free particle spectrum for $R_{\text{Wall}} = 10 + \epsilon$

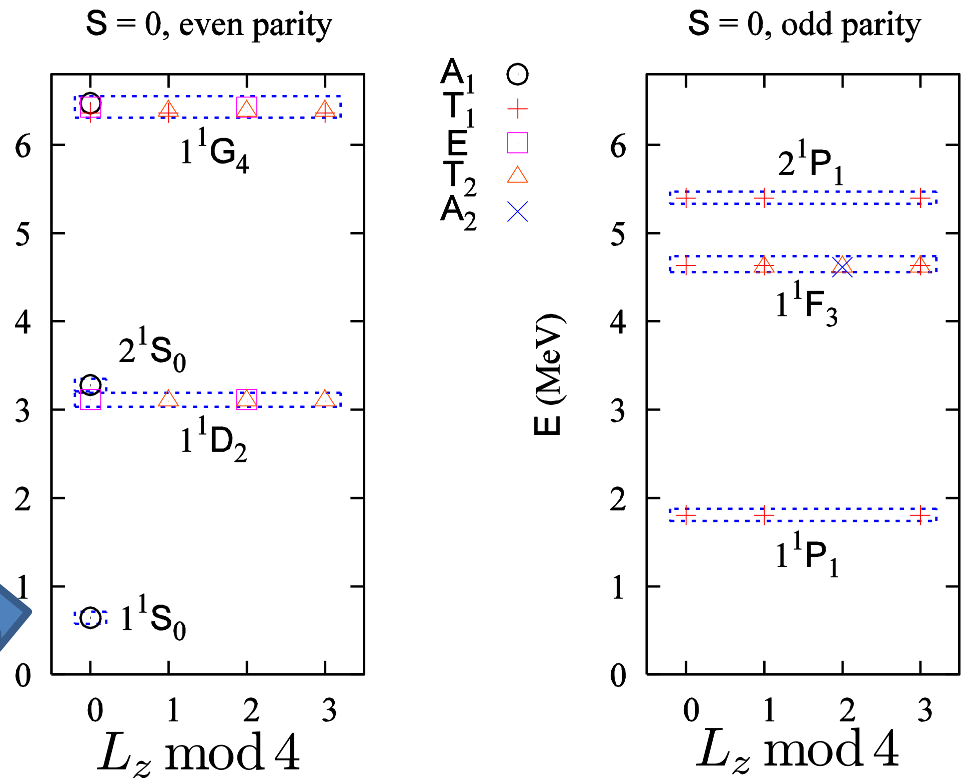


1^1S_0 energy $= 0.9280\,\text{MeV}$

$$k_{\text{free}} = 29.52\,\text{MeV}, j_0(k_{\text{free}}R_{\text{Wall}}) = 0$$

$$R_{\text{Wall}} = \frac{\pi}{k_{\text{free}}} = 0.1064\,\text{MeV}^{-1}$$

Interacting spectrum for $S = 0$



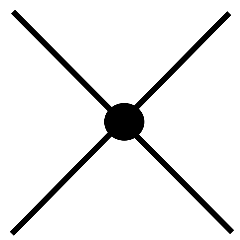
1^1S_0 energy $= 0.6445\,\text{MeV}$

$$k = 24.60\,\text{MeV}$$

$$\delta(^1S_0) = \tan^{-1}\left[\frac{j_0(kR_{\text{Wall}})}{y_0(kR_{\text{Wall}})}\right] = 30.0^o$$

Chiral EFT at NLO: Mixing angles

Borasoy, Epelbaum, H.K., Lee, Meißner, Eur. Phys. J. A35 (2008) 343



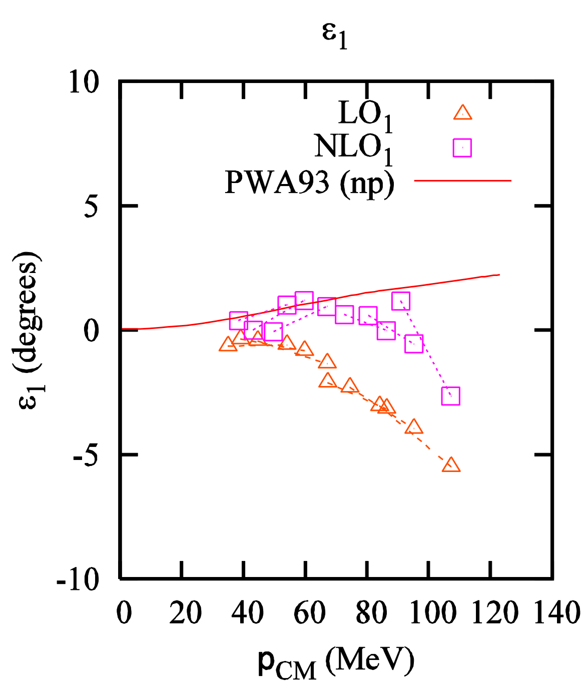
9 NLO LECs fitted to the various S(D) and P(F)-wave phase-shifts
+ Quadrupole moment of the deuteron

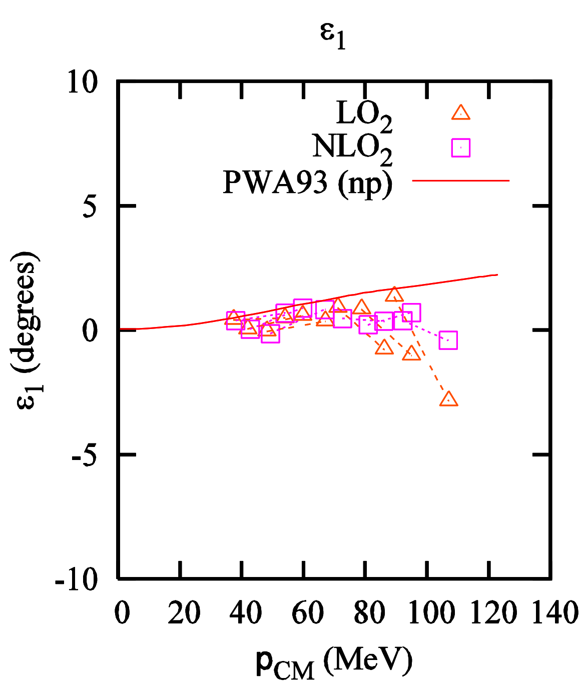
- NLO results with different actions ⟹ estimate errors

Two different LO contact interactions in momentum space

$$\mathcal{L}_C^{\mathrm{LO}_{1,2}} \sim \sum_{\vec{q}} f_{1,2}(\vec{q}) \left[C N^\dagger(\vec{q}) N(\vec{q}) N^\dagger(-\vec{q}) N(-\vec{q}) + C_I N^\dagger(\vec{q}) \vec{\tau} N(\vec{q}) \cdot N^\dagger(-\vec{q}) \vec{\tau} N(-\vec{q}) \right]$$

- $f_1(\vec{q}) = 1$ ⟹ original repr.
- $f_2(\vec{q}) \sim \exp(-b \sum_{l=1,2,3} (1 - \cos q_l))$ ⟹ Gaussian smearing





- Fairly accurate description for momenta $\leq M_\pi$
- Mixing angle changes sign by LO ⟹ NLO
- Deviations appear consistent with higher-order effects

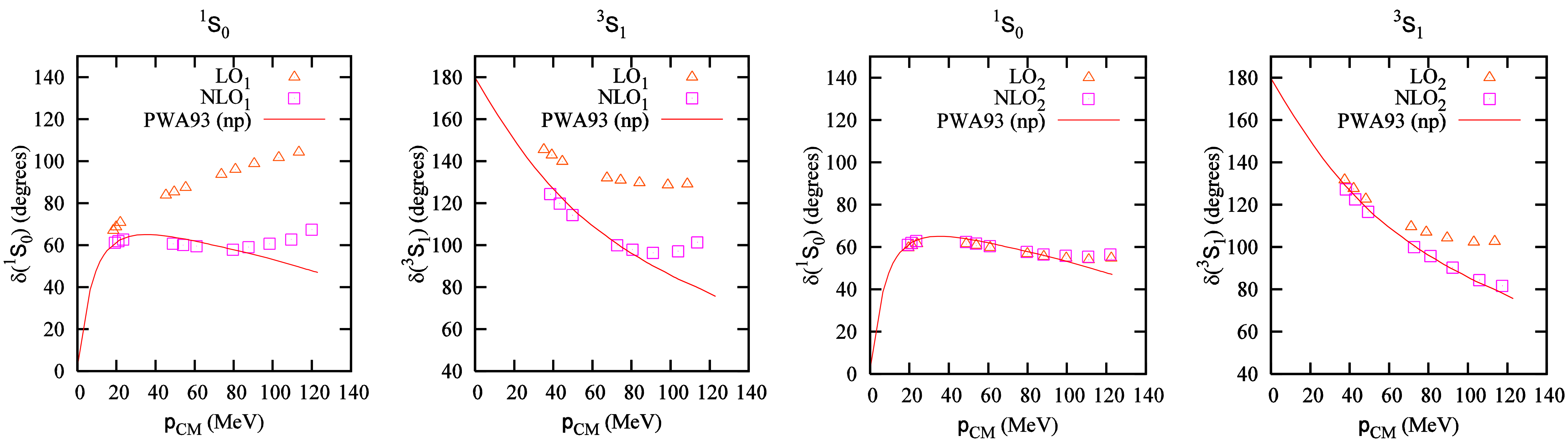
S-wave phase shifts

Two different LO contact interactions in momentum space

$$\mathcal{L}_C^{\mathrm{LO}_{1,2}} \sim \sum_{\vec{q}} f_{1,2}(\vec{q}) \left[C N^\dagger(\vec{q}) N(\vec{q}) N^\dagger(-\vec{q}) N(-\vec{q}) + C_I N^\dagger(\vec{q}) \vec{\tau} N(\vec{q}) \cdot N^\dagger(-\vec{q}) \vec{\tau} N(-\vec{q}) \right]$$

- $f_1(\vec{q}) = 1$ ➡ original repr.
- $f_2(\vec{q}) \sim \exp(-b \sum_{l=1,2,3} (1 - \cos q_l))$ ➡ Gaussian smearing

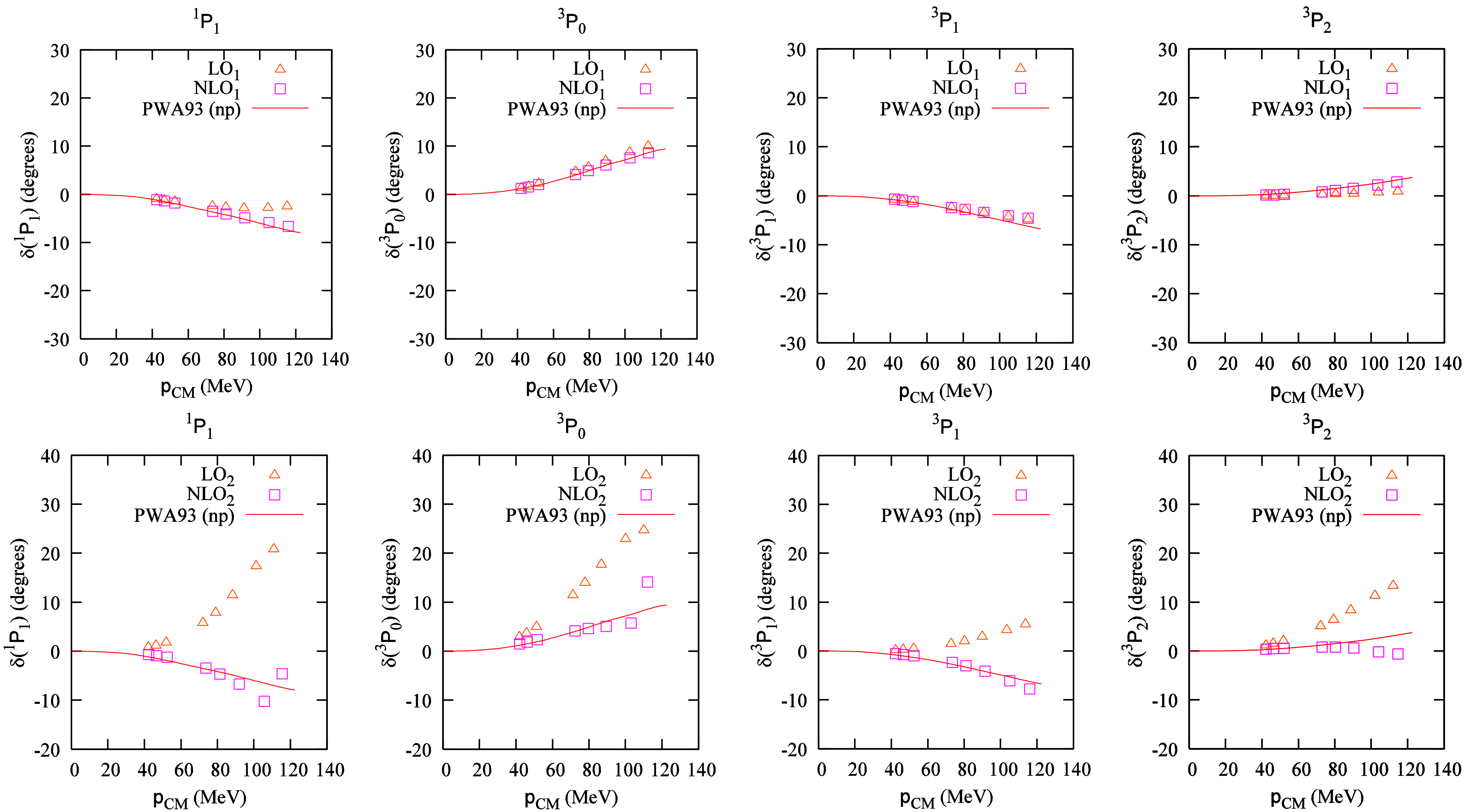
- S-wave phase shifts with different actions



- Accurate NLO-description in both cases for momenta $\leq 80\,\mathrm{MeV}$
- Better convergence with smeared action

P-wave phase shifts

- P-wave phase shifts with different actions



- **Better convergence with non-smeared action**

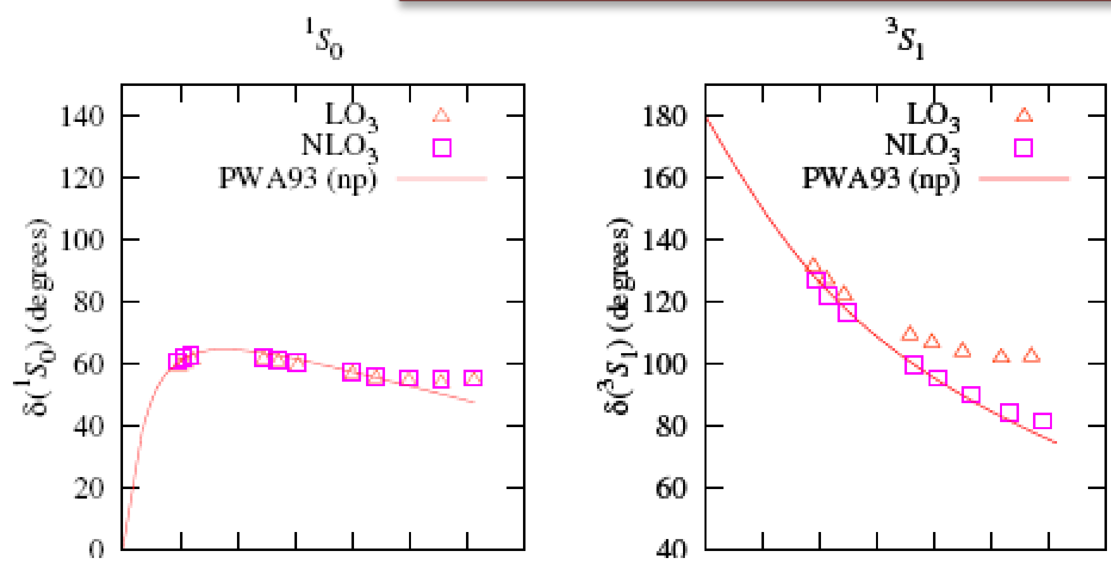
Improving convergence

- Project the smearing out of the P-waves

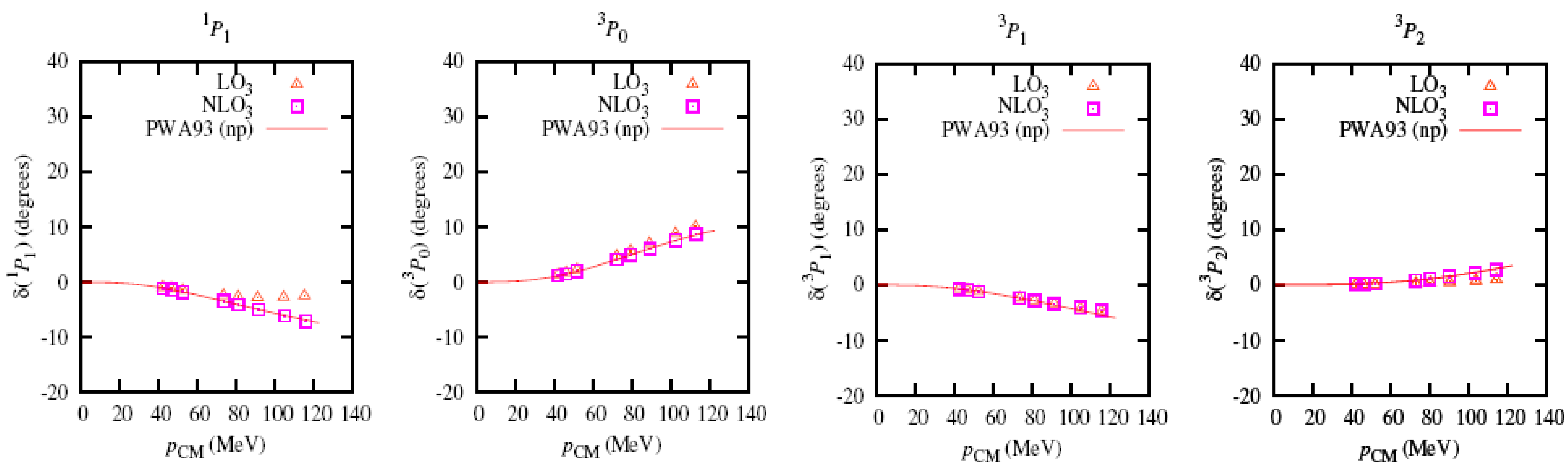
$$V_{\mathrm{LO}_3} = C_{^1S_0} f(\vec{q}) \left(\frac{1}{4} - \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\left(\frac{3}{4} + \frac{1}{4}\vec{\tau}_1 \cdot \vec{\tau}_2\right) + C_{^3S_1} f(\vec{q}) \left(\frac{3}{4} + \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\left(\frac{1}{4} - \frac{1}{4}\vec{\tau}_1 \cdot \vec{\tau}_2\right) + V_{\mathrm{OPE}}$$

Spin 0/Isospin 1 projector

Spin 1/Isospin 0 projector

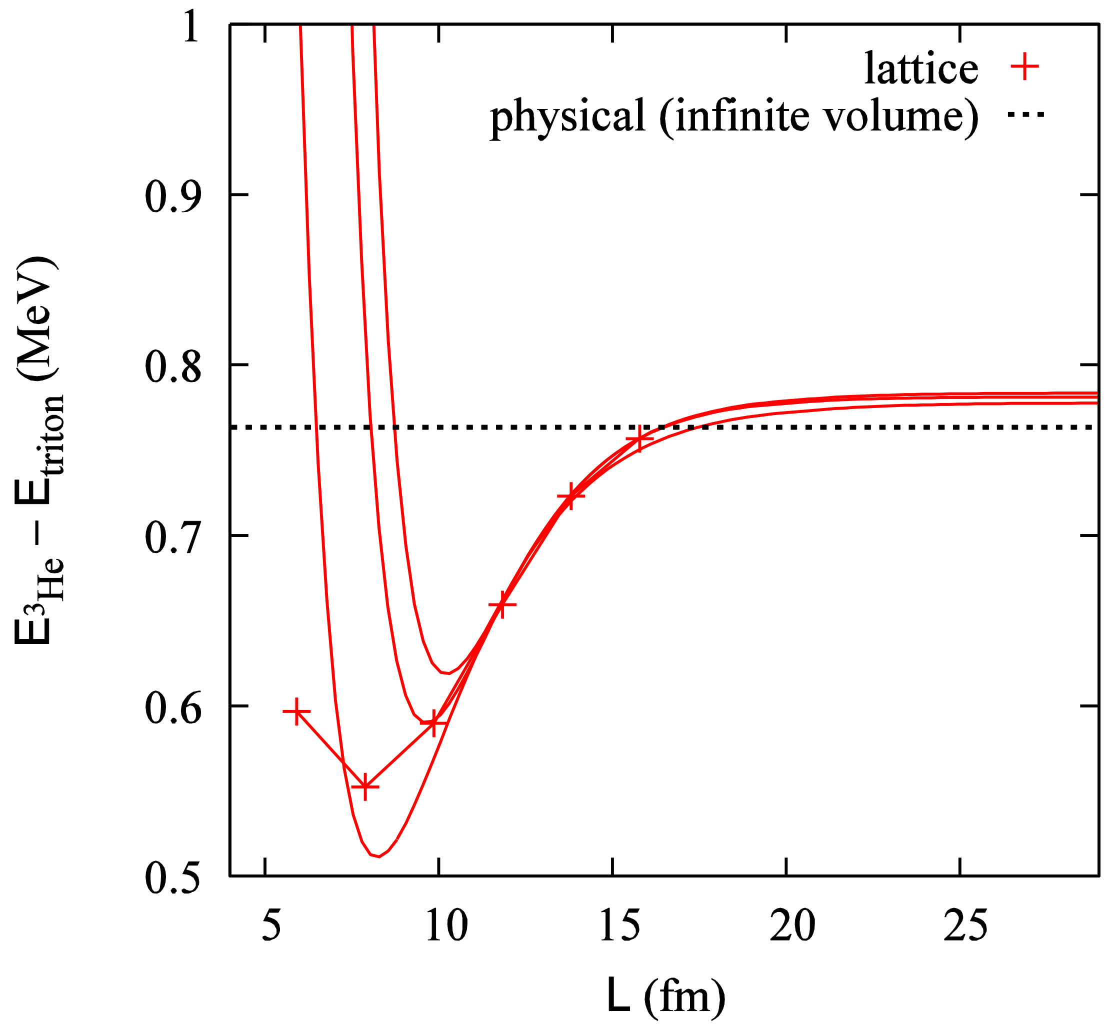


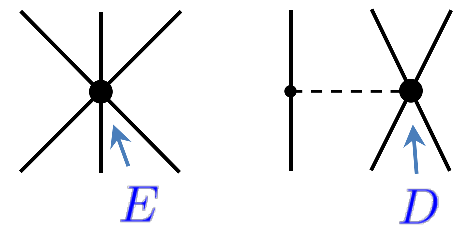
- Fast convergence in all partial waves
  - NLO can be treated perturbatively
- Small systematic errors at NLO



Triton and Helium-3

- Binding energy difference between Triton and Helium-3 is a testable prediction





$$D = \frac{C_D}{F_\pi^2 \Lambda_\chi}, E = \frac{C_E}{F_\pi^4 \Lambda_\chi}$$

- C_E as a function of C_D fitted to Triton binding energy $-8.48\,\mathrm{MeV}$
- C_D loosely constrained by spin-doublet nucleon-deuteron phase shift:
- We determine C_D from Triton β-decay

Different asymptotic fits with different subsets of data points: Lüscher '86

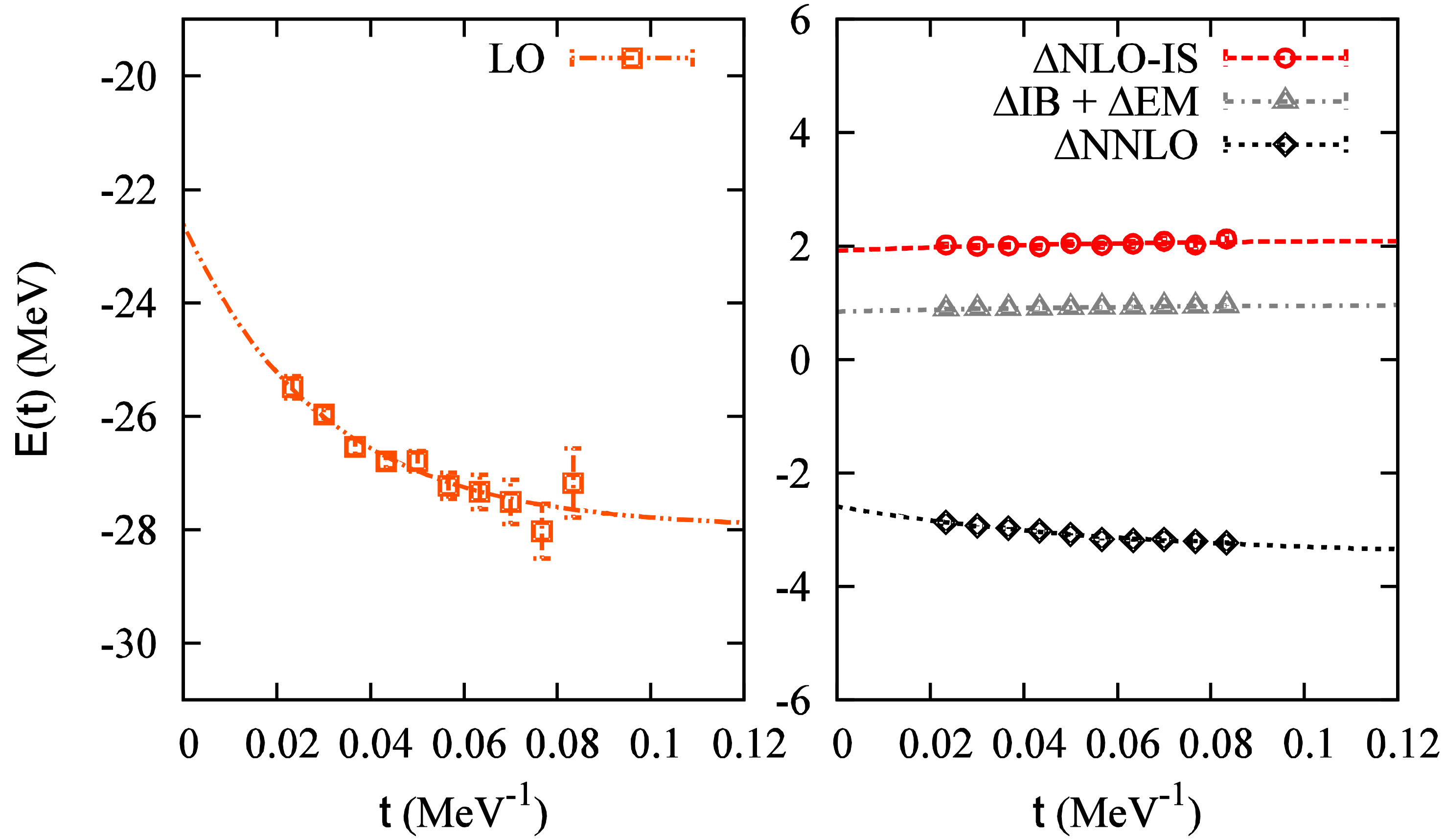
$$E(L) = E(\infty) - \frac{C}{L} e^{-L/L_0} + O\left(e^{-\sqrt{2}L/L_0}\right)$$

Infinite volume extrapolation	Experimental value
$0.78(5)\,\mathrm{MeV}$	$0.76\,\mathrm{MeV}$

Results for Helium-4

Epelbaum, H.K. Lee, Meißner: Phys. Rev. Lett. 106 (2011) 192501,
Phys. Rev. Lett 104 (2010) 142501, Eur. Phys. J. A45 (2010) 335

- Ground state energy of Helium-4 in a box of length $L = 11.8\,\mathrm{fm}$



LO	+ NLO + IB + EM	+ NNLO	Exp. Value
$-28.0(3)\,\mathrm{MeV}$	$-24.9(5)\,\mathrm{MeV}$	$-28.3(6)\,\mathrm{MeV}$	$-28.3\,\mathrm{MeV}$

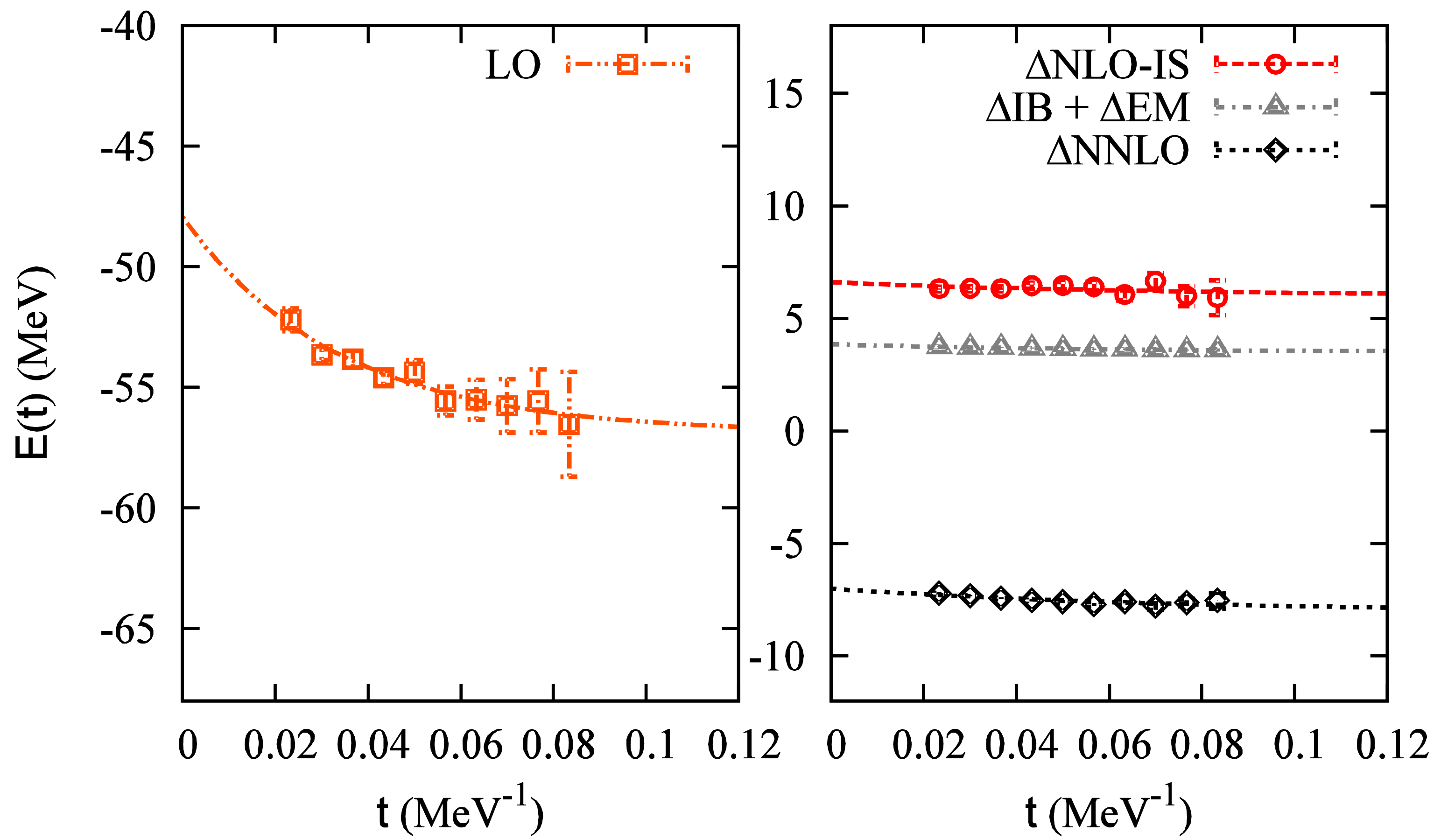
- Accurate description of experimental value already at LO

- Cancelations betw. NLO + IB + EM and NNLO

Results for Beryllium-8

Epelbaum, H.K. Lee, Meißner: Phys. Rev. Lett. 106 (2011) 192501,
Phys. Rev. Lett 104 (2010) 142501, Eur. Phys. J. A45 (2010) 335

- Ground state energy of Beryllium-8 in a box of length $L = 11.8\,\mathrm{fm}$



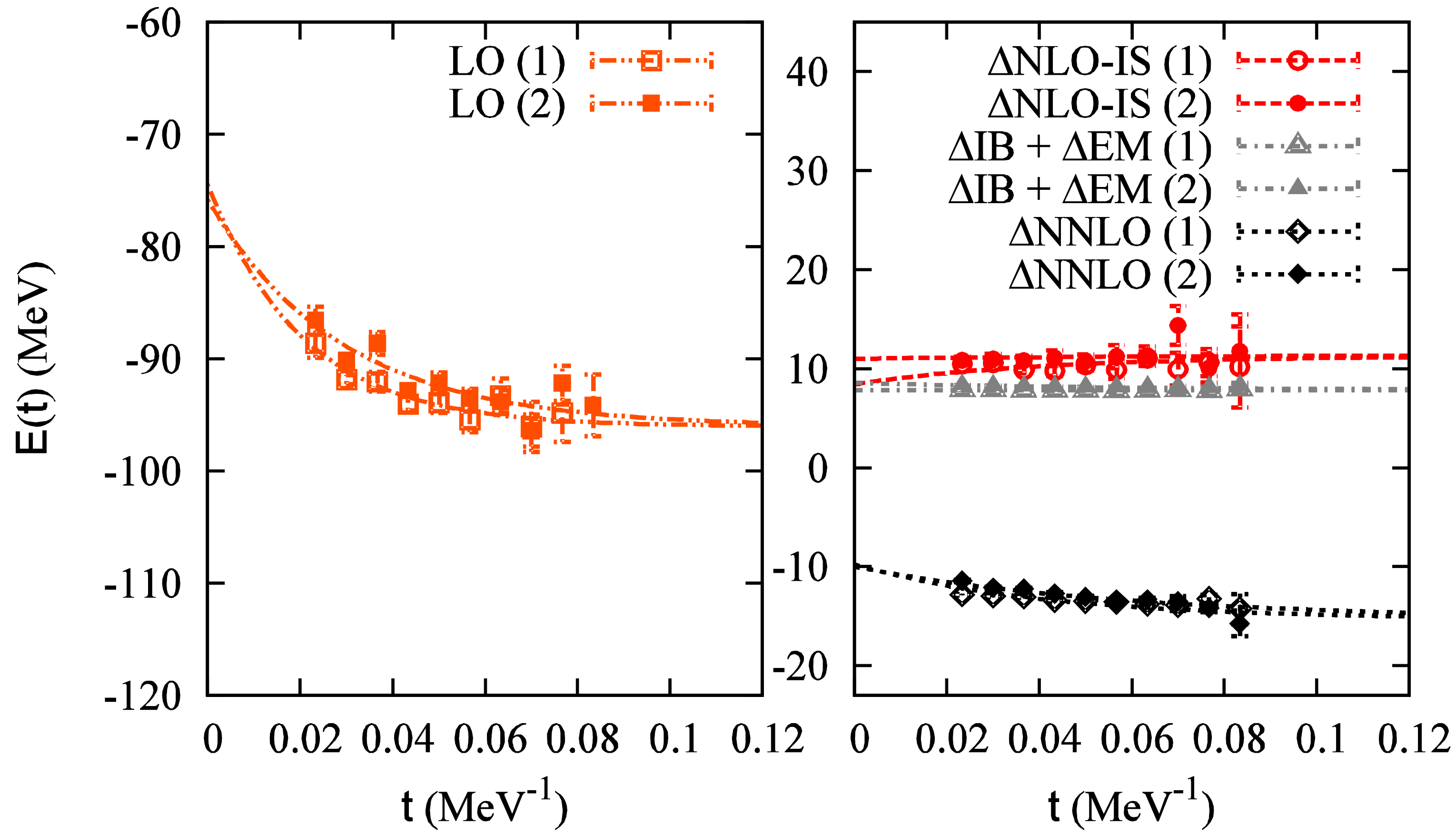
LO	+ NLO + IB + EM	+ NNLO	Exp. Value
$-57(2)$ MeV	$-47(2)$ MeV	$-55(2)$ MeV	-56.5 MeV

- Accurate description of experimantal value already at LO
- Cancelations betw. NLO + IB + EM and NNLO

Results for Carbon-12

Epelbaum, H.K. Lee, Meißner: Phys. Rev. Lett. 106 (2011) 192501,
Phys. Rev. Lett 104 (2010) 142501, Eur. Phys. J. A45 (2010) 335

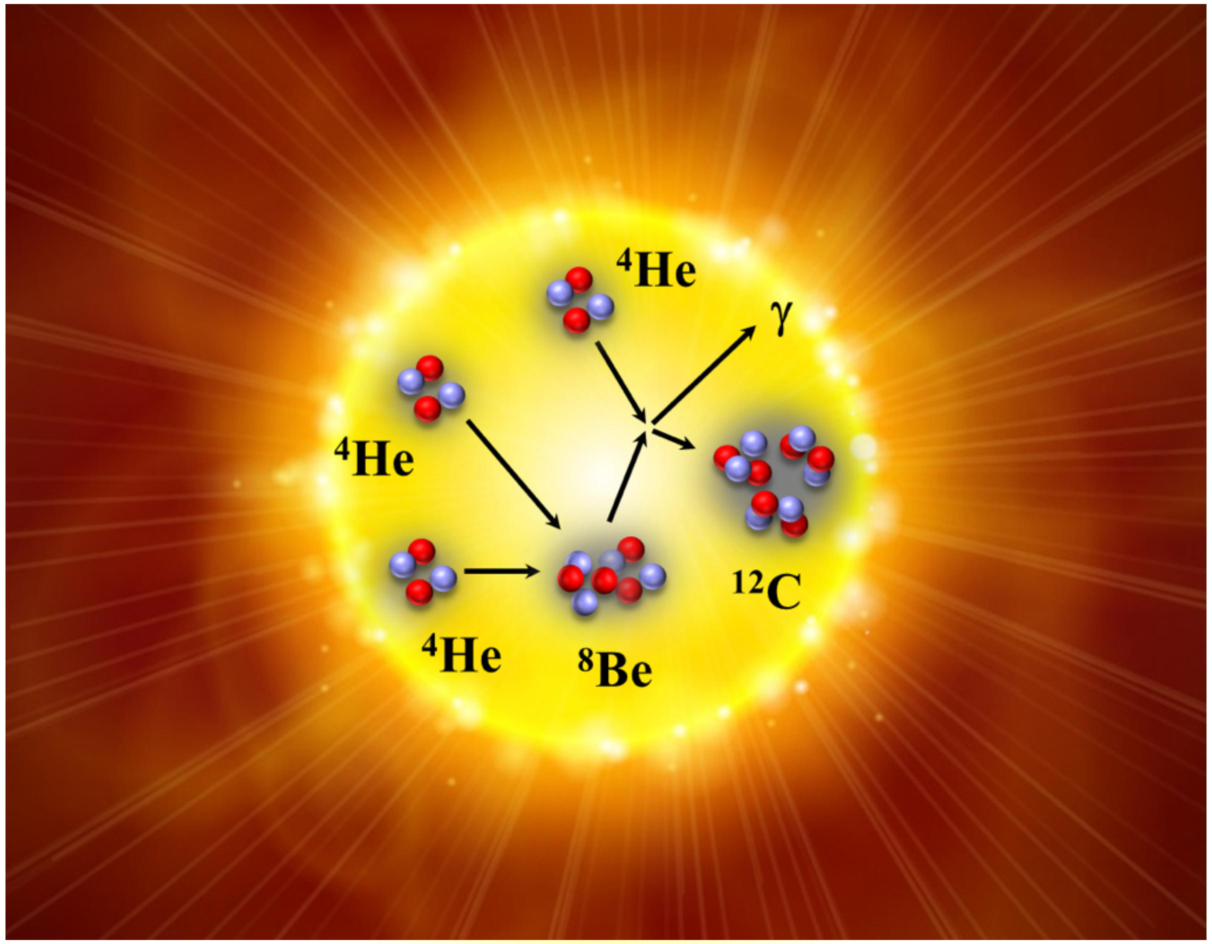
- Ground state energy of Carbon-12 in a box of length $L = 11.8\,\mathrm{fm}$



LO	+ NLO + IB + EM	+ NNLO	Exp. Value
$-96(2)\,\mathrm{MeV}$	$-77(3)\,\mathrm{MeV}$	$-92(3)\,\mathrm{MeV}$	$-92.2\,\mathrm{MeV}$

- Results for two different SU(4) couplings ⇒ no dependence on SU(4) filter
- Finite volume corrections smaller than combined statistical and extrapolation error bars

Production of ^{12}C in stellar environment



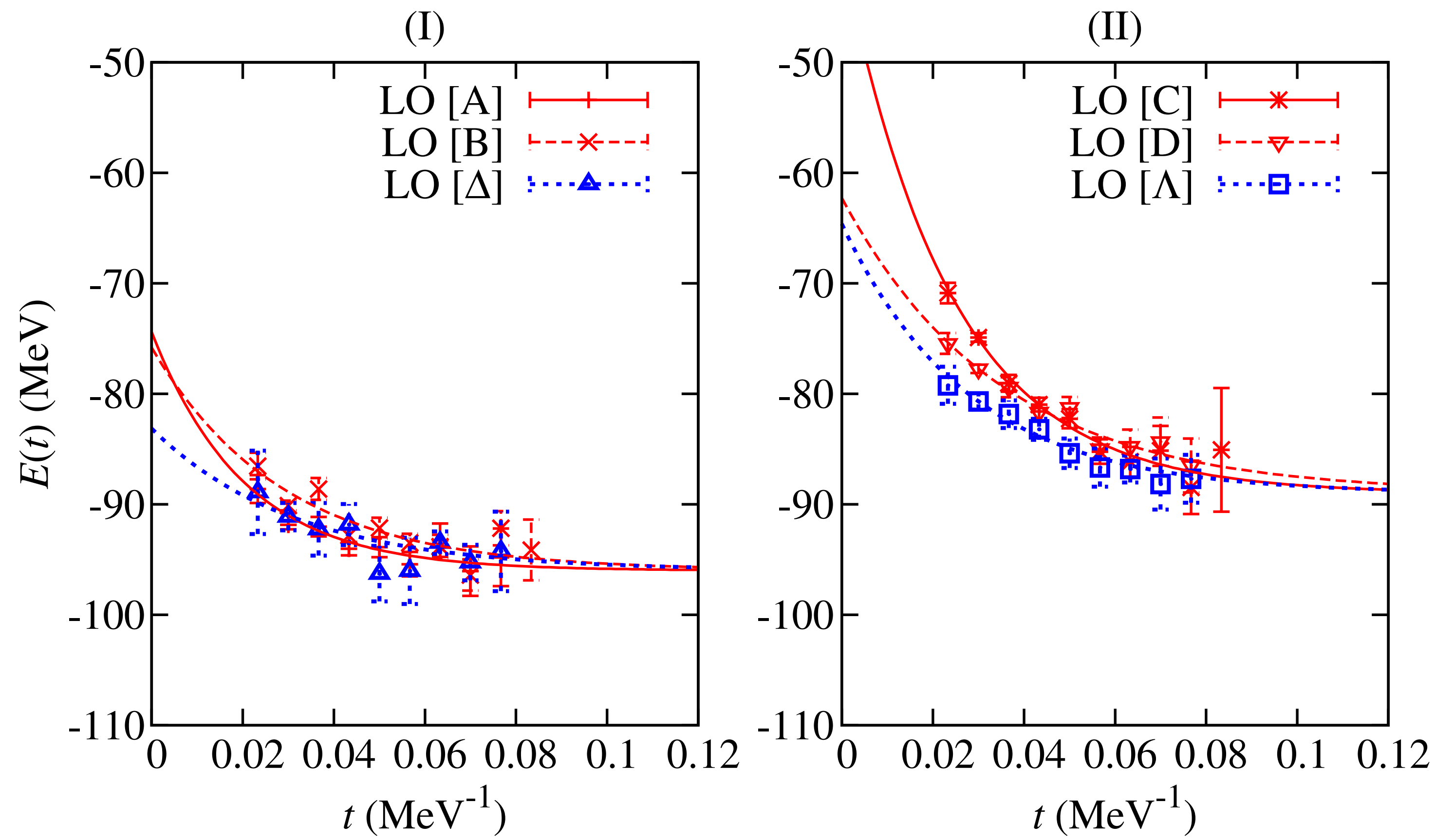
An excited state of $^{12}\mathrm{C}$ was postulated by Hoyle '54

Necessary ingredient for production of $^{12}\mathrm{C}$ at stellar temperatures by fusion of three α–particles

- Existence of the Hoyle state was experimentally confirmed: Cook et al. '57
- First principle calculations of the Hoyle state has been one of the biggest challenges in nuclear structure theory.

Hoyle state 0_2^+

Epelbaum, H.K., Lähde, Lee, Meißner, Phys. Rev. Lett 109 (2012) 252501

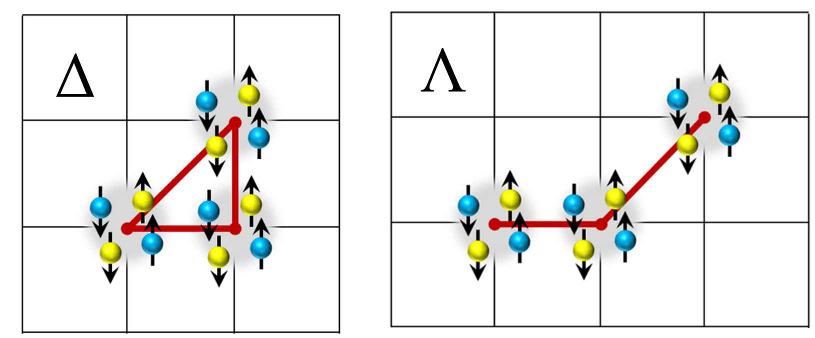


Spectroscopic notation

Total spin → J_n^π ← Parity

← Excitation

Two different Initial/final states configurations



- No signal for 0_2^+ if initial/final states with all three alpha clusters at rest

Hoyle state is not a loosly-bound BEC of three alpha clusters

- Strong signal for 0_2^+ if there is some relative motion excited betw. two alpha clusters

Hoyle state as bent-arm or obtuse triangular configuation

- No visible relaxation to the ground state for Euclidean time probed

Carbon-12: summary

Low-lying even parity states of ^{12}C in units of MeV

	LO	+ NLO + IB + EM	+ NNLO	Exp. Value
0_1^+	$-96(2)$	$-77(3)$	$-92(3)$	-92.16
2_1^+	$-94(2)$	$-74(3)$	$-89(3)$	-87.72
0_2^+	$-89(2)$	$-72(3)$	$-85(3)$	-84.51
2_2^+	$-88(2)$	$-70(3)$	$-83(3)$	$-82.6(1)$

Root-mean-square radii and quadrupole moments of ^{12}C

	$r(0_1^+)$ [fm]	$r(2_1^+)$ [fm]	$Q(2_1^+)$ [e fm^2]	$r(0_2^+)$ [fm]	$r(2_2^+)$ [fm]	$Q(2_1^+)$ [e fm^2]
LO	$2.2(2)$	$2.2(2)$	$6(2)$	$2.4(2)$	$2.4(2)$	$-7(2)$
Exp.	$2.47(2)$	—	$6(3)$	—	$3.07(13)$	—

- Interpretation of 2_2^+-state as rotational excitation of alpha chain

Electromagnetic transitions of ^{12}C

	$B(E2, 2_1^+ \to 0_1^+)$ [e^2 fm^4]	$B(E2, 2_1^+ \to 0_2^+)$ [e^2 fm^4]	$B(E2, 2_2^+ \to 0_1^+)$ [e^2 fm^4]	$B(E2, 2_2^+ \to 0_2^+)$ [e^2 fm^4]	$m(E0, 0_2^+ \to 0_1^+)$ [e fm^2]
LO	$5(2)$	$1.5(7)$	$2(1)$	$6(2)$	$3(1)$
Exp.	$7.6(4)$	$2.6(4)$	$0.73(13)$	—	$5.5(1)$

Summary

- Lattice EFT is a promising tool for a quantitative description of light nuclei
- LECs are fitted to binding energies and to phase-shifts by spherical wall method
- First simulations of ground (and excited) states for light nuclei with A=4,8,12
- Interpretation of the Hoyle state in Carbon-12 as bent-arm chain of alpha clusters
- Prediction of 2_2^+ state in Carbon-12

Outlook

- Spectrum of Oxygen-16
- Ground states of Nitrogen-14 and Neon-20

Multi-channel projection Monte Carlo

- Construct several Slater determinants for A single nucleons

$$|\Psi_A^1\rangle, |\Psi_A^2\rangle, \ldots, |\Psi_A^K\rangle$$

- Correlator function matrix for A nucleons $Z_A^{ij}(t) = \langle \Psi_A^i | \exp(-tH) | \Psi_A^j \rangle$

Slater Determinants for A free Nucleons

- Diagonalize $Z_A^{ij}(t)$ at large t to isolate the different energy eigenstates

- Use the eigenvectors of $Z_A^{ij}(t)$ to calculate expectation values of some operator *O* for each energy eigenstate