Scattering Amplitudes Interpolating between Instant form and Front form of Relativistic Dynamics

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“Return of Prodigal Son”

Longitudinal Boost

from dynamic

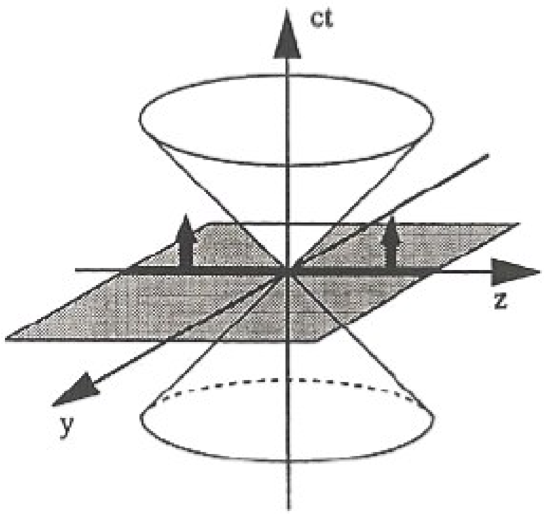
to kinematic

Outline

- Interpolation between Instant and Front Forms
  - Just Formal or Physical Consequences?
  - Physical Meaning of Stability Group
  - "Return of Prodigal Son": Longitudinal Boost
- Landscape of Interpolating Helicity Amplitudes
  - Chiral Spinors
  - Jacob and Wick Helicity vs. LF Helicity
  - Physical Consequences in DVCS and Ang. Con.
- Conclusion

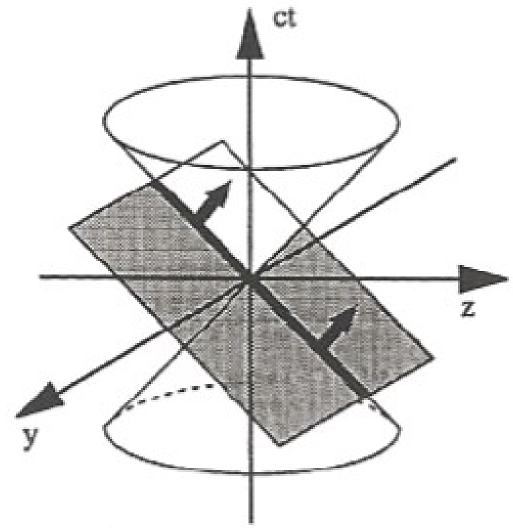
C.Ji and Z. Li, in preparation;
C.Ji and A. Suzuki, PRD87, 065015 (2013);
C.Ji and B. Bakker, IJMPE22 (Review), 1330002 (2013)

Dirac's Proposition

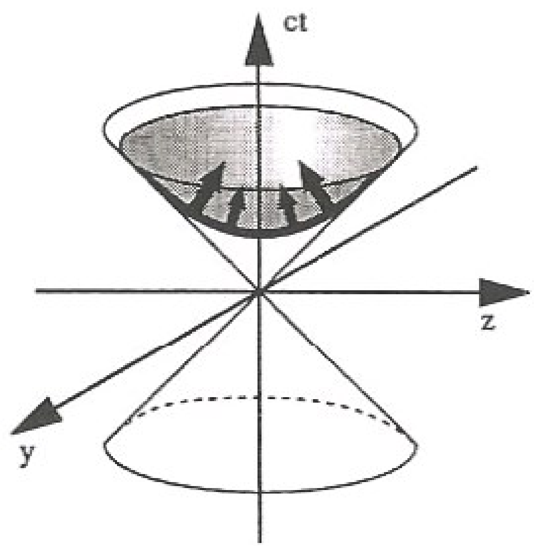


The instant form

1949



The front form

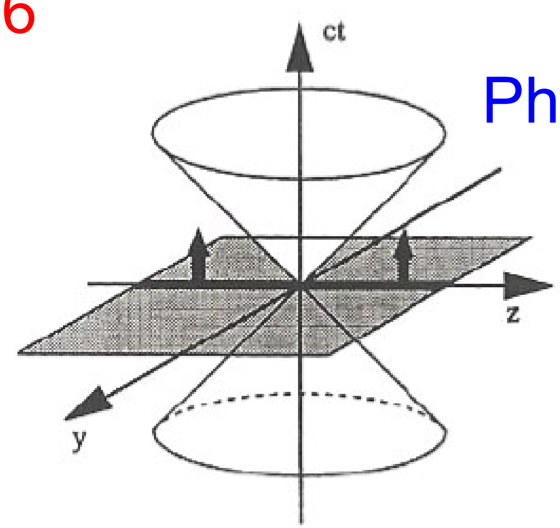


The point form

Stability Group

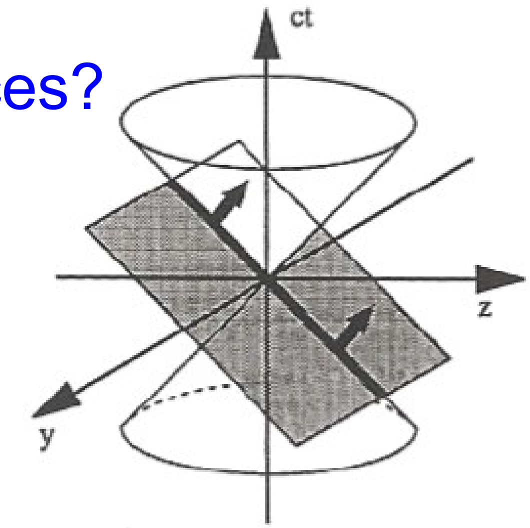
Just Formal?
or
Physical Consequences?

6

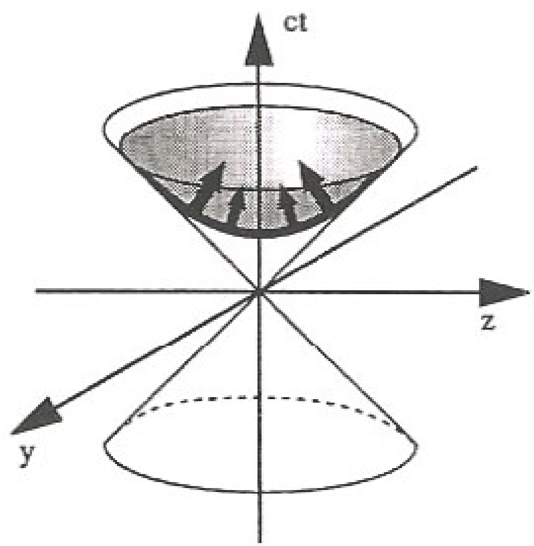


The instant form

7



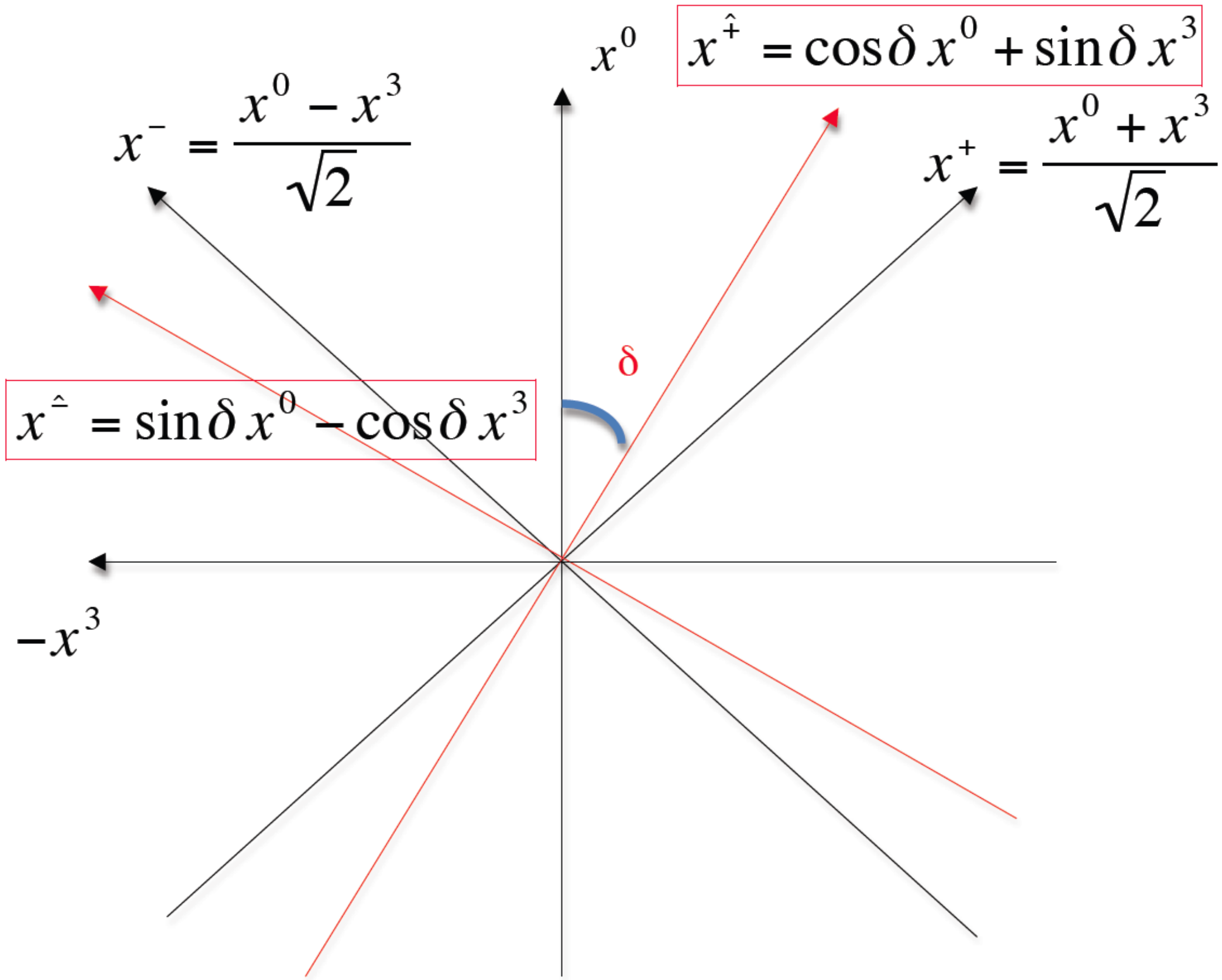
The front form



The point form

6

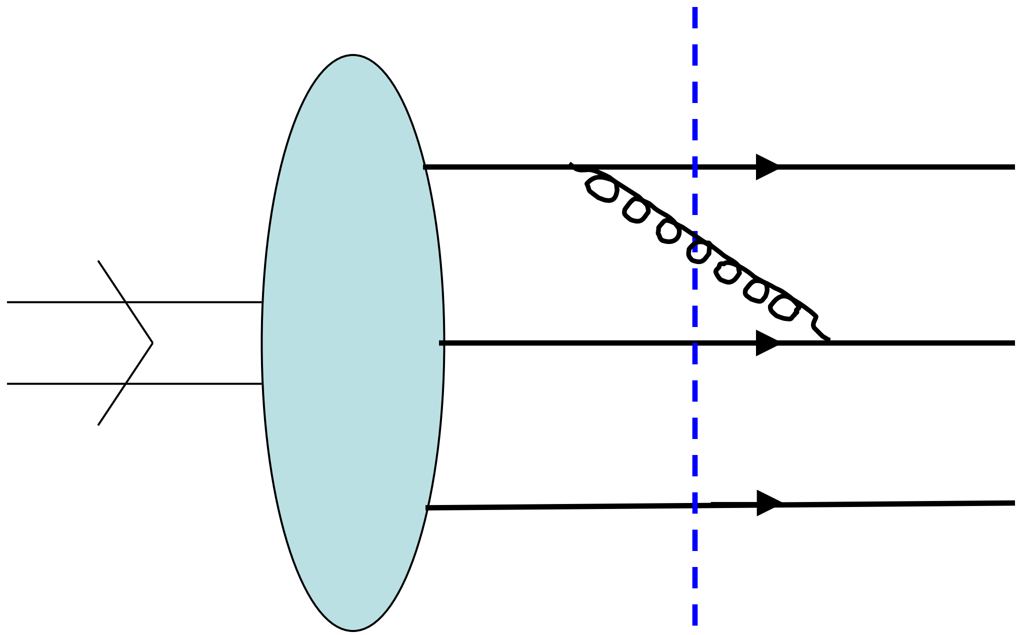
Interpolation between Instant and Front Forms



K. Hornbostel, PRD45, 3781 (1992)
C.Ji and C. Mitchell, PRD64,085013 (2001)
C.Ji and A. Suzuki, PRD87,065015 (2013)

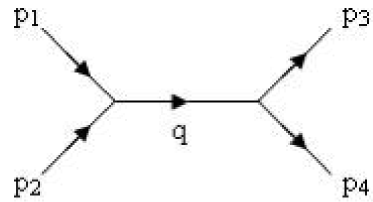
Interpolating Hadronic Wavefunction

$$x^{0} \leftarrow x^{\hat{+}} \rightarrow x^{+}$$



Invariant under kinematic transformations

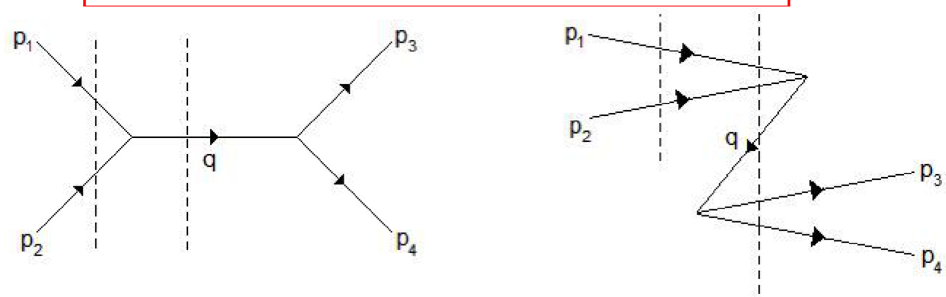
$$(\vec{P}, \vec{J}) \qquad 0 \leftarrow \delta \rightarrow \pi/4 \qquad (P^{+}, \vec{P}_{\perp}, J^{3}, \vec{E}_{\perp}, K^{3})$$



$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

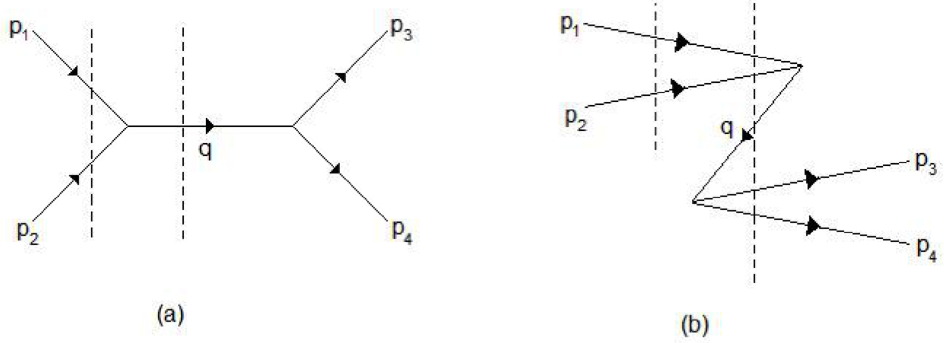
$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_\perp^2 + m^2\right)}$$

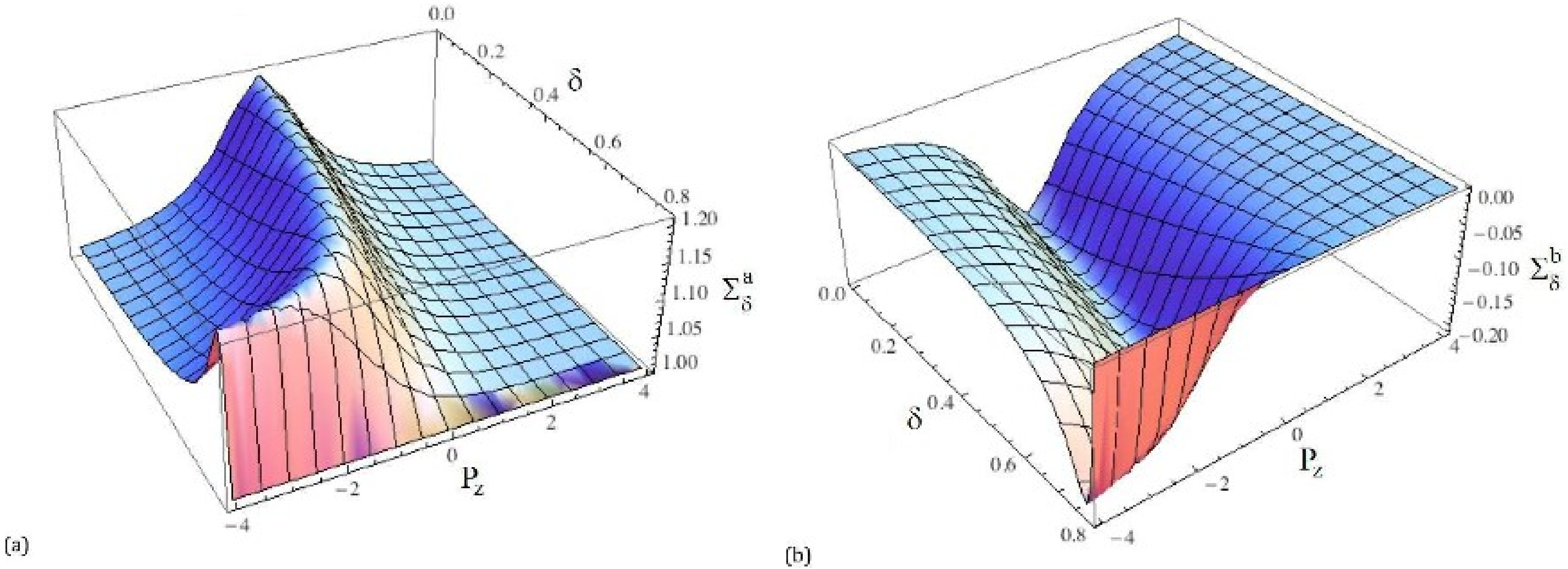
$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_\perp^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_\perp^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\to \infty \quad \text{as } \mathbb{C} \to 0$$





Σ(a)+Σ(b)=1/(s-m²) ; s=2 GeV, m=1GeV

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \longrightarrow g^{\hat\mu\hat\nu} = \begin{pmatrix} \cos 2\delta & 0 & 0 & \sin 2\delta \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin 2\delta & 0 & 0 & -\cos 2\delta \end{pmatrix}$$

$$\int d^3\vec{x}\, T^{0\mu} = P^{\mu} = \begin{pmatrix} P^0 \\ P^1 \\ P^2 \\ P^3 \end{pmatrix} \longrightarrow P^{\hat\mu} = \begin{pmatrix} P^{\hat +} = \cos\delta\, P^0 + \sin\delta\, P^3 \\ P^{\hat 1} = P^1 \\ P^{\hat 2} = P^2 \\ P^{\hat -} = \sin\delta\, P^0 - \cos\delta\, P^3 \end{pmatrix} = \int dx^{\hat -} d^2\vec{x}_{\perp}\, T^{\hat + \hat\mu}$$

$$\int d^3\vec{x}\,(T^{0\mu}x^{\nu} - T^{0\mu}x^{\nu}) = J^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix} \longrightarrow J^{\hat\mu\hat\nu} = \begin{pmatrix} 0 & \hat{E}^1 & \hat{E}^2 & -K^3 \\ -\hat{E}^1 & 0 & J^3 & -\hat{F}^1 \\ -\hat{E}^2 & -J^3 & 0 & -\hat{F}^2 \\ K^3 & \hat{F}^1 & \hat{F}^2 & 0 \end{pmatrix}$$

$$\hat{E}^1 = J^2 \sin\delta + K^1 \cos\delta$$
$$\hat{E}^2 = K^2 \cos\delta - J^1 \sin\delta$$
$$\hat{F}^1 = K^1 \sin\delta - J^2 \cos\delta$$
$$\hat{F}^2 = J^1 \cos\delta + K^2 \sin\delta$$

$$T^{\mu\nu} = \sum_k \frac{\partial L}{\partial(\partial_\mu \phi_k)} \partial^{\nu} \phi_k - g^{\mu\nu} L \quad ; \quad \partial_\mu T^{\mu\nu} = 0$$

Poincaré Algebra

$$[P^{\hat{\mu}}, P^{\hat{\nu}}] = 0 \qquad [P^{\hat{\mu}}, J^{\hat{\rho}\hat{\sigma}}] = i(g^{\hat{\mu}\hat{\rho}} P^{\hat{\sigma}} - g^{\hat{\mu}\hat{\sigma}} P^{\hat{\rho}})$$

$$[J^{\hat{\mu}\hat{\nu}}, J^{\hat{\rho}\hat{\sigma}}] = i(g^{\hat{\mu}\hat{\sigma}} J^{\hat{\nu}\hat{\rho}} + g^{\hat{\nu}\hat{\rho}} J^{\hat{\mu}\hat{\sigma}} - g^{\hat{\mu}\hat{\rho}} J^{\hat{\nu}\hat{\sigma}} - g^{\hat{\nu}\hat{\sigma}} J^{\hat{\mu}\hat{\rho}})$$

e.g. $$[P^{\hat{+}}, J^{\hat{+}\hat{-}}] = i(g^{\hat{+}\hat{+}} P^{\hat{-}} - g^{\hat{+}\hat{-}} P^{\hat{+}})$$

$$[P^{\hat{+}}, -K^3] = i(P^{\hat{-}} \cos 2\delta - P^{\hat{+}} \sin 2\delta)$$

"Return of Prodigal Son" $\delta \to \pi / 4$

$$[K^3, P^+] = -iP^+$$

$$Exp\left(-i\omega K^3\right) | x^+ > \propto | x^+ >$$

One more kinematic generator appears only in the front form.

Maximum number (7) of members in the stability group.

Kinematic Operators
(Members of Stability Group)

$$Exp\left(-i\omega\hat{\square}^{i}\right)|\,x^{\hat{+}}> \propto |\,x^{\hat{+}}>$$

$$[\hat{\square}^{i}, P^{\hat{+}}] = 0$$

$$\hat{\square}^{i} = \hat{F}^{i}\cos 2\delta - \hat{E}^{i}\sin 2\delta$$

$\delta = 0$

$-J^{2}$

J^{1}

←

$$\hat{\square}^{1} = -J^{2}\cos\delta - K^{1}\sin\delta$$

$$\hat{\square}^{2} = J^{1}\cos\delta - K^{2}\sin\delta$$

→

$\delta = \pi/4$

$-E^{1} = -(J^{2} + K^{1})/\sqrt{2}$

$E^{2} = (J^{1} - K^{2})/\sqrt{2}$

$$(J^{3}, P^{1}, P^{2}, P_{\hat{-}})$$

particle at rest

$$p^0 = M, \quad p^1 = p^2 = p^3 = 0$$

$$(p_{\hat{+}} = M\cos\delta, \; p_{\hat{-}} = M\sin\delta)$$

same p^0 ← Under $\hat{\square}^{i}$ transformation → $p^0 + p^3$ same

remain at rest

$$P^0 = M; \; p^3 = 0$$

can move

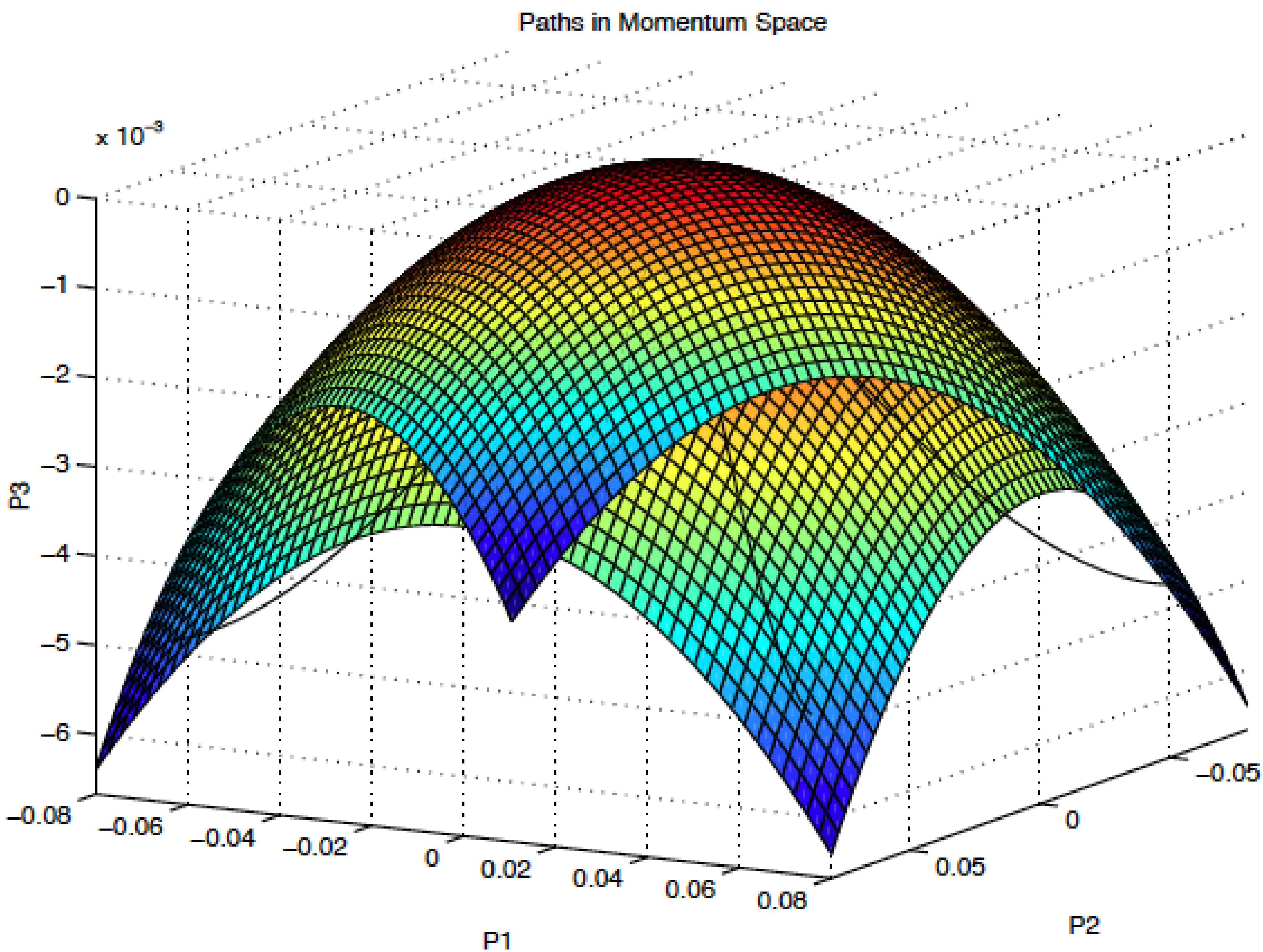
$$P^0 = M + \frac{\vec{p}_\perp^{\,2}}{2M}; \; p^3 = -\frac{\vec{p}_\perp^{\,2}}{2M}$$

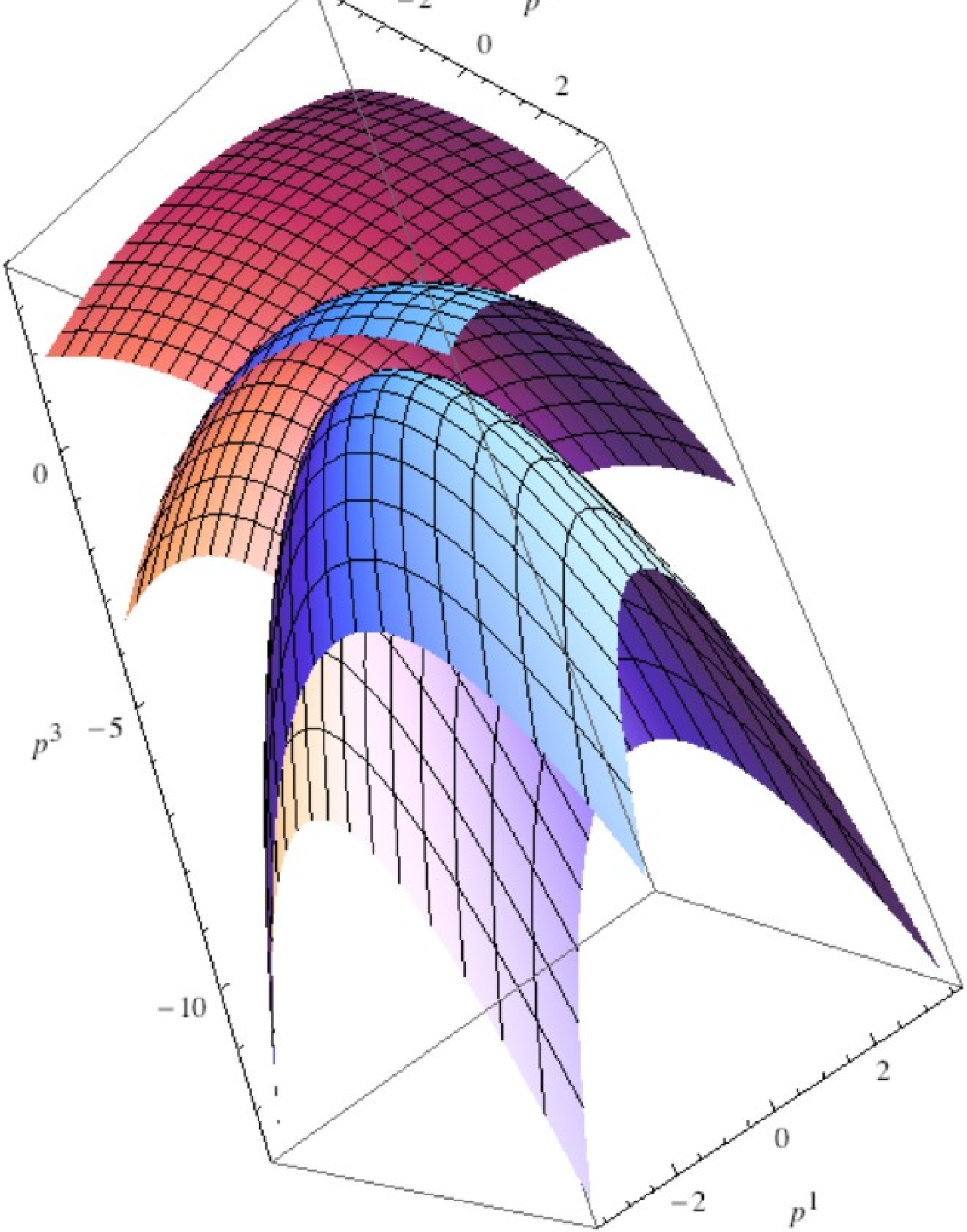
↓

$$(p^0)^2 - (p^3)^2 = \left(M + \frac{\vec{p}_\perp^{\,2}}{2M}\right)^2 - \left(-\frac{\vec{p}_\perp^{\,2}}{2M}\right)^2 = M^2 + \vec{p}_\perp^{\,2} = 2p^+p^- > 0$$

Rational Energy-Momentum Dispersion Relation

Vacuum gets simpler in LFD.


Paths in Momentum Space
x 10^-3
0
−1
−2
−3
−4
−5
−6
P3
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
P1
0.05
0
−0.05
P2



p^2
−2
0
2
0
−5
p^3
−10
2
0
−2
p^1

Angular Momentum

$$[J^i, J^j] = i\varepsilon_{ijk}J^k, [J^i, M] = 0$$

$$T = Exp\left\{-i(\beta_1 \hat{\Box}^1 + \beta_2 \hat{\Box}^2)\right\}$$

$$[\hat{|}^i, \hat{|}^j] = i\varepsilon_{ijk}\hat{|}^k, [\hat{|}^i, M] = 0$$

$$T | n >=| p, n >$$

$$\hat{|}^i | p, n >= TJ^i | n >$$

$$\hat{|}^i = TJ^i T^+$$

$$\hat{\Gamma}^3 = \left[J^3 p_{\hat{-}} + \hat{z} \cdot (\vec{p}_\perp \times \vec{\hat{\Gamma}}_\perp) \right] / M \sin\delta$$

$$\vec{\hat{\Gamma}}_\perp = \vec{J}_\perp + \left[\vec{p}_\perp \cos\delta \left(J^3 + \frac{\hat{z} \cdot (\vec{p}_\perp \times \vec{\hat{\Gamma}}_\perp)}{p_{\hat{-}} + M\sin\delta} \right) - (\hat{z} \times \vec{p}_\perp)\sin\delta \left(K^3 + \frac{\vec{p}_\perp \cdot \vec{\hat{E}}_\perp}{p_{\hat{-}} + M\sin\delta} \right) \right]$$

$$\delta \to \pi/4$$

$$\Gamma^3 = J^3 + \hat{z} \cdot (\vec{E}_\perp \times \vec{p}_\perp) / p^+$$

$$\vec{\Gamma}_\perp = \left[\hat{z} \times (p^- \vec{E}_\perp - p^+ \vec{F}_\perp + \vec{p}_\perp K^3) - \frac{\vec{p}_\perp}{p^+} \left(p^+ J^3 + \hat{z} \cdot \vec{E}_\perp \times \vec{p}_\perp \right) \right] / M$$

$$\Gamma^3 = \frac{W^+}{p^+} \qquad W^{\hat{\mu}} = \frac{1}{2} \varepsilon^{\hat{\mu}\hat{\nu}\hat{\alpha}\hat{\beta}} p_{\hat{\nu}} J_{\hat{\alpha}\hat{\beta}}$$

$$[\Gamma^3, \mathit{Stability\ Group\ Members}] = 0$$

Interpolating Spinors

$$\hat{\Gamma}^{3}\hat{u}_{CR}^{(1)} = (+1)\hat{u}_{CR}^{(1)}$$

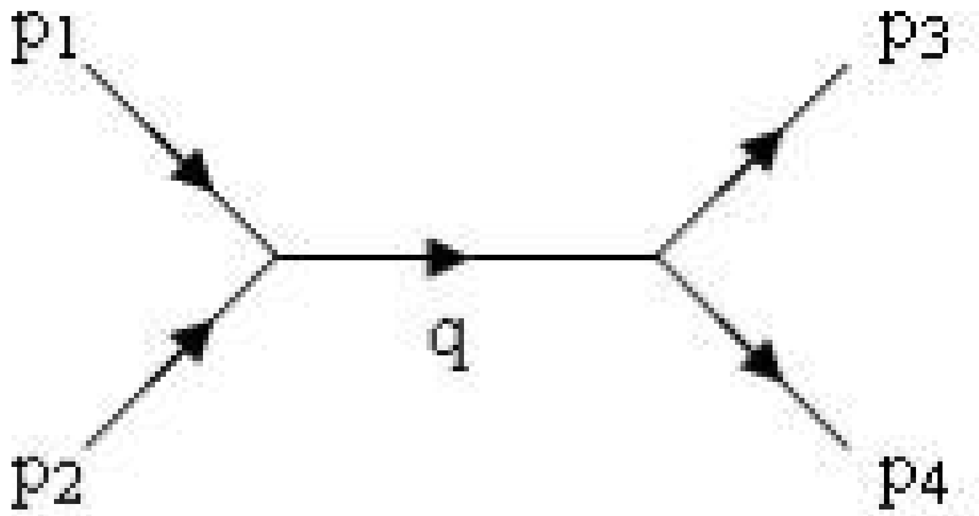
$$\hat{u}_{CR}^{(1)} = \sqrt{M}\begin{bmatrix} \sqrt{\frac{P_{\pm} + \sqrt{P^{\ddagger 2} - M^2 C}}{2\sqrt{P^{\ddagger} - M^2 C}}}\sqrt{\frac{P^{\ddagger} + \sqrt{P^{\ddagger 2} - M^2 C}}{M\left(\sin\delta + \cos\delta\right)}} \\ \frac{P_R\left(\sin\delta + \cos\delta\right)}{\sqrt{CP_{\perp}^2}}\sqrt{\frac{\sqrt{P^{\ddagger 2} - M^2 C} - P_{\pm}}{2\sqrt{P^{\ddagger 2} - M^2 C}}}\sqrt{\frac{P^{\ddagger} + \sqrt{P^{\ddagger 2} - M^2 C}}{M\left(\sin\delta + \cos\delta\right)}} \\ \sqrt{\frac{P_{\pm} + \sqrt{P^{\ddagger 2} - M^2 C}}{2\sqrt{P^{\ddagger 2} - M^2 C}}}\sqrt{\frac{P^{\ddagger} - \sqrt{P^{\ddagger 2} - M^2 C}}{M\left(\cos\delta - \sin\delta\right)}} \\ \frac{P_R\left(\cos\delta - \sin\delta\right)}{\sqrt{CP_{\perp}^2}}\sqrt{\frac{\sqrt{P^{\ddagger 2} - M^2 C} - P_{\pm}}{2\sqrt{P^{\ddagger 2} - M^2 C}}}\sqrt{\frac{P^{\ddagger} - \sqrt{P^{\ddagger 2} - M^2 C}}{M\left(\cos\delta - \sin\delta\right)}} \end{bmatrix}$$

Interpolating Spinors

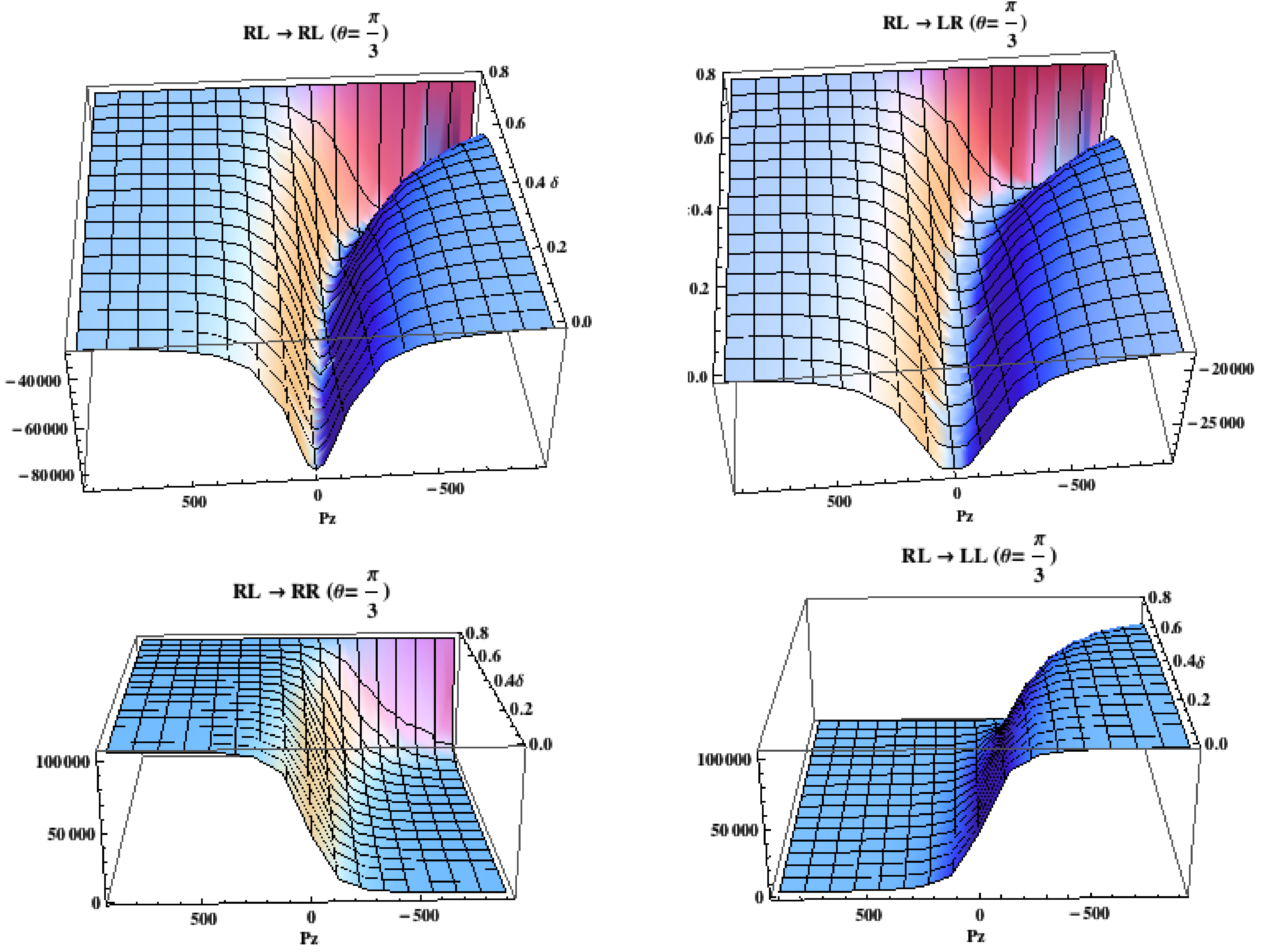
$$\hat{\Gamma}^{3}\hat{u}_{CR}^{(2)} = (-1)\hat{u}_{CR}^{(2)}$$

$$\hat{u}_{CR}^{(2)} = \sqrt{M}\begin{bmatrix} -\dfrac{P_L(\cos\delta - \sin\delta)}{\sqrt{CP_\perp^2}}\sqrt{\dfrac{\sqrt{P^{+2} - M^2C} - P_\pm}{2\sqrt{P^{+2} - M^2C}}}\sqrt{\dfrac{P^{+} - \sqrt{P^{+2} - M^2C}}{M(\cos\delta - \sin\delta)}} \\ \sqrt{\dfrac{P_\pm + \sqrt{P^{+2} - M^2C}}{2\sqrt{P^{+2} - M^2C}}}\sqrt{\dfrac{P^{+} - \sqrt{P^{+2} - M^2C}}{M(\cos\delta - \sin\delta)}} \\ -\dfrac{P_L(\sin\delta + \cos\delta)}{\sqrt{CP_\perp^2}}\sqrt{\dfrac{\sqrt{P^{+2} - M^2C} - P_\pm}{2\sqrt{P^{+2} - M^2C}}}\sqrt{\dfrac{P^{+} + \sqrt{P^{+2} - M^2C}}{M(\sin\delta + \cos\delta)}} \\ \sqrt{\dfrac{P_\pm + \sqrt{P^{+2} - M^2C}}{2\sqrt{P^{+} - M^2C}}}\sqrt{\dfrac{P^{+} + \sqrt{P^{+2} - M^2C}}{M(\sin\delta + \cos\delta)}} \end{bmatrix}$$

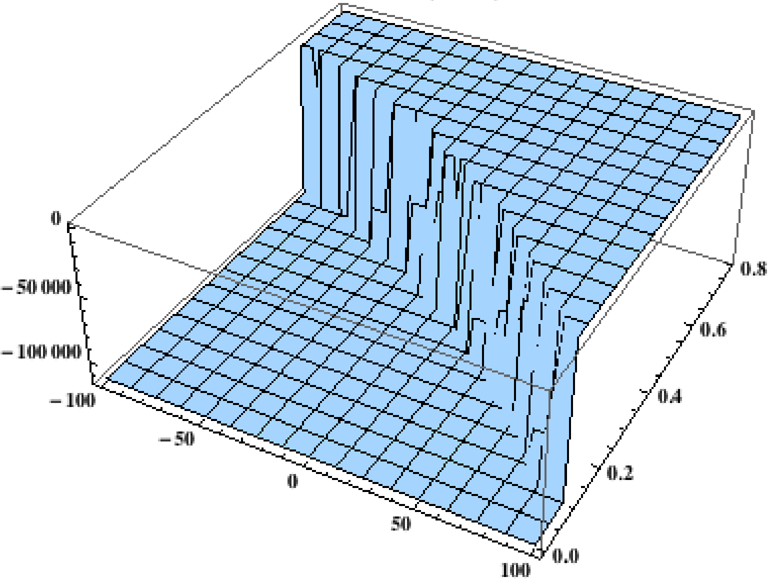
Interpolating Helicity Amplitude



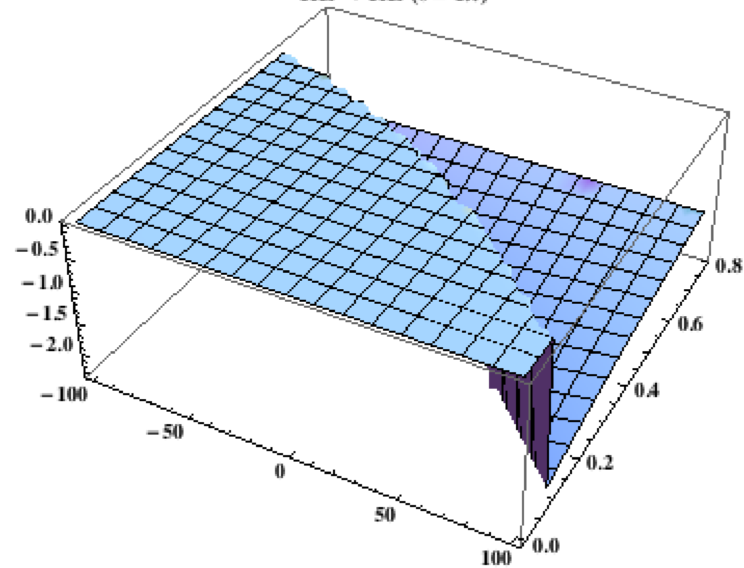
$$\hat{M}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\bar{\hat{\upsilon}}(\hat{p}_2,\lambda_2)\gamma^{\hat{\mu}}\hat{u}(\hat{p}_1,\lambda_1)\bar{\hat{u}}(\hat{p}_3,\lambda_3)\gamma_{\hat{\mu}}\hat{\upsilon}(\hat{p}_4,\lambda_4)$$


RL → RL (θ= π/3)
0.8
0.6
0.4 δ
0.2
0.0
−40000
−60000
−80000
500
0
−500
Pz
RL → LR (θ= π/3)
0.8
0.6
0.4
0.2
0.0
−20000
−25000
500
0
−500
Pz
RL → RR (θ= π/3)
0.8
0.6
0.4δ
0.2
0.0
100000
50000
0
500
0
−500
Pz
RL → LL (θ= π/3)
0.8
0.6
0.4δ
0.2
0.0
100000
50000
0
500
0
−500
Pz

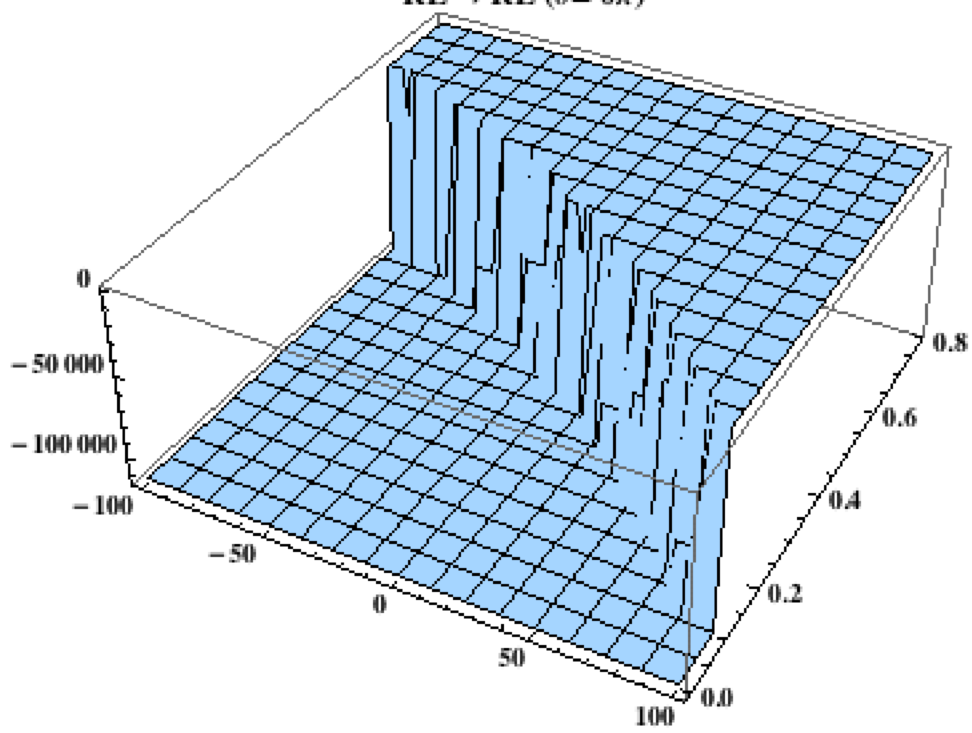
RL → RL (θ= 0π)

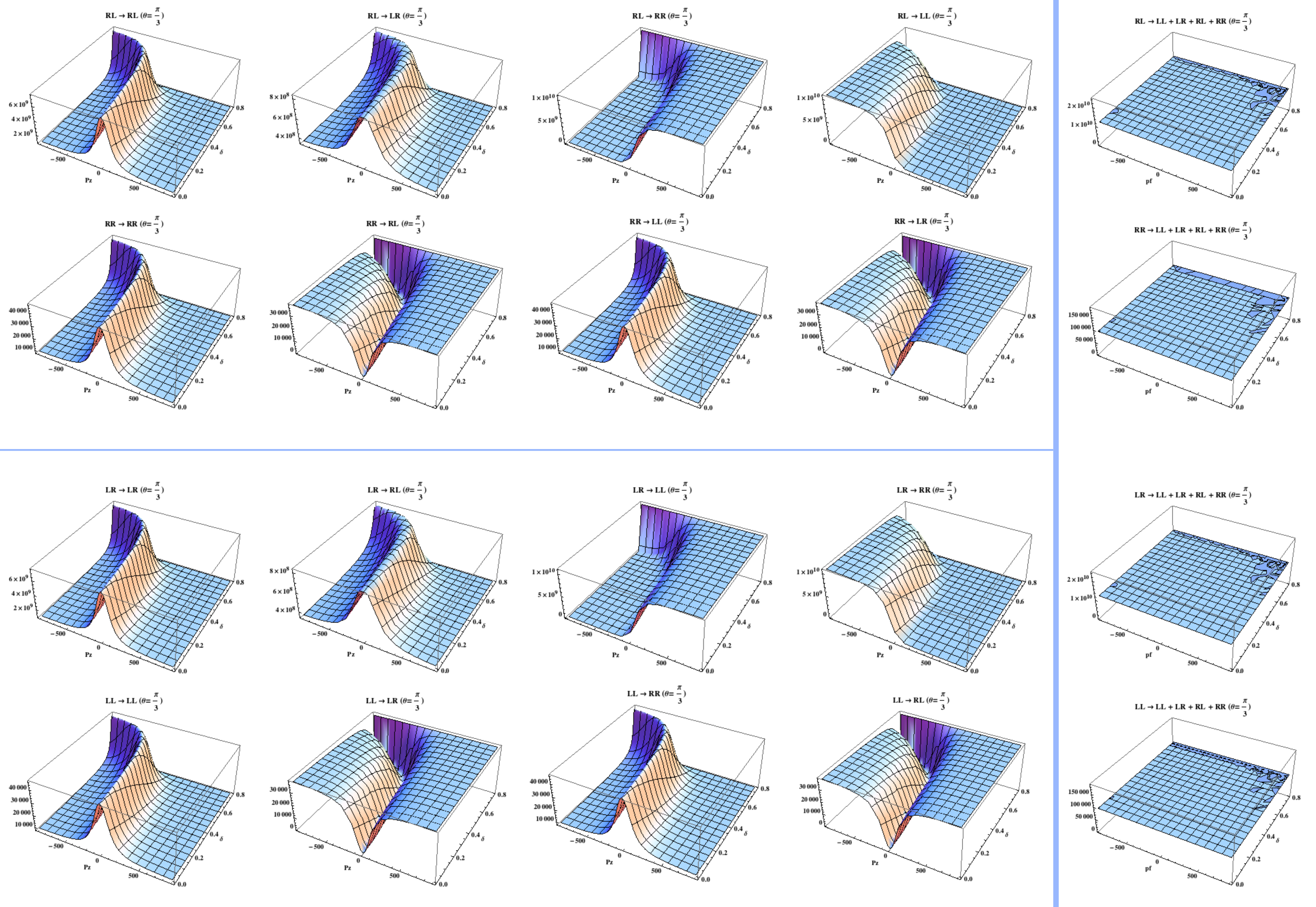


RL → RL (θ= 1π)



RL → RL (θ= 0π)




RL → RL (θ= π/3)
RL → LR (θ= π/3)
RL → RR (θ= π/3)
RL → LL (θ= π/3)
RL → LL + LR + RL + RR (θ= π/3)
RR → RR (θ= π/3)
RR → RL (θ= π/3)
RR → LL (θ= π/3)
RR → LR (θ= π/3)
RR → LL + LR + RL + RR (θ= π/3)
LR → LR (θ= π/3)
LR → RL (θ= π/3)
LR → LL (θ= π/3)
LR → RR (θ= π/3)
LR → LL + LR + RL + RR (θ= π/3)
LL → LL (θ= π/3)
LL → LR (θ= π/3)
LL → RR (θ= π/3)
LL → RL (θ= π/3)
LL → LL + LR + RL + RR (θ= π/3)
Pz
pf
δ

Jacob-Wick Helicity vs. Light-Front Helicity

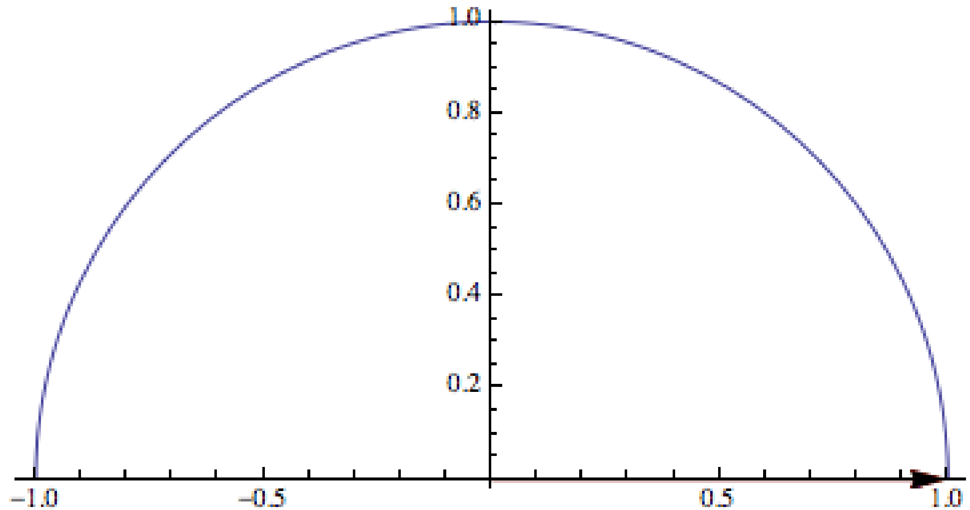
Invariant under kinematic transformations

$$|p', \mu'\rangle_B = R_y(\pi - \theta) e^{-iK_3\xi} |rest, \mu'\rangle_z$$

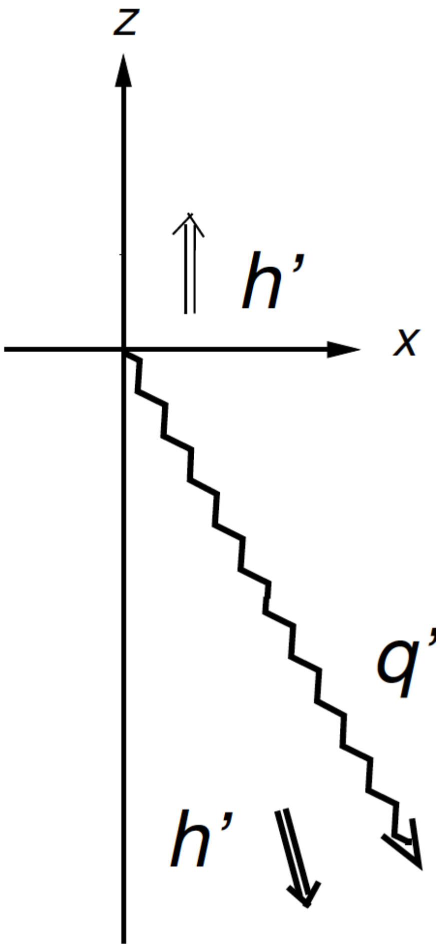
$$|p', \lambda'\rangle_L = e^{-iQE_1/m} e^{-iK_3\xi'} |rest, \lambda'\rangle_z$$

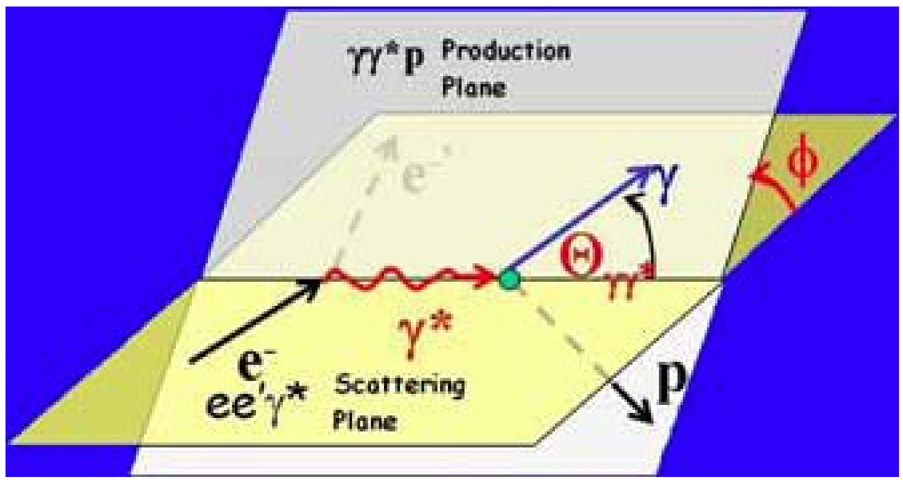
Related by a rotation

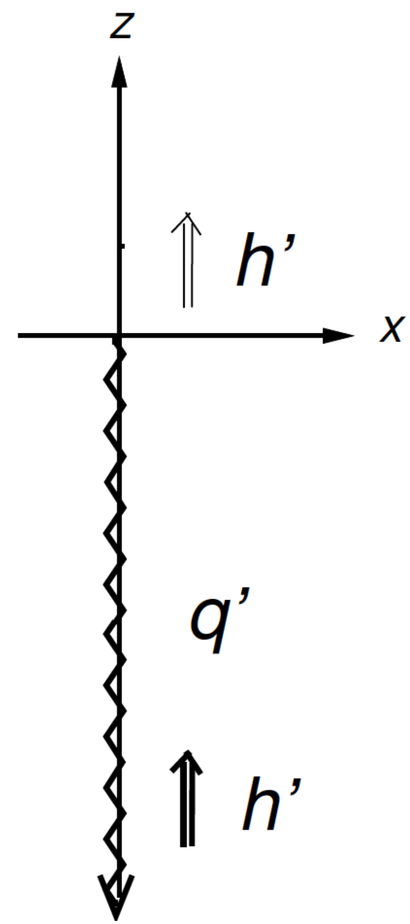
$$_B\langle p', \mu' | p', \lambda' \rangle_L \;\; = \;\; d^{j'}_{\mu'\lambda'}(-\theta')$$



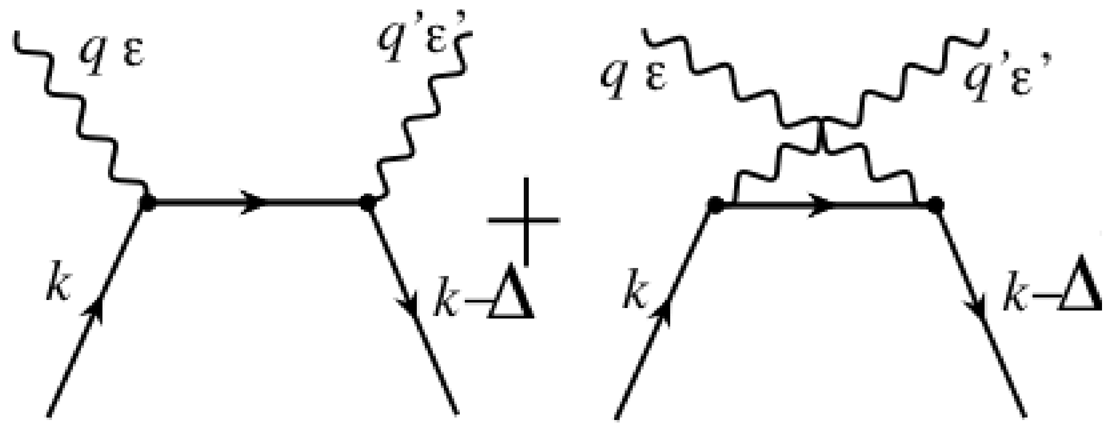
Treacherous Limits in DVCS







“Bare Bone” VCS Amplitude at Tree Level

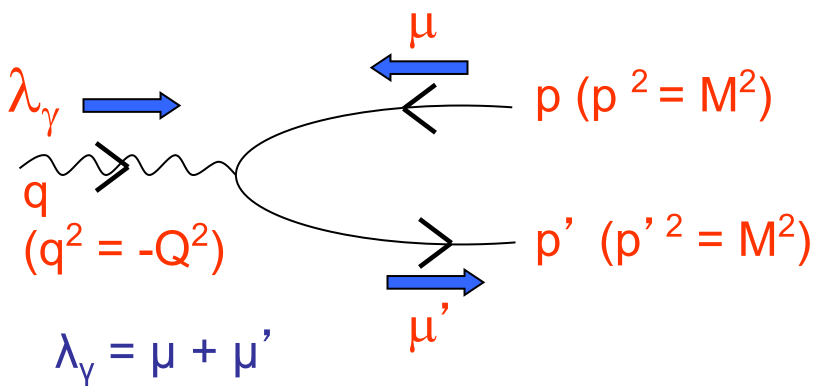


Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda', \lambda\}, h)\frac{1}{q^2}\mathcal{H}(\{h', h\}\{s', s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda' = \lambda$ and $s' = s$.

λ	h'	s	this work	AVR	XJ
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$

Angular Condition

Angular Momentum Conservation in Breit Frame



e.g. Spin-1 Form Factors

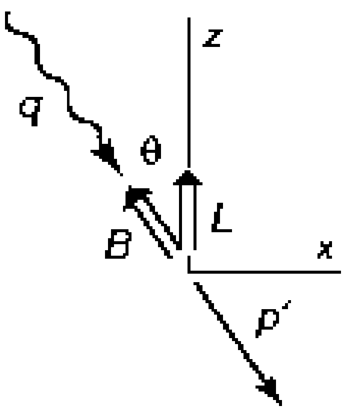
$$G^{\nu}_{B\mu'\mu} = {}_B\langle p', \mu' \mid J^{\nu} \mid p, \mu \rangle_B$$

$$= \quad 0 \quad \textit{if} \quad |\mu + \mu'| \geq 2$$

Light-front Helicity Amplitude in $q^+ = 0$ Frame

$$G^{\nu}_{L\lambda'\lambda} = {}_L\langle p', \lambda' \mid J^{\nu} \mid p, \lambda \rangle_L$$

$$= d^{j'}_{\mu'\lambda'}(\theta)\, G^{\nu}_{B\mu'\mu}\, d^{j}_{\lambda\mu}(-\theta)$$



$$(2\eta + 1) G^{+}_{++} + \sqrt{8\eta}\, G^{+}_{0+} + G^{+}_{+-} - G^{+}_{00} = 0$$

$$\textit{where} \quad \eta = \cot^2\theta = \frac{Q^2}{4M^2}$$

C.Carlson and C.Ji, Phys.Rev.D67,116002(03)

High Q Scaling in QCD (modulo logarithm)

$$G^{+}_{\lambda'\lambda} = \left(\frac{\Lambda}{Q}\right)^{|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|} \qquad G^{+}_{\lambda_{\min}\lambda_{\min}} \propto \left(\frac{\Lambda}{Q}\right)^{2(n-1)+|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|}$$

n = the number of quarks in the state;

$\lambda_{\min}$ = 0 (bosons) or 1/2 (fermions); Λ = QCD scale;

e.g. X.Ji, J.-P.Ma, and F.Yuan, PRL90, 241601 (2003)

$$G^{+}_{\lambda_{\min}\lambda_{\min}} = G^{+}_{00}\ ;\ G^{+}_{+0} = a\left(\frac{\Lambda}{Q}\right)G^{+}_{00}\ ;\ G^{+}_{+-} = \left(b\frac{\Lambda}{Q}\right)^{2} G^{+}_{00}\ ;\ G^{+}_{++} = \left(c\frac{\Lambda}{Q}\right)^{2} G^{+}_{00}$$

$$(2\eta+1)G^{+}_{++} + \sqrt{8\eta}\,G^{+}_{0+} + G^{+}_{+-} - G^{+}_{00} = 0$$

$$1+\sqrt{2}\,\frac{a\Lambda}{M} - \frac{1}{2}\left(\frac{c\Lambda}{M}\right)^{2} = 0\ ;\ a \text{ or } c \text{ must be } O\left(\frac{M}{\Lambda}\right) \approx 20 \text{ for Deuteron}$$

Leading PQCD postponed to a larger Q region; 12GeV upgrade anticipated.

Helicity Amplitudes in N-Δ Transition

$$\mu + \mu' = 2;\ \mu = \frac{1}{2},\ \mu' = \frac{3}{2}$$

$$d^{\frac{3}{2}}_{\lambda' \frac{3}{2}}(-\theta')\, G^{+}_{\lambda'\lambda}\, d^{\frac{1}{2}}_{\frac{1}{2}\lambda}(\theta) = 0$$

$$G^{+}_{\frac{1}{2}-\frac{1}{2}} = G^{+}_{-\frac{1}{2}\frac{1}{2}} \qquad -G^{+}_{\frac{3}{2}-\frac{1}{2}} = G^{+}_{-\frac{3}{2}\frac{1}{2}}$$

$$G^{+}_{\frac{3}{2}\frac{1}{2}} \quad G^{+}_{\frac{1}{2}\frac{1}{2}} \quad G^{+}_{-\frac{1}{2}\frac{1}{2}} \quad G^{+}_{-\frac{3}{2}\frac{1}{2}}$$

$$G^{+}_{\frac{3}{2}-\frac{1}{2}} \quad G^{+}_{\frac{1}{2}-\frac{1}{2}} \quad G^{+}_{-\frac{1}{2}-\frac{1}{2}} \quad G^{+}_{-\frac{3}{2}-\frac{1}{2}}$$

$$G^{+}_{-\frac{1}{2}-\frac{1}{2}} = -G^{+}_{\frac{1}{2}\frac{1}{2}} \qquad G^{+}_{-\frac{3}{2}-\frac{1}{2}} = G^{+}_{\frac{3}{2}\frac{1}{2}}$$

Angular Condition

$$G^{+}_{\frac{3}{2}-\frac{1}{2}} = \frac{Q[Q^2 - m(M-m)]G^{+}_{\frac{3}{2}\frac{1}{2}} + \sqrt{3}MQ^2 G^{+}_{\frac{1}{2}\frac{1}{2}} + \sqrt{3}MQ(M-m)G^{+}_{\frac{1}{2}-\frac{1}{2}}}{[(M-m)(M^2-m^2)+mQ^2]}$$

High Q Scaling in QCD (modulo logarithm)

$$G^{+}_{\lambda'\lambda} = \left(\frac{\Lambda}{Q}\right)^{|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|} \qquad G^{+}_{\lambda_{\min}\lambda_{\min}} \propto \left(\frac{\Lambda}{Q}\right)^{2(n-1)+|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|}$$

n = the number of quarks in the state;

$\lambda_{\min}$ = 0 (bosons) or 1/2 (fermions); Λ = QCD scale;

e.g. X.Ji, J.-P.Ma, and F.Yuan, PRL90, 241601 (2003)

$$G^{+}_{\frac{1}{2}\frac{1}{2}} \sim \frac{1}{Q^4}\,;\; G^{+}_{-\frac{1}{2}\frac{1}{2}} = a\left(\frac{\Lambda}{Q}\right)G^{+}_{\frac{1}{2}\frac{1}{2}}\,;\; G^{+}_{\frac{3}{2}\frac{1}{2}} = b\left(\frac{\Lambda}{Q}\right)G^{+}_{\frac{1}{2}\frac{1}{2}}\,;\; G^{+}_{-\frac{3}{2}\frac{1}{2}} = \left(c\frac{\Lambda}{Q}\right)^{2}G^{+}_{\frac{1}{2}\frac{1}{2}}$$

$$G^{+}_{\frac{3}{2}-\frac{1}{2}} = \frac{Q[Q^2-m(M-m)]G^{+}_{\frac{3}{2}\frac{1}{2}} + \sqrt{3}MQ^2G^{+}_{\frac{1}{2}\frac{1}{2}} + \sqrt{3}MQ(M-m)G^{+}_{\frac{1}{2}-\frac{1}{2}}}{[(M-m)(M^2-m^2)+mQ^2]}$$

$$\sqrt{3}+\frac{b\Lambda}{M}=0\,;\; b=-\frac{\sqrt{3}M}{\Lambda}\approx -20$$

Leading PQCD postponed to a larger Q region; 12GeV upgrade anticipated.

Conclusion

- LFD is not just formal but consequential in the analysis of physical observables.
- Longitudinal boost joins stability group in LFD.
- LF helicity amplitudes are independent of all references frames that are related by front-form boosts.
- Model independent constraints can be made using LFD and LF helicity.
- More careful investigation on treacherous points is necessary for successful hadron physics.