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Fermion mass mixing in vacuum

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Renormalization group procedure for effective particles (RGPEP) is applied to theory of fermions that interact only through a mass mixing term in the Hamiltonian. Problems with virtual pair production in vacuum are avoided by using the front form of Hamiltonian dynamics. Masses and states of physical fermions emerge at the end of an exact quantum calculation for arbitrary strength of the mass mixing. An a priori infinite set of renormalization group equations for all momentum modes of fermionic fields is reduced to just one equation for a two-by-two mass matrix. In distinction from scalars, fermions never become tachyons but appear chirally rotated when the interaction is sufficiently strong.

S. D. Głazek, arXiv:1305.3702[hep-th].

S. D. Głazek, Phys. Rev. D **85**, 125018 (2012).

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Outline

1. Exact solution of an elementary fermion mass mixing model
2. using the **R**enormalization **G**roup **P**rocedure for **E**ffective **P**articles (**RGPEP**)
3. including the issues of vacuum and Poincaré symmetry
4. and fermions never becoming tachyons, as bosons can.

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$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \mu)\psi + \bar{\phi}(i\partial\!\!\!/ - \nu)\phi$$

$$(i\partial\!\!\!/ - \mu)\,\psi = 0 \qquad (i\partial\!\!\!/ - \nu)\,\phi = 0$$

$$H = \int d^3x\, \mathcal{T}^{00}$$

$$\mathcal{T}^{\rho\sigma} = \frac{\partial \mathcal{L}}{\partial \partial_\rho \psi_\alpha}\,\partial^\sigma \psi_\alpha + \frac{\partial \mathcal{L}}{\partial \partial_\rho \phi_\alpha}\,\partial^\sigma \phi_\alpha - g^{\rho\sigma}\,\mathcal{L}$$

.

$$H = \int d^3x \, \left[\psi^\dagger (i\vec{\alpha}\vec{\partial} + \beta\mu)\psi + \phi^\dagger (i\vec{\alpha}\vec{\partial} + \beta\nu)\phi \right]$$

$$\psi(\vec{x}\,) = \sum\!\!\!\!\!\!\int_{\mu ps} \left[u_{\mu ps}\, b_{\mu ps}\, e^{i\vec{p}\,\vec{x}} + v_{\mu ps}\, d^\dagger_{\mu ps}\, e^{-i\vec{p}\,\vec{x}} \right]$$

$$\phi(\vec{x}\,) = \sum\!\!\!\!\!\!\int_{\nu ps} \left[u_{\nu ps}\, b_{\nu ps}\, e^{i\vec{p}\,\vec{x}} + v_{\nu ps}\, d^\dagger_{\nu ps}\, e^{-i\vec{p}\,\vec{x}} \right]$$

$$\sum\!\!\!\!\!\!\int_{\mu ps} = \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3 2E_{\mu p}} \qquad E_{\mu p} = \sqrt{\mu^2 + \vec{p}^{\,2}} \quad \mu \to \nu \quad etc.$$

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$$u_{\mu p s} = B(\mu, \vec{p}\,)\, u_{\mu 0 s} \qquad v_{\mu p s} = B(\mu, \vec{p}\,)\, v_{\mu 0 s}$$

$$B(\mu, \vec{p}\,) = \frac{1}{\sqrt{2\mu(E_{\mu p} + \mu)}} \, (\not{p}\beta + \mu) \qquad \mu \to \nu \quad etc.$$

Quantization: $\psi, \phi \to \hat{\psi}, \hat{\phi}$

$$\left\{ \hat{\psi}(\vec{x}\,), \hat{\psi}^\dagger(\vec{x}\,') \right\} = \delta^3(\vec{x} - \vec{x}\,')$$

$$\left\{ b_{\mu p s}, b^\dagger_{\mu p' s'} \right\} = \left\{ d_{\mu p s}, d^\dagger_{\mu p' s'} \right\} = 2E_{\mu p}(2\pi)^3 \delta^3(\vec{p} - \vec{p}\,')\, \delta_{s s'}$$

Quantum Hamiltonian for free fermions

$$\hat{H}_0 = \int d^3x : \left[\hat{\psi}^\dagger (i\vec{\alpha}\vec{\partial} + \beta\mu)\hat{\psi} + \hat{\phi}^\dagger (i\vec{\alpha}\vec{\partial} + \beta\nu)\hat{\phi} \right] :$$

$$\hat{H}_0 = \sum\!\!\!\!\!\!\int_{\mu ps} E_{\mu p} \left(b^\dagger_{\mu ps} b_{\mu ps} + d^\dagger_{\mu ps} d_{\mu ps} \right) \sum\!\!\!\!\!\!\int_{\nu ps} E_{\nu p} \left(b^\dagger_{\nu ps} b_{\nu ps} + d^\dagger_{\nu ps} d_{\nu ps} \right)$$

Add interactions (for simplicity, just mass mixing)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \mu)\psi + \bar{\phi}(i\partial\!\!\!/ - \nu)\phi - m\left(\bar{\psi}\phi + \bar{\phi}\psi\right)$$

.

$$\hat{H} = \hat{H}_0 + \hat{H}_I$$

$$\hat{H}_I = m \int d^3x \; : \left(\hat{\psi}^\dagger \gamma^0 \hat{\phi} + \hat{\phi}^\dagger \gamma^0 \hat{\psi} \right) :$$

$$\hat{H}_I = m \; \sum\!\!\!\!\!\!\!\int_{\mu p s} \sum_{s'} \frac{1}{2E_{\nu p}} \Big[\bar{u}_{\mu p s} \, u_{\nu p s'} \, b^\dagger_{\mu p s} \, b_{\nu p s'} + \bar{u}_{\mu p s} \, v_{\nu -p s'} \, b^\dagger_{\mu p s} \, d^\dagger_{\nu -p s'}$$
$$+ \; \bar{v}_{\mu p s} \, u_{\nu -p s'} \, d_{\mu p s} \, b_{\nu -p s'} - \bar{v}_{\mu p s} \, v_{\nu p s'} \, d^\dagger_{\nu p s'} \, d_{\mu p s} \Big] + (\mu \leftrightarrow \nu)$$

Vacuum problem

$$\hat{h} = m \; \sum\!\!\!\!\!\!\!\int_{\mu p s} \sum_{s'} \frac{1}{2E_{\nu p}} \, \bar{u}_{\mu p s} \, v_{\nu -p s'} \, b^\dagger_{\mu p s} \, d^\dagger_{\nu -p s'}$$

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$$\bar{u}_{\mu p s}\, v_{\nu - p s'} = \left(\sqrt{\frac{E_{\nu p} + \nu}{E_{\mu p} + \mu}} + \sqrt{\frac{E_{\mu p} + \mu}{E_{\nu p} + \nu}} \right) \chi_s^\dagger\, \vec{\sigma} \vec{p}\, i \sigma^2\, \chi_{s'}$$

$$|h\rangle = \hat{h}|0\rangle = ?$$

$$\langle h|h\rangle = \langle 0|\hat{h}^\dagger \hat{h}|0\rangle = V m^2 \sum\!\!\!\!\!\!\!\int_{\mu p s} \sum_{s'} \frac{1}{2E_{\nu p}} |\bar{u}_{\mu p s}\, v_{\nu - p s'}|^2$$

$$\sim V\, 4 m^2 \int \frac{d^3 p}{(2\pi)^3 2 E_{\mu p}} \frac{\vec{p}^{\,2}}{E_{\nu p}}$$

Cutoff on $|\vec{p}|$? Lorentz symmetry ? Schrödinger picture ?

Instant form re-quantization

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$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - M)\Psi \qquad \Psi = \begin{bmatrix} \psi \\ \phi \end{bmatrix} \qquad M = \begin{bmatrix} \mu & m \\ m & \nu \end{bmatrix}$$

$$m_{1,2} = [\,\mu + \nu \,\pm\, (\mu - \nu)\,\epsilon\,]\,/2$$

$$v_1 = \begin{bmatrix} \cos\varphi \\ -\sin\varphi \end{bmatrix} \qquad v_2 = \begin{bmatrix} \sin\varphi \\ \cos\varphi \end{bmatrix}$$

$$\epsilon = \sqrt{1 + [2m/(\mu - \nu)]^2} \qquad \varphi = -\arctan\sqrt{\frac{\epsilon - 1}{\epsilon + 1}}$$

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$$\Psi = \psi_1 \, v_1 + \psi_2 \, v_2$$

$$\psi_1 = \cos\varphi \ \psi - \sin\varphi \ \phi \qquad \psi_2 = \sin\varphi \ \psi + \cos\varphi \ \phi$$

$$\mathcal{L} = \bar{\psi}_1 (i\partial\!\!\!/ - m_1)\psi_1 + \bar{\psi}_2 (i\partial\!\!\!/ - m_2)\psi_2$$

$$\hat{H} = \sum\!\!\!\!\!\!\!\int_{m_1 ps} E_{m_1 p} \left(b^\dagger_{m_1 ps} b_{m_1 ps} + d^\dagger_{m_1 ps} d_{m_1 ps} \right) + \sum\!\!\!\!\!\!\!\int_{m_2 ps} E_{m_2 p} \left(b^\dagger_{m_2 ps} b_{m_2 ps} + d^\dagger_{m_2 ps} d_{m_2 ps} \right)$$

What would we do with $|\Omega\rangle$ if we did not know m_1 and m_2 ?

RGPEP in the FF of Hamiltonian dynamics

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$$(i\not\partial - M)\,\Psi = 0 \qquad \Psi_\pm = \Lambda_\pm \Psi \qquad \Lambda_\pm = \gamma^0\gamma^\pm/2$$

$$i\partial^-\Psi_+ + i\partial^+\Psi_- - (i\alpha^\perp\partial^\perp + \beta M)(\Psi_+ + \Psi_-) = 0$$

$$i\partial^-\Psi_+ = (i\alpha^\perp\partial^\perp + \beta M)\Psi_-$$

$$i\partial^+\Psi_- = (i\alpha^\perp\partial^\perp + \beta M)\Psi_+$$

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$$P^- = \frac{1}{2} \int dx^- d^2x^\perp \, \mathcal{T}^{+-}$$

$$\frac{1}{2}\mathcal{T}^{+-} = \Psi_+^\dagger i\partial^- \Psi_+ = \Psi_+^\dagger (i\alpha^\perp \partial^\perp + \beta M) \frac{1}{i\partial^+} (i\alpha^\perp \partial^\perp + \beta M) \Psi_+$$

$$P^- = \int dx^- d^2x^\perp \, \Psi_+^\dagger \, \frac{-\partial^{\perp\, 2} + M^2}{i\partial^+} \, \Psi_+ \qquad M^2 = \begin{bmatrix} \mu^2 + m^2 & m(\mu+\nu) \\ m(\mu+\nu) & \nu^2 + m^2 \end{bmatrix}$$

Quantization

$$\hat{P}^- = \int dx^- d^2x^\perp \; : \hat{\Psi}_+^\dagger \, \frac{-\partial^{\perp\, 2} + M^2}{i\partial^+} \, \hat{\Psi}_+ :$$

$$\psi(x) = \begin{bmatrix} \zeta(x) \\ \xi(x) \end{bmatrix} \qquad \phi(x) = \begin{bmatrix} \omega(x) \\ \rho(x) \end{bmatrix}$$

$$\psi_+(x) = \begin{bmatrix} \zeta(x) \\ 0 \end{bmatrix} \qquad \phi_+(x) = \begin{bmatrix} \omega(x) \\ 0 \end{bmatrix} \qquad \Psi_+(x) = \begin{bmatrix} \zeta(x) \\ 0 \\ \omega(x) \\ 0 \end{bmatrix}$$

$$x^+ = x'^+ = 0 \qquad \left\{\hat{\zeta}(x), \hat{\zeta}^\dagger(x')\right\} = \left\{\hat{\omega}(x), \hat{\omega}^\dagger(x')\right\} = \delta^3(x - x')$$

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$$\hat{\zeta}(x) = \sum\!\!\!\!\!\!\int_{ps} \sqrt{p^+}\ \left[b_{\zeta ps}\, e^{-ipx} - d^\dagger_{\zeta ps}\, e^{ipx} \sigma^1 \right]\ \chi_s$$

$$\hat{\omega}(x) = \sum\!\!\!\!\!\!\int_{ps} \sqrt{p^+}\ \left[b_{\omega ps}\, e^{-ipx} - d^\dagger_{\omega ps}\, e^{ipx} \sigma^1 \right]\ \chi_s$$

$$\sum\!\!\!\!\!\!\int_{ps} = \sum_{s=\pm 1} \int_{-\infty}^{+\infty} \frac{d^2 p^\perp}{(2\pi)^2} \int_0^{+\infty} \frac{dp^+}{2(2\pi)p^+}$$

$$\left\{ b_{\zeta ps}, b^\dagger_{\zeta p' s'} \right\} = \left\{ d_{\zeta ps}, d^\dagger_{\zeta p' s'} \right\} = \left\{ b_{\omega ps}, b^\dagger_{\omega p' s'} \right\} = \left\{ d_{\omega ps}, d^\dagger_{\omega p' s'} \right\}$$

$$= 2p^+ (2\pi)^3 \delta^3(p - p')\, \delta_{ss'}$$

no mass dependence

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$$\hat{P}^- = \hat{P}^-_f + \hat{P}^-_I$$

$$\hat{P}^-_f = \int dx^- d^2x^\perp \; : \left(\hat{\zeta}^\dagger \, \frac{-\partial^{\perp\,2} + \mu^2}{i\partial^+} \, \hat{\zeta} + \hat{\omega}^\dagger \, \frac{-\partial^{\perp\,2} + \nu^2}{i\partial^+} \, \hat{\omega} \right) :$$

$$\hat{P}^-_I = \int dx^- d^2x^\perp \; : \Big[\hat{\zeta}^\dagger \, \frac{m(\mu+\nu)}{i\partial^+} \, \hat{\omega} + \hat{\omega}^\dagger \, \frac{m(\mu+\nu)}{i\partial^+} \, \hat{\zeta} + \; \hat{\zeta}^\dagger \, \frac{m^2}{i\partial^+} \, \hat{\zeta} + \hat{\omega}^\dagger \, \frac{m^2}{i\partial^+} \, \hat{\omega} \Big] :$$

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$$\hat{P}^- = \sum\!\!\!\!\!\!\!\int_{ps} \Bigg[\left(p_\mu^- + \frac{m^2}{p^+} \right) \left(b^\dagger_{\zeta ps}\, b_{\zeta ps} + d^\dagger_{\zeta ps}\, d_{\zeta ps} \right)$$

$$+ \left(p_\nu^- + \frac{m^2}{p^+} \right) \left(b^\dagger_{\omega ps}\, b_{\omega ps} + d^\dagger_{\omega ps}\, d_{\omega ps} \right)$$

$$+ \frac{m(\mu + \nu)}{p^+} \left(b^\dagger_{\zeta ps}\, b_{\omega ps} + d^\dagger_{\omega ps}\, d_{\zeta ps} + b^\dagger_{\omega ps}\, b_{\zeta ps} + d^\dagger_{\zeta ps}\, d_{\omega ps} \right) \Bigg]$$

$$p_\mu^- = \frac{p^{\perp\, 2} + \mu^2}{p^+} \qquad p_\nu^- = \frac{p^{\perp\, 2} + \nu^2}{p^+}$$

$$\hat{P}^- |0\rangle = 0 \qquad |\Omega\rangle = |0\rangle$$

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RGPEP equation

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$$t = s^4 \qquad \mathcal{P}_0^- = \hat{P}^- \qquad \mathcal{P}_t^{-\prime} = \Big[[\mathcal{P}_f^-, \mathcal{P}_{Pt}^-], \mathcal{P}_t^- \Big]$$

$$\mathcal{P}_t^- = \sum\!\!\!\!\!\!\!\int_{ps} \Big[A_{tp} \left(b^\dagger_{\zeta ps} b_{\zeta ps} + d^\dagger_{\zeta ps} d_{\zeta ps} \right) + B_{tp} \left(b^\dagger_{\omega ps} b_{\omega ps} + d^\dagger_{\omega ps} d_{\omega ps} \right)$$
$$+ \ C_{tp} \left(b^\dagger_{\zeta ps} b_{\omega ps} + b^\dagger_{\omega ps} b_{\zeta ps} + d^\dagger_{\zeta ps} d_{\omega ps} + d^\dagger_{\omega ps} d_{\zeta ps} \right) \Big]$$

$$A_{tp} = \frac{p^{\perp\, 2} + \mu_t^2}{p^+} \qquad B_{tp} = \frac{p^{\perp\, 2} + \nu_t^2}{p^+} \qquad C_{tp} = \frac{m_t^2}{p^+}$$

$$\mu_0^2 = \mu^2 + m^2 \qquad \nu_0^2 = \nu^2 + m^2 \qquad m_0^2 = m(\mu + \nu)$$

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$$\mathcal{P}_t^{-\,\prime} = \Big[[\mathcal{P}_f^-, \mathcal{P}_{Pt}^-], \mathcal{P}_t^- \Big]$$

$$\mathcal{P}_f^- = \sum\!\!\!\!\!\!\int_{ps} \left[p_\mu^- \left(b_{\zeta ps}^\dagger\, b_{\zeta ps} + d_{\zeta ps}^\dagger\, d_{\zeta ps} \right) + p_\nu^- \left(b_{\omega ps}^\dagger\, b_{\omega ps} + d_{\omega ps}^\dagger\, d_{\omega ps} \right) \right]$$

$$\mathcal{P}_{Pt}^- = \sum\!\!\!\!\!\!\int_{ps} p^{+\,2} \left[A_{tp} \left(b_{\zeta ps}^\dagger\, b_{\zeta ps} + d_{\zeta ps}^\dagger\, d_{\zeta ps} \right) + B_{tp} \left(b_{\omega ps}^\dagger\, b_{\omega ps} + d_{\omega ps}^\dagger\, d_{\omega ps} \right) \right.$$

$$\left. + \; C_{tp} \left(b_{\zeta ps}^\dagger\, b_{\omega ps} + b_{\omega ps}^\dagger\, b_{\zeta ps} + d_{\zeta ps}^\dagger\, d_{\omega ps} + d_{\omega ps}^\dagger\, d_{\zeta ps} \right) \right]$$

$$[\mathcal{P}_f^-, \mathcal{P}_{Pt}^-] = \sum\!\!\!\!\!\!\int_{ps} C_{tp}\, p^{+\,2} (p_\mu^- - p_\nu^-)$$

$$\times \left(b_{\zeta ps}^\dagger\, b_{\omega ps} - b_{\omega ps}^\dagger\, b_{\zeta ps} + d_{\zeta ps}^\dagger\, d_{\omega ps} - d_{\omega ps}^\dagger\, d_{\zeta ps} \right)$$

$$A'_{tp} = 2p^{+\,2}\,(p^-_\mu - p^-_\nu)\,C^2_{tp}$$

$$B'_{tp} = -2p^{+\,2}\,(p^-_\mu - p^-_\nu)\,C^2_{tp}$$

$$C'_{tp} = -p^{+\,2}\,(p^-_\mu - p^-_\nu)\,(A_{tp} - B_{tp})\,C_{tp}$$

$$\left(\frac{p^{\perp\,2} + \mu_t^2}{p^+}\right)' = 2p^{+\,2}\,\left(\frac{p^{\perp\,2} + \mu^2}{p^+} - \frac{p^{\perp\,2} + \nu^2}{p^+}\right)\,\left(\frac{m_t^2}{p^+}\right)^2$$

$$\left(\frac{p^{\perp\,2} + \nu_t^2}{p^+}\right)' = -2p^{+\,2}\,\left(\frac{p^{\perp\,2} + \mu^2}{p^+} - \frac{p^{\perp\,2} + \nu^2}{p^+}\right)\,\left(\frac{m_t^2}{p^+}\right)^2$$

$$\left(\frac{m_t^2}{p^+}\right)' = -p^{+\,2}\,\left(\frac{p^{\perp\,2} + \mu^2}{p^+} - \frac{p^{\perp\,2} + \nu^2}{p^+}\right)\,\left(\frac{p^{\perp\,2} + \mu_t^2}{p^+} - \frac{p^{\perp\,2} + \nu_t^2}{p^+}\right)\,\left(\frac{m_t^2}{p^+}\right)$$

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$$\left(\mu_t^2\right)' = 2\ \left(\mu^2 - \nu^2\right)\ \left(m_t^2\right)^2$$

$$\left(\nu_t^2\right)' = -2\ \left(\mu^2 - \nu^2\right)\ \left(m_t^2\right)^2$$

$$\left(m_t^2\right)' = -\left(\mu^2 - \nu^2\right)\ \left(\mu_t^2 - \nu_t^2\right)\ m_t^2$$

$$\begin{bmatrix} \mu_t^2 & m_t^2 \\ m_t^2 & \nu_t^2 \end{bmatrix}' = \left[\left[\begin{bmatrix} \mu^2 & 0 \\ 0 & \nu^2 \end{bmatrix}, \begin{bmatrix} 0 & m_t^2 \\ m_t^2 & 0 \end{bmatrix}\right], \begin{bmatrix} \mu_t^2 & m_t^2 \\ m_t^2 & \nu_t^2 \end{bmatrix}\right]$$

$$\mu_t^2 = m^2 + \frac{1}{2}(\mu^2 + \nu^2) + \frac{1}{2}\,\delta\mu_t^2$$

$$\nu_t^2 = m^2 + \frac{1}{2}(\mu^2 + \nu^2) - \frac{1}{2}\,\delta\mu_t^2$$

$$\delta\mu_t^2 = (\mu^2 - \nu^2)\,\frac{\cosh x_t + \epsilon \sinh x_t}{\cosh x_t + \epsilon^{-1} \sinh x_t}$$

$$m_t^2 = \frac{m(\mu + \nu)}{\cosh x_t + \epsilon^{-1} \sinh x_t}$$

$$x_t = (\mu^2 - \nu^2)^2\,\epsilon\,t \qquad \epsilon = \sqrt{1 + [2m/(\mu - \nu)]^2}$$

$$M_t^2 = \begin{bmatrix} \mu_t^2 & m_t^2 \\ m_t^2 & \nu_t^2 \end{bmatrix} = \begin{bmatrix} m_{1t} & m_{It} \\ m_{It} & m_{2t} \end{bmatrix}^2$$

$$\lim_{t\to\infty} \begin{bmatrix} m_{1t} & m_{It} \\ m_{It} & m_{2t} \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$m_{1,2}$ are the mass eigenvalues of the IF re-quantization.

Quantum solution:

States of free fermions with masses m_1 and m_2

in empty vacuum form a Poincaré invariant spectrum.

What did the RGPEP do?

$$\psi_t = \mathcal{U}_t \, \psi_0 \, \mathcal{U}_t^\dagger \qquad b_{tp} = \mathcal{U}_t \, b_{0p} \, \mathcal{U}_t^\dagger \qquad d_{tp} = \mathcal{U}_t \, d_{0p} \, \mathcal{U}_t^\dagger$$

$$\mathcal{H}_t(b_t, d_t) = \mathcal{H}_0(b_0, d_0) \qquad b, d \leftrightarrow q$$

$$\mathcal{H}_0(q_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_0(i_1, \ldots, i_n) \; q^\dagger_{0i_1} \cdots q_{0i_n}$$

$$\mathcal{H}_t(q_t) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_t(i_1, \ldots, i_n) \; q^\dagger_{ti_1} \cdots q_{ti_n}$$

$$\mathcal{H}_t(q_0) = \mathcal{U}_t^\dagger \, \mathcal{H}_0(q_0) \, \mathcal{U}_t \qquad \mathcal{H}'_t(q_0) = [\mathcal{G}_t(q_0), \mathcal{H}_t(q_0)]$$

$$\mathcal{G}_t = -\mathcal{U}_t^\dagger \mathcal{U}'_t = [\mathcal{H}_f, \mathcal{H}_{Pt}] \qquad \mathcal{U}_t = T \exp\left(- \int_0^t d\tau \, \mathcal{G}_\tau \right)$$

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The RGPEP in general QFT

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$$\mathcal{G}_t = [\mathcal{H}_f, \mathcal{H}_{Pt}]$$

$$\mathcal{H}_f = \sum_i p_i^- \, q_{0i}^\dagger q_{0i} \qquad\qquad p_i^- = \frac{p_i^{\perp\,2} + m_i^2}{p_i^+}$$

$$\mathcal{H}_t(q_0) = \sum_{n=2}^{\infty} \sum_{i_1,i_2,...,i_n} c_t(i_1,...,i_n) \; q_{0i_1}^\dagger \cdots q_{0i_n}$$

$$\mathcal{H}_{Pt}(q_0) = \sum_{n=2}^{\infty} \sum_{i_1,i_2,...,i_n} c_t(i_1,...,i_n) \; \left(\frac{1}{2}\sum_{k=1}^{n} p_{i_k}^+\right)^2 \; q_{0i_1}^\dagger \cdots q_{0i_n}$$

$$\boxed{\mathcal{H}_t' = [[\mathcal{H}_f, \mathcal{H}_{Pt}], \mathcal{H}_t] \qquad \mathcal{H}_0 = \mathcal{H}_{canonical} + \mathcal{H}_{CT}}$$

Absence of fermion tachyons

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$$\text{bosons} \quad \to \quad \text{fermions}$$

$$\begin{bmatrix} \mu^2 & m^2 \\ m^2 & \nu^2 \end{bmatrix} \to \begin{bmatrix} \mu & m \\ m & \nu \end{bmatrix}^2 = \begin{bmatrix} m^2+\mu^2 & m(\mu+\nu) \\ m(\mu+\nu) & m^2+\nu^2 \end{bmatrix}$$

$$\lim_{t\to\infty} \begin{bmatrix} m_{1t} & m_{It} \\ m_{It} & m_{2t} \end{bmatrix}^2 = \begin{bmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{bmatrix} > 0$$

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Conclusion

- The RGPEP is a new tool for studying QFT.
- It passes tests of solving elementary boson and fermion theories.
- In the tests, the RGPEP works around the vacuum problem, $|\Omega\rangle = |0\rangle$, and provides information about what happens when mass mixing interaction terms are large, $|m| > \sqrt{\mu\nu}$.
- Boson theories collapse, fermions are chirally rotated.

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