Heavy meson decay in three-mesons and FSI

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Outline

CP Violation and FSI

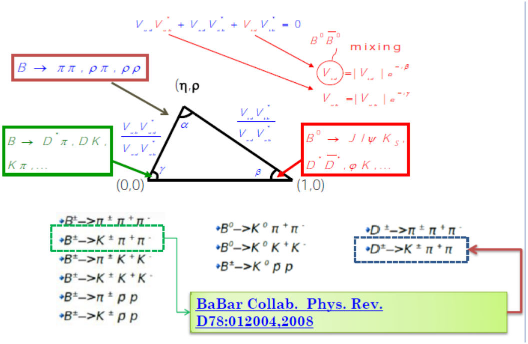


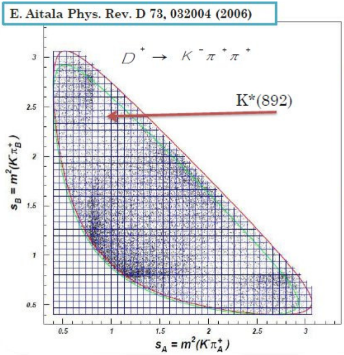
Figure: CKM angles and decays.

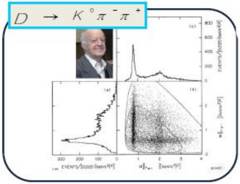
Violation of CP in $B^{\pm} \to \pi^{\pm} K^{\pm} K^{\mp}$ (J. Miranda for the LHCb Collaboration, 7th conference in the CKM series, arXiv:1301.0283)

Outline

Dalitz Plot: $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$

E791 Collaboration





$$s_A = (p_K + p_{\pi_A})^2$$

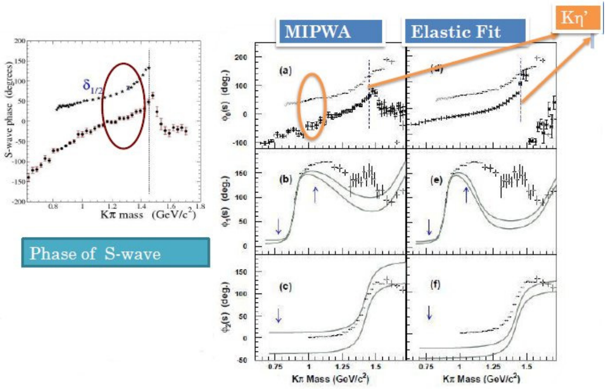
$$s_B = (p_K + p_{\pi_B})^2$$

$$s_C = (p_{\pi_A} + p_{\pi_B})^2$$

$$\mathcal{A} = \sum_L \left(a_L(s_{K\pi}) e^{i\phi_L(s_{K\pi})} P_L + a_L(s_{K\pi'}) e^{i\phi_L(s_{K\pi'})} P_L \right)$$

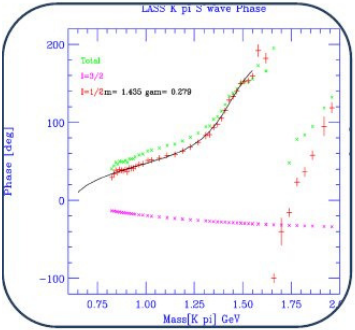
E791 and LASS

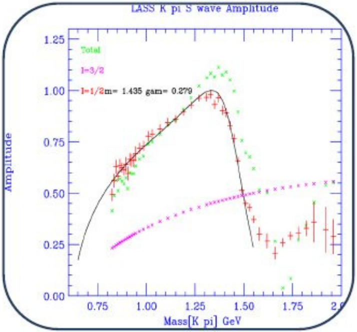
D. Aston et al., Nucl. Phys. B296(1988)493 (LASS): $K^+ p \to K^+ \pi^+ n$ and $K^- p \to K^- \pi^- \Delta^{++}$
E.M. Aitala et al. (E791 Collaboration), Phys. Rev. D73,032004(2006): $D^\pm \to K^\mp \pi^\pm \pi^\pm$



$$A_L(s_A, s_B) = a_L(s_A)e^{i\phi_L(s_A)} + a_L(s_B)e^{i\phi_L(s_B)}$$

$K\pi$ s-wave phase-shift: LASS and Isospin

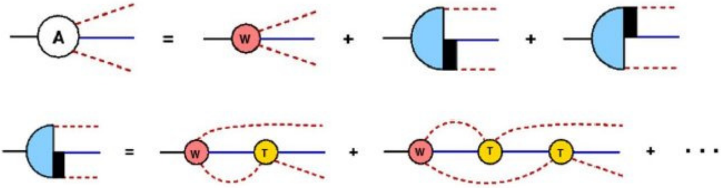




D. Aston et al., Nucl. Phys. B296(1988)493 (LASS)

Outline

Three-Body Rescattering Model: $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$



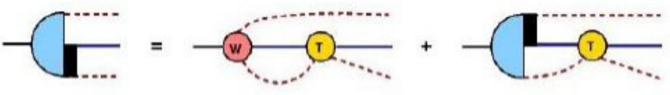
$$\mathcal{A}(k_\pi, k_{\pi'}) = a_0(s_A)e^{i\phi_0(s_A)} + a_0(s_B)e^{i\phi_0(s_B)} = D(k_\pi, k_{\pi'}) + a(m_{12}^2) + a(m_{23}^2) =$$

$$= D(k_\pi, k_{\pi'}) + \int \frac{d^4q_\pi d^4q_{\pi'}}{(2\pi)^8} T_{3,3}(k_\pi, k_{\pi'}; q_\pi, q_{\pi'}) S_\pi(q_\pi) S_\pi(q_{\pi'}) S_K(K - q_{\pi'} - q_\pi) D(q_\pi, q_{\pi'})$$

- Partonic decay amplitude: $W \equiv D(k_\pi, k_{\pi'})$
- S-wave $K\pi$ scatt. amplitude depends only on $s_{K\pi} = m_{K\pi}^2$
- Separable model for the $K\pi$ T-matrix $\Rightarrow a(m_{12}^2) = \tau(m_{12}^2)\xi(p_3)$

$D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$ 4d covariant Model with FSI

$$\tau\left((K-q)^2\right)\xi(q)$$



Spectator equations

$$\xi(k) = \xi_0(k) + \int \frac{d^4q}{(2\pi)^4}\tau\left((K-q)^2\right) S_K(K-k-q)\, S_\pi(q)\,\xi(q)$$

$$\xi_0(k) = \int \frac{d^4q}{(2\pi)^4} S_\pi(q) S_K(K-k-q) D(k,q)$$

K. S. F. F. Guimarães et al, Nucl. Phys. B Proc. Suppl 199, 341 (2010)

Chiral Model - Perturbative Solution - 2-Loops - $I_T = 3/2$

$D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$ [P.C. Magalhães et al, PRD 84, 094001 (2011)]

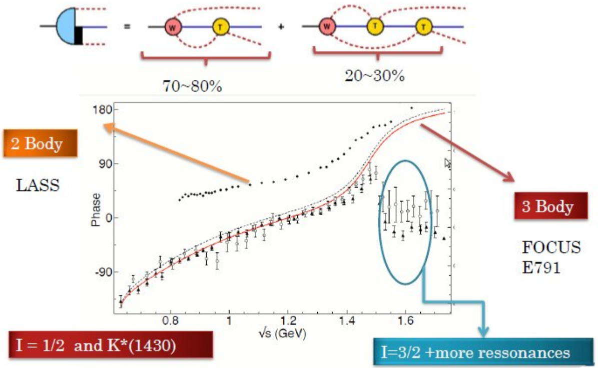
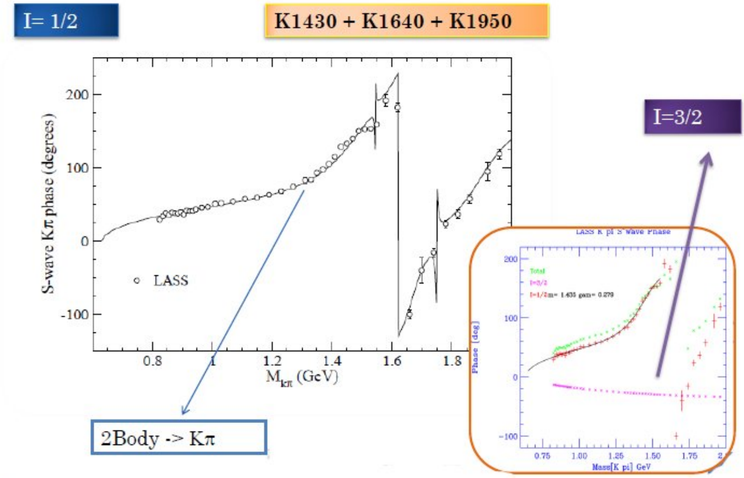


Figure: E.M.Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. **86** (2001) 765; Phys. Rev. Lett. **86** (2001) 770; Phys. Rev. Lett. **89** (2002) 121801. J. M. Link *et al.* (FOCUS Collaboration), Phys. Lett. **B585** (2004) 200; Phys. Lett. **B681** (2009) 14.

Asumption: partonic amplitude has small overlap with the final state

$$\Rightarrow a_0(s_{12})e^{i\phi_0(s_{12})} = \tau(m_{12}^2)\xi(p_3)$$

Improving the $K\pi$ scattering amplitude model



D. Aston et al., Nucl. Phys. B296(1988)493 (LASS)

Improving the $K\pi$ scattering amplitude model

$K\pi$ Isospin 1/2 S-wave S-matrix:

$$S^{1/2}_{K\pi} = \frac{k\cot\delta + i\,k}{k\cot\delta - i\,k} \prod_{r=1}^{3} \frac{M_r^2 - M_{K\pi}^2 + iz_r\bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - iz_r\Gamma_r} \qquad z_r = k\,M_r^2/(k_r\,M_{K\pi})$$

$$k\cot\delta = \frac{1}{a} + \frac{1}{2}r_0\,k^2 \qquad a = 1.6\text{GeV}^{-1} \qquad r_0 = 3.32GeV^{-1}$$

Table: Resonance parameters. († Omitted from Summary Table)

$\frac{1}{2}(0^+)$	M_r (GeV)	PDG (GeV)	Γ_r (GeV)	$\bar{\Gamma}_r$ (GeV)	PDG (GeV)
$K_0^*(1430)$	1.48	$1.425 \pm .050$	0.25	0.25	$0.27 \pm .08$
$K_0^*(1630)$	1.67	$1.629 \pm .007^\dagger$	0.1	0.1	$< .025$
$K_0^*(1950)$	1.9	$1.945 \pm 0.22^\dagger$	0.2	0.14	0.201 ± 0.086

Outline

Light-front projection of the heavy-meson decay model

- **LF projection and Quasi-Potential approach:**

Sales, TF, Carlson, Sauer, Phys. Rev. C61, 044003 (2000); C63, 064003 (2001); Marinho, TF, PoS(LC2008)036; Marinho, TF, Pace, Salmè, Sauer, Phys. Rev. **D 77**, 116010 (2008); TF, Salmè, Few-Body Syst. **49**, 163 (2011)

TRUNCATION AT THE VALENCE STATE (LO in QP expansion)

Kinematics: Decay plane transverse to z-direction (rotational invariance is preserved!)

$$\xi^i(y,\vec{k}_\perp) = \xi^i_0(y,\vec{k}_\perp) + \frac{i}{2}\int_0^{1-y}\frac{dx}{x(1-x-y)}\int\frac{d^2q_\perp}{(2\pi)^3}\left[\frac{\tau_j\left(M^2_{ik}(x,q_\perp)\right)\xi^j(x,\vec{q}_\perp)}{M^2-M^2_0(x,\vec{q}_\perp;y,\vec{k}_\perp)+i\varepsilon}+(j\leftrightarrow k)\right]$$

TF, Phys. Lett. B282, 409 (1992) (3B bound-state)

Spectator functions: isospin 1/2 and 3/2 interactions

$I_T = 3/2$ ($I_{K\pi} = 1/2, 3/2$) or $5/2$ ($I_{K\pi} = 3/2$)

Faddeev-like coupled integral equations for $D^\pm \to K^\mp \pi^\pm \pi^\pm$:

$$\xi^{I_T^z}_{I_T, I_{K\pi}}(y, k_\perp) = \langle I_T, I_{K\pi}, I_T^z | D \rangle \, \xi_0(y, k_\perp) +$$
$$+ \frac{i}{2} \sum_{I_{K\pi'}} R^{I_T^z}_{I_T, I_{K\pi}, I_{K\pi'}} \int_0^{1-y} \frac{dx}{x(1-y-x)} \int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{I_{K\pi'}}(y, k_\perp; x, q_\perp) \, \xi^{I_T^z}_{I_T, I_{K\pi'}}(x, q_\perp),$$

Recoupling coefficient: $R^{I_T^z}_{I_T, I_{K\pi}, I_{K\pi'}} = \langle I_T, I_{K\pi}, I_T^z | I_T, I_{K\pi'}, I_T^z \rangle$

Kernel: $K_{I_{K\pi'}}(y, k_\perp; x, q_\perp) = \int_0^{2\pi} d\theta \, \frac{q_\perp \, \tau_{I_{K\pi'}}\left(M^2_{K\pi'}(x, q_\perp)\right)}{M_D^2 - M^2_{0,K\pi\pi}(x, q_\perp, y, k_\perp) + i\varepsilon}.$

Driving term:

$$\xi_0(y, k_\perp) = \lambda(\mu^2) + \frac{i}{2} \int_0^1 \frac{dx}{x(1-x)} \int_0^{2\pi} d\theta \int_0^\infty \frac{dq_\perp \, q_\perp}{(2\pi)^3}$$
$$\times \left[\frac{1}{M^2_{K\pi}(y, k_\perp) - M^2_{0,K\pi}(x, q_\perp) + i\varepsilon} - \frac{1}{\mu^2 - M^2_{0,K\pi}(x, q_\perp)} \right],$$

$\lambda(0) = 0.12 + \imath 0.06$ from the fitting of $K\pi$ $I = 1/2$ S-wave phase-shift with χ-model

Faddeev-like coupled equations for the spectator functions

$D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$: $I_T = 3/2$ and $I_T = 5/2$

$$\xi^{3/2}_{3/2,1/2}(y,k_\perp) = A_w \xi_0(y,k_\perp) - \frac{i}{3}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{1/2}(y,k_\perp;x,q_\perp)\,\xi^{3/2}_{3/2,1/2}(x,q_\perp) + i\frac{\sqrt{5}}{6}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{3/2}(y,k_\perp;x,q_\perp)\,\xi^{3/2}_{3/2,3/2}(x,q_\perp)$$

$$\xi^{3/2}_{3/2,3/2}(y,k_\perp) = B_w \xi_0(y,k_\perp) + i\frac{\sqrt{5}}{6}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{1/2}(y,k_\perp;x,q_\perp)\,\xi^{3/2}_{3/2,1/2}(x,q_\perp) + \frac{i}{3}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{3/2}(y,k_\perp;x,q_\perp)\,\xi^{3/2}_{3/2,3/2}(x,q_\perp)$$

$$\xi^{3/2}_{5/2,3/2}(y,k_\perp) = C_w \xi_0(y,k_\perp) + \frac{i}{2}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{3/2}(y,k_\perp;x,q_\perp)\,\xi^{3/2}_{5/2,3/2}(x,q_\perp),$$

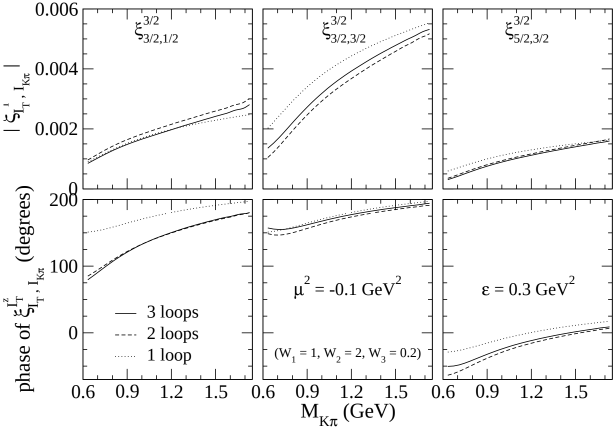
$$A_w = \frac{1}{2}\left\langle I_T = 3/2, I_{K\pi} = 1/2, I^z_T = 3/2 \right| D\rangle = \sqrt{\frac{1}{54}}(W_1 - W_2)$$

$$B_w = \frac{1}{2}\left\langle I_T = 3/2, I_{K\pi} = 3/2, I^z_T = 3/2 \right| D\rangle = \sqrt{\frac{5}{54}}(W_1 - W_2)$$

$$C_w = \frac{1}{2}\left\langle I_T = 5/2, I_{K\pi} = 3/2, I^z_T = 3/2 \right| D\rangle = \frac{W_3}{\sqrt{5}}$$

Spectator functions: 1, 2 and 3-Loops calculations

1



Outline

Motivation

Dalitz Plot

E791 and LASS

$K\pi$ s-wave phase-shift: LASS and Isospin

Three-Body Rescattering Model

$D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$ 4d Model with FSI

Chiral Model - Perturbative Solution - 2-Loops

Improving the $K\pi$ scattering amplitude model

Light-front projection of the heavy-meson decay model

Spectator functions in $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$: isospin 1/2 and 3/2 interactions

Coupled equations for the spectator functions in $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$

Results for $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$

Single-channel $I_T = 3/2$ (2-Loops)

Coupled-channel $I_T = 3/2$ and $5/2$ (2-Loops)

Summary and Outlook

Phase and amplitude separation

$$a_0(M^2_{K\pi})e^{i\Phi_0(M^2_{K\pi})} = C_1\left[\frac{A_w}{2} + \tau_{1/2}(M^2_{K\pi})\xi^{3/2}_{3/2,1/2}(k_{\pi'})\right] + C_2\left[\frac{B_w}{2} + \tau_{3/2}(M^2_{K\pi})\xi^{3/2}_{3/2,3/2}(k_{\pi'})\right] + C_3\left[\frac{C_w}{2} + \tau_{3/2}(M^2_{K\pi})\xi^{3/2}_{5/2,3/2}(k_{\pi'})\right]$$

$$C_1 = \langle K^-\pi^+\pi^+ |\, I_T = 3/2, I_{K\pi} = 1/2, I^z_T = 3/2\rangle$$

$$C_2 = \langle K^-\pi^+\pi^+ |\, I_T = 3/2, I_{K\pi} = 3/2, I^z_T = 3/2\rangle$$

$$C_3 = \langle K^-\pi^+\pi^+ |\, I_T = 5/2, I_{K\pi} = 3/2, I^z_T = 3/2\rangle$$

$$A_w = \frac{1}{2}\langle I_T = 3/2, I_{K\pi} = 1/2, I^z_T = 3/2 |\, D\rangle = \sqrt{\frac{1}{54}}(W_1 - W_2)$$

$$B_w = \frac{1}{2}\langle I_T = 3/2, I_{K\pi} = 3/2, I^z_T = 3/2 |\, D\rangle = \sqrt{\frac{5}{54}}(W_1 - W_2)$$

$$C_w = \frac{1}{2}\langle I_T = 5/2, I_{K\pi} = 3/2, I^z_T = 3/2 |\, D\rangle = \frac{W_3}{\sqrt{5}}$$

Single-channel $I_T = 3/2$ (2-Loops): $I_{K\pi} = 1/2$ + more resonances [1]

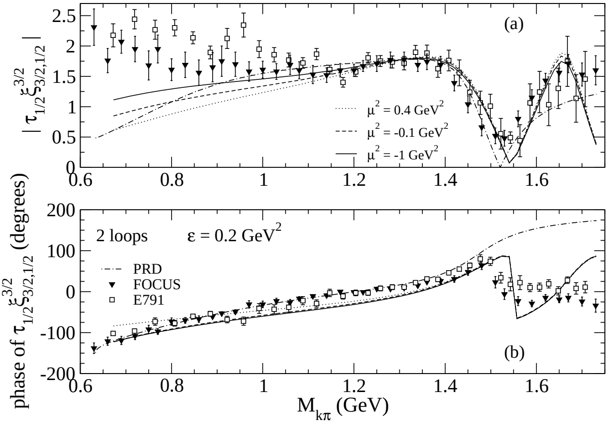


Figure: LF model with $K_0^*(1430)$, $K_0^*(1630)$, $K_0^*(1950)$: $\mu^2 = -1$GeV (solid line), $\mu^2 = -0.4$GeV (dashed line), $\mu^2 = -0.1$GeV (dotted line). Chiral-model: P.C. Magalhães et al, PRD 84, 094001 (2011) (dot-dashed line).

Coupled-channel $I_T = 3/2$ and 5/2 (2-Loops): $I_{K\pi} = 1/2$ and 3/2 + more resonances

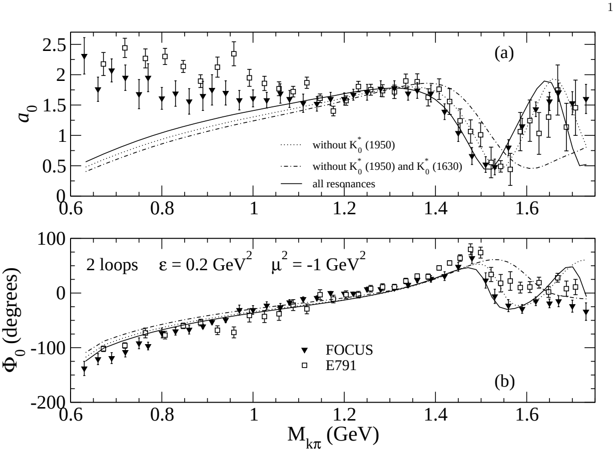


Figure: $W_1 = 1$, $W_2 = 2$ and $W_3 = 0.2$.

Outline

Summary and Outlook

Summary

- Formulation of the three-body FSI in heavy meson decay with Faddeev-like equations;
- Projection onto the LF: Dominance of the valence state;
- Importance of $I_{K\pi} = 3/2$ and $1/2$ interactions ($I_{K\pi} = 1/2$ is dominant) in $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$;
- Calculations up to 2-loops are sufficient for $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$;
- $K_0^*(1630)\ \frac{1}{2}(0^+)$ necessary with $\Gamma \lessapprox 100$MeV.

Outlook

- Introduction of interactions in L=1 and 2;
- Partonic amplitude?
- Application to $B^{\pm} \to \pi^{\pm}K^{\pm}K^{\mp}$, $B^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$, $B^{\pm} \to K^{\pm}K^{\pm}K^{\mp}$...
- CP violation and CPT constraints with 3-body FSI in the decay...