On Nucleon Spin Structure and Drell-Yan

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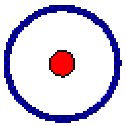
Skiathos May 21, 2013

- Introduction. Parton model spin structure.
- Quark Models Results. Interesting relations.
- PDFs. Some experimental results.
- Pretzelosity and other PDFs?
- Evolution Eq. and Interference Fragmentation.
- Drell-Yan process.
- Conclusions.

Main (twist-2) parton characteristics of hadron (integrated over k_T)

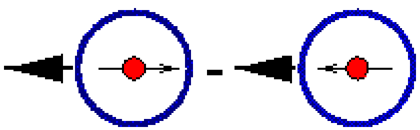


- Non-polarized PDF $f_1^a(x, Q^2)$.

-Measured for decades. Rather well known.

-Q^2-evolution, $\alpha_s(Q^2)$ extraction

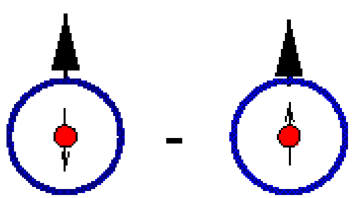
-Problem of very small *x*-behavior (bare Pomeron, BFKL-equation)



- Longitudinal spin distribution.

$$\Delta f^a \equiv f_{\rightarrow}^{a\rightarrow} - f_{\rightarrow}^{a\leftarrow} \equiv g_1^a(x, Q^2) \ .$$

-Parton content of proton spin. Main problem ΔG. Dedicated experiments (e.g. COMPASS, RHIC). -Sea spin flavour asymmetry (spectacular in DY).



- Transverse spin distribution. (transversity)

$$\delta f^a \equiv f_{\uparrow}^{a\uparrow} - f_{\uparrow}^{a\downarrow} \equiv h_1^a(x, Q^2)$$

Not measured in DIS (χ–odd).

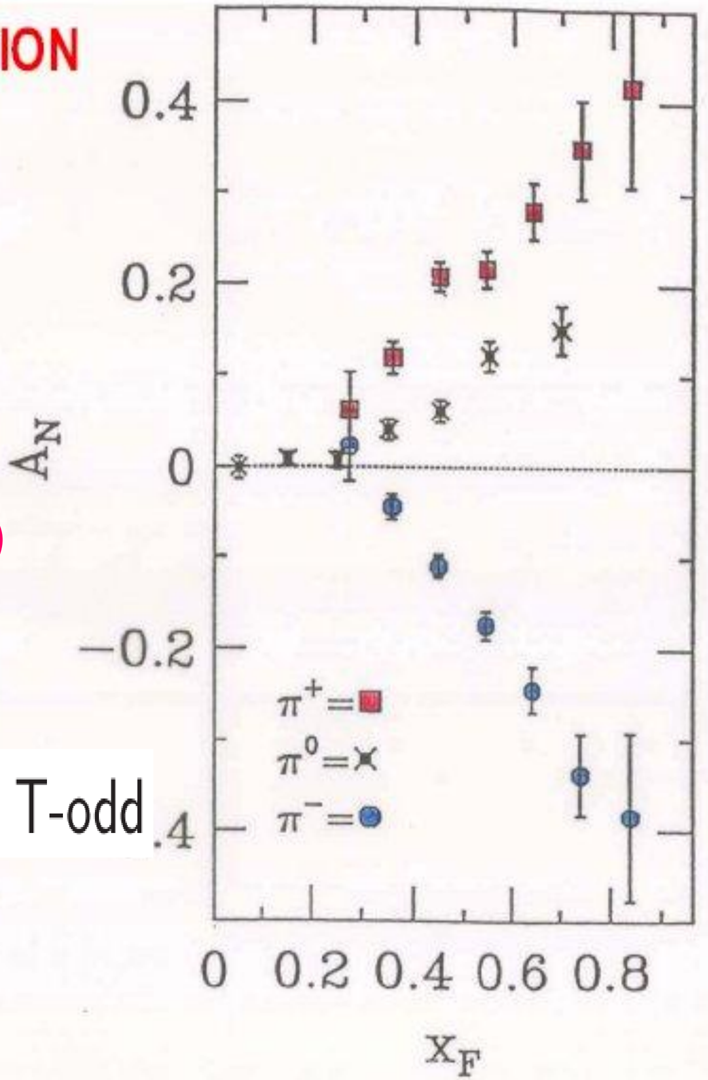
Why k_T is necessary?

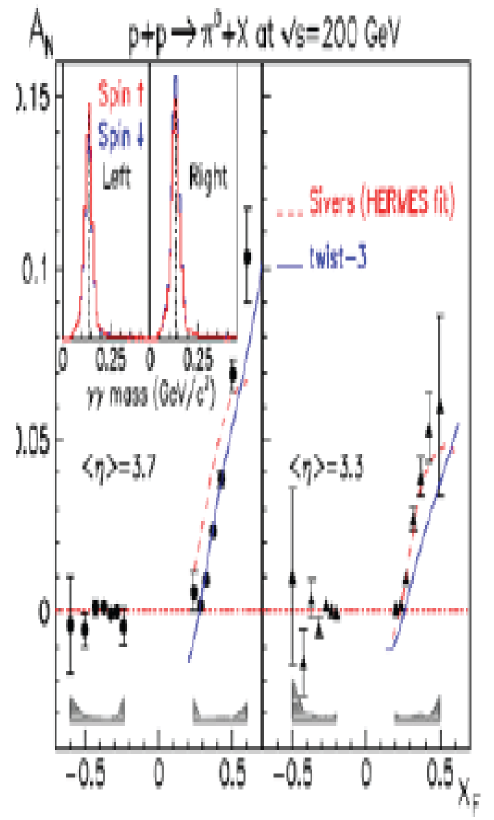
INCLUSIVE PION PRODUCTION

200 GeV Polarized Proton Beam
from Polarized Hyperon Decay
1990s Fermilab E-704
Yokosawa *et al.*
Phys Lett B264, 462 (1991)

$A_n \sim 40\%$

QCD said $A_n \sim 0$ **(naïve)**





$\sqrt{s}$ = 200 GeV [*STAR coll. (2008)*]

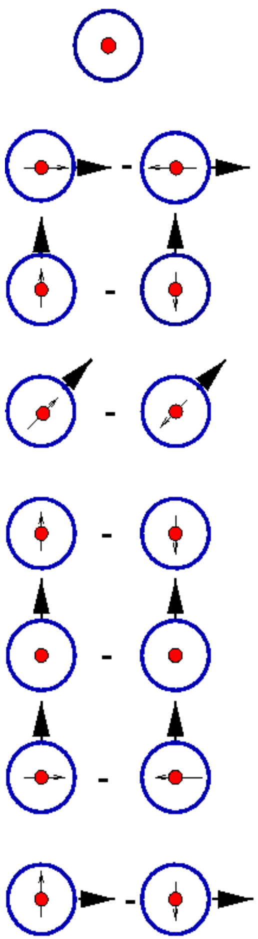
Collins PFF

$H_1^{\perp}\ \boldsymbol{\sigma}[\boldsymbol{k}\boldsymbol{P}]$ χ-odd, T-odd

Sivers PDF

$(f_{1T}^{\perp})$ $\boldsymbol{S}[\boldsymbol{k}\boldsymbol{P}]$ χ–even, T-odd **+ETQS - functions...**

New possibilities with k_T account.



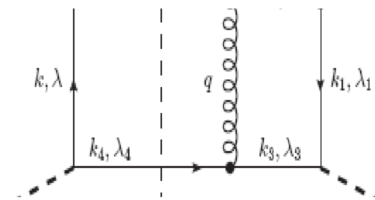
$(f_1)\ D_1$	
$(g_{1L})\ G_{1L}\ (\boldsymbol{\sigma P})(\boldsymbol{SP})/P^2$	Helicity
$(h_{1T})\ H_{1T}\ [\boldsymbol{\sigma S} - (\boldsymbol{\sigma P})(\boldsymbol{SP})/P^2]$, χ-odd	Transversity
$(h_{1T}^{\perp})\ H_{1T}^{\perp}\ \boldsymbol{\sigma}[\boldsymbol{kP}]\boldsymbol{S}[\boldsymbol{kP}]/k_T^2 P^2$, χ-odd	Pretzelosity
$(h_1^{\perp})$ $H_1^{\perp}\ \boldsymbol{\sigma}[\boldsymbol{kP}]/k_T P$, χ-odd, T-odd	Boer-Mulders Collins
$(f_{1T}^{\perp})$ $D_{1T}^{\perp}\ \boldsymbol{S}[\boldsymbol{kP}]/k_T P$, T-odd	Sivers
$(g_{1T}^{\perp})\ G_{1T}^{\perp}\ (\boldsymbol{\sigma P})(\boldsymbol{S}\boldsymbol{k}_T)/P k_T$	Worm-gear-T
$(h_{1L}^{\perp})\ H_{1L}^{\perp}\ (\boldsymbol{SP})(\boldsymbol{\sigma}\boldsymbol{k}_T)/P k_T$, χ-odd	Worm-gear-L

(T-odd **Boer-Mulders** and **Sivers** were "forbidden" by T-parity and hermiticity but reanimated by Brodsky and Collins.)

Light-front correlators $(z^+ = 0,\ p^+ = xP^+)$:

$$\phi(x,\vec{p}_T)_{ij} = \int \frac{dz^- d^2\vec{z}_T}{(2\pi)^3}\, e^{ipz}\, \langle N(P,S)|\bar{\psi}_j(0)\,\{\text{gauge link}\}\,\psi_i(z)|N(P,S)\rangle$$

$$\{\text{gauge link}\} = \mathrm{P}\exp\left[-ig\int_0^z d\xi_j \hat{A}_j(\xi)\right]_{\text{path}}$$



Sivers

helicity

Worm-gear

transversity

pretzelosity

Boer-Mulders

$$\frac{1}{2}\mathrm{tr}[\gamma^+\,\phi(x,\vec{p}_T)] = f_1 - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N} f_{1T}^\perp$$

$$\frac{1}{2}\mathrm{tr}[\gamma^+\gamma_5\,\phi(x,\vec{p}_T)] = S_L\, g_1 + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N} g_{1T}^\perp$$

$$\frac{1}{2}\mathrm{tr}[i\sigma^{j+}\gamma_5\phi(x,\vec{p}_T)] = S_T^j h_1 + \frac{\varepsilon^{jk}p_T^k}{M_N} h_1^\perp + S_L\frac{p_T^j}{M_N} h_{1L}^\perp + \boxed{\frac{(p_T^j p_T^k - \frac{1}{2}\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2} h_{1T}^\perp}$$

$f_1/g_1/h_1$ 'collinear' well/known/models, lattice, first data & extractions (Anselmino et all.)

$f_{1T}^\perp/h_1^\perp$ 'T-odd' hot!, models, data, extractions (many authors/Drell-Yan)

$g_{1T}^\perp/h_{1L}^\perp$ certain interest, related to g_1/h_1 in Wandzura-Wilczek-type relations (next slides)

$h_{1T}^\perp$ modest interest, undeserved in my view. What is that?

$\otimes D_1$ $\quad \otimes H_1^\perp$ all accessible in SIDIS and e^+e^- (Boer, Mulders, Tangerman, Kotzinian 1996-1998)

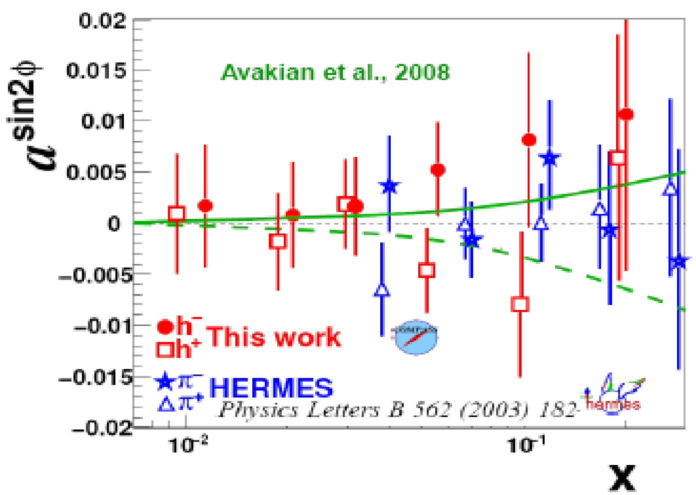
"Deeper into forest, more firewood!"

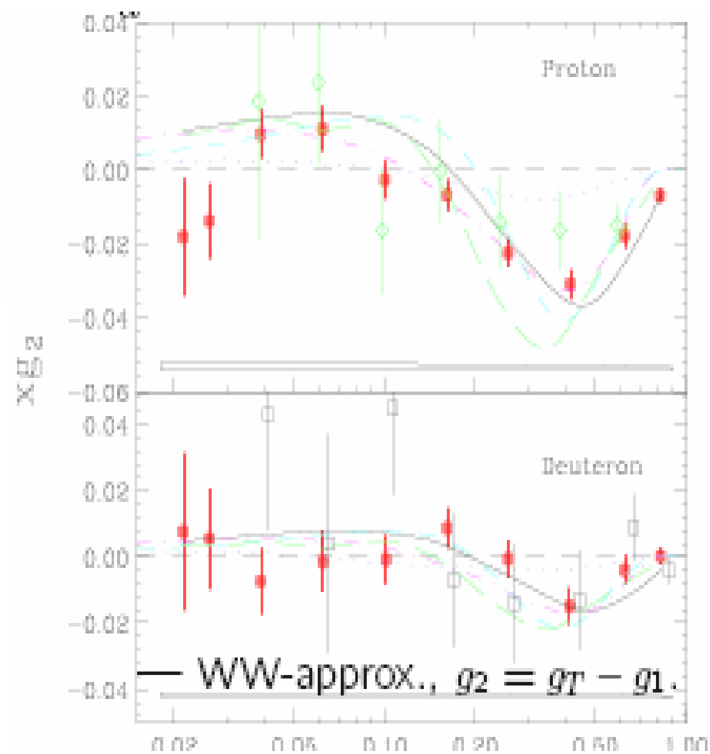
- All 8 leading twist TMDs f_1, g_1, h_1, $f_{1T}^\perp$, $h_1^\perp$, $g_{1T}^\perp$, $h_{1L}^\perp$, $h_{1T}^\perp$ **but** also 16 subleading twist TMDs g_T, h_L, e, … etc. contain independent information on the nucleon structure.
- There are *no exact relations* among TMDs!
- But having well-motivated "approximations" is valuable! At initial stage important (motivations, proposals for experiments).
- For example, Wandzura-Wilczek-type approximations (neglect pure-twist-3 & mass terms).

$$g_{1T}^{\perp(1)a}(x) \overset{!?}{\approx} x\int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y) \approx x g_T^a(x)$$

$$h_{1L}^{\perp(1)a}(x) \overset{!?}{\approx} -x^2\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^a(y) \approx -\frac{1}{2}\, x h_L^a(x)$$

+ 7 other relation twist-3 with twist-2





Specifc model relations in twist-2

(1) $f_1^q(x,\vec{p}_T^2) = N_q\, f_1(x,\vec{p}_T^2)$ with $N_u = 2,\ N_d = 1$

(2) $g_1^q(x,\vec{p}_T^2) = P_q\, g_1(x,\vec{p}_T^2)$ $P_u = \frac{4}{3},\ P_d = -\frac{1}{3}$ from SU(6)

(h_1, $h_{1T}^{\perp}$ analog)

"Bare" distributions satisfy:

(1) $f_1(x,\vec{p}_T^2) + g_1(x,\vec{p}_T^2) = 2h_1(x,\vec{p}_T^2)$

(2) $h_1(x,p_T^2) - h_{1T}^{\perp(1)}(x,p_T^2) = f_1(x,p_T^2)$

(3) $h_{1L}^{\perp(1)}(x,p_T^2) = -g_{1T}(x,p_T^2)$

All T-odd = 0

(4) $$\frac{1}{2}\left[h_{1L}^{\perp q}(x,k_\perp)\right]^2 = -\,h_1^q(x,k_\perp)\, h_{1T}^{\perp q}(x,k_\perp)$$

Hold in LCQM, bag, spectator and also in "Zavada Model"

(Pasquini et al. PRD72(2005); hep-ph:0806.2298; Avakian et al. arXiv:0805.3355; AE, Schweitzer, Teryaev, Zavada, PRD80(2009)014021, arXiv:0903.3490)

In spectator (1) and (2) only if $M^a_{qq}=M^s_{qq}$ (Jakob at al. NPA626(1997))

More general and exciting relation:

In **all** mentioned models:

$$g_1^q(x) - h_1^q(x) = h_{1T}^{\perp(1)q}(x)$$

'measure' of relativistic effects **= pretzelosity!**

Valid at low scale in large class of relativistic models,
not valid in models with gluons (Meissner, Metz, Goeke 2007),
not valid in QCD (all TMDs independent, not preserved by evolution).

More important is possible access to quark orbital momentum!

(J.She, J.Zhu,B.Ma, PRD79 (09)054008,
Bag model (Avakian, AE, Schweitzer,Yuan PRD81:074035,2010), Zavada model PoS DIS2010 253

$$L^q(x,\vec{p}_T^{\,2}) = h_1^q(x,\vec{p}_T^{\,2}) - g_1^q(x,\vec{p}_T^{\,2}) = -h_{1T}^{\perp(1)q}(x,\vec{p}_T^{\,2})$$

B.Pasquini et al. (LCQCModel) – true only for P_T-integrated

Nucleon spin balance

(Bag model)

Avakian, AE, Schweitzer, Yuan

PRD81:074035,2010

$$2L_u^3 \equiv -2\int \mathrm{d}x\, h_{1T}^{\perp(1)u}(x) = 0.46\,, \quad 2L_d^3 \equiv -2\int \mathrm{d}x\, h_{1T}^{\perp(1)d}(x) = -0.11\,,$$

$$2S_u^3 \equiv \int \mathrm{d}x\, g_1^u(x) = 0.87\,, \quad 2S_d^3 \equiv \int \mathrm{d}x\, g_1^d(x) = -0.22\,,$$

$$2J_u^3 = 2L_u^3 + 2S_u^3 = \frac{4}{3}\,, \quad 2J_d^3 = 2L_d^3 + 2S_d^3 = -\frac{1}{3}\,,$$

$$2L_Q^3 = 2L_u^3 + 2L_d^3 = 0.35\,,$$

$$2S_Q^3 = 2S_u^3 + 2S_d^3 = 0.65\,,$$

$$2J_Q^3 = 2L_Q^3 + 2S_Q^3 = 1\,.$$

1. Whether there exists connection between pretzelosity and GPDs?
2. Whether QM relation may inspire a way to rigorous connection between TMDs and OAM in QCD?
3. Could insights from models and lattice QCD be helpfull?

Rich azimuthal structure

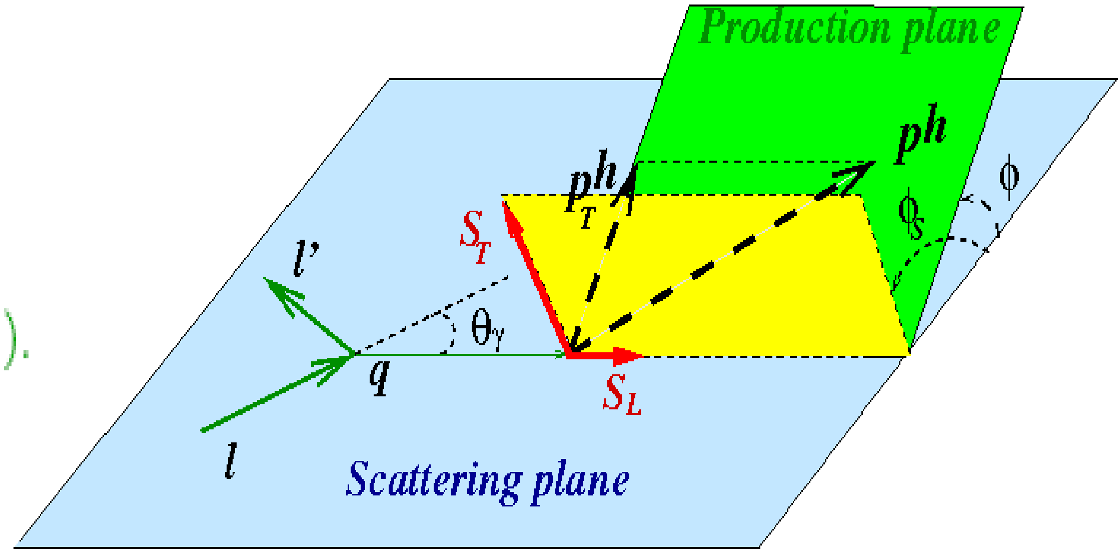
Nucleon spin structure much more complicated than thought!

SIDIS $lN \to l'hX$

Rich azimuthal structure

even at twist-2

review: Bacchetta et al., JHEP (2007).



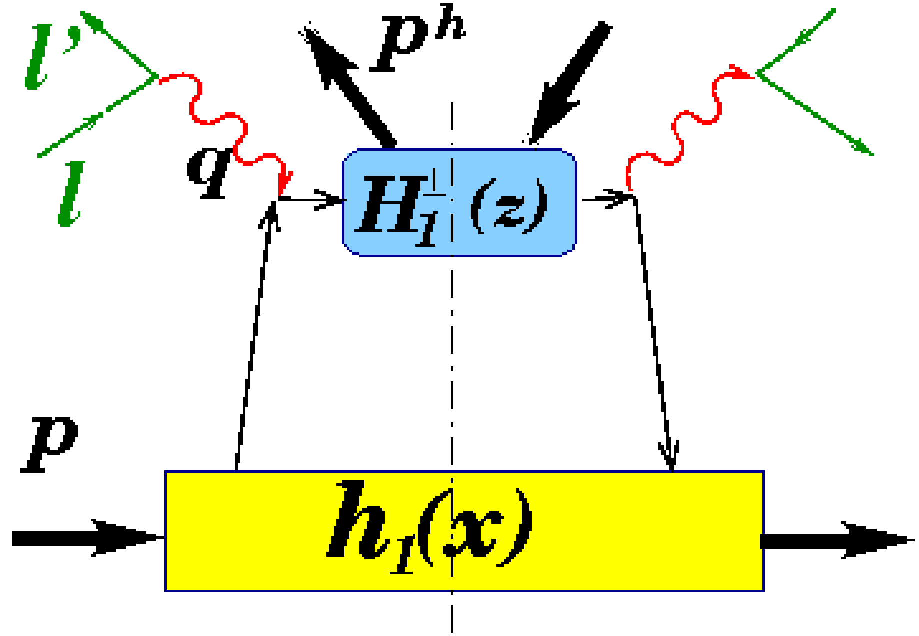
$$\frac{d\sigma}{d\phi_h} = F_{UU} + \lambda_e S_L F_{LL}$$

$$+ \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + S_L \sin(2\phi) F_{UL}^{\sin(2\phi)} + \lambda_e S_T \cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)}$$

$$+ S_T[\sin(\phi-\phi_S) F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)}]$$

+ twist-3 terms.

Leading twist approximation



$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

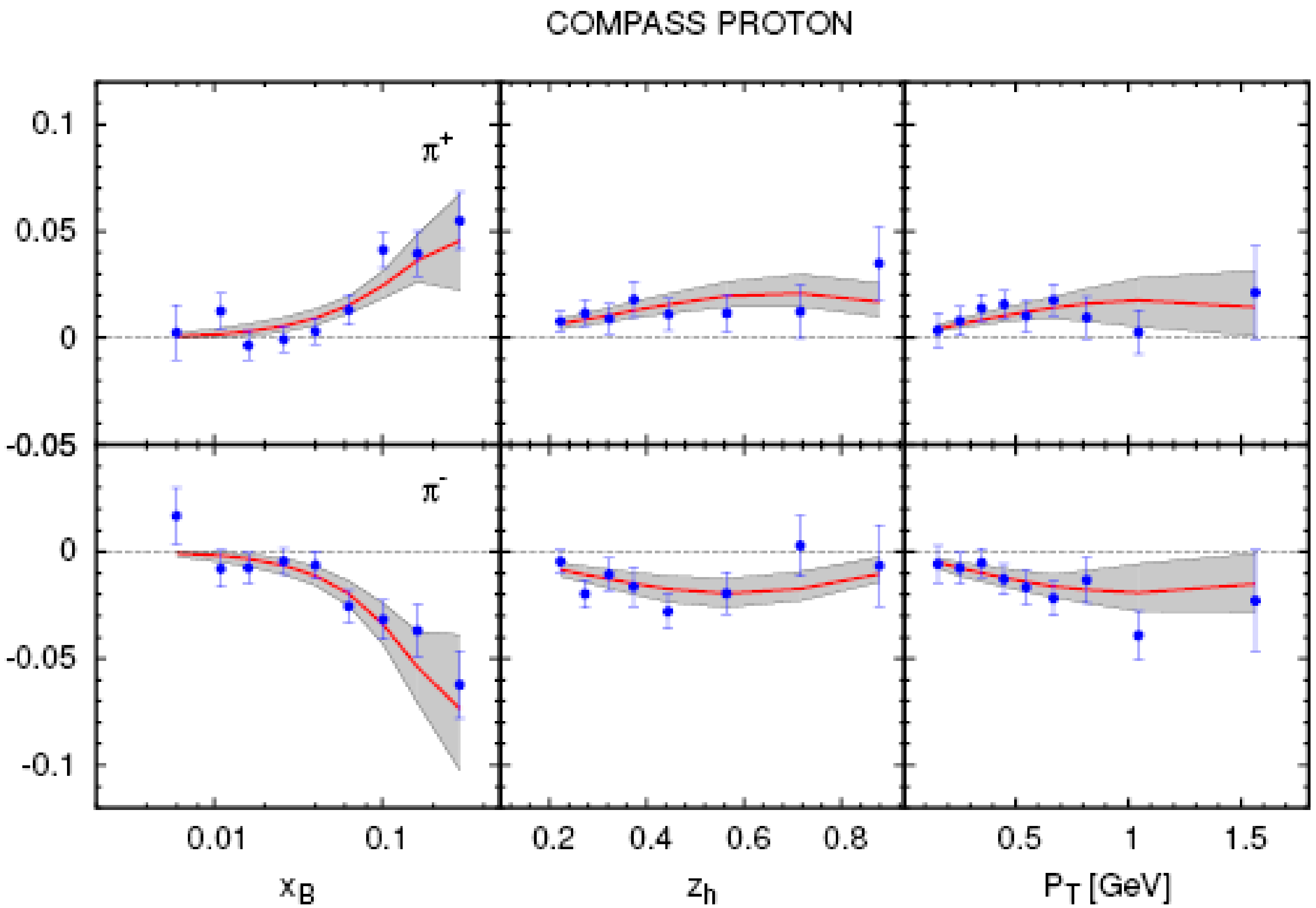
chiral-odd TMDs

e.g.

$$f \otimes D = x \sum_a e_a^2 \int d^2 p_T \, d^2 k_T \, \delta^{(2)}\left(p_T - k_T - P_{h\perp}/z\right) w(p_T, k_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

PDFs. Some experimental results

Collins asymmetry for proton from COMPASS and Belle



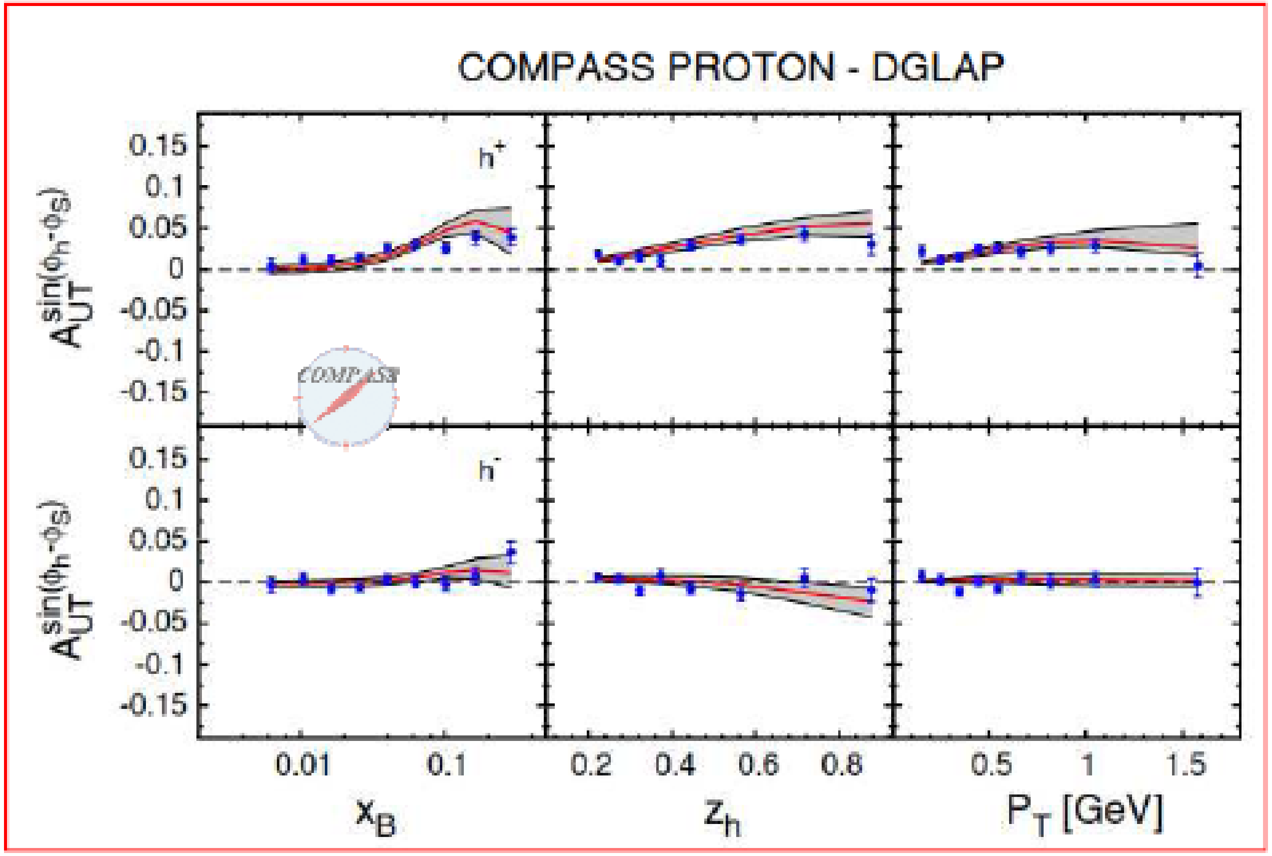
M.Anselmino at al. 1303.3822 But DGLAP evolution Eq?!

Sivers asymmetry on proton

charged hadrons, 2010 data - Q^2 evolution

M. Anselmino, M. Boglione, S. Melis PRD86 (2012) 014028

fit to HERMES p and COMPASS d and p 2010 data

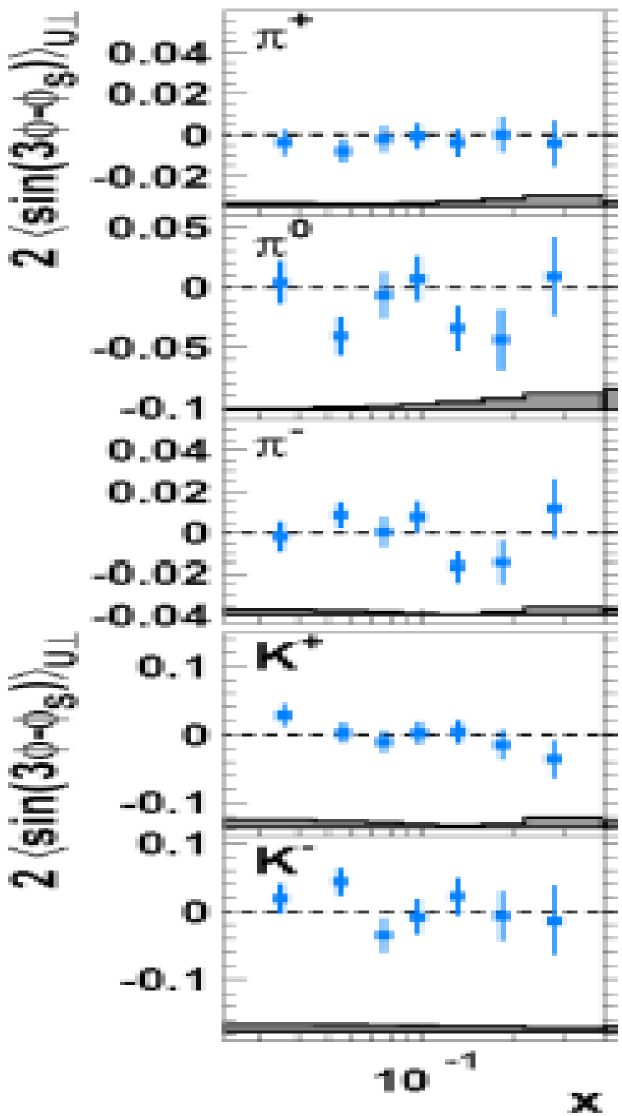


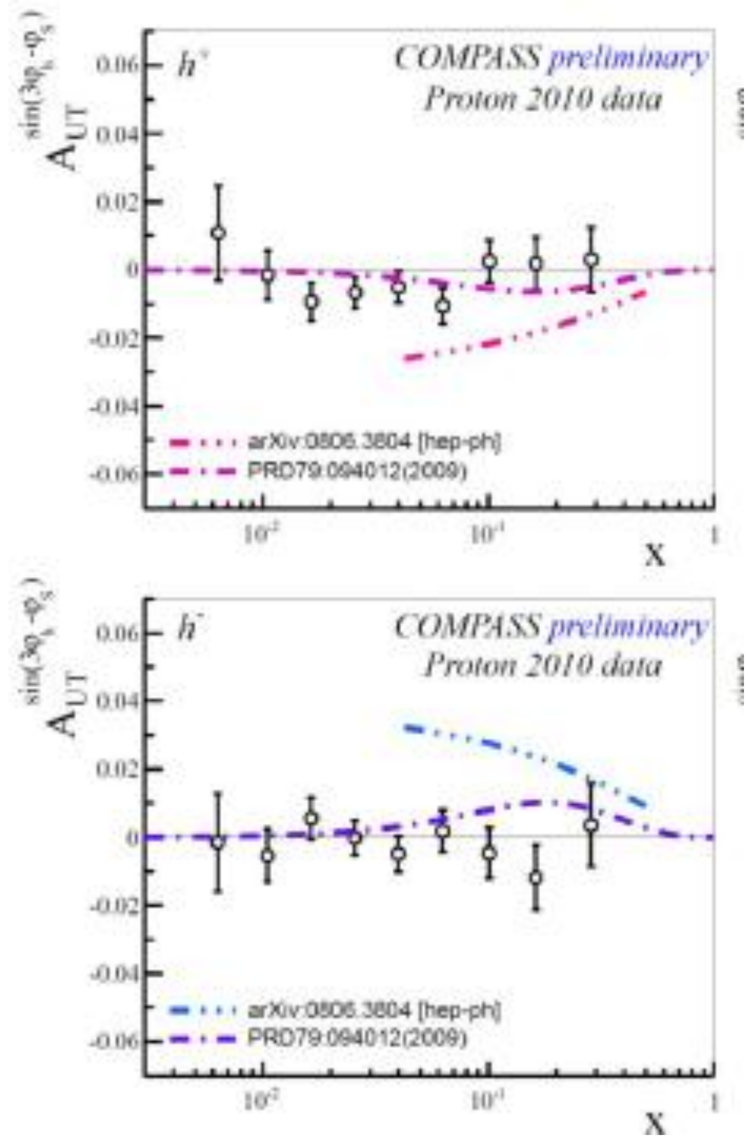
Pretzelosity in SIDIS and thory predictions

Boffi, A.E, Pasquini,Schweitzer PRD79:094012(2009)
Kotzinian arXiv:0806.3804[hep-ph]

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto \sum_q e_q^2 h_{1T}^{\perp q} \otimes H_1^{\perp q}$$

- Proton & deuteron (HERMES & COMPASS) data consistent with zero
- Or just the suppression by third power of $1/p_{hT}$?
- Experiment planned at CLAS12



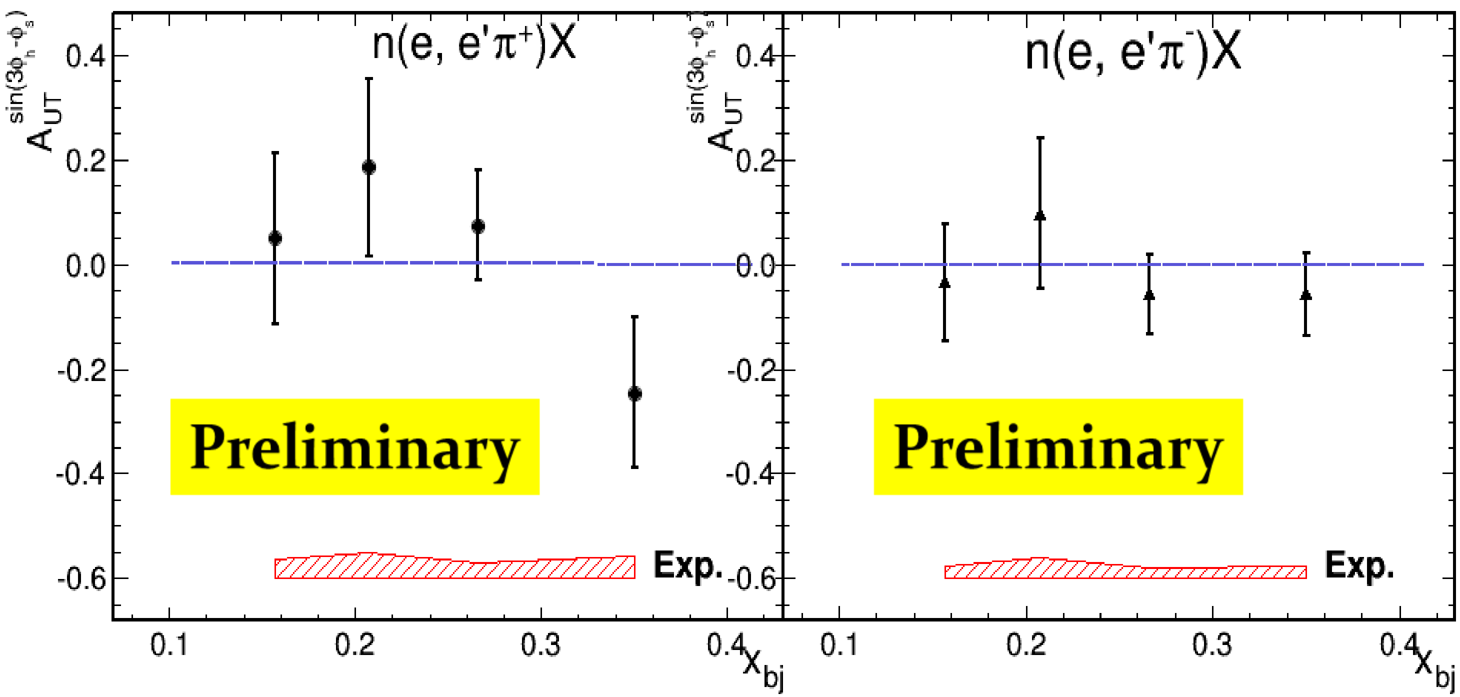


Extracted Results on Neutron

Extracted Pretzelosity Asymmetries,

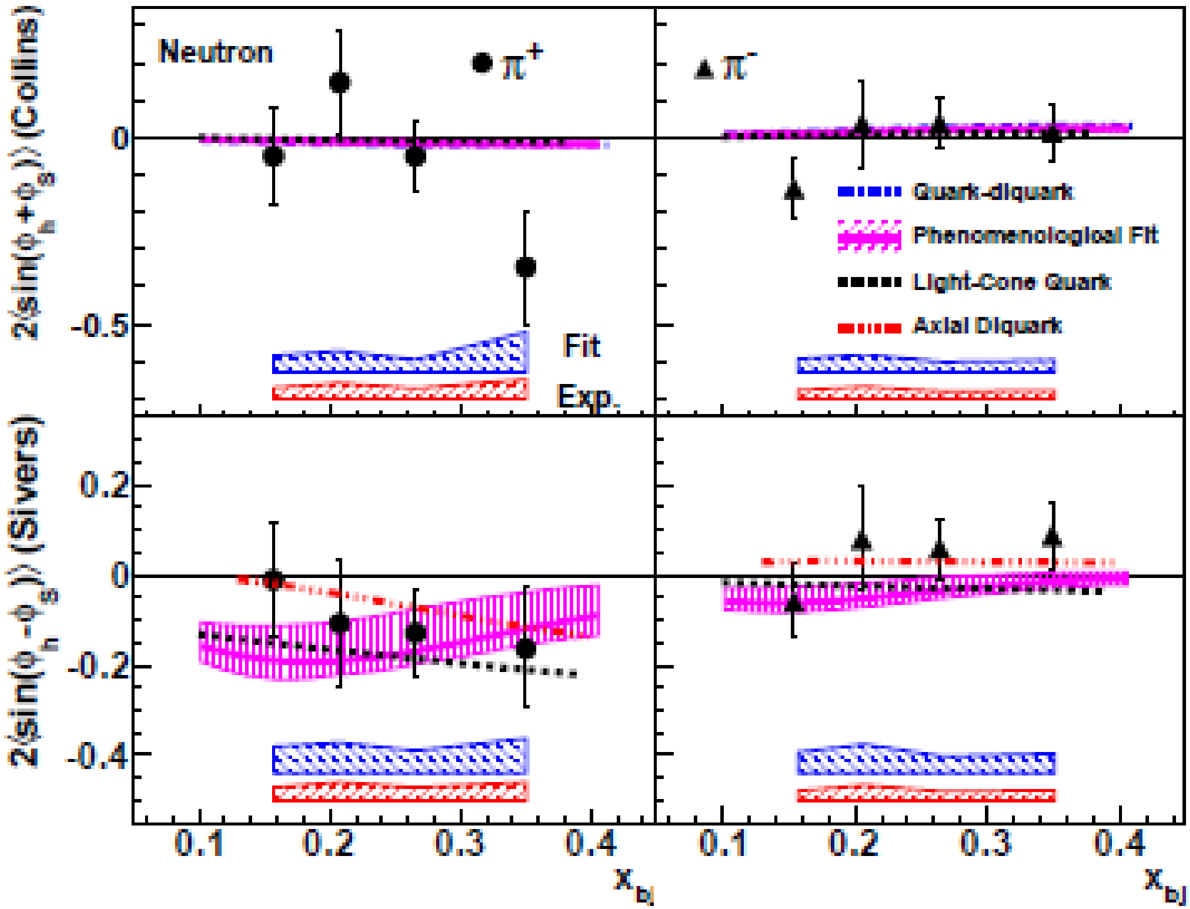
$A_{UT}\sin(\varphi_h - \varphi_s)$, on the neutron

For both π+ and π-, **consistent with zero** within uncertainties.



(Jian-Ping Chen, JLab Hall A E06-010 with a Transversely Polarized ^{3}He (n), QCD Evolution Workshop May 6-10, 2013**)**

Neutron Results with Polarized ^{3}He from JLab



Collins

asymmetries are not large, except at x=0.34

Sivers

p^{+} $(u\bar{d})$ negative

Blue band: model (fitting) uncertainties

Red band: other systematic uncertainties

Pretzelosity in SIDIS: prospects

There will be data from COMPASS proton target (small x)

Most preferable conditions:

- intermediate $x \sim (0.2 - 0.4)$
- high luminosity

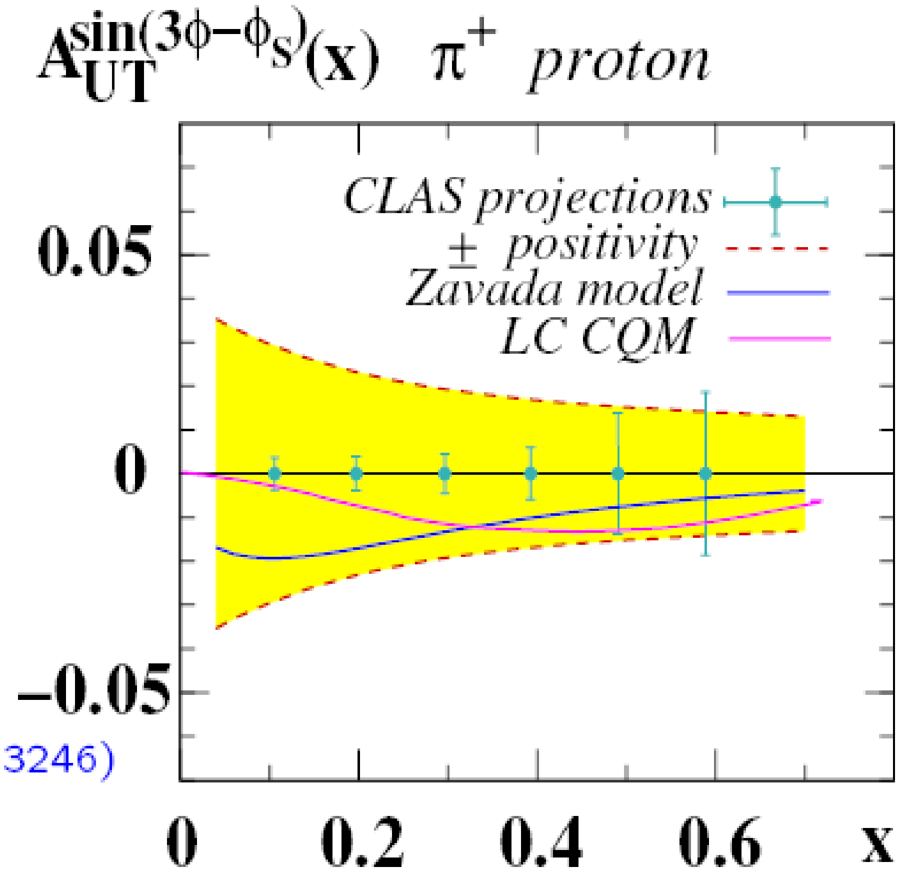
  $\rightarrow$ JLab

CLAS with 12 GeV (H.Avakian et al, LOI 12-06-108). Error projections for 2000 hours run time at CLAS12

— $|h_{1T}^{(1)\perp q}| < f_1^q - g_1^q$

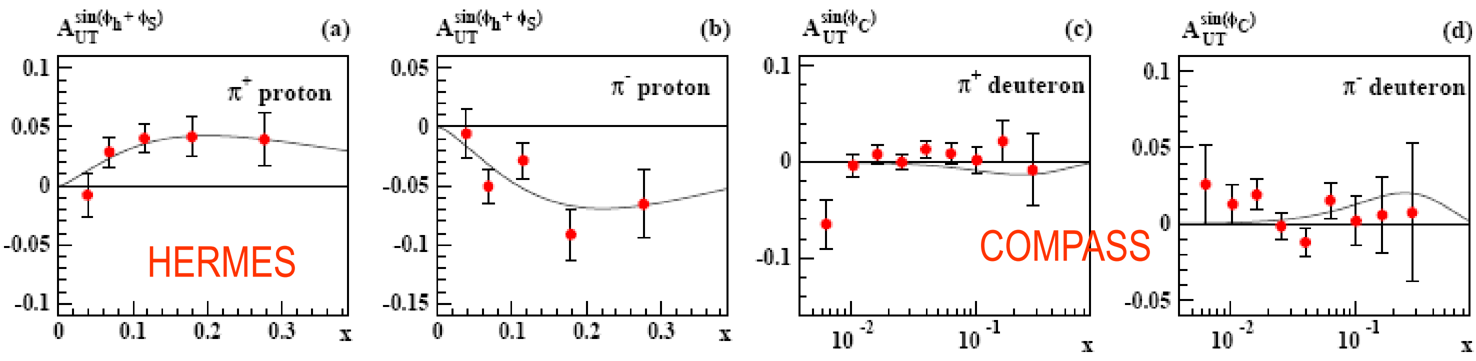
— Covariant parton model (AE,Schweitzer, Teryaev, Zavada; arXiv:0812.3246)

— Light-Cone CQM (Boffi, AE, Pasquini, Schweitzer – [PRD79(2009)094012]



Can be accessed in Drell-Yan in certain azimuthal spin asymmetries.

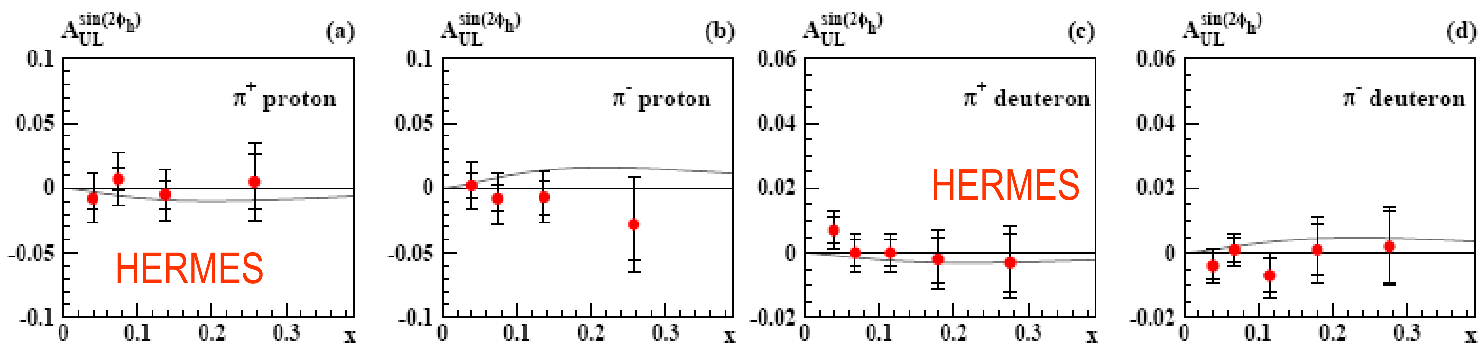
T-even asymmetries in SIDIS (LCQCModel)



Transversity

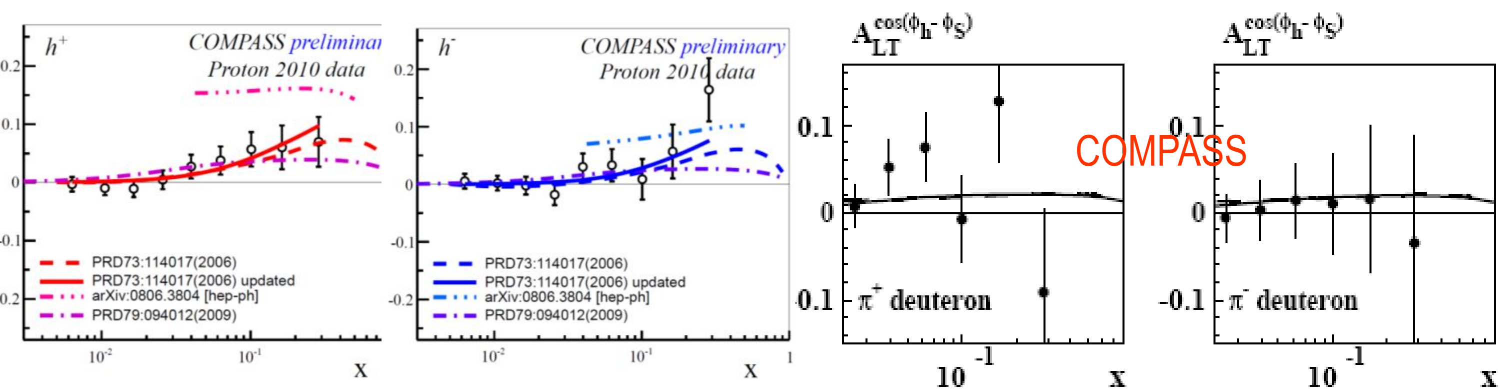
$F_{UT}^{\sin(\phi+\phi_S)} \propto \sum_a e_a^2\, h_1^a \otimes H_1^{\perp a}$

$A_{UT}^{\sin\phi_C} = -A_{UT}^{\sin(\phi_h-\phi_S)}$



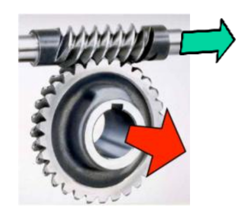
Worm-gear-L

$F_{UL}^{\sin(2\phi)} \propto \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a}$



Worm-gear-T

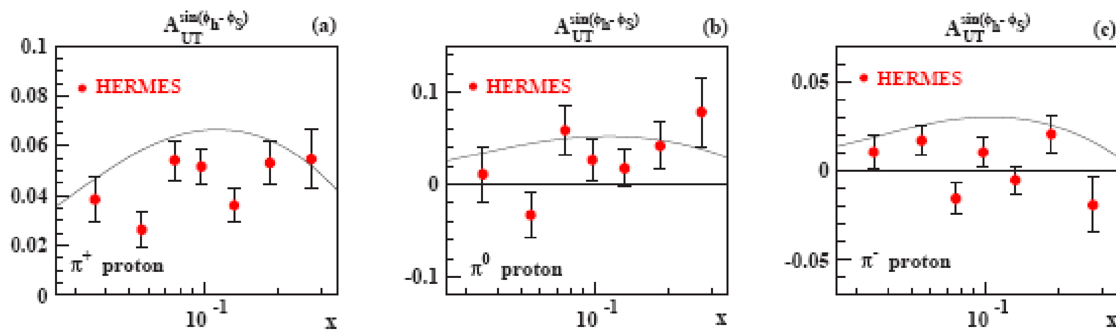
$F_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a$



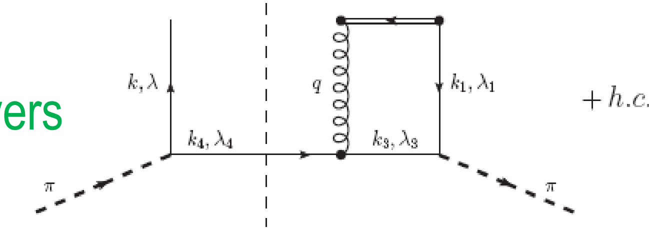
(Boffi, AE, Pasquini, Schweitzer [PRD79(2009)094012]);
Kotzinian, Parsamyan, Prokudin PRD73 (2006)114017;Kotzinian arXiv:0806.3804[hep-ph]

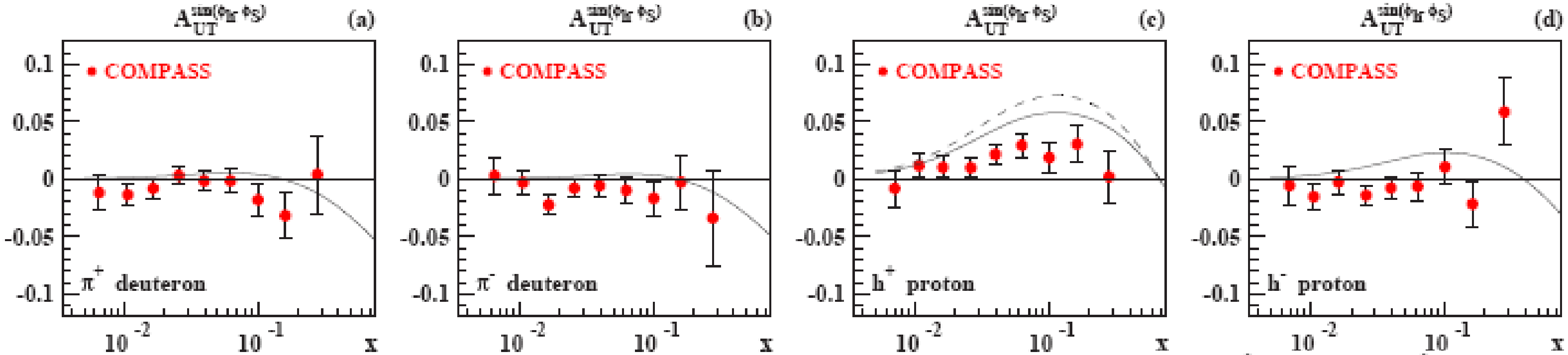
T-odd asymmetries in SIDIS (LCQCModel)

Pasquini, Schweitzer ArXiv:1103.5977



Sivers

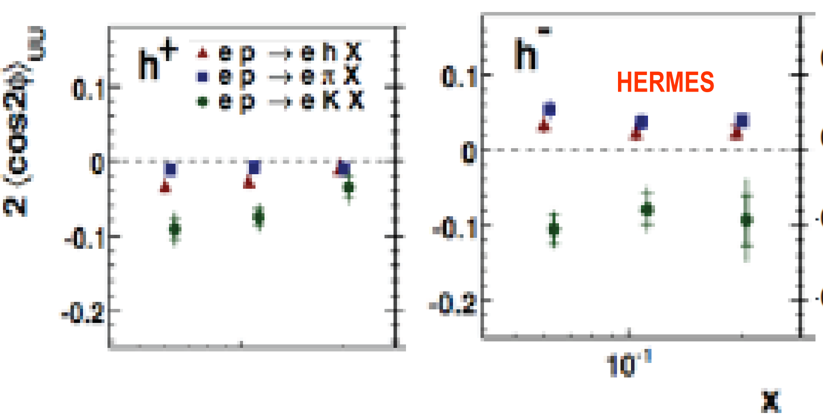


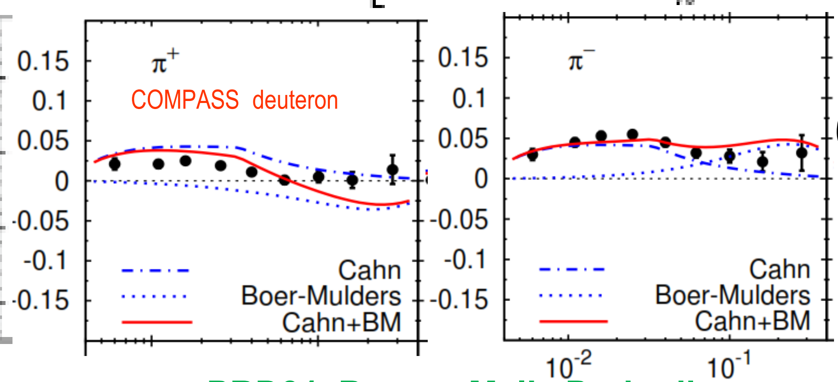


$$F_{UT}^{\sin(\phi_h-\phi_S)} = -\mathcal{C}\left[\frac{\hat{h}\cdot p_T}{M} f_{1T}^{\perp} D_1\right]$$

$$F_{UU}^{\cos(2\phi_h)} = \mathcal{C}\left[\frac{2\,(\hat{h}\cdot K_T)\,(\hat{h}\cdot p_T) - K_T\cdot p_T}{z\, m_h M} h_1^{\perp} H_1^{\perp}\right]$$

Boer-Mulders



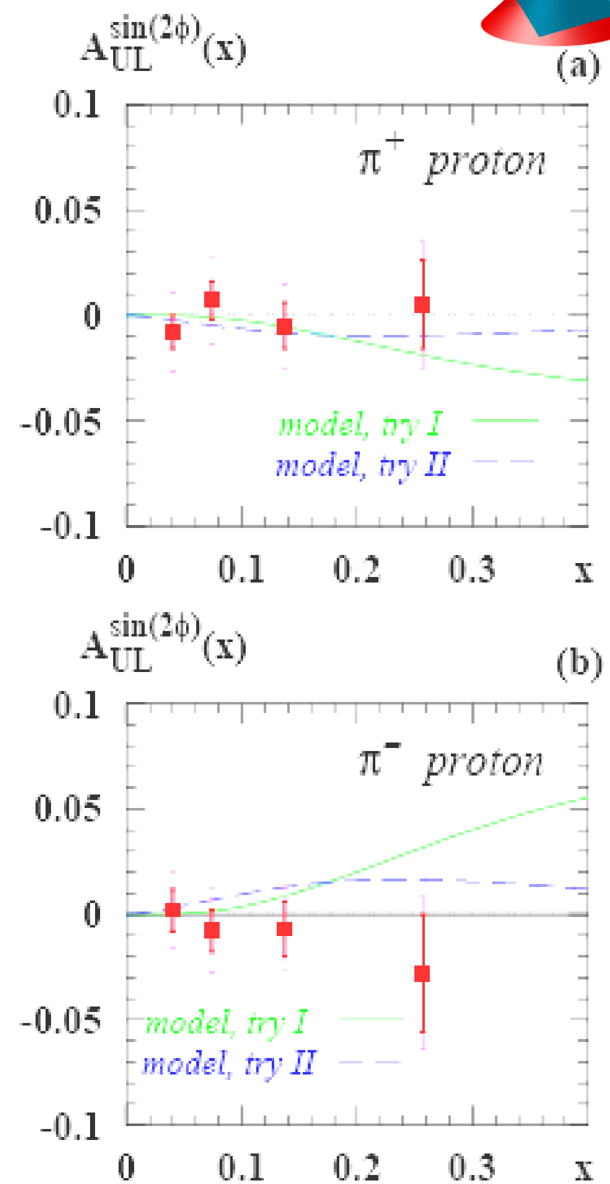


$$\mathcal{C} \equiv \otimes$$

PRD81, Barone,Melis,Prokudin

Problem with evolution.

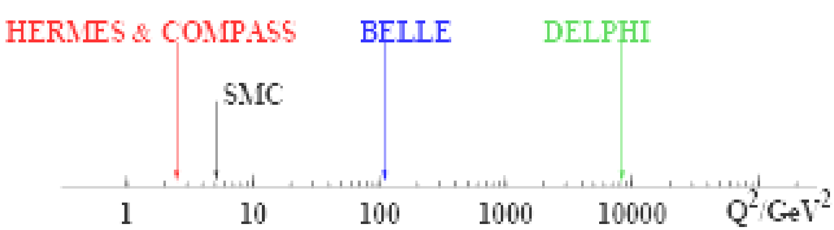
LCQM (and others models) gives TMD functions at low scale μ_0^2.
Evolution equation for $h_{1L,T}^{(1)\perp}(x,Q^2)$ yet unknown.
Two possibilities:



Model I —— no evolution
(chiral odd, no mixture with gluon)
Model II - - - evolution similar to $h_1(x,Q^2)$, i.e.

$$h_{1L}^{(1)\perp}(x,Q^2) = h_{1L}^{(1)\perp}(x,\mu_0^2)\frac{h_1(x,Q^2)}{h_1(x,\mu_0^2)}$$

Data HERMES: PRL84(00); NP.Proc.Suppl.79(99).
Seems better agrees with experiment.
Similar problem with Collins PFF $H_1^\perp$



Singlet evolution is usually assumed.
Good important problem for RG-community!

Historically TMD factorization is formulated as Collins-Soper-Sterman resummation

Collins, Soper, Sterman 1985

Proven for polarized case

Ji, Ma, Yuan 2004

Collins 2011

Alternative formulations

Cherednikov, Stefanis 2008

Echevarria, Idilbi, Scimemi 2011

Trentadue, Ceccoperi, 2008

Hautman, 2008

Equivalence with some approaches was shown in

Collins, Rogers 2012

New trend: Generalize Bessel

$$\tilde{f}(x, \boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \, f(x, \boldsymbol{p}_T^2)$$

$$= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f^a(x, \boldsymbol{p}_T^2) \; ,$$

Boer, Gamberg,Musch,Prokudin JHEP 2011
Boer, Gamberg,Musch,Prokudin

$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) \equiv n! \left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x, \boldsymbol{b}_T^2)$$

$$= \frac{2\pi \; n!}{(M^2)^n} \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x, \boldsymbol{p}_T^2)$$

News

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\, J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) = $$

$$2\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,(d\sigma^\uparrow - d\sigma^\downarrow)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;(d\sigma^\uparrow + d\sigma^\downarrow)}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) = $$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z, \mathcal{B}_T^2)}{\tilde{S}(\mathcal{B}_T^2)\,H_{UU,T}(Q^2)\sum_a e_a^2\,\tilde{f}_1^a(x, z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z, \mathcal{B}_T^2)}$$

1. Cancelation of soft factor!

Advantages of Bessel Weighting

1.“Deconvolution”-SIDIS structure function simple products $\mathcal{P}[\ldots]$ instead of $\mathcal{C}[\ldots]$;

2. Soft Factor cancels in asymmetries;

3. Circumvents the problem of ill-defined p_T-moments when $\mathcal{B}_T$ is non-zero;

4. Bessel Weight asymmetries sensitive to low P_{hT}-region

5. Cancellation of perturbative Sudakov broadening mentioned by D. Boer;

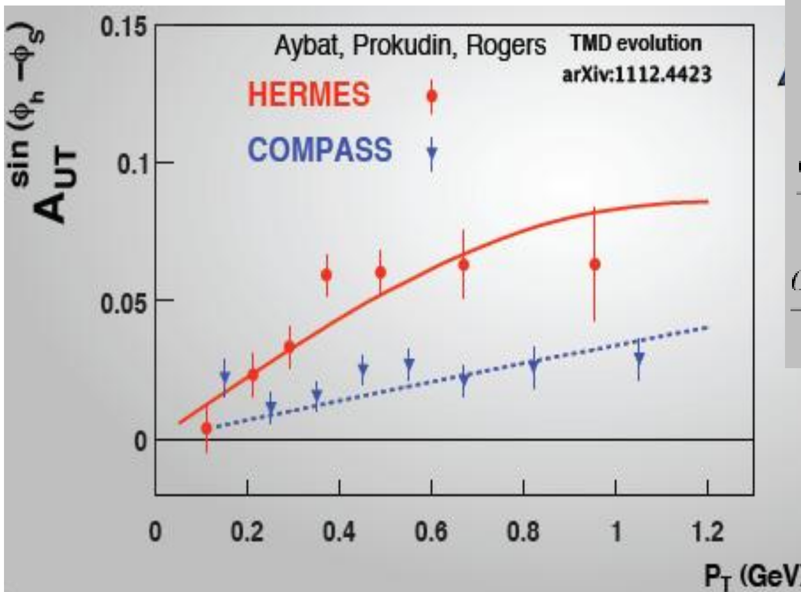
6. Possible to compare observables at different $\mathcal{B}_T$ scales.... could be useful for an EIC.

Comment: Traditional weighted asymmetry recovered but UV divergent.

$$\lim_{\mathcal{B}_T \to 0} w_1 = 2J_1(|P_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |P_{h\perp}|/zM$$

2. New evolution equation for TMDs

$$\tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F) = -(2\pi)\int_0^\infty dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$



$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu), \zeta)$$

Collins-Soper kernel in coordinate space. Process independent.

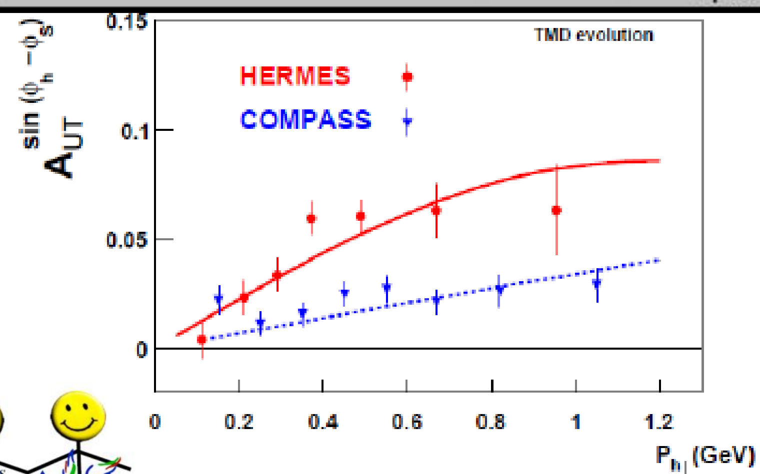
TMD: Collins 2011 Rogers, Aybat 2011 Aybat, Collins, Qiu, Rogers 2012, PRD 85, 034043

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) \times \exp\left[-g_K(b_T)\ln\frac{Q}{Q_0}\right]$$

Non perturbative

$$\times \exp\left[\ln\frac{Q}{Q_0}\tilde{K}(b_*; \mu_b) + \int_{Q_0}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right] + \int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{Q}{Q_0}\gamma_K(g(\mu'))\right]$$

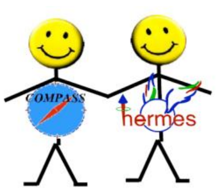
Perturbative



$$\tilde{K}(b_T, \mu) = \tilde{K}(b_*, \mu) - g_K(b_T) \quad b_{max} = 0.5\ (\text{GeV}^{-1})$$

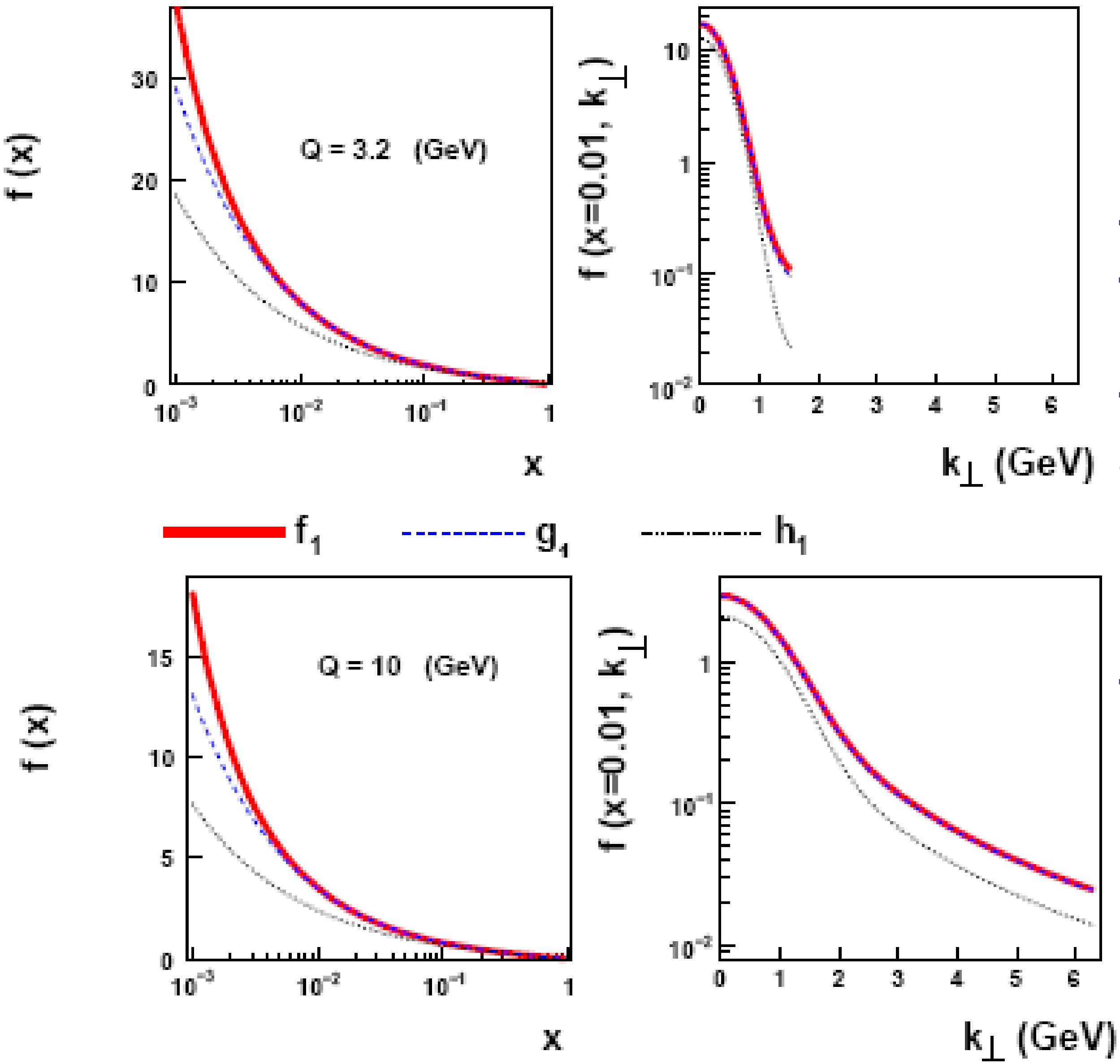
$$g_K(b_T) = \frac{1}{2} g_2 b_T^2 \qquad b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

$$g_2 \simeq 0.68\ (GeV^2)$$



COMPASS data is at <Q²>=3.6 GeV², HERMES data is at <Q²>=2.4 GeV²

Evolutions for f_1, g_1, h_1



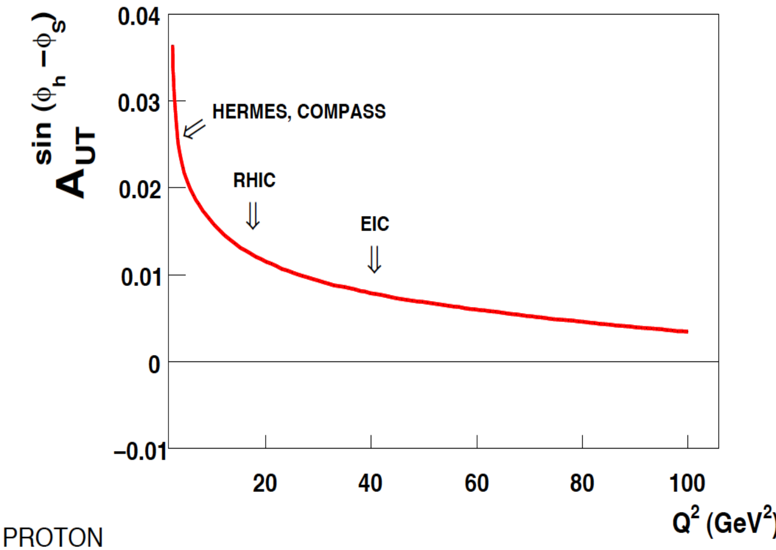
- Evolution for TMD transversity and helicity functions is calculated
- Form resemblant much to that used in phenomenology. at fixed scale
- Results are checked with CSS formalism
- Soffer bound on transversity is not violated numerically

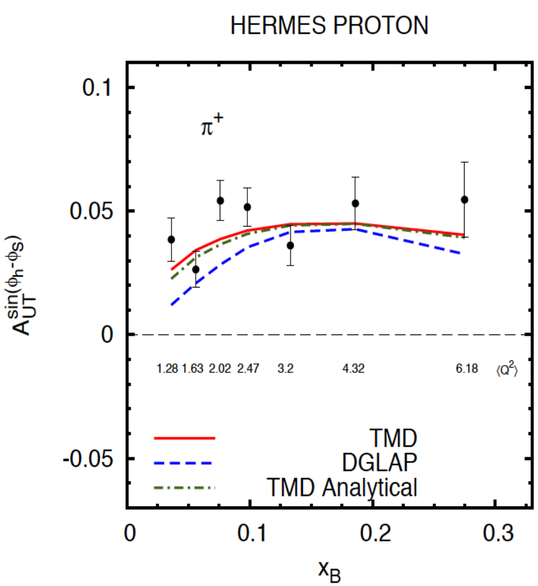
Bacchetta, Prokudin, 1303.2129

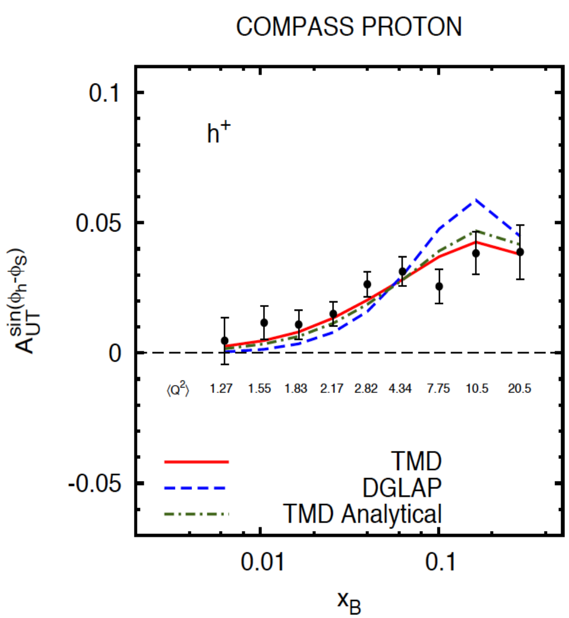
Takes this field to new level ! However, a number of caveats:

- various simplifying assumptions
- results will depend on large-b prescription: b_* only one possible choice of parameters such as g_2,b_{max} (Cancels in asymmetries.)
- matching to large-$k_T$tail.

Aybat,Prokudin,Rogers



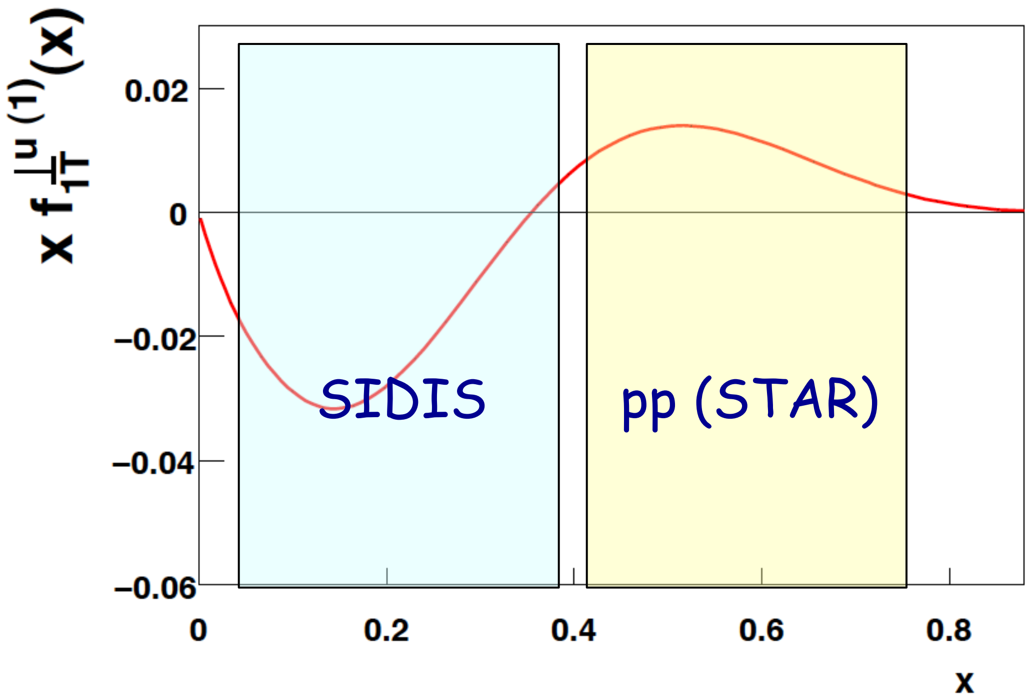


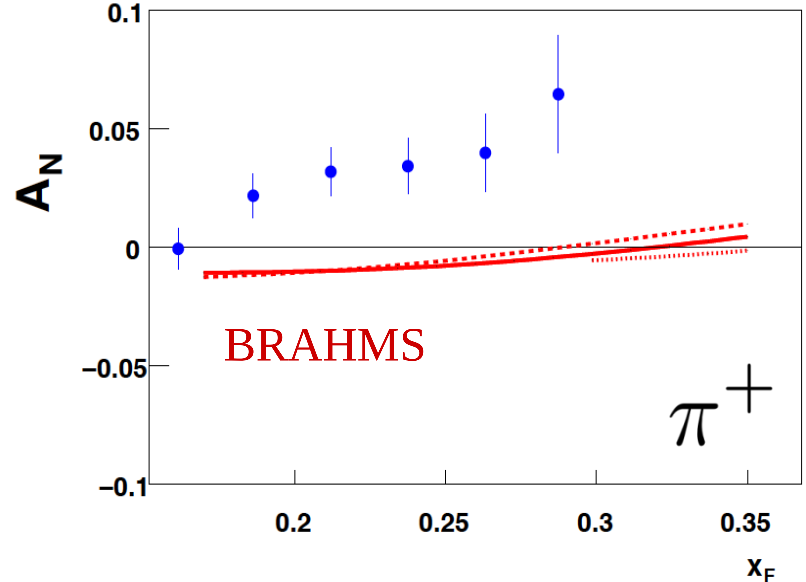


Anselmino, Boglione, Melis, PRD66(12)014028

Joint fit to SIDIS and pp data: mismatch problem

other (than Sivers) effects dominant? ep-data: Sivers function only constraint for x<0.4: Nodes? ETQS? Other?





Works (reasonably) well for SIDIS and STAR,

But fails for BRAHMS!

Strengthens case **for study of DY "sign change" !**

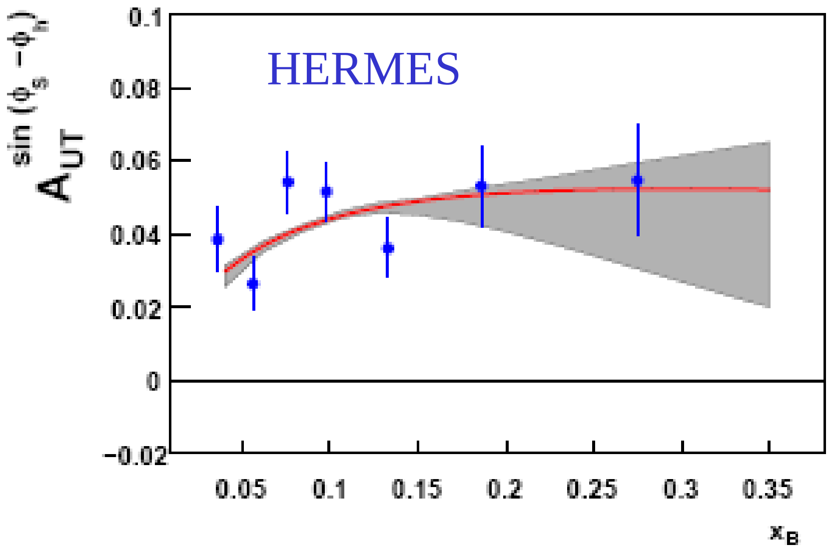
NICA, COMPASS, E906, W bosons at RHIC

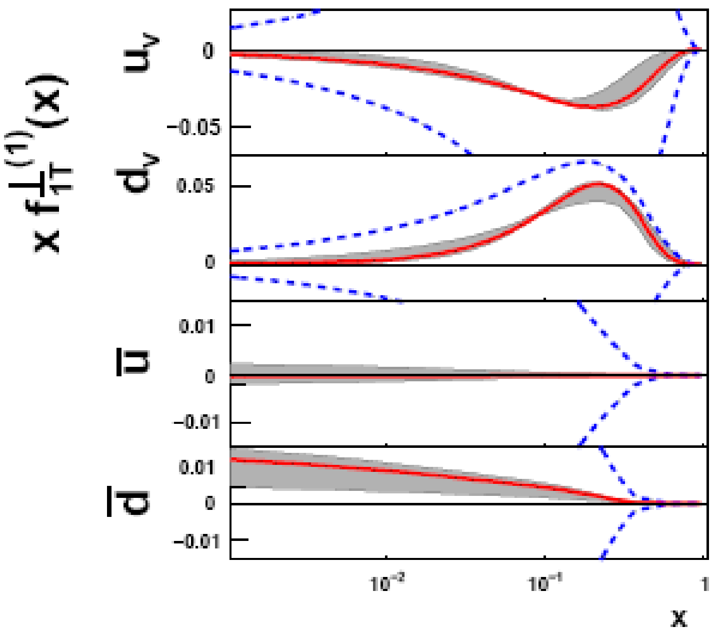
Kang,Prokudin PRD85(2012)07408

Process-dependence of Sivers effect

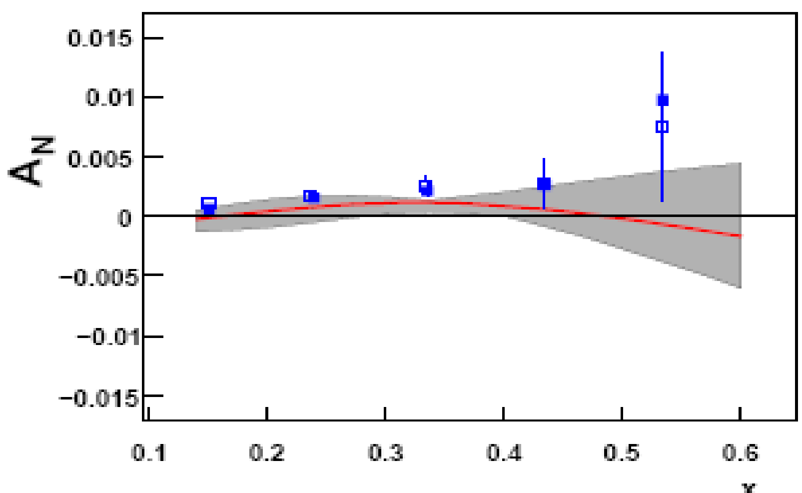
Gamberg, Kang, Prokudin: 1302.3218

Fit HERMES and COMPASS data

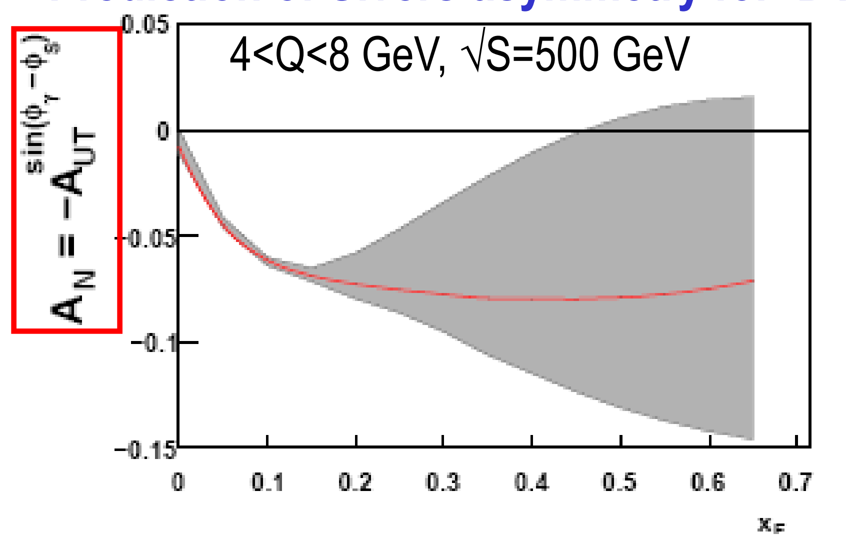


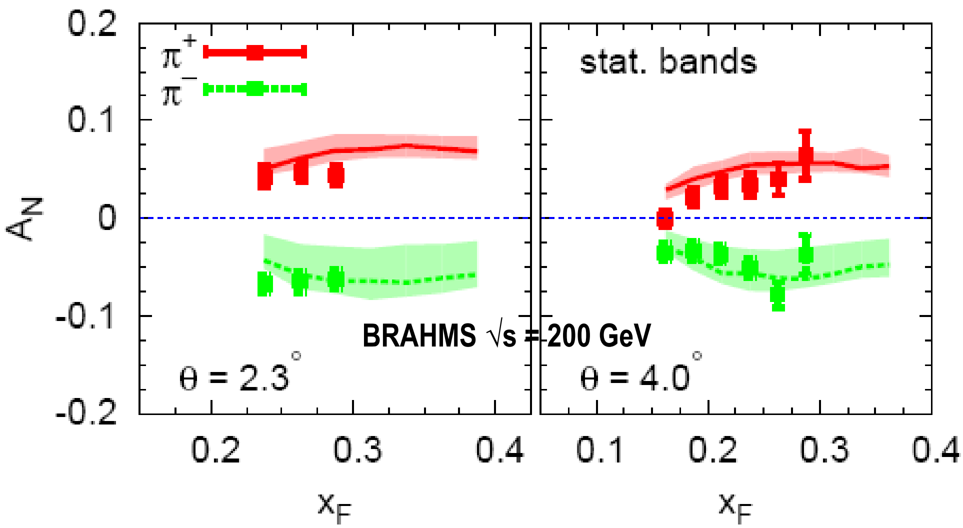


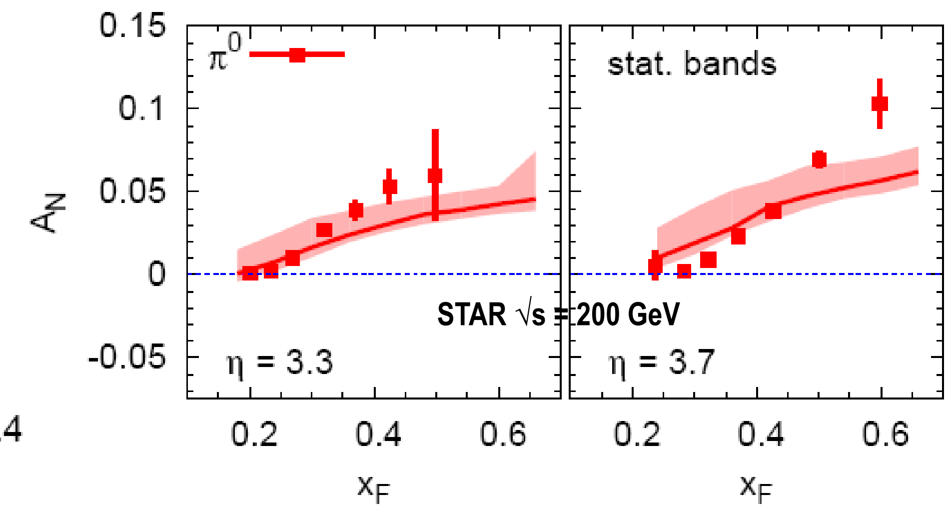
Single jet AnDY experiment seems <u>compatible</u> with Sivers SIDIS!

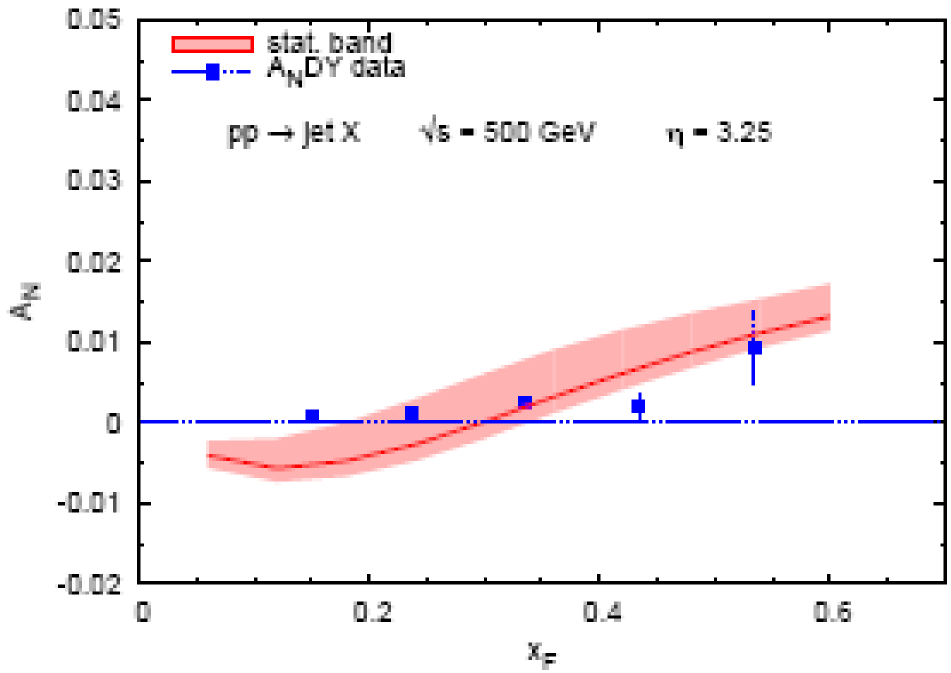


Prediction of Sivers asymmetry for DY





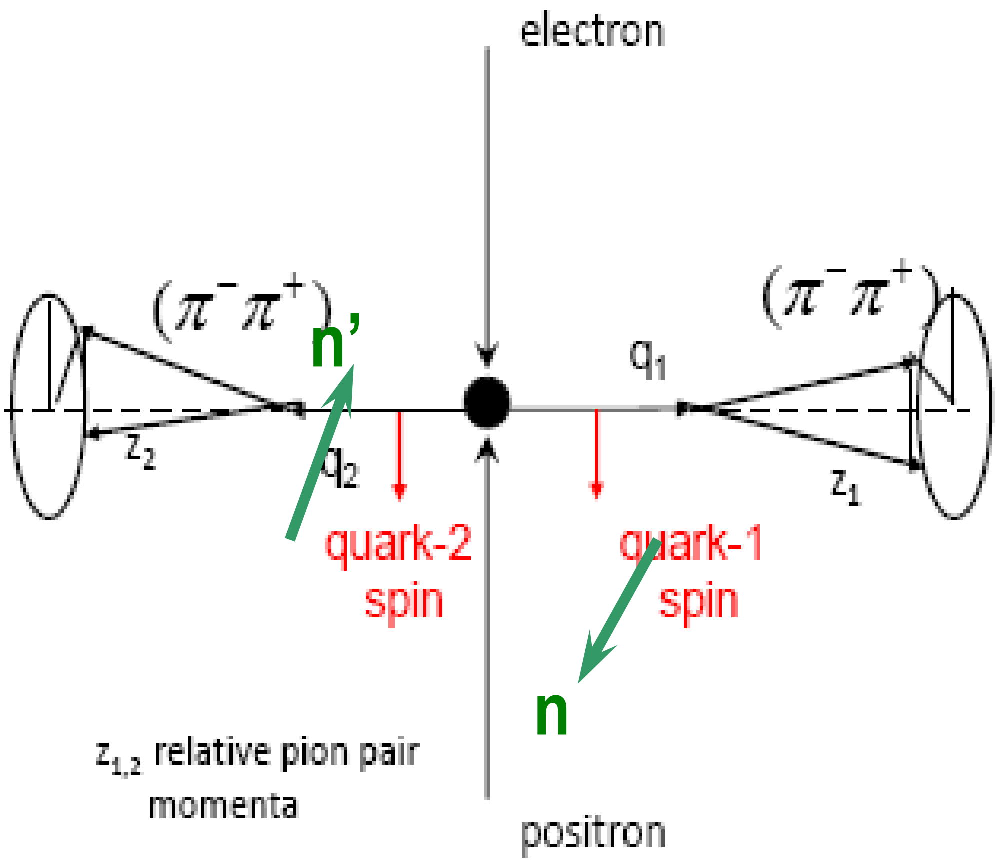




M. Anselmino et al. 1304.7691

- GPM model. Generalises the usual collinear factorisation scheme with DGLAP evolution.
- No mismatch problem, the same sign as in SIDIS. But predict change sign for pp --> jet X!

Measuring di-Hadron Correlations In e^+e^- Annihilation into Quarks



Interference effect in e^+e^- quark fragmentation will lead to azimuthal asymmetries in di-hadron correlation measurements!

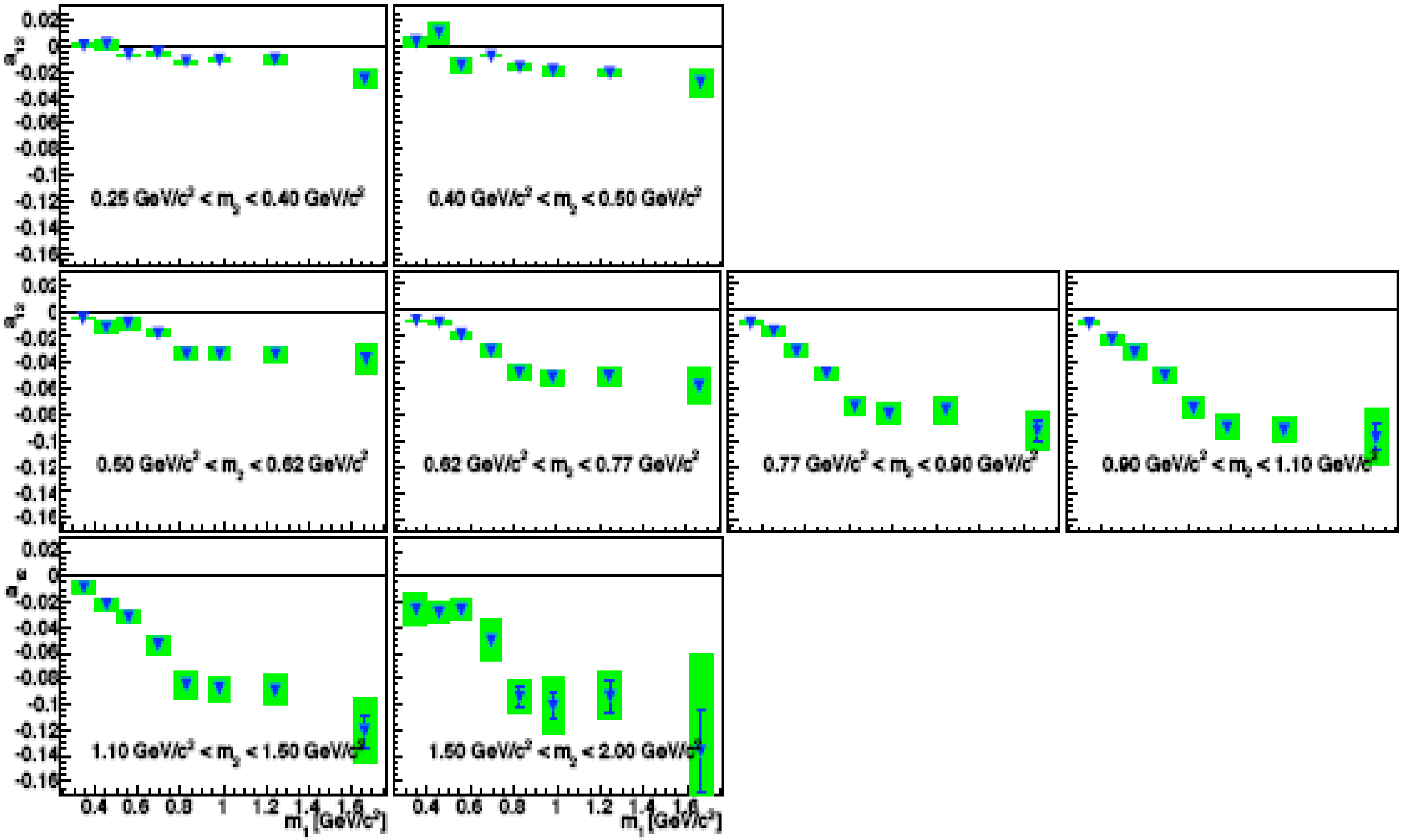
(Handedness correlation!)

Experimental requirements:

- Small asymmetries ➔ very large data sample!
- Good particle ID to high momenta.
- Hermetic detector
- Observable: $\cos(\varphi_{R1}+\varphi_{R2})$

modulation measures $H_1^{\angle}\overline{H}_1^{\angle}$

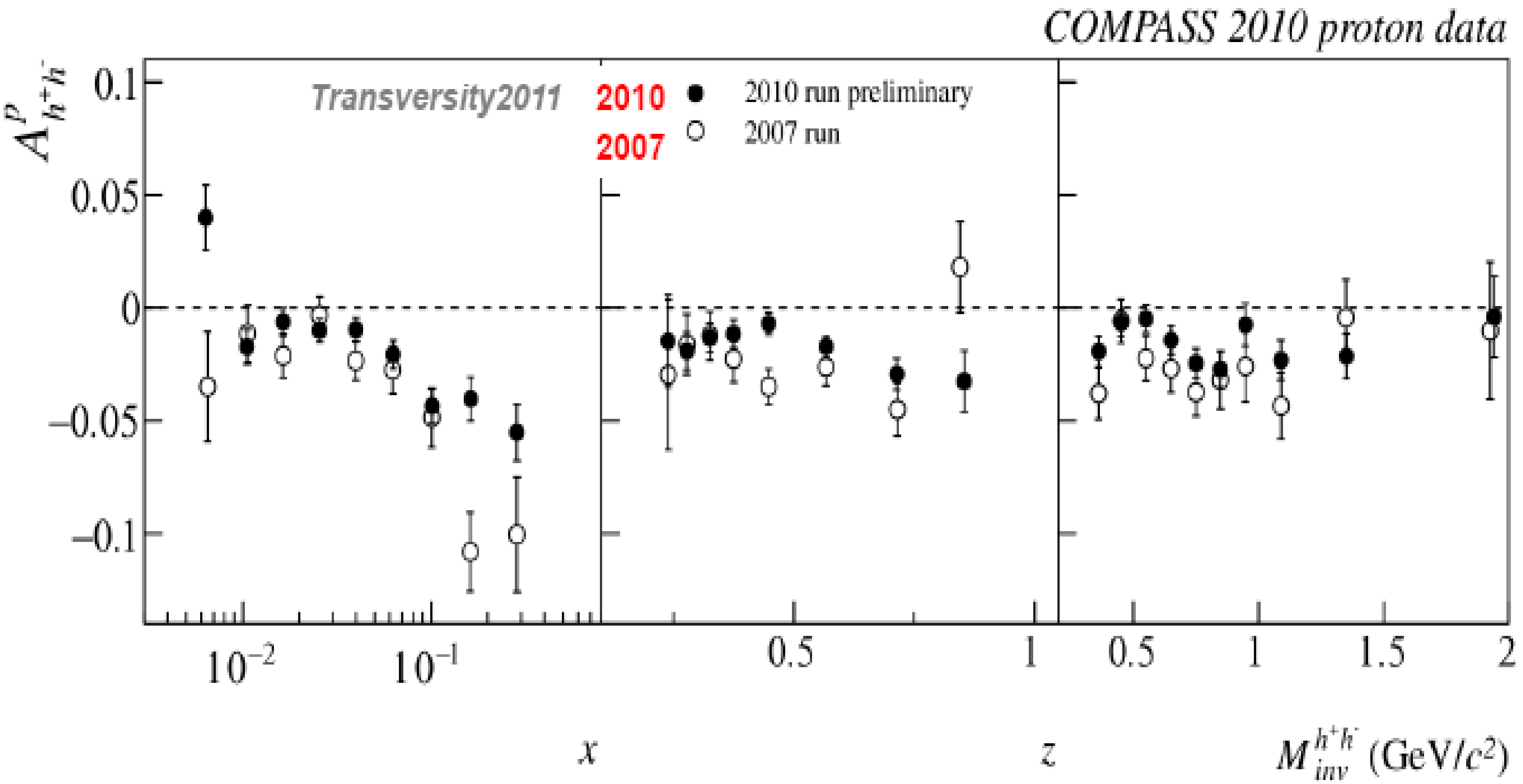
BELLE data

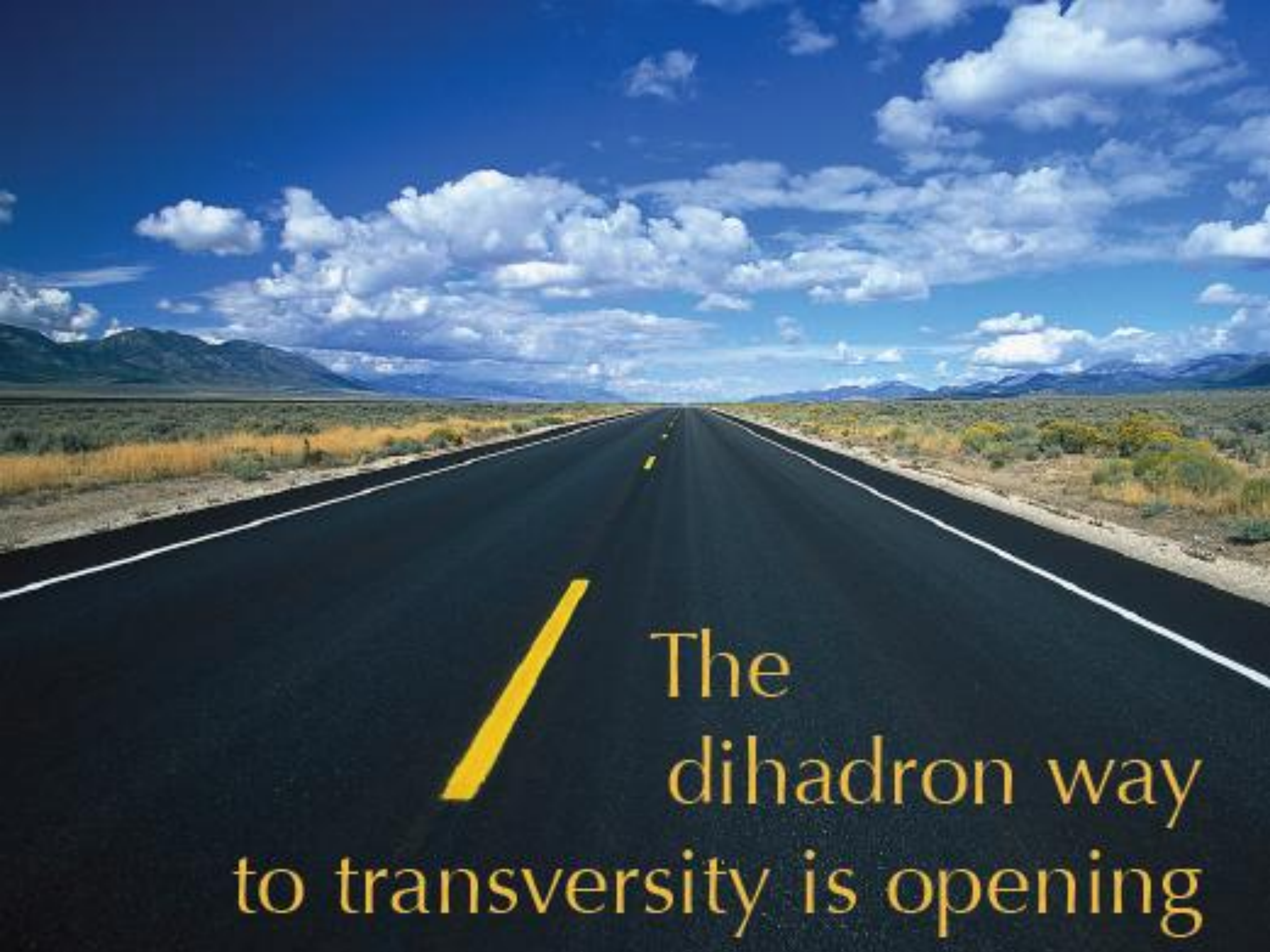


Vossen, Seidl et al. (Belle), PRL 107 (2011)

Two hadron asymmetries in SIDIS

New results from 2010 run of COMPASS




The
dihadron way
to transversity is opening

$$A_{DIS}(x,z,M_h^2) = -\langle C_y\rangle \frac{\sum_q e_q^2\, h_1^q(x)\ \frac{|\boldsymbol{R}|}{M_h} H_{1,q}^{\sphericalangle}(z,M_h^2)}{\sum_q e_q^2\, f_1^q(x)\ D_{1,q}(z,M_h^2)}$$

$$A_{e+e-}(z,M_h^2,\bar{z},\bar{M}_h^2) = -\frac{\langle\sin^2\theta_2\rangle\langle\sin\theta\rangle\,\langle\sin\bar{\theta}\rangle}{\langle 1+\cos^2\theta_2\rangle} \frac{\sum_q e_q^2\,\frac{|\boldsymbol{R}|}{M_h} H_{1,q}^{\sphericalangle}(z,M_h^2)\,\frac{|\bar{\boldsymbol{R}}|}{M_h} H_{1,\bar{q}}^{\sphericalangle}(\bar{z},\bar{M}_h^2)}{\sum_q e_q^2\, D_{1,q}(z,M_h^2)\, D_{1,\bar{q}}(\bar{z},\bar{M}_h^2)}$$

Simplified expressions $H_{1q}^{S} = -H_{1\bar{q}}^{S},\ H_{1u}^{S} = -H_{1d}^{S},\ \ D_1^{u} = D_1^{\bar{u}} = D_1^{d}$

Courtoy, Bacchetta, Radici, Bianconi, arXiv:1202.0323, 1202.6150, 1206.1836,1212.3568

$$\frac{n_u^{\uparrow}}{n_u} = \frac{\iint \frac{|\boldsymbol{R}|}{M_h} H_{1,u}^{\sphericalangle}(z,M_h^2)}{\iint D_{1,u}(z,M_h^2)}$$

=-21±2%

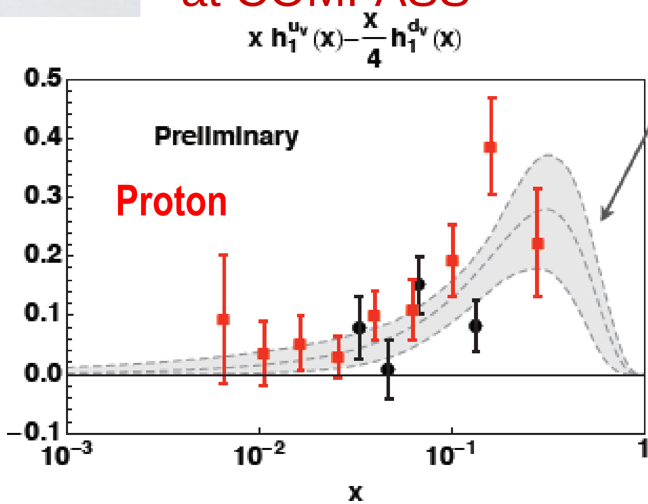
at COMPASS

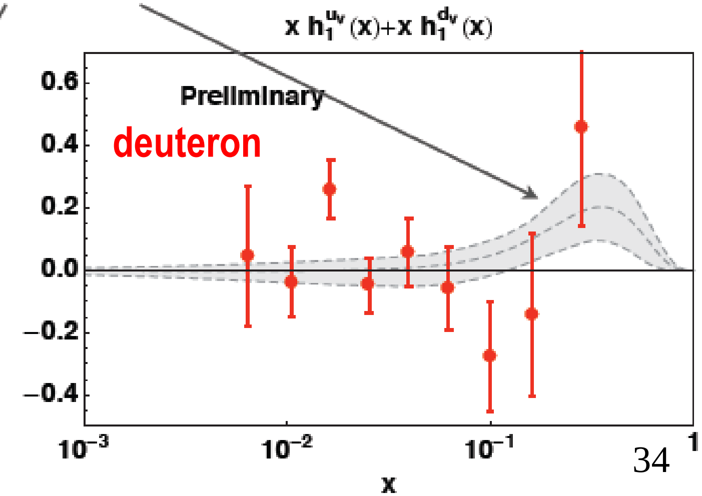
$$A_{DIS}(x) \approx -\langle C_y\rangle \frac{(h_1^{u_v}(x) - h_1^{d_v}(x)/4)}{(f_1^{u+\bar{u}}(x) + f_1^{d+\bar{d}}(x)/4)}\,\frac{n_u^{\uparrow}}{n_u}$$

Torino's fit

↑

Transversal HANDEDNESS!

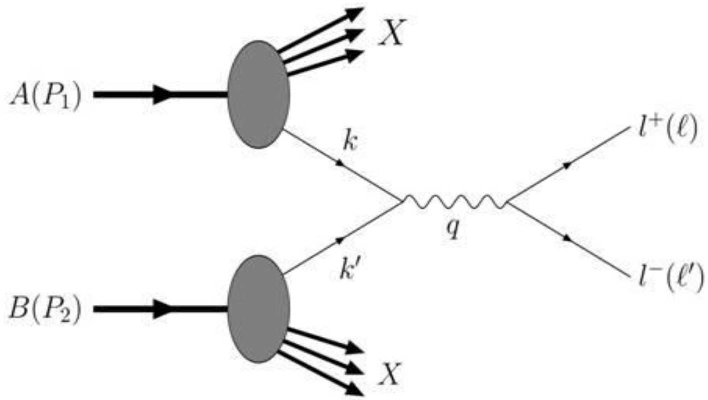


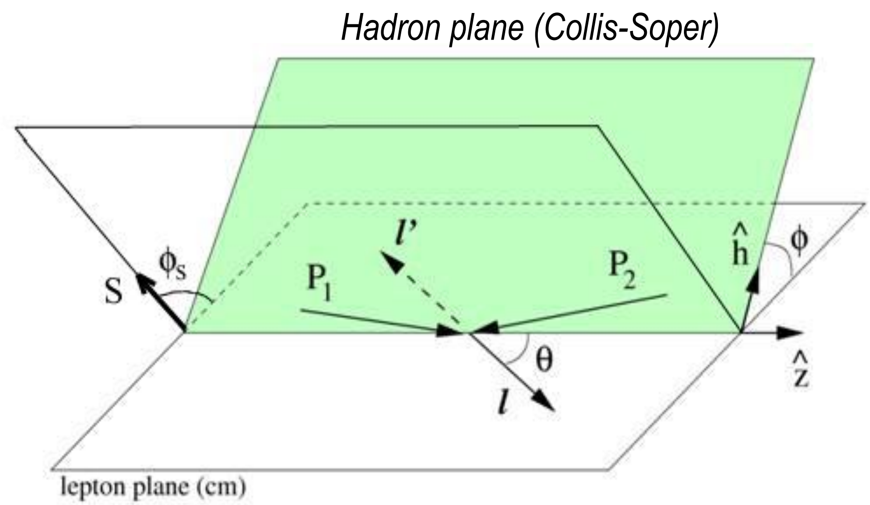


Summary on SIDIS

- transversity is non-zero and quite sizable
  - can be measured, e.g., via Collins effect or interference in 2-hadron fragmentation
- Sivers and Boer-Mulders effects are also non-zero
  - direct probe of "physics of the QCD Wilson line"
  - possibly large evolution effects
- so far no sign of a non-zero pretzelosity distribution
- first evidences for non-vanishing worm-gear functions
- let's prepare for
  - precision measurements at ongoing and future SIDIS facilities
  - fundamental QCD tests in Drell-Yan experiments

Drell-Yan processes (single-spin)





Arnold, Metz, Schlegel PRD79(2009)034005

$$x_1 = \frac{q^2}{2P_1 \cdot q}, \quad x_2 = \frac{q^2}{2P_2 \cdot q}$$

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{\alpha_{em}^2}{F\, q^2} \times$$

$$\Big\{ \Big((1+\cos^2\theta)\, F_{UU}^1 + (1-\cos^2\theta)\, F_{UU}^2 + \sin 2\theta \cos\phi\, F_{UU}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{UU}^{\cos 2\phi} \Big)$$

$$+\ S_{aL} \Big(\sin 2\theta \sin\phi\, F_{LU}^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_{LU}^{\sin 2\phi} \Big)$$

$$+\ |\vec{S}_{aT}| \Big[\sin\phi_a \Big((1+\cos^2\theta)\, F_{TU}^1 + (1-\cos^2\theta)\, F_{TU}^2 + \sin 2\theta \cos\phi\, F_{TU}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{TU}^{\cos 2\phi} \Big)$$

$$+ \cos\phi_a \Big(\sin 2\theta \sin\phi\, F_{TU}^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_{TU}^{\sin 2\phi} \Big) \Big]$$

Parton model (leading twist, single-spin)

$$F_{UU}^{1} = \mathcal{C}\left[f_1 \bar{f}_1\right],$$

$$F_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_1^{\perp}\bar{h}_1^{\perp}\right],$$

$$F_{LU}^{\sin 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_{1L}^{\perp}\bar{h}_1^{\perp}\right],$$

$$F_{TU}^{1} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} f_{1T}^{\perp}\bar{f}_1\right],$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} h_1 \bar{h}_1^{\perp}\right],$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})\left[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}\right] - \vec{k}_{aT}^{2}(\vec{h}\cdot\vec{k}_{bT})}{2M_a^2 M_b} h_{1T}^{\perp}\bar{h}_1^{\perp}\right]$$

For g_{1L} and g_{1T} one needs F^1_{LL} and F^1_{LT}.

For $p\bar{p}$ Drell-Yan all $\bar{f}, \bar{h} \rightarrow f, h$

Allows uniquely measure most PDFs!

Cornestoun QCD prediction

$$f_{1T}^{\perp q}\,|_{SIDIS} = -f_{1T}^{\perp q}\,|_{DY}$$

$$\mathcal{C}\left[w(\vec{k}_{aT},\vec{k}_{bT}) f_1 \bar{f}_2\right] \equiv \frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})\, w(\vec{k}_{aT},\vec{k}_{bT}) \times \left[f_1^q(x_a,\vec{k}_{aT}^2)\, f_2^{\bar{q}}(x_b,\vec{k}_{bT}^2) + f_1^{\bar{q}}(x_a,\vec{k}_{aT}^2)\, f_2^q(x_b,\vec{k}_{bT}^2)\right].$$

Parton model (leading twist, double spin, LL and TL)

$$F_{LL}^{1} = -\mathcal{C}\left[g_{1L}\,\bar{g}_{1L}\right],$$

$$F_{LL}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}\,h_{1L}^{\perp}\bar{h}_{1L}^{\perp}\right],$$

$$F_{LT}^{1} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}\,g_{1L}\,\bar{g}_{1T}\right],$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}\,h_{1L}^{\perp}\,\bar{h}_{1}\right],$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})\left[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}\right] - \vec{k}_{bT}^{2}(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^{2}}\,h_{1L}^{\perp}\,\bar{h}_{1T}^{\perp}\right],$$

$$F_{TL}^{1} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}\,g_{1T}\,\bar{g}_{1L}\right],$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}\,h_{1}\,\bar{h}_{1L}^{\perp}\right],$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})\left[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}\right] - \vec{k}_{aT}^{2}(\vec{h}\cdot\vec{k}_{bT})}{2M_a^{2} M_b}\,h_{1T}^{\perp}\,\bar{h}_{1L}^{\perp}\right],$$

Parton model (leading twist, double spin, TT)

$$F_{TT}^{1} = C\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}\left(f_{1T}^{\perp}\,\bar{f}_{1T}^{\perp} - g_{1T}\,\bar{g}_{1T}\right)\right],$$

$$F_{TT}^{1} = -C\left[\frac{\vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}\left(f_{1T}^{\perp}\,\bar{f}_{1T}^{\perp} + g_{1T}\,\bar{g}_{1T}\right)\right],$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = C\left[h_1\,\bar{h}_1\right],$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = C\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})^2 - \vec{k}_{bT}^2}{2M_b^2}\,h_1\,\bar{h}_{1T}^{\perp}\right],$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = C\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})^2 - \vec{k}_{aT}^2}{2M_a^2}\,h_{1T}^{\perp}\,\bar{h}_1\right],$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = C\left[\left(\frac{4(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT})\left[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}\right]}{4M_a^2 M_b^2} + \frac{\vec{k}_{aT}^2\vec{k}_{bT}^2 - 2\vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})^2 - 2\vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})^2}{4M_a^2 M_b^2}\right) h_{1T}^{\perp}\,\bar{h}_{1T}^{\perp}\right]$$

Prediction for RHIC

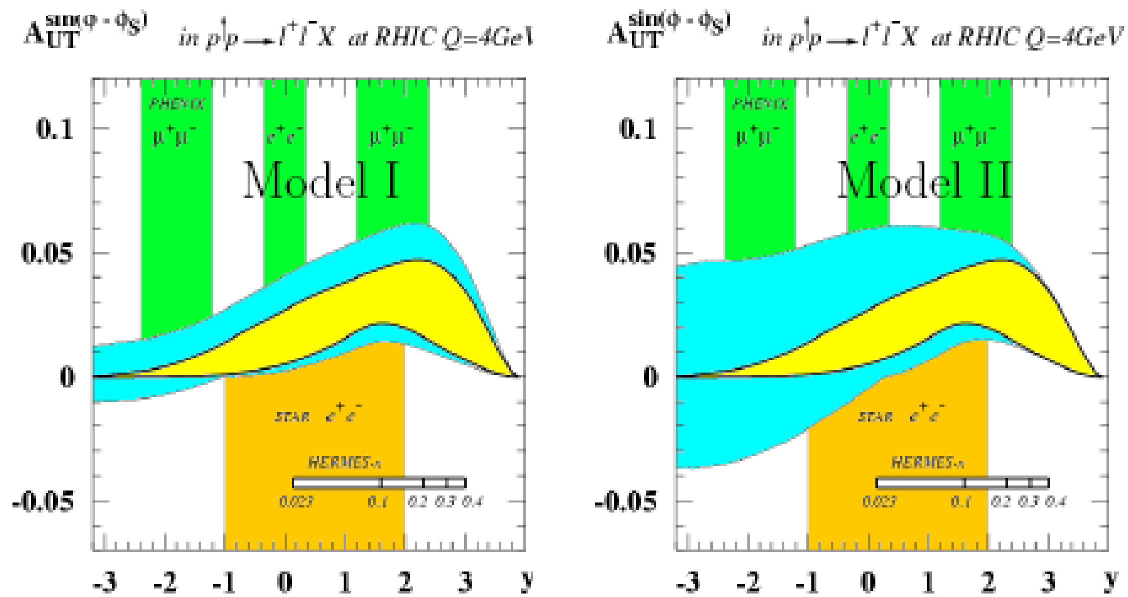
Let us simulate Sivers-$\bar{q}$:

$$f_{1T}^{\perp(1)\bar{q}}(x) = \underbrace{f_{1T}^{\perp(1)q}(x)}_{\text{best fit}} \times \begin{cases} \pm 25\% \\ \pm \dfrac{(f_1^{\bar{u}} + f_1^{\bar{d}})(x)}{(f_1^{u} + f_1^{d})(x)} \end{cases}$$

Collins, AE et al. Phys.Rev. D73 (2006) 094023

RHIC

- $p^{\uparrow}p \rightarrow l^{+}l^{-}X$
  Valence q and sea $\bar{q}$ on equal footing. Sensitive to Sivers-$\bar{q}$ in certain y-region.
- RHIC can test "change of sign" & provide information on Sivers-$\bar{q}$!

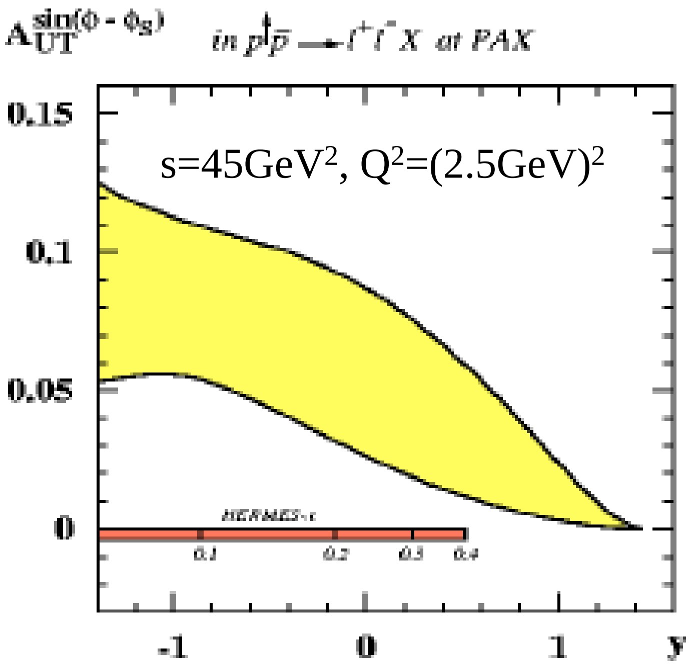


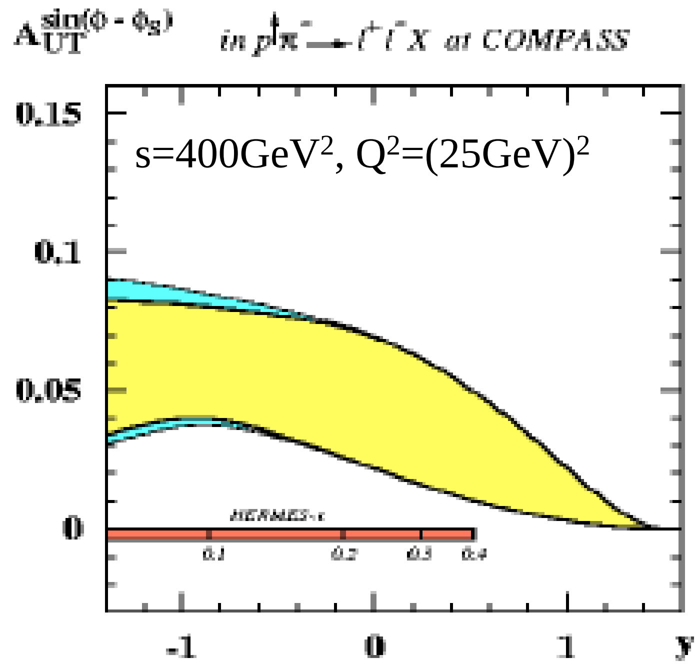
yellow = 1-σ region, blue = effects due to Sivers-$\bar{q}$

Accuracy ($\int Ldt = 125\,pb^{-1}$): $\delta A = \begin{cases} 0.7\% \text{ (STAR, PHENIX)} \\ 0.1\% \text{ (RHIC II)} \end{cases}$

Predictions for Sivers in PAX and COMPASS

$$A_{UT}^{\sin(\phi-\phi_S)} = + \frac{a^{\text{DY}}_{\text{Gauss}} \sum_a e_a^2 f_{1T\,\text{DY}}^{\perp(1)a}(x_1) f_1^{\bar{a}}(x_2)}{\sum_a e_a^2 f_1^a(x_1) f_1^{\bar{a}}(x_2)}$$





- **PAX at GSI**

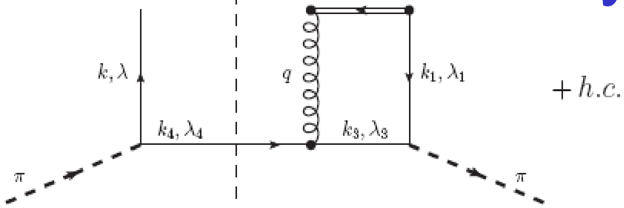
$p^{\uparrow}\bar{p} \rightarrow l^+ l^- X$ (byproduct)

- **COMPASS**

$p^{\uparrow}\pi^- \rightarrow l^+ l^- X$

Annihilations of valence dominate.

The BM-Pretzelosity Asymmetry in π p Drell Yan



$\alpha_s(\mu_0^2)/4\pi\,|_{NLO} = 0.100;\ \langle p_{pT,\text{unp}}^2 \rangle = 0.08\ \text{GeV}^2$

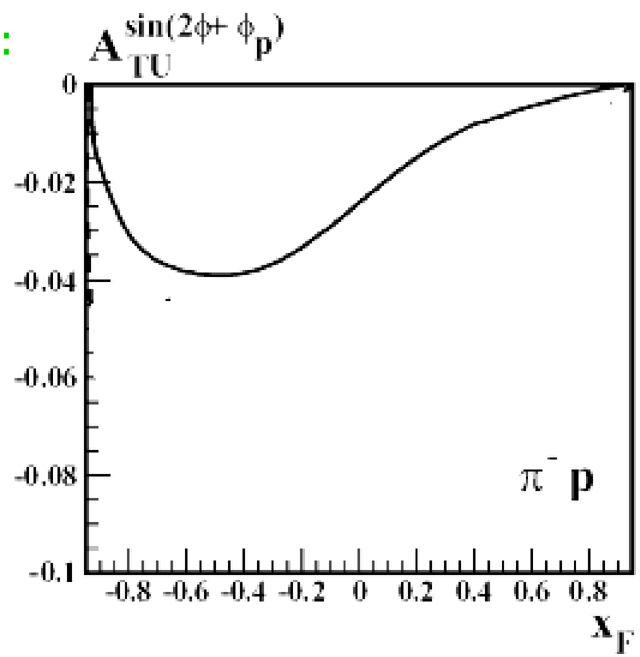
$\langle p_{pT}^2 \rangle = 0.05\ \text{GeV}^2,\ \langle p_{\pi T}^2 \rangle = 0.092\ \text{GeV}^2$

1. Light Cone Model Boer-Mulders function of pion generated from S-P wave interference in one-gluon exchange approximation,
2. COMPASS kinematics: $x_p\, x_\pi = Q^2/s$ with Q^2 =20 GeV^2 and s=400 GeV^2,
3. Evolution equations for $h_{1T}^{\perp}$ and $h_1^{\perp}$ are not yet used → we include "approximate" evolution effects using transversity evolution,

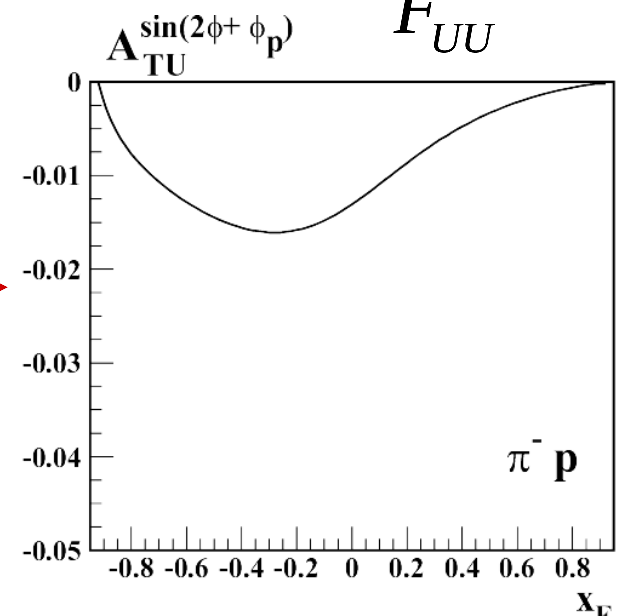
[AE, Pasquini, Schweitzer, Yuan, in preparation]

$$A_{TU}^{\sin(2\phi+\phi_s)} = \frac{F_{TU}^{\sin(2\phi+\phi_s)}}{F_{UU}^{1}}$$

Also Lu, Ma, She: arXiv:1101.2702



"Approximate" evolution



TMD evolution phenomenology

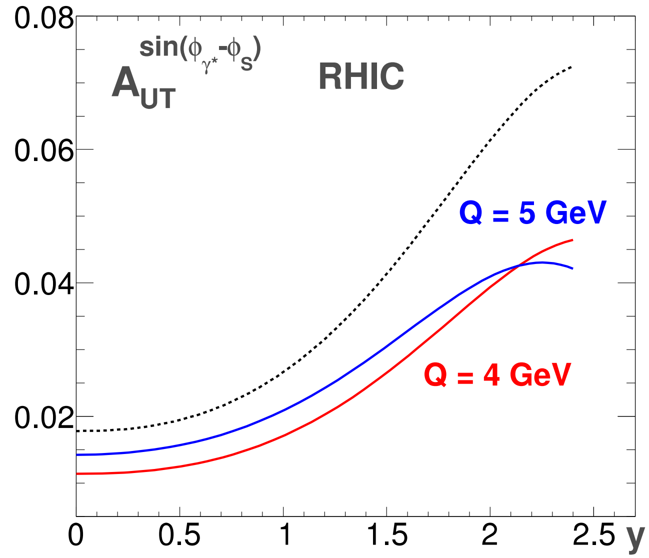
Sun and Yuan, 1304.5037 and Feng Yuan 1304.5037

- *recently applied the CSS original evolution scheme at one loop to account for TMD evolution of the unpolarized TMD PDFs, and extended this formalism to the Sivers function as well.*

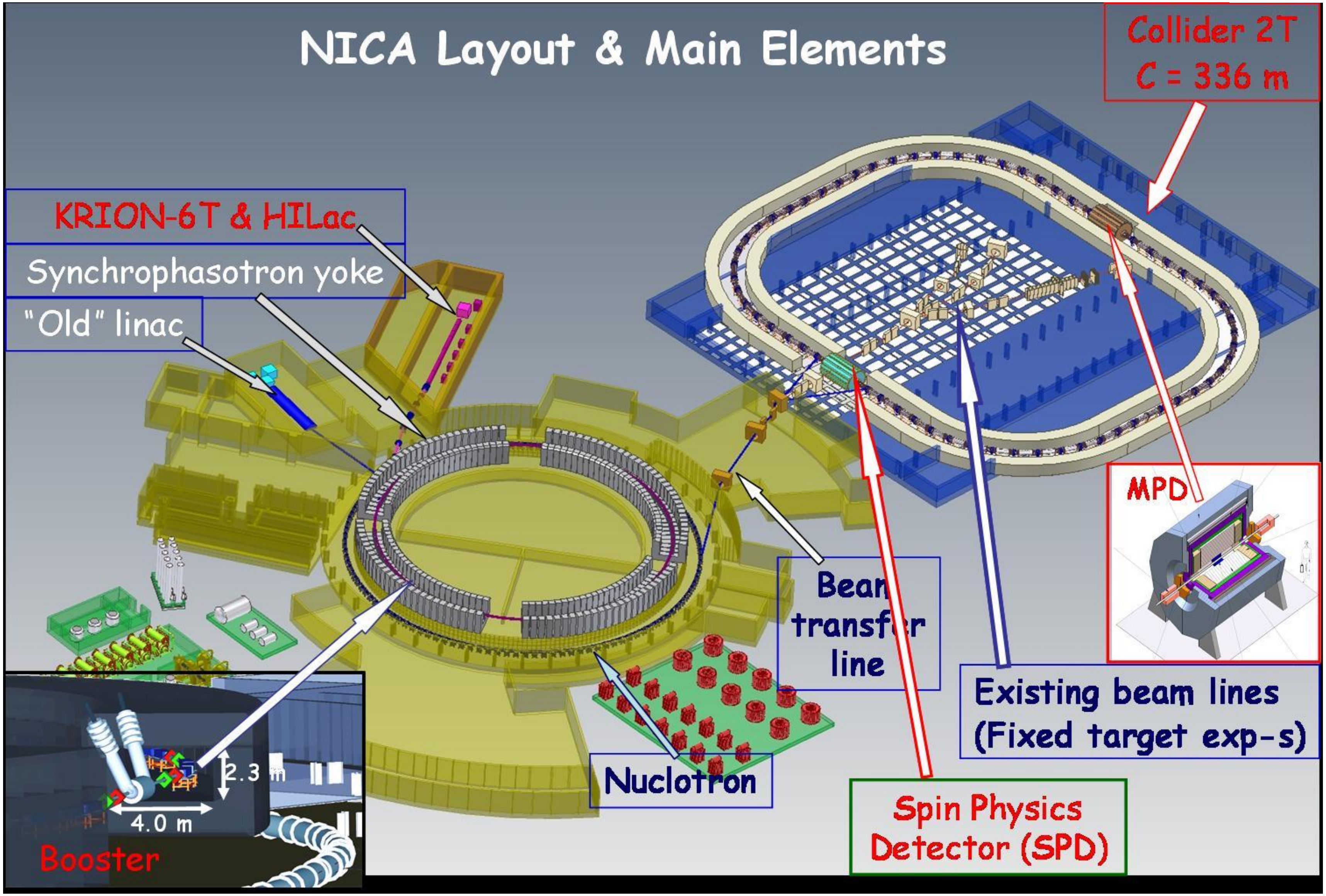
$$\widetilde{F}^{\alpha}_{\rm sivers}(Q;b) = \widetilde{F}^{\alpha}_{\rm sivers}(Q_0;b)e^{-\mathcal{S}_{Sud}(Q,Q_0,b)}$$

$$\mathcal{S}_{Sud} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

- **There is no Landau pole singularity in the integral**
- **Almost parameter-free**
- **No Q-dependent non-perturbative form factor**
- **Gaussian assumption at lower scale Q_0**
- **Fit to Sivers asymmetries in SIDIS**
- **Predictions at RHIC**
- **This can be used to calculate the asymmetries up to W/Z boson production**
- **EIC will be perfect, because Q coverage**

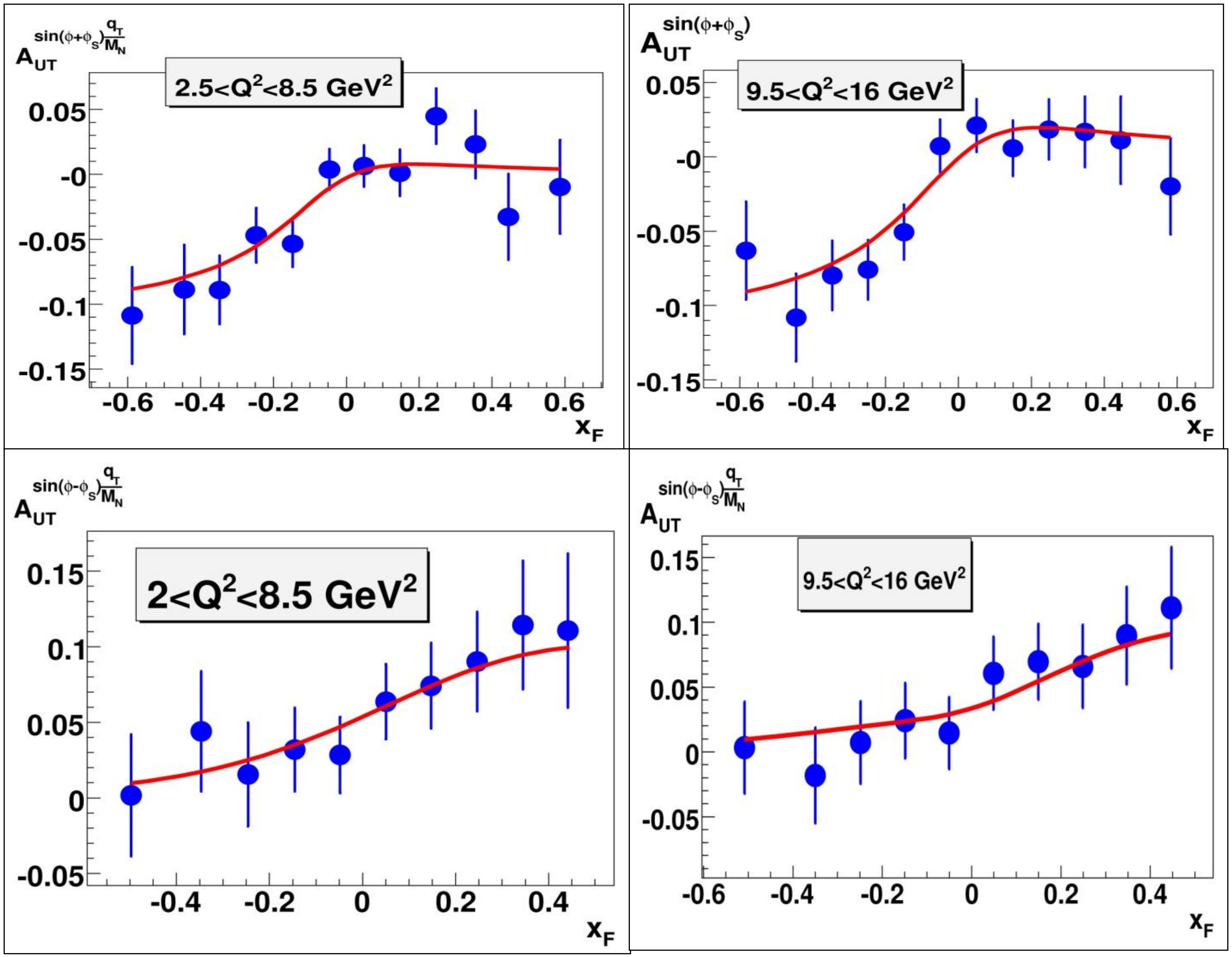


Drell-Yan program at SPD



MC estimation of precision for 100Kevents

(~2 years data taking *s~670 GeV²*)



Transversity and B-M

Sivers

Sissakian at al. EPJC46(2006)147

Conclusions

- TMD PDFs are necessary for complet hadron spin structure description.
- Well motivated approximate model relation are valuable (estimation, proposal motivations for experiments).
- WW-type relations supported by existing experimental data.
- Exiting relation: pretzelosity = - quark angular momentum!
- Experimental information about all TMDs are now available.
- Evolution schemes (not one!) & first attempts to phenomenological study of TMDs are in progress.
- More data on TMDs from SIDIS and DY are necessary and planed (COMPASS, JLab, RHIC, FemiLab, JPARK, PAX, NICA). **Difficult but possible!**

Thank You!