

Pion Electromagnetic and Transition Form Factors with the Light-Front Approach

Venturing off the Light-Cone - Local Versus Global Features
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J. Pacheco B. C. de Melo

^aLaboratório de Física Teórica e Computacional-LFTC, UNICSUL (Brazil) and

^bInstituto de Física Teórica, IFT, UNESP, (Brazil)

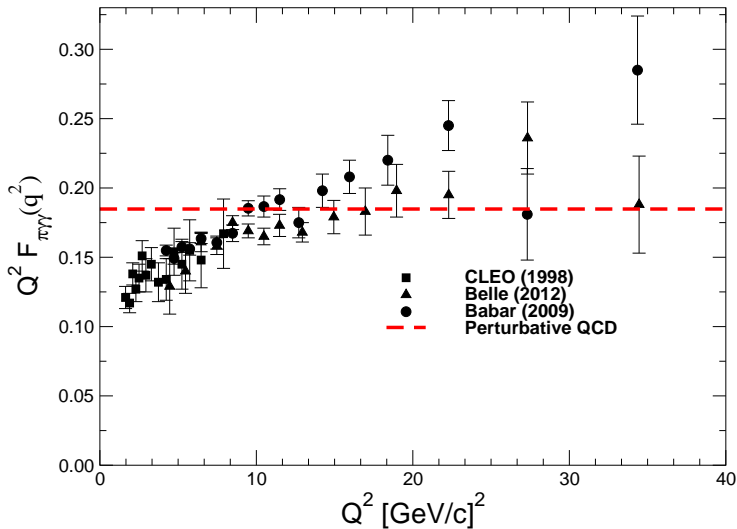
Collaborators: Bruno El-Bennich (LFTC) and T. Frederico (CTA-ITA)

May 24, 2013

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Motivations

- $\pi^0 \rightarrow \gamma\gamma$ **Most Important Example of the Triangle Anomaly**
- π^0 **Meson is the Lightest Meson cannot Decay to Another Hadronic State**
- $\pi^0 \rightarrow \gamma\gamma$ **Is Connected to the Adler-Bell-Jackiw Anomaly**
- **Babar Experiment (2009)**
- **Belle Experiment (2012)**



• **QCD:** $2f_\pi \implies$ **Brodsky & Lepage (1980)**

Light-Front Coordinates

$$\text{Four-Vector} \implies x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$$

$$x^+ = t + z \quad x^+ = x^0 + x^3 \implies \text{Time}$$

$$x^- = t - z \quad x^- = x^0 - x^3 \implies \text{Position}$$

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3$$

$$p^- = p^0 - p^3$$

$$p^\perp = (p^1, p^2)$$

Dirac Matrix and Electromagnetic Current

$$\begin{aligned} \gamma^+ &= \gamma^0 + \gamma^3 \implies \text{Electr. Current} & J^+ &= J^0 + J^3 \\ \gamma^- &= \gamma^0 - \gamma^3 \implies \text{Electr. Current} & J^- &= J^0 - J^3 \\ \gamma^\perp &= (\gamma^1, \gamma^2) \implies \text{Electr. Current} & J^\perp &= (J^1, J^2) \end{aligned}$$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, x_\perp \implies p^+, p^-, \vec{p}_\perp$$

$$p^- \implies \text{Light-Front Energy}$$

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+} \quad \text{On-shell}$$

$$\text{Bosons} \implies S_F(p) = \frac{1}{p^2 - m^2 + i\epsilon}$$

$$\text{Fermions} \implies S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} + \frac{\gamma^+}{2p^+}$$

Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky

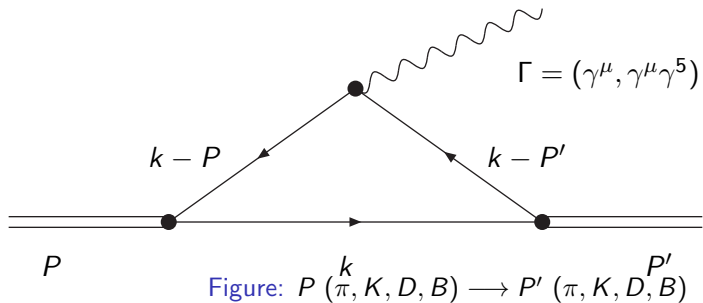


Figure: $P \xrightarrow{k} P'$ (π, K, D, B) \rightarrow (π, K, D, B)

- **Three-point Function** \Rightarrow **The General Matrix Element**

$$\langle p' | J_q^\mu (q^2) | p \rangle = \frac{N_c}{(2\pi)^4} \int d^4 k \text{Tr} [\Lambda_{M'} (k, p') S_F (k - p') J_q^\mu S(k - p) \Lambda_M (k, p) S_F (k)]$$

- **PS - $q\bar{q}$ Vertex:** γ^5
- **Fermi Propagator** $\Rightarrow S_F (p) = \frac{i}{\not{p} - m_i + i\epsilon}$

Electromagnetic Form Factor and Pseudoscalar Constant

$$\langle p' | J_q^\mu(q^2) | p \rangle = (p + p')^\mu F_{PS}^{em}(q^2).$$

- Weak Decay Constant of the Pseudoscalar Mesons

$$\langle 0 | A^\mu(0) | p \rangle = i\sqrt{2}f_{ps}p^\mu$$

$$\Rightarrow \mathbf{A}^\mu = \bar{\mathbf{q}}\gamma^\mu\gamma^5\frac{\tau}{2}\mathbf{q}(x)$$

$$i\mathbf{p}^\mu f_\pi = \frac{m}{f_\pi} N_c \int \frac{d\mathbf{k}^4}{(2\pi)^4} \text{Tr} \left[\gamma^\mu \gamma^5 \mathbf{S}(\mathbf{k}) \gamma^5 \mathbf{S}(\mathbf{k} - \mathbf{p}) \right] \Lambda_M(\mathbf{k}, \mathbf{p})$$

- Plus Component E.M. Current: $\gamma^+ = \gamma^0 + \gamma^3$

Some QCD REMARKS

$$\bullet R = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^-\mu^+)} = N_c \sum Q_q^2 = N_c \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 \right] = \frac{2}{3} N_c = 2$$

For $E_{cm} > 2 \text{ GeV} \gg 2m_u, 2m_d, 2m_s$, for (u,d,s)

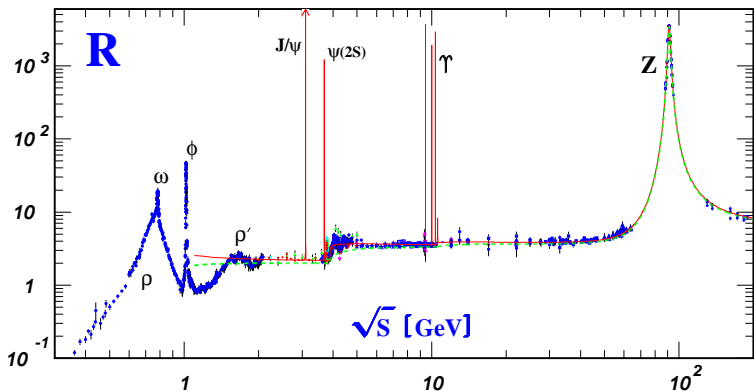
$$\bullet R = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^-\mu^+)} = N_c \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right] = \frac{10}{9} N_c$$

For $E_{cm} > 3 \text{ GeV} \gg 2m_c$, for (u,d,s,c)

$$\bullet R = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^-\mu^+)} = N_c \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 \right] = \frac{11}{9} N_c$$

For $E_{cm} > 10 \text{ GeV} \gg 2m_b$, for (u,d,s,c,b)

★ If $N_c = 3$!!!



Ref.: J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012)

- $\pi^0 \rightarrow \gamma\gamma \implies \Gamma = \frac{g_{\pi^0\gamma\gamma}^2 \alpha_{em}^2 m_\pi^3}{16\pi^3 f_\pi^2}$

- Exp.: $\Gamma^{Exp} \simeq 7.80 \iff N_c = 3, g_{\pi\gamma\gamma} = 0.5, f_\pi \simeq 92 \text{ MeV}$

- Effective Theory:

$$\Gamma = \frac{m_\pi^3}{64\pi} |A_{\gamma\gamma}|^2$$

Where: $|A_{\gamma\gamma}|^2 = \frac{\alpha N_c}{3\pi f_\pi}$

Again: $\implies N_c = 3 \text{ and } f_\pi \simeq 92 \text{ MeV}$

Ref. **Dynamics of the Standard Model** by
J. F. Donoghue, E. Golowich and B. R. Holstein

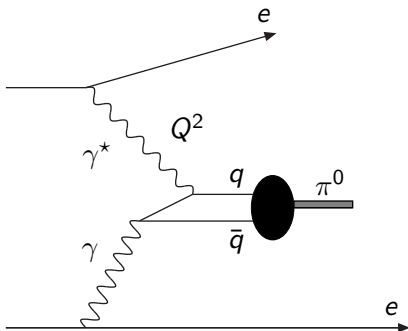
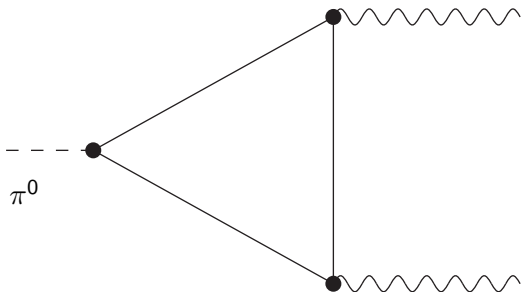
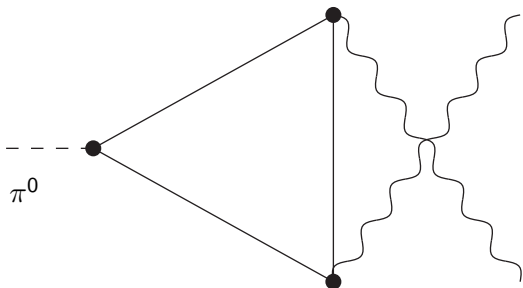


Figure: Feynman Diagram for $\gamma\gamma^* \rightarrow q\bar{q} \rightarrow \pi^0$



(a)

Figure: Pion Decay Diagram (a).



(b)

Figure: Pion Decay Diagram (b).

Effective Interaction Lagrangian

$$\mathcal{L}_{\pi q}^{int} = -i \frac{M}{f_\pi} \vec{\pi} \cdot \vec{q} \gamma^5 \vec{\tau} q ;$$

Where:

- M : **Constituent Quark Mass**
- f_π : **Weak Decay Constant**
- π : **Pion Field**
- q : **Quark Field**
- $\hbar=c=1$

$T^{\mu\nu}$ Tensor : Amplitude (a) and (b)

$$T^{\mu\nu} = t_{\mu\nu}(k_1, k_2) + t_{\mu\nu}(k_2, k_1)$$

- **After Calculation of Trace in Spinor and Flavour Basis:**

$$t_{\mu\nu} = \frac{4}{3} \frac{M^2}{f_\pi} e_0^2 N_c \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta I(k_1^2) ;$$

$$I(k_1^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{((k_2 - k)^2 - M^2 + i\epsilon)} \frac{1}{(k^2 - M^2 + i\epsilon)} \frac{1}{((k_\pi - k)^2 - M^2 + i\epsilon)} .$$

- $k_\pi^\mu = k_1^\mu + k_2^\mu \iff$ **Pion Momentum**
- $k_1^\mu = q^\mu \iff$ **Momentum Transfer**

$$I(k_1^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{((k_2 - k)^2 - M^2 + i\epsilon)} \times \frac{1}{(k^2 - M^2 + i\epsilon)((k_\pi - k)^2 - M^2 + i\epsilon)} .$$

- **Ref. Frame:** $q^+ = q^- = 0$ and $q_\perp \neq 0$

k^- Integration:

$$I(k_1^2) = \frac{1}{2(2\pi)^4} \int dk^+ d^2k^+ dk^- \frac{1}{(k^+(k_\pi^+ - k^+)(k_2^+ - k^+)(k^- - \frac{k_\perp^2 + M^2 - i\epsilon}{k^+}))} \times \frac{1}{(k_2^- - k^- - \frac{(\vec{k}_2 - \vec{k})_\perp^2 + M^2 - i\epsilon}{(k_2^+ - k^+)}) (k_\pi^- - k^- - \frac{(\vec{k}_\pi - \vec{k})_\perp^2 + M^2 - i\epsilon}{(k_\pi^+ - k^+)})},$$

- $k_\pi^+ = k_2^+$ and $k_\pi^- = k_2^-$.

$$I(q^2) = -\frac{1}{2(2\pi)^3} \int \frac{dx d^2 k_{\perp}}{x(1-x)^2} \frac{1}{(k_2^- k_2^+ - \frac{k_{\perp}^2 + M^2}{x} - \frac{(\vec{k}_2 - \vec{k})_{\perp}^2 + M^2}{1-x})} \\ \times \frac{1}{(k_{\pi}^- k_{\pi}^+ - \frac{k_{\perp}^2 + M^2}{x} - \frac{(\vec{k}_{\pi} - \vec{k})_{\perp}^2 + M^2}{1-x})}$$

Integral Contribution

- i) $k^+ < 0 \implies$ **Not Contribute**
- ii) $k^+ > k_{\pi}^+ \implies$ **Not Contribute**
- iii) $0 < k^+ < k_{\pi}^+ \implies$ **Contribute** (**On-Shell Quark**)

- Integral Results

$$I(q^2) = -\frac{1}{2(2\pi)^3} \int \frac{dx d^2 k_{\perp}}{x(1-x)^2} \frac{1}{(k_2^- k_2^+ - \frac{k_{\perp}^2 + M^2}{x} - \frac{(\vec{k}_2 - \vec{k})_{\perp}^2 + M^2}{1-x})} \\ \times \frac{1}{(k_{\pi}^- k_{\pi}^+ - \frac{k_{\perp}^2 + M^2}{x} - \frac{(\vec{k}_{\pi} - \vec{k})_{\perp}^2 + M^2}{1-x})} ,$$

- $\implies 0 < \mathbf{k}^+ < \mathbf{k}_{\pi}^+ \iff 0 < \mathbf{x} < 1$

- **Relative momentum:** $\vec{K}_\perp = (1-x)\vec{k}_\perp - x(\vec{k}_\pi - \vec{k})_\perp$

$$I(q^2) = \frac{1}{2(2\pi)^3} \int \frac{dx d^2 K_\perp}{(1-x)} \frac{1}{((\vec{K} + x\vec{q})_\perp^2 + M^2)(m_\pi^2 - M_0^2)}$$

- **Free Mass Operator**

$$M_0^2 = \frac{K_\perp^2 + M^2}{x(1-x)}$$

Pion Electromagnetic Transition Form Factor

$$j_\mu = e_0^2 \epsilon_{\mu\nu\alpha\beta} \epsilon_\gamma^\nu q^\alpha k_\pi^\beta F_{\pi^0}(q^2)$$

$$F_{\pi^0}(-q^2) = \frac{N_c}{6\pi^3} \frac{M^2}{f_\pi} \int \frac{dx d^2 K_\perp}{(1-x)} \frac{1}{((\vec{K} + x\vec{q})_\perp^2 + M^2)(M_0^2 - m_\pi^2)}$$

- $\epsilon_\gamma^\nu \Rightarrow$ **Polarization // Real Photon**

Wave Function

- The Wave Function // Replaced

$$\frac{1}{-m_\pi^2 + M_0^2} \rightarrow \frac{\pi^{\frac{3}{2}} f_\pi}{M \sqrt{M_0 N_c}} \Phi_\pi(K^2)$$

- Wave Function $\Phi(k^2)$ Normalization

$$\int d^3K \Phi_\pi^2(k^2) = 1 ,$$

Ref.

- T. Frederico and G. Miller, *Phy. Rev. D*45 (1992) 071901
*ibid. Phy. Rev. D*50 (1994) 210
- J.P.B.C. de Melo, T. Frederico and H.L. Naus
*Phy.Rev. C*59 (1999) 2278

- Final Pion Transition Form Factor

$$F_{\pi^0}(-q^2) = \frac{\sqrt{N_c} M}{6\pi^{\frac{3}{2}}} \int \frac{dx d^2 K_{\perp}}{(1-x)\sqrt{M_0}} \frac{\Phi_{\pi}(K^2)}{((\vec{K} - x\vec{q})_{\perp}^2 + M^2)}$$

- Soft Pion Limit \star :

$$F_{\gamma\pi^0}(0) = \frac{1}{4\pi^2 f_{\pi}}$$

\star J. S. Schwinger, *Phys. Rev.* 82, (1951) 664.

S. L. Adler, *Phys. Rev. Phys. Rev.* 177, (1969) 2426.

J. S. Bell, R. Jackiw, *Nuovo Cimento*, 60, (1969) 47.

Wave Function

i) Gaussian Wave Function

$$\Phi_{\pi} = \left(\frac{8r_{NR}^2}{3\pi} \right)^{3/4} \exp \left[- \left(\frac{4}{3} \right) (r_{NR}k)^2 \right]$$

ii) Hydrogen-Atom Wave Function

$$\Phi_{\pi} = \frac{1}{2\pi} \left(\frac{\sqrt{3}}{r_{NR}} \right)^{5/2} \left[\frac{1}{\left(\frac{3}{4}r_{NR}^{-2} + k^2 \right)^2} \right]$$

- **Two Independent Parameters**

i) Quark Mass: M

ii) Non-Relativistic Charge Radius: r_{NR}

$$r_{NR}^2 = -6 \frac{d}{dq^2} \int d^3K \Phi(|\vec{K} + \frac{\vec{q}}{2}) \Phi_{\pi}(K)$$

- **Neutral Pion Radius**

$$r_{\pi^0}^2 = \frac{\sqrt{N_c} M}{F_{\pi^0}(0) \pi^{\frac{3}{2}}} \int \frac{dx d^2 K_{\perp}}{(1-x) \sqrt{M_0}} \frac{-\vec{K}_{\perp}^2 + M^2}{(-\vec{K}_{\perp}^2 + M^2)^3} \Phi_{\pi}(K^2) .$$

- **Limit:** $-q^2 \rightarrow \infty \implies \sim q^2$

$$\Lambda_{\pi^0} = \lim_{-q^2 \rightarrow \infty} [-q^2 F_{\pi^0}(-q^2)] = \frac{\sqrt{N_c} M}{6\pi^{\frac{3}{2}}} \int \frac{dx d^2 K_{\perp}}{(1-x) \sqrt{M_0}} \Phi_{\pi}(K^2)$$

Some Results

Table-I: f_π : 92.4 MeV Fixed

model	$m_{u,d}$ [GeV]	r_{nr} [fm]	$\langle r^2 \rangle$ [fm^2]	$\langle r_{\pi^0}^2 \rangle$ [fm^2]
Gaussian	0.220	0.345	0.637	0.683
	0.330	0.472	0.655	0.552
Hydrogen	0.220	0.593	0.795	0.782
	0.330	0.708	0.807	0.582
Exp.[PDG]			0.672 \pm 0.008	

Table-II: Quark Mass fixed : $m_{u,d} = 0.220 \text{ GeV}$

model	f_π [MeV]	r_{nr} [fm]	$\langle r^2 \rangle$ [fm^2]	$\langle r_{\pi^0}^2 \rangle$ [fm^2]
Gaussian	92.4	0.345	0.637	0.683
	97.0	0.303	0.589	0.657
	110.0	0.172	0.406	0.664
Hydrogen	92.4	0.593	0.795	0.782
	97.0	0.543	0.750	0.767
	110.0	0.410	0.626	0.720
Exp.[PDG]	92.2 ± 0.021		0.672 ± 0.008	

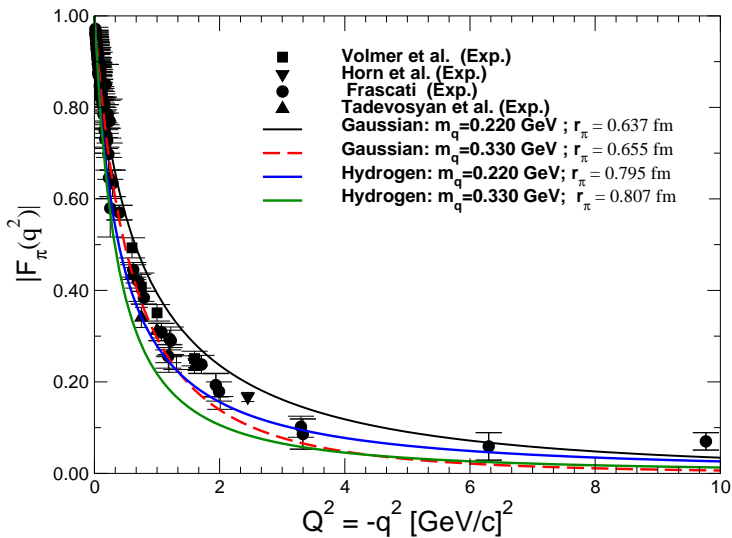
Exp. References:

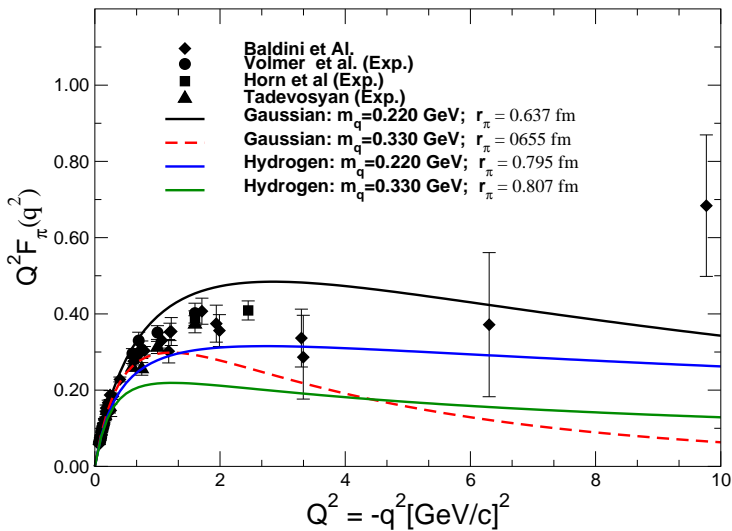
- **Pion Electromagnetic Form Factor**

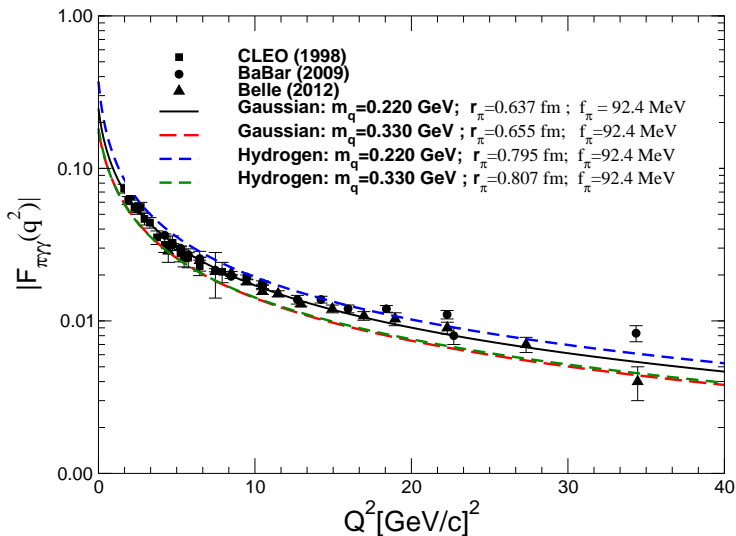
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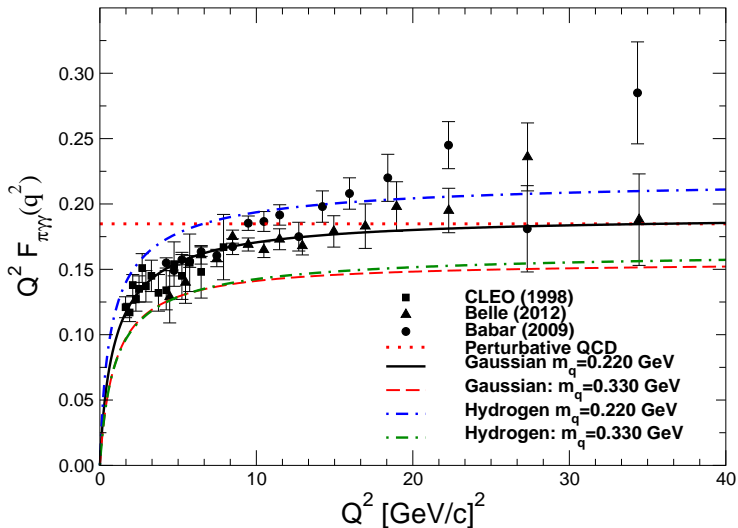
- **Pion Transition Form Factor**

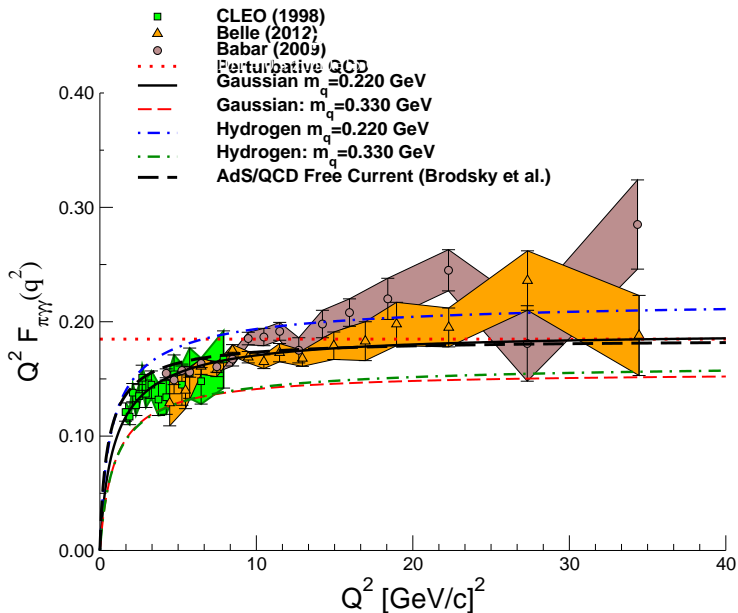
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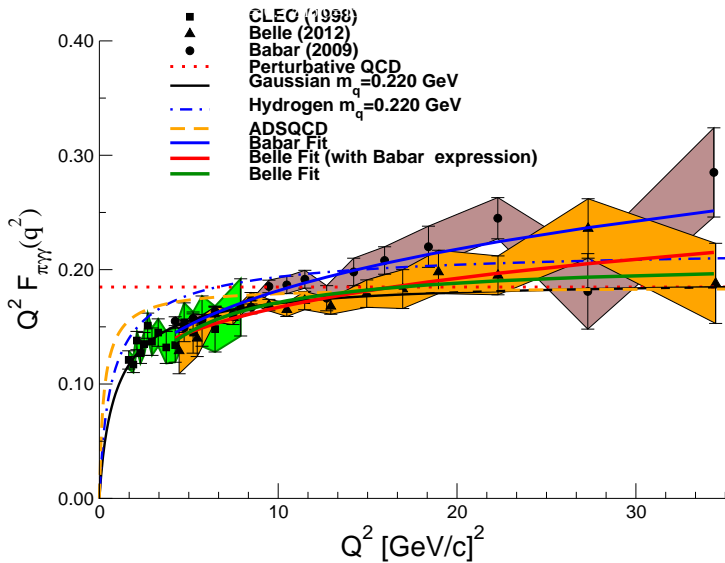






Fit Curves:

- **Babar:** $Q^2|F(Q^2)| = A \left(\frac{Q^2}{10 \text{ GeV}^2}\right)^\beta \implies \begin{cases} A = 0.182 \text{ GeV} \\ \beta = 0.250 \end{cases}$
- **Belle:** $Q^2|F(Q^2)| = A_1 \left(\frac{Q^2}{10 \text{ GeV}^2}\right)^{\beta_1} \implies \begin{cases} A_1 = 0.167 \text{ GeV} \\ \beta_1 = 0.204 \end{cases}$
- **Belle:** $Q^2|F(Q^2)| = \frac{B Q^2}{Q^2 + C} \implies \begin{cases} B = 0.209 \text{ GeV} \\ C = 2.2 \text{ GeV}^2 \end{cases}$



Some Coments

- **Theoretical Analyses:** Explain Babar Data and Not Explain

Models try reproduce Babar:

- **Alteration of the asymptotic pion wave function or distribution amplitude**
- **Dressing $\gamma - q\bar{q}$ -vertex with Phenomological Interactions, ie., like VMD**

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Some Remarks

Ligth-Front Approach:

- \implies **Computation of Form-Factors and Decay Constants**
- \implies **Easy to Test Different Analytical Models**
- \implies **Correct Asymptotic Form-Factors**
- \implies **Agreement with Experiments (CLEO and Belle) and Others Models** (but, also with Babar for small Q^2)
- \implies **The Large Babar data is Not Consistent with QCD**

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