

Light-Front Zero-Mode Issue on the Vector Meson Decay Constant

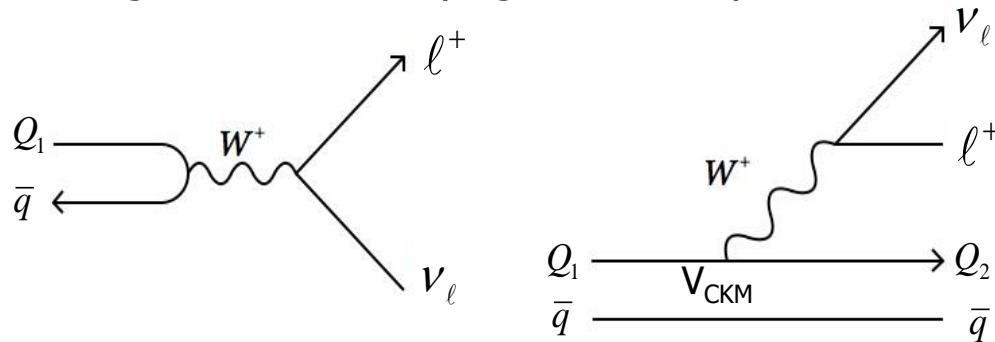
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Outline

- 1. Motivation**
 - Zero-mode issue on the vector meson decay constant
- 2. Comparison of the vector meson decay constant between manifestly covariant model and Light-Front Quark Model(LFQM)**
- 3. LF covariant description of a vector meson decay constant within LFQM**
- 4. Conclusion**

1. Motivation

- Decay constants(f) and weak transition form factors(F_i) are two of the most important ingredients in studying weak decays of mesons.

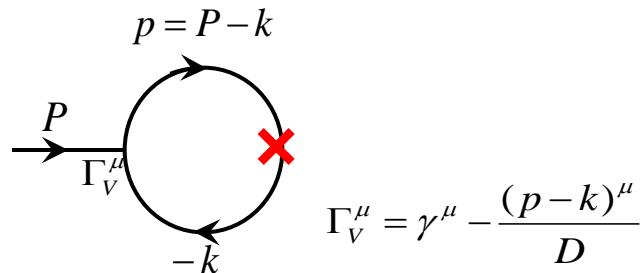


- Light-Front Quark Model (LFQM) based on the LF quantization (Brodsky,Pauli,Pinsky 98) has been quite successful in describing such meson exclusive processes.
- One of the central problems in extracting (f, F_i) is the **zero-mode issue** in the calculation of the current matrix elements.
e.g.) some weak form factors for $P \rightarrow P(V)$ and a **decay constant of a vector meson** [Jaus 99; Cheng,Chua,Hwang (CCH)04; de Melo, Frederico 97,12; Choi&Ji,PRD80,054016(09);NPA865,95(11); PLB695,518(11)]

In this work, we discuss the zero-mode issues raised by Jaus and find the **covariant description of a vector meson decay constant in LFQM**.

- Zero-mode issues raised by Jaus[PRD60, 054026 (99)] & Cheng, Chua, and Hwang(CCH) [PRD69, 074025 (04)]

$$\langle 0 | V^\mu | V(P, h) \rangle = f_V M \varepsilon^\mu(h)$$



$$\Gamma_V^\mu = \gamma^\mu - \frac{(p-k)^\mu}{D}$$

$$\varepsilon^\mu(\pm 1) = \left[0, 2 \frac{\varepsilon_\perp(\pm 1) \cdot \vec{P}_\perp}{P^+}, \varepsilon_\perp(\pm 1) \right],$$

$$\varepsilon_\perp(\pm 1) = \mp \frac{(1, \pm i)}{\sqrt{2}},$$

$$\varepsilon^\mu(0) = \frac{1}{M} \left[P^+, \frac{\vec{P}_\perp^2 - M^2}{P^+}, \vec{P}_\perp \right]$$

- f_V receives a zero-mode(z.m.) when the plus current($\mu=+$) with $\varepsilon(0)$ is used [Standard LF (SLF) method].
- One needs to use $\varepsilon(\pm)$ to obtain LF covariant f_V .
- "It is not clear why the result of f_V depends on the polarization vector."

- In our previous work [Few Body Syst. DOI 10.1007/s00601-012-0535-7 (2013)] , we resolved those issues within the manifestly covariant Bethe-Salpeter(BS) model!

1. f_v receives a zero-mode when the plus current($\mu=+$) with $\epsilon(0)$ is used [SLF method in LFQM].
→ depends on the choice of vertex factor(e.g. immune to the z.m. for $\Gamma^\mu=\gamma^\mu$!)
2. One needs to use $\epsilon(\pm)$ to obtain LF covariant f_v .
→ using $\epsilon(\pm)$ cannot avoid the zero-mode (even at the level of $\Gamma^\mu=\gamma^\mu$!)
3. "It is not clear why the result of f_v depends on the polarization vector."
→ $f_v [\epsilon(0)] = f_v [\epsilon(\pm)]$ if the zero-mode is properly included!
→ Find the exact zero-mode operators in this covariant BS model.

- In this works: we study the applicability of the zero-mode operators found in BS model to a more realistic LFQM.

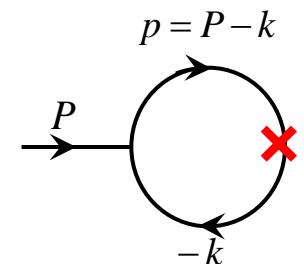
$$[f_V^{(h=0)}]_{BS} = [f_V^{(h=1)}]_{BS} \quad \longrightarrow \quad [f_V^{SLF}]_{LFQM}$$

2. Correspondence between BS model and LFQM

- LF covariant method (BS model)
 - Manifestly covariant!
 - Semi-realistic!
 - Off-shell quark propagator!
 - Know how to find the z.m.

- Standard LF(SLF) method (LFQM)

- Not manifestly covariant!
- More realistic!
- On-shell propagator (e.g. Melosh transformation) !
- Don't know how to find the z.m.



$$\int d^4k H_V(p^2) \dots$$

\downarrow e.g.) $H_V = \frac{g}{(p^2 - \Lambda^2 + i\epsilon)^n}$

$$\int [d^3\vec{k}] \chi(x, \vec{k}_\perp) \times [spin]$$

$$\int d^4k \text{ (4D form of } \varphi \text{ ??)}$$

\downarrow \rightarrow $\phi(x, k_\perp) = \alpha \exp(-\beta \vec{k}^2)$

$$\int [d^3\vec{k}] \phi(x, \vec{k}_\perp) \times [spin]$$

or

$$\phi(x, \vec{k}_\perp) = \frac{\alpha}{(1 + \beta \vec{k}^2)^n}$$

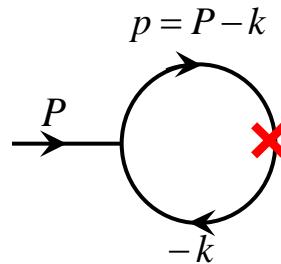
Application LFBS model to LFQM by matching
(i.e. Semi-realistic BS model \rightarrow More Realistic LFQM)

$$\chi \rightarrow \phi$$

e.g.) Decay constant of a **pseudoscalar meson** $\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(P) \rangle = i f_P P^\mu$

- LF covariant method
(BS model)

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_h^\mu$$



$$\begin{aligned} H_V &= \frac{g}{(p^2 - \Lambda^2 + i\varepsilon)^2} \\ N_p &= p^2 - m_1^2 + i\varepsilon \\ N_k &= k^2 - m_2^2 + i\varepsilon \end{aligned}$$

$$f_p^{LFBS} = 2N_c \int \frac{dx d^2 \vec{k}_\perp}{8\pi^3} \frac{A}{(1-x)} \chi(x, \vec{k}_\perp) \quad \longleftrightarrow \quad f_p^{SLF} = \sqrt{2N_c} \int \frac{dx d^2 \vec{k}_\perp}{8\pi^3} \frac{A}{\sqrt{\vec{k}_\perp^2 + A^2}} \phi(x, \vec{k}_\perp)$$

$$A = (1-x)m_1 + xm_2$$

Correspondence:

$$\sqrt{2N_c} \frac{\chi}{1-x} \rightarrow \frac{\phi}{\sqrt{\vec{k}_\perp^2 + A^2}}$$

- SLF method
(LFQM)

$$\begin{aligned} \int [d^3 \vec{k}] \phi(x, \vec{k}_\perp) \times [spin] \\ \phi(x, \vec{k}_\perp) = \alpha \exp(-\beta \vec{k}^2) \end{aligned}$$



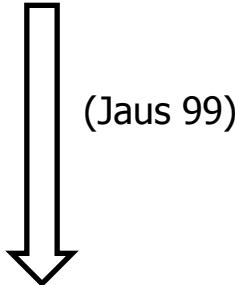
$$\begin{aligned} R_{\lambda\bar{\lambda}}^{00} &= -\frac{\bar{u}_\lambda(p_1)\lambda_5 v_{\bar{\lambda}}(p_2)}{\sqrt{2} [M_0^2 - (m_1 - m_2)^2]^{1/2}} \\ \sum_{\lambda_q \bar{\lambda}_{\bar{q}}} R_{\lambda_q \bar{\lambda}_{\bar{q}}}^{J J_z} R_{\lambda_{\bar{q}} \bar{\lambda}_{\bar{q}}}^{J J_z} &= 1 \end{aligned}$$

$f_p^{LFBS} = f_p^{SLF}$ \Rightarrow No zero mode!

What about a **vector meson**? $\langle 0 | V^\mu | V(P, h) \rangle = f_V M \epsilon^\mu(h)$

- LF covariant method (BS model)

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_h^\mu$$



$$\Gamma_V^\mu = \gamma^\mu - \frac{(p_1 - p_2)^\mu}{D}$$

- SLF method (LFQM)

$$\int [d^3 \vec{k}] \phi(x, \vec{k}_\perp) \times [\text{spin}]$$

$$R_{\lambda\bar{\lambda}}^{IJ_z} = -\frac{\bar{u}_\lambda(p_1) \left[\not{e}(J_z) - \frac{\varepsilon \cdot (p_1 - p_2)}{D_{LF}} \right] v_{\bar{\lambda}}(p_2)}{\sqrt{2} [M_0^2 - (m_1 - m_2)^2]^{1/2}}$$

$$\sum_{\lambda_q \bar{\lambda}_{\bar{q}}} R_{\lambda_q \bar{\lambda}_{\bar{q}}}^{IJ_z} R_{\lambda_q \bar{\lambda}_{\bar{q}}}^{IJ_z} = 1$$

$$\begin{aligned} \left[f_V^{(h=1)} \right]_{Full} &= \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \\ &\times \frac{1}{M} \left\{ xM_0^2 - m_1(m_1 - m_2) - \vec{k}_\perp^2 + \frac{(m_1 + m_2)}{D_{con}} \vec{k}_\perp^2 \right\} \\ D_{con} &= M + m_1 + m_2 \end{aligned}$$

(Jaus 99)

Type I
Correspondence:

$$\begin{aligned} f_V^{SLF(h=0)} &= \sqrt{2N_c} \int \frac{dx d^2 \vec{k}_\perp}{8\pi^3} \frac{\phi(x, \vec{k}_\perp)}{\sqrt{\vec{k}_\perp^2 + A^2}} \left[A + \frac{2\vec{k}_\perp^2}{D_{LF}} \right] \\ D_{LF} &= M_0 + m_1 + m_2 \end{aligned}$$

(Jaus 91, Choi&Ji 99)

$$\begin{aligned} \sqrt{2N_c} \frac{\chi}{1-x} &\rightarrow \frac{\phi}{\sqrt{\vec{k}_\perp^2 + A^2}} \\ D_{con} &\rightarrow D_{LF} \end{aligned}$$

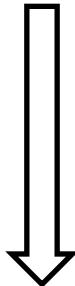
$[f_V^{(h=1)}]_{Full} \neq f_V^{SLF(h=0)}$
Right? Wrong?

We shall show in LFQM using φ that

- LF covariant method (BS model)

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_h^\mu$$

$$\Gamma_V^\mu = \gamma^\mu - \frac{(p_1 - p_2)^\mu}{D}$$



Wrong!

- SLF method (LFQM)

$$\int [d^3 \vec{k}] \phi(x, k_\perp) \times [spin]$$



$$R_{\lambda\bar{\lambda}}^{1J_z} = -\frac{\bar{u}_\lambda(p_1) \left[\not{\phi}(J_z) - \frac{\varepsilon \cdot (p_1 - p_2)}{D_{LF}} \right] v_{\bar{\lambda}}(p_2)}{\sqrt{2} [M_0^2 - (m_1 - m_2)^2]^{1/2}}$$

$$\sum_{\lambda_q \lambda_{\bar{q}}} R_{\lambda_q \lambda_{\bar{q}}}^{JJ_z} {}^\dagger R_{\lambda_q \lambda_{\bar{q}}}^{JJ_z} = 1$$

$$\begin{aligned} \left[f_V^{(h=1)} \right]_{Full} &= \frac{N_c}{4\pi^3 M} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \\ &\times \left\{ xM_0^2 - m_1(m_1 - m_2) - \vec{k}_\perp^2 + \frac{(m_1 + m_2)}{D_{con}} \vec{k}_\perp^2 \right\} \\ D_{con} &= M + m_1 + m_2 \end{aligned}$$



$$\begin{aligned} f_V^{SLF(h=0)} &= \sqrt{2N_c} \int \frac{dx d^2 \vec{k}_\perp}{8\pi^3} \frac{\phi(x, \vec{k}_\perp)}{\sqrt{\vec{k}_\perp^2 + A^2}} \left[A + \frac{2\vec{k}_\perp^2}{D_{LF}} \right] \\ D_{LF} &= M_0 + m_1 + m_2 \end{aligned}$$

Right!

(Jaus 99, CCH 04)

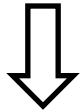
and in addition... find a new covariant form of f_v

3. Self-consistent covariant form of f_V in LFQM

(Step 1) LF covariant method (BS model)

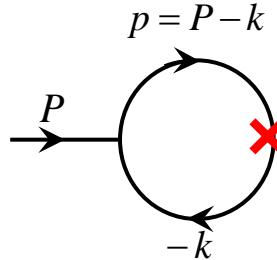
$$\Gamma_V^\mu = \gamma^\mu - \frac{(p-k)^\mu}{D}$$

- LF cal.



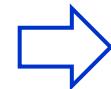
$$f_V^{(h=0)} = \frac{A_{h=0}^+}{M\epsilon^+(0)}$$

$$f_V^{(h=1)} = \frac{A_{h=1}^\perp \cdot \epsilon_\perp^*(+)}{M}$$



$$S_h^\mu = Tr[\gamma^\mu (p + m_1) \Gamma \cdot \epsilon(h) (-k + m_2)]$$

$$\Gamma^\alpha = \gamma^\alpha - \frac{(p-k)^\alpha}{D}$$



- Manifestly covariant cal.

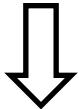
$$f_V^{Cov}$$

3. Self-consistent covariant form of f_V in LFQM

(Step 1) LF covariant method (BS model)

$$\Gamma_V^\mu = \gamma^\mu - \frac{(p-k)^\mu}{D}$$

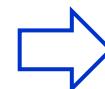
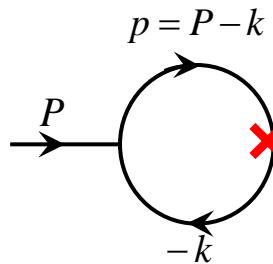
- LF cal.



$$f_V^{(h=0)} = \frac{A_{h=0}^+}{M \epsilon^+(0)}$$

$$f_V^{(h=1)} = \frac{A_{h=1}^\perp \cdot \epsilon_\perp^*(+)}{M}$$

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_h^\mu$$



- Manifestly covariant cal.

$$f_V^{Cov}$$

$$S_h^\mu = Tr[\gamma^\mu (p + m_1) \Gamma \cdot \epsilon(h) (-k + m_2)]$$

$$\Gamma^\alpha = \gamma^\alpha - \frac{(p-k)^\alpha}{D}$$

- Valence contribution ($0 < k^+ < P^+$, i.e. $k^- = k^-_{on}$)

$$(i) [f_V^{(h=0)}]_{val} = f_V^{Cov} \quad \text{when } (\Gamma^\mu = \gamma^\mu) \quad (i) [f_V^{(h=1)}]_{val} \neq f_V^{Cov} \quad \text{when } (\Gamma^\mu = \gamma^\mu)$$

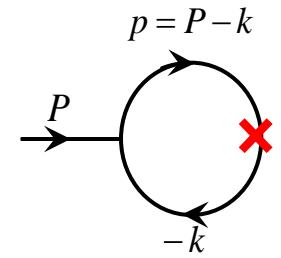
$$(ii) [f_V^{(h=0)}]_{val} \neq f_V^{Cov} \quad \text{when } (D = D_{con}) \quad (ii) [f_V^{(h=1)}]_{val} \neq f_V^{Cov} \quad \text{when } (D = D_{con})$$

$$\rightarrow f_V^{Cov} - [f_V^{(h=0)}]_{val} \equiv [f_V^{(h=0)}]_{Z.M.}$$

$$\rightarrow f_V^{Cov} - [f_V^{(h=1)}]_{val} \equiv [f_V^{(h=1)}]_{Z.M.}$$

- Method of finding zero-mode operator [Choi & Ji, PRD80,054016(09); NPA 856, 95(11); PLB696, 518(11)]:

$$[f_V^{(h=0)}]_{Full} = \frac{[A_{h=0}^+]_{val} + [A_{h=0}^+]_{Z.M.}}{M\mathcal{E}^+(0)} = [f_V^{(h=0)}]_{val} + [f_V^{(h=0)}]_{Z.M.}$$



$$[A_{h=0}^+]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ [S_{h=0}^+]_{val} + [S_{h=0}^+]_{Z.M.} \right\}$$

$$\chi(x, k_\perp) = \frac{g}{[x(M^2 - M_0^2)][x^n(M^2 - M_\Lambda^2)^n]}$$

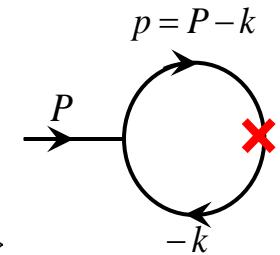
Zero-mode operator!

$$M_{0(\Lambda)}^2 = \frac{k_\perp^2 + m_1^2(\Lambda^2)}{x} + \frac{k_\perp^2 + m^2}{1-x}$$

$$D_{con} = M + m_1 + m_2$$

- Method of finding zero-mode operator [Choi & Ji, PRD80,054016(09); NPA 856, 95(11); PLB696, 518(11)]:

$$[f_V^{(h=0)}]_{Full} = \frac{[A_{h=0}^+]_{val} + [A_{h=0}^+]_{Z.M.}}{M\varepsilon^+(0)} = [f_V^{(h=0)}]_{val} + [f_V^{(h=0)}]_{Z.M.}$$



$$[A_{h=0}^+]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ [S_{h=0}^+]_{val} + [S_{h=0}^+]_{Z.M.} \right\}$$

$$\chi(x, k_\perp) = \frac{g}{[x(M^2 - M_0^2)][x^n(M^2 - M_\Lambda^2)^n]}$$

Zero-mode operator!

$$M_{0(\Lambda)}^2 = \frac{k_\perp^2 + m_1^2(\Lambda^2)}{x} + \frac{k_\perp^2 + m^2}{1-x}$$

Step1) Find the (singular) term proportional to p^- (or $1/x$) in S_h^μ as $x \rightarrow 0$ when $p^- = p^-_{on}$.

$$D_{con} = M + m_1 + m_2$$

$$\lim_{x \rightarrow 0} S_{h=0}^+(p^- = p^-_{on}) = 4 \frac{\varepsilon^+(0)}{D_{con}} m_1 p^-$$

Step2) Replace $p^- \rightarrow -Z_2$ [Jaus 99, Choi&Ji 09,11]

$$[S_{h=0}^+]_{Z.M.} = 4 \frac{\varepsilon^+(0)}{D_{con}} m_1 (-Z_2)$$

where

$$Z_2 = x(M^2 - M_0^2) + m_1^2 - m^2 + (1-2x)M^2$$

- Method of finding zero-mode operator [Choi & Ji, PRD80,054016(09); NPA 856, 95(11); PLB696, 518(11)]:

$$[f_V^{(h=1)}]_{Full} = \frac{([A_{h=1}^\perp]_{val} + [A_{h=0}^+]_{Z.M.}) \cdot \mathcal{E}_\perp^*(+)}{M} = [f_V^{(h=1)}]_{val} + [f_V^{(h=1)}]_{Z.M.}$$

$$[A_{h=1}^\perp]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ [S_{h=1}^\perp]_{val} + [S_{h=1}^\perp]_{Z.M.} \right\}$$

Zero-mode operator!

$$[S_{h=1}^\perp]_{Z.M.} = 2(-Z_2) \mathcal{E}_\perp(+)$$

We find from the covariant BS model that

$$(a) [f_V^{(h=0)}]_{Full} = [f_V^{(h=1)}]_{Full} = f_V^{Cov}$$

$$(b) [f_V^{(h=0)}]_{val} = [f_V^{(h=0)}]_{on}$$

$$(c) [f_V^{(h=1)}]_{val} = [f_V^{(h=1)}]_{on} + [f_V^{(h=1)}]_{off}$$

$\rightarrow 0$ as $M \rightarrow M_0$

(Step 2) Check if the zero mode found in BS model can be applicable to LFQM with φ

$$\left[f_V^{(h=0)} \right]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ \left[S_{h=0}^+ \right]_{val} + \left[S_{h=0}^+ \right]_{Z.M.} \right\}$$

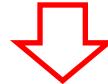
$$\left[f_V^{(h=1)} \right]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \frac{1}{M} \left\{ \left[S_{h=1}^\perp \right]_{val} + \left[S_{h=1}^\perp \right]_{Z.M.} \right\} \cdot \varepsilon_\perp^*(+)$$

Type I
Correspondence
(Jaus's prescription):

$$\boxed{\begin{aligned} \sqrt{2N_c} \frac{\chi}{1-x} &\rightarrow \frac{\phi}{\sqrt{k_\perp^2 + A^2}} \\ D_{con} &\rightarrow D_{LF} \end{aligned}}$$



$$\left[f_V^{(h=1)} \right]_{Full} \neq f_V^{SLF}$$



$$\left[f_V^{(h=0)} \right]_{Full} \stackrel{?}{=} \left[f_V^{(h=1)} \right]_{Full}$$

(Step 2) Check if the zero mode found in BS model can be applicable to LFQM with φ

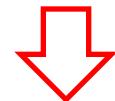
$$[f_V^{(h=0)}]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ [S_{h=0}^+]_{val} + [S_{h=0}^+]_{Z.M.} \right\}$$

$$[f_V^{(h=1)}]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \frac{1}{M} \left\{ [S_{h=1}^\perp]_{val} + [S_{h=1}^\perp]_{Z.M.} \right\} \cdot \varepsilon_\perp^*(+)$$

Type I
Correspondence
(Jaus's prescription):

$$\sqrt{2N_c} \frac{\chi}{1-x} \xrightarrow{\text{No}} \frac{\phi}{\sqrt{k_\perp^2 + A^2}}$$

$$D_{con} (= M + m_1 + m_2) \rightarrow D_{LF} (= M_0 + m_1 + m_2)$$



$$[f_V^{(h=0)}]_{Full} \neq [f_V^{(h=1)}]_{Full} \neq f_V^{SLF}$$

Jaus's formula
doesn't satisfy
the covariance!

(Step 2) Check if the zero mode found in BS model can be applicable to LFQM with φ

$$[f_V^{(h=0)}]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \left\{ [S_{h=0}^+]_{val} + [S_{h=0}^+]_{Z.M.} \right\}$$

$$[f_V^{(h=1)}]_{Full} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \frac{1}{M} \left\{ [S_{h=1}^\perp]_{val} + [S_{h=1}^\perp]_{Z.M.} \right\} \cdot \varepsilon_\perp^*(+)$$

Type II
Correspondence
(Our prescription):

$$\sqrt{2N_c} \frac{\chi}{1-x} \rightarrow \frac{\phi}{\sqrt{k_\perp^2 + A^2}}$$

$$B.E. = (M - M_0) \rightarrow 0$$



$$\begin{aligned} [f_V^{(h=0)}]_{Full} &= [f_V^{(h=1)}]_{Full} = f_V^{SLF} \\ &= [f_V^{(h=1)}]_{on} \end{aligned}$$

$$\left[f_V^{(h=0)} \right]_{Full} = \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \\ \times \frac{1}{M} \left\{ x(1-x) M^2 + \vec{k}_\perp^2 + m_1 m_2 + x(m_1 + m_2) \frac{[\vec{k}_\perp^2 + m_2^2 - (1-x)^2 M^2]}{(1-x) D_{con}} \right\}$$

$$\left[f_V^{(h=1)} \right]_{on} = \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \\ \times \frac{1}{M} \left\{ \frac{\vec{k}_\perp^2 + A^2}{2x(1-x)} - \vec{k}_\perp^2 + \frac{(m_1 + m_2)}{D_{con}} \vec{k}_\perp^2 \right\}$$

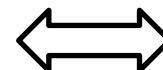
$$\left[f_V^{(h=1)} \right]_{Full} = \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \vec{k}_\perp \chi(x, \vec{k}_\perp) \\ \times \frac{1}{M} \left\{ x M_0^2 - m_1(m_1 - m_2) - \vec{k}_\perp^2 + \frac{(m_1 + m_2)}{D_{con}} \vec{k}_\perp^2 \right\}$$

$$f_V^{SLF(h=0)} = \sqrt{2N_c} \int \frac{dx d^2 \vec{k}_\perp}{8\pi^3} \frac{\phi(x, \vec{k}_\perp)}{\sqrt{\vec{k}_\perp^2 + A^2}} \left[A + \frac{2\vec{k}_\perp^2}{D_{LF}} \right]$$

• Decay constants(in unit of MeV)

		$f_{full}^{(h=0)}$	$f_{on}^{(h=1)}$	$f_{full}^{(h=1)}$	f_{SLF}
ρ	Type I	304	369	369	246
	Type II	[246]	[246]	[246]	
K^*	I	322	389	389	256
	II	[256]	[256]	[256]	
D^*	I	274	322	322	239
	II	[239]	[239]	[239]	
J/ψ	I	429	486	486	360
	II	[360]	[360]	[360]	

Jaus's prescription: $f_{full}^{(h=1)}$ with Type I
[PRD60,054026(99)]



Our LFQM: f_{SLF}
[PRD75,034019(07); PRD75,073016(07)]

• Decay constants(in unit of MeV)

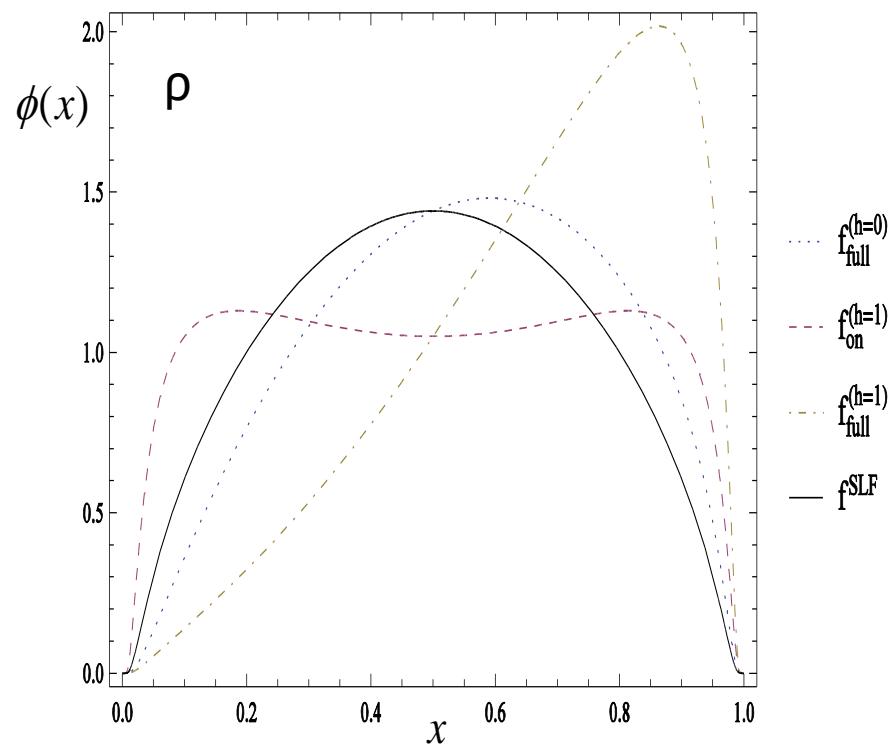
	$f_{full}^{(h=0)}$	$f_{on}^{(h=1)}$	$f_{full}^{(h=1)}$	f_{SLF}
ρ Type I 	304	369	369 	246
	[246]	[246]	[246]	
K^* 	322	389	389 	256
	[256]	[256]	[256]	
D^* 	274	322	322 	239
	[239]	[239]	[239]	
J/ψ 	429	486	486 	360
	[360]	[360]	[360]	

(Step 3) Further constraint

	$f_{\text{full}}^{(h=0)}$	$f_{\text{on}}^{(h=1)}$	$f_{\text{full}}^{(h=1)}$	f^{SLF}
ρ	Type I 304	369	369 →	246
	Type II [246]	[246]	[246]	
K^*	I 322	389	389 →	256
	II [256]	[256]	[256]	
D^*	I 274	322	322 →	239
	II [239]	[239]	[239]	
J/ψ	I 429	486	486 →	360
	II [360]	[360]	[360]	

Quark Distribution Amplitudes(DAs)

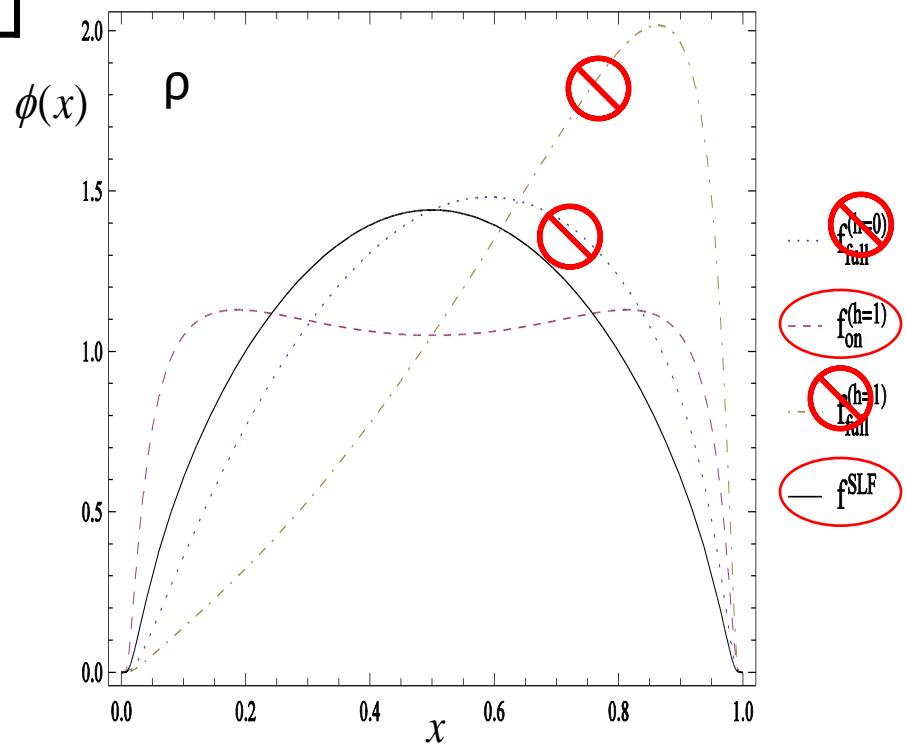
$$\int_0^1 \phi(x) dx = \frac{f_V}{2\sqrt{6}}$$



	$f_{\text{full}}^{(h=0)}$	$f_{\text{on}}^{(h=1)}$	$f_{\text{full}}^{(h=1)}$	f^{SLF}
ρ	Type I 304	369	369 →	246
	Type II [246]	[246]	[246]	
K^*	I 322	389	389 →	256
	II [256]	[256]	[256]	
D^*	I 274	322	322 →	239
	II [239]	[239]	[239]	
J/ψ	I 429	486	486 →	360
	II [360]	[360]	[360]	

Quark Distribution Amplitudes(DAs)

$$\int_0^1 \phi(x) dx = \frac{f_V}{2\sqrt{6}}$$

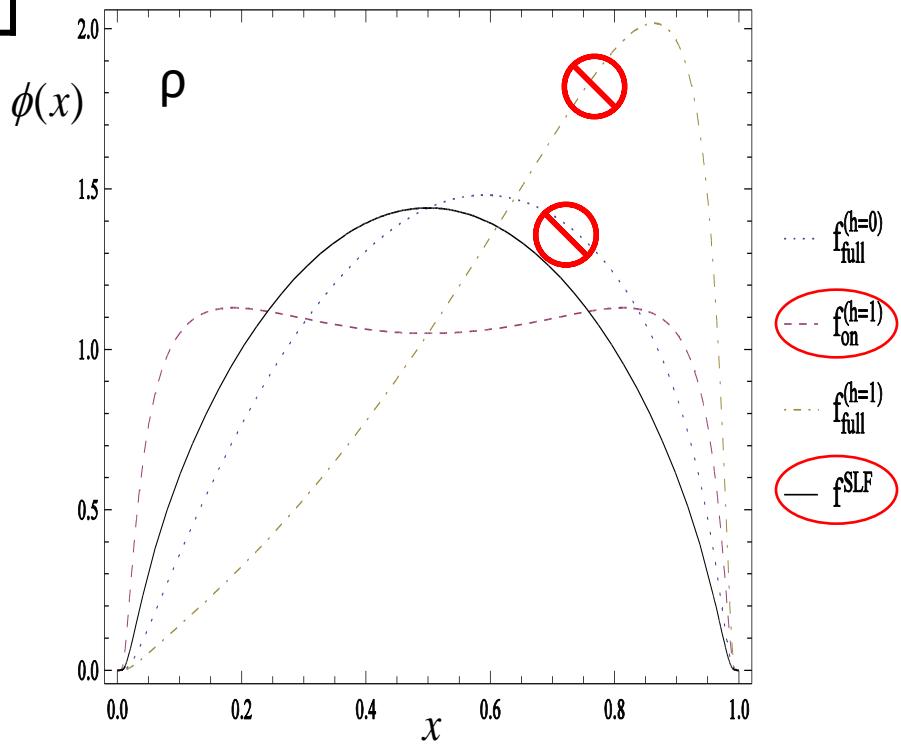


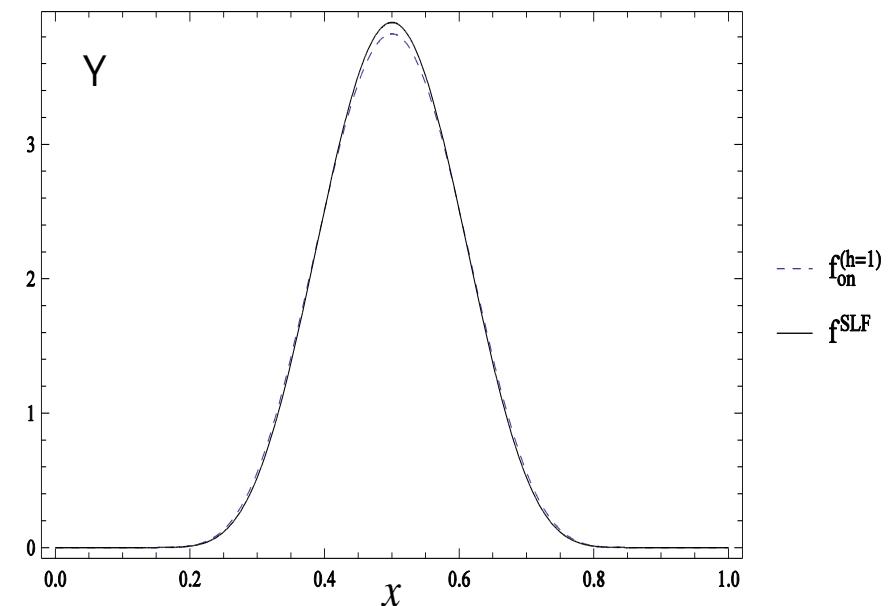
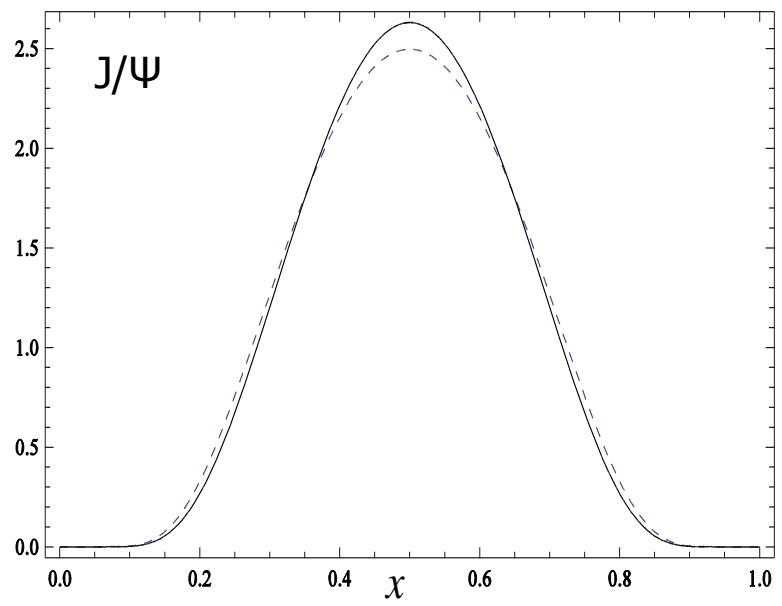
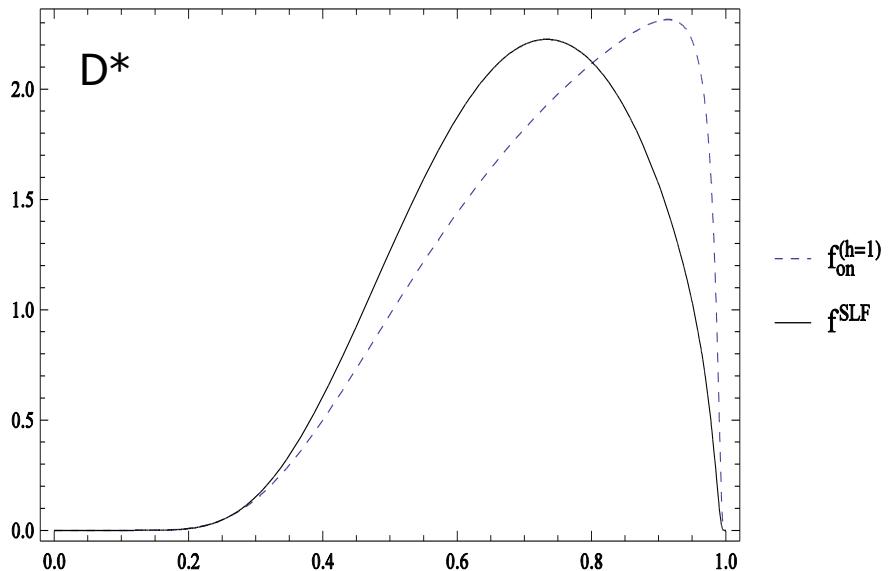
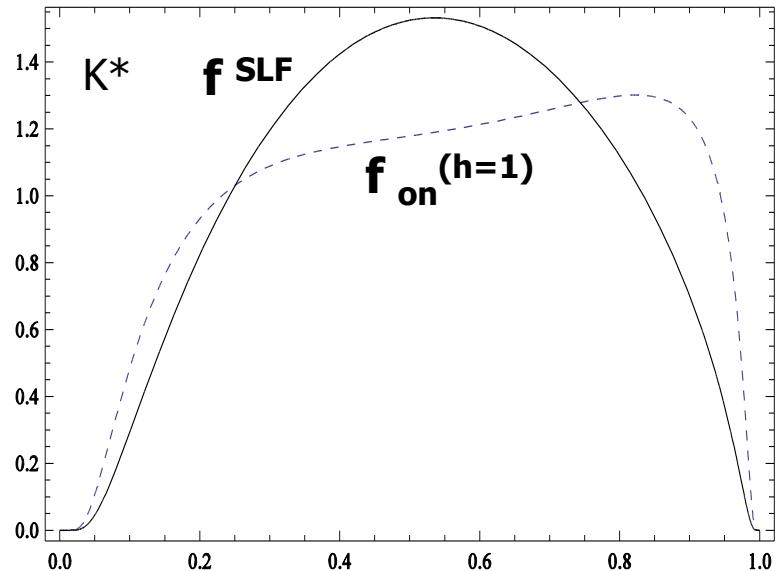
	$f_{\text{full}}^{(h=0)}$	$f_{\text{on}}^{(h=1)}$	$f_{\text{full}}^{(h=1)}$	f^{SLF}
ρ	Type I	304	369	369 → 246
	Type II	[246]	[246]	[246]
K^*	I	322	389	389 → 256
	II	[256]	[256]	[256]
D^*	I	274	322	322 → 239
	II	[239]	[239]	[239]
J/ψ	I	429	486	486 → 360
	II	[360]	[360]	[360]

Quark Distribution Amplitudes(DAs)

$$\int_0^1 \phi(x) dx = \frac{f_V}{2\sqrt{6}}$$

→ Play the role of very good Constraints!





4. Conclusions

In this work, we investigated a vector meson decay constant f_V in both manifestly covariant model and more realistic LFQM.

1. We computed f_V using $(J^+, \varepsilon(0))$ and $(J^\perp, \varepsilon(+))$ and check the LF covariance of it.
 - We have found in the manifestly covariant model that
 - *f_V is independent of the polarization vectors if the zero-modes are properly included!*
 - Jaus's Type I replacement of the LF vertex function in covariant model with the gaussian w.f. shows that the two combinations give different results.
 - Jaus's form of decay constant is incorrect for more realistic LFQM using gaussian w.f.
2. We find a self-consistent covariant description of f_V in a more realistic LFQM
 - $f^{(h=1)}_{\text{on}}$ with type II replacement and f^{SLF} satisfy the consistency of the model
 - the quark DA plays an important constraint to check the validity of the model