The Nucleon Spin Sum Rule

Matthias Burkardt

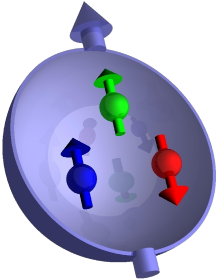
New Mexico State University

May 23, 2013

spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\, [q_\uparrow(x) - q_\downarrow(x)] =$ fraction of the nucleon spin due to quark spins
- $\Delta G =$ contribution from gluon spin to the nucleon spin
- $\mathcal{L} =$ quark & gluon orbital angular momentum



EMC collaboration (1987):

only small fraction of the proton spin due to quark spins

$$\Delta\Sigma \sim 30\%$$

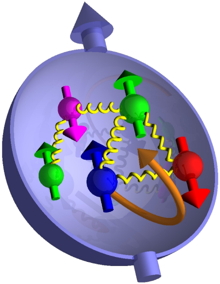
↪ was called 'spin crisis', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

↪ quest for the remaining 70%

spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\, [q_\uparrow(x) - q_\downarrow(x)] =$ fraction of the nucleon spin due to quark spins
- $\Delta G =$ contribution from gluon spin to the nucleon spin
- $\mathcal{L} =$ quark & gluon orbital angular momentum



EMC collaboration (1987):

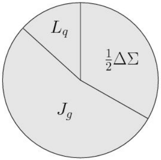
only small fraction of the proton spin due to quark spins

$$\Delta\Sigma \sim 30\%$$

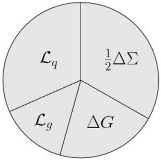
↪ was called 'spin crisis', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

↪ quest for the remaining 70%

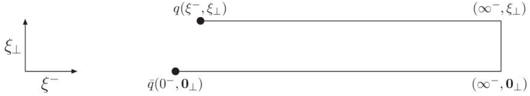
- the nucleon spin pizzas
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link $\longrightarrow L^q$ ('Ji-OAM')
- light-cone staple- gauge link $\longrightarrow \mathcal{L}_+^q$ ('JM-OAM')
- $\mathcal{L}_+^q - L^q =$ change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$ gauge (with anti-symmetric boundary condition) $\mathcal{L}_+^q \to$ canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD

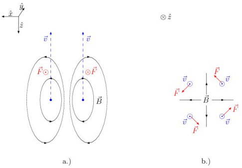


'pizza tre stagioni'



'pizza quattro stagioni'



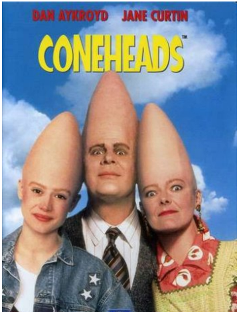


What could possibly be simpler:

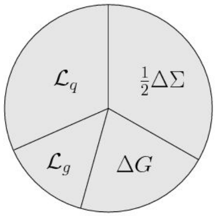
- take light-cone wave function for each Fock component
- evaluate expectation value of $-i\sum_k \left(\vec{r}_k \times \vec{\nabla}_k\right)_z$ for each Fock component

$\hookrightarrow$ sum over all Fock components

- result equivalent to OAM in JM decomposition



Jaffe-Manohar decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

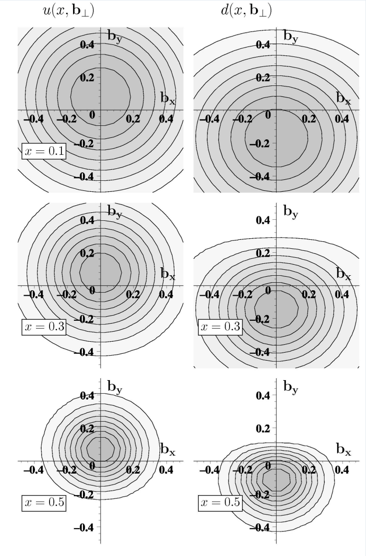
$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+\left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle$$

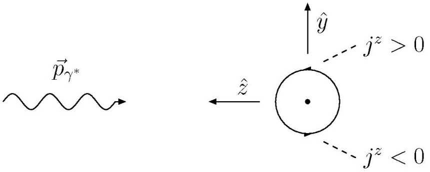
$$\Delta G = \varepsilon^{+-ij}\int d^3r\, \langle P,S|\, \mathrm{Tr} F^{+i} A^j\, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S|\mathrm{Tr} F^{+j}\left(\vec{x} \times i\vec{\partial}\right)^z A^j|P,S\rangle$$

manifestly gauge invariant definition for each term exists ($\rightarrow$ Hatta)

Impact Parameter Dependent Quark Distributions





proton polarized in $+\hat{x}$ direction

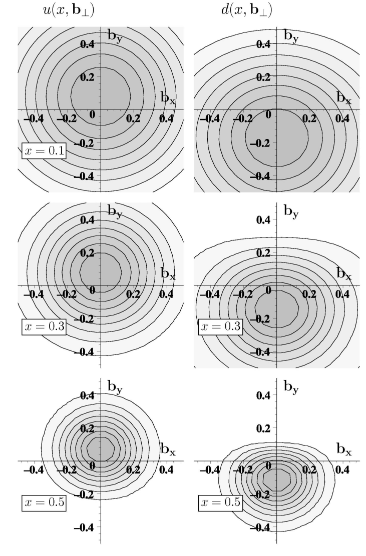
no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

Impact Parameter Dependent Quark Distributions



proton polarized in $+\hat{x}$ direction

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

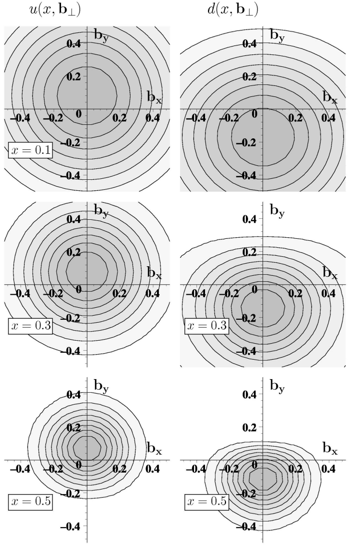
sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

Impact Parameter Dependent Quark Distributions



sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

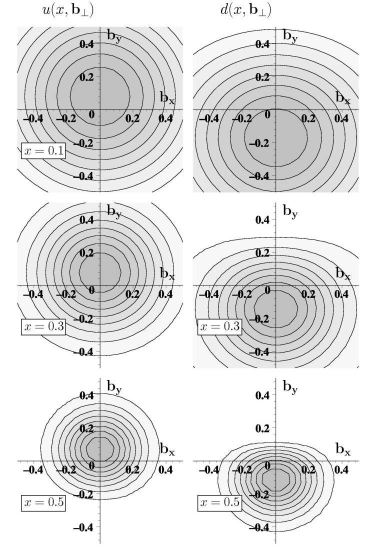
↪ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

↪ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!

Impact Parameter Dependent Quark Distributions

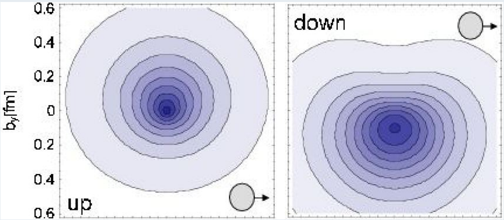


sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

lattice QCD (QCDSF): lowest moment



distribution in delocalized wave packet (pol. in $+\hat{x}$ direction)

$$q_\psi(x,\mathbf{b}_\perp) = \int d^2r_\perp q(x,b_\perp - r_\perp) \left(|\psi(\mathbf{r}_\perp)|^2 - \frac{1}{2M}\frac{\partial}{\partial r_y}|\psi(\mathbf{r}_\perp)|^2\right) \text{ with}$$

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

two contributions to $\perp$ shift

- intrinsic shift relative to center of momentum $\mathbf{R}_\perp$
- overall shift of $\mathbf{R}_\perp$ for $\perp$ polarized nucleon

insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$ (here: derived for $\vec{p} = \vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components of $\vec{J}_q$!
- Fourier trafo of $\frac{1}{2}\int dx\, x\left[H(x,0,t) + E(x,0,t)\right]$ not distribution of angular momentum [L.Adhikari+MB 2013]

insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right]$ (here: derived for $\vec{p}=\vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components of $\vec{J}_q$!
- Fourier trafo of $\frac{1}{2}\int dx\, x\left[H(x,0,t)+E(x,0,t)\right]$ not distribution of angular momentum [L.Adhikari+MB 2013]

gauge invariance

- matrix element of $T_q^{++} = \bar{q}\gamma^+ i\partial^+ q$ in $A^+=0$ gauge same as that of $\bar{q}\gamma^+\left(i\partial^+ - gA^+\right)q$ in any gauge

$\hookrightarrow$ identify $\frac{1}{2}\int dx\, x\left[H(x,0,0)+E(x,0,0)\right]$ with J_q in decomposition where $\vec{L}_q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)q(\vec{x})|P,S\rangle$

insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right]$ (here: derived for $\vec{p}=\vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components of $\vec{J}_q$!
- Fourier trafo of $\frac{1}{2}\int dx\, x\left[H(x,0,t)+E(x,0,t)\right]$ not distribution of angular momentum [L.Adhikari+MB 2013]

caution!

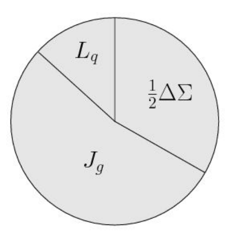
- made heavily use of rotational invariance

$\hookrightarrow$ itentification $\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H(x,0,0)+E(x,0,0)\right]$ does not apply to unintegrated quantities

  - $\int d^2\Delta_\perp e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}\frac{x}{2}\left[H(x,0,-\Delta_\perp^2)+E(x,0,-\Delta_\perp^2)\right]$ not equal to $J^z(\mathbf{b})_\perp$
  - $J_q(x)\equiv\frac{x}{2}\left[H_q(x,0,0)+E_q(x,0,-\Delta_\perp^2)\right]$ not x-distribution of angular momentum $J_q^z(x)$ in long. pol. target

regardless whether one takes gauge covariant definition or not

Ji decomposition



'pizza tre stagioni'

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$

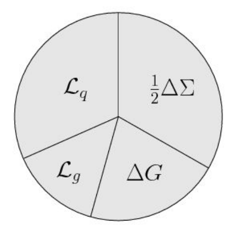
$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\, |P,S\rangle$

$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$

$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$

$\mathcal{L}_q = \int d^3r \langle P,S| \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$

$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$

$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$

manifestly gauge invariant definition for each term exists ($\rightarrow$ Hatta)

QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right] \\ &= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right] \\ &= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]\end{aligned}$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of $\vec{A} \to$ Chen/Goldman, Wakamatsu

Ji decomposition

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \,|P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S|\, \mathrm{Tr} F^{+i} A^j \,|P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

manifestly gauge invariant definitions for each term exist ($\rightarrow$ Hatta)

- GPDs $\longrightarrow L^q$
- $\vec{p}\,\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i\in q,g} \mathcal{L}^i$
- QED: $\mathcal{L}^e \neq L^e$ [M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
  - can we calculate/predict the difference?
  - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

- (quasi) probabilty distribution for $\mathbf{b}_\perp$ and $\mathbf{k}_\perp$
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$

OAM from Wigner (Lorcè, Pasquini, ...)

$$L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$$
$$= \int d^3r \langle P,S|\,\bar{q}(\vec{r})\gamma^+ \left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle = \mathcal{L}^q$$

Gauge Invariance?

need to include Wilson-line gauge link to connect 0 and ξ (Ji, Yuan; Hatta; Lorcè;...)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ \mathcal{U}_{0\xi} q(\xi)|PS\rangle.$$

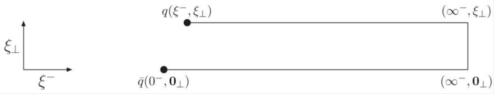
$\langle\vec{k}_\perp\rangle \equiv \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)\vec{k}_\perp$ depends on choice of path!

straight-line gauge link

$\langle\vec{k}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D} q(\vec{x})|P,S\rangle$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle\vec{k}_\perp\rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

$\langle\vec{\mathcal{K}}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{\mathcal{D}} q(\vec{x})|P,S\rangle$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$ $\quad A^+ = 0$
- $\langle\vec{\mathcal{K}}_\perp\rangle \neq 0$ (FSI! Brodsky,Hwang,Schmidt)

Quark $\perp$ momentum from Wigner Distributions

difference $\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$

$$\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x})|P,S\rangle$$

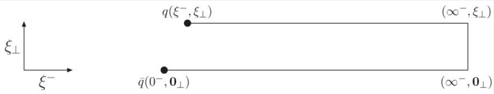
$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D} q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle \vec{k}_\perp \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{\mathcal{D}} q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp) + g \int_{x^-}^{\infty} dr^- \vec{\partial} A^+$
- $i\mathcal{D}^j = i\partial^j - gA^j(x^-, \mathbf{x}_\perp) - g \int_{x^-}^{\infty} dr^- F^{+j}$

difference $\langle \vec{\mathcal{K}}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{\mathcal{K}}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x})|P,S\rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Impulse due to FSI

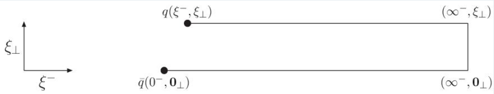
$\Delta \vec{k}_\perp^q \equiv \langle \vec{\mathcal{K}}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$
$=$ (average) change in ⊥ momentum due to FSI!

straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D} q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle \vec{k}_\perp \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{\mathcal{D}} q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp) + g\int_{x^-}^{\infty} dr^- \vec{\partial} A^+$
- $i\mathcal{D}^j = i\partial^j - gA^j(x^-, \mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$

difference $\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$

$$\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x})|P,S\rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Corollary: $d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

matrix element defining d_2 $\leftrightarrow$ 1^{st} integration point in QS-integral

difference $\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$

$$\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x})|P,S\rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Corollary: $d_2 \leftrightarrow$ average ⊥ force on quark in DIS from ⊥ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

sign of d_2 $\leftrightarrow$ ⊥ deformation of quark distributions

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ \mathcal{U}_{0\xi} q(\xi)|PS\rangle.$$

W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields

$$L^q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- same as Ji-OAM
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$ not the TMDs relevant for SIDIS (missing FSI!)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

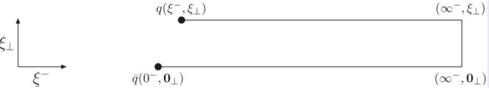
$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ \mathcal{U}_{0\xi} q(\xi)|PS\rangle.$$

W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

Light-Cone Staple for $\mathcal{U}^{\pm LC}_{0\xi}$ (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated $d^2\mathbf{b}_\perp$

$\hookrightarrow$ path for gauge link $\longrightarrow$ 'light-cone staple' $\longrightarrow \mathcal{U}^{+LC}_{0\xi}$



$$\mathcal{L}^q_+ = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^3 q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty,\mathbf{x}_\perp) \quad (\quad A^+ = 0)$$

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

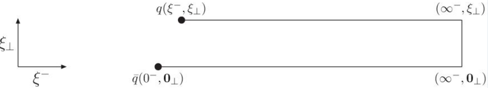
$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle.$$

W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated $d^2\mathbf{b}_\perp$

↪ path for gauge link $\longrightarrow$ 'light-cone staple' $\longrightarrow \mathcal{U}_{0\xi}^{+LC}$



$$\mathcal{L}_+^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^3 q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty,\mathbf{x}_\perp) + g\int_{x^-}^{\infty} dr^- \vec{\partial}A^+$$

$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$$

straight line ($\rightarrow$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ($\rightarrow$ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^{\infty} dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple (→ Jaffe-Manohar)

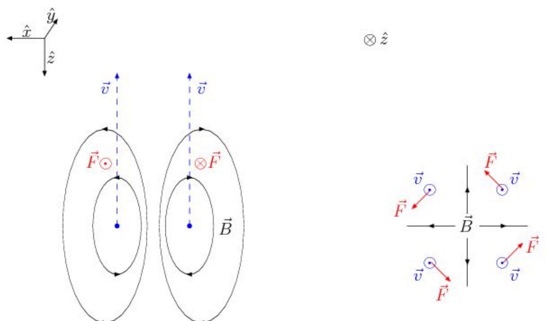
$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ (→ Wakamatsu: $-L^q_{pot}$)

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



PT (Hatta)

- PT$\longrightarrow \mathcal{L}_+ = \mathcal{L}_-$

(differend than SSAs due to factor $\vec{x}$ in OAM)

antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv. $A^\mu \rightarrow A^\mu + \partial^\mu \phi(\vec{x}_\perp)$ by imposing $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$ gauge inv.
- $\mathcal{L}_+$ involves $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_-$ involves $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \rightarrow$ no contribution from $\vec{A}(\infty, \mathbf{x}_\perp)$

$\hookrightarrow$ 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \quad \rightarrow \quad \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv. $A^\mu \to A^\mu + \partial^\mu \phi(\vec{x}_\perp)$ by imposing $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$ gauge inv.
- $\mathcal{L}_+$ involves $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_-$ involves $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \to$ no contribution from $\vec{A}(\infty, \mathbf{x}_\perp)$

$\hookrightarrow$ 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

PT (Hatta)

- PT$\longrightarrow \mathcal{L}_+ = \mathcal{L}_-$

(differend than SSAs due to factor $\vec{x}$ in OAM)

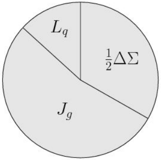
alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \quad \to \quad \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2}\left(\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp) \right)$

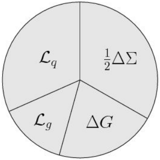
$\hookrightarrow \mathcal{L}_{JB} = \frac{1}{2}\left(\mathcal{L}_+ + \mathcal{L}_-\right) = \mathcal{L}_+ = \mathcal{L}_-$

Summary

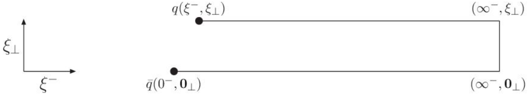
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link $\longrightarrow L^q$ ('Ji-OAM')
- light-cone staple- gauge link $\longrightarrow \mathcal{L}^q_+$ ('JM-OAM')
- $\mathcal{L}^q_+ - L^q =$ change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$ gauge (with anti-symmetric boundary condition) $\mathcal{L}^q_+ \to$ canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD

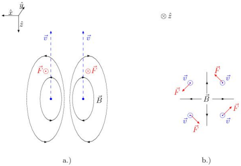


'pizza tre stagioni'

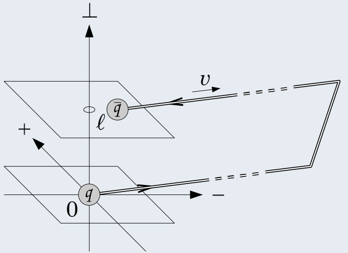


'pizza quattro stagioni'





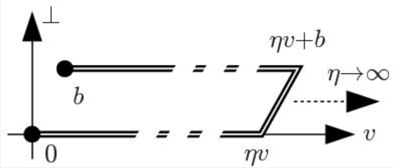
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

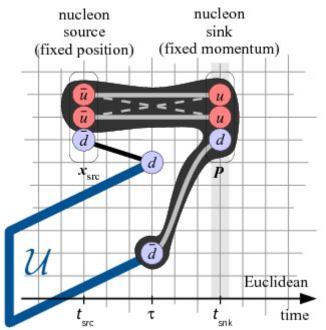
solution: 'Infinite Momentum Frame'

B. Musch, P. Hägler, M. Engelhardt

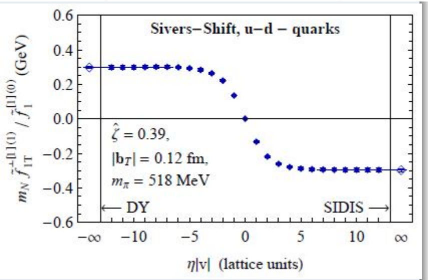


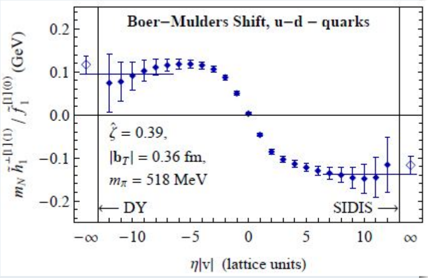
- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by dividing with nonperturbatively calculated 'soft factor'

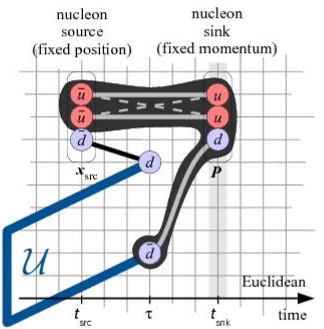
↪ numerically extrapolate to $P_z \to \infty$

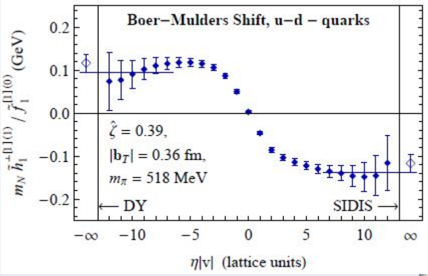


$f^{\perp}_{1T,SIDIS} = -f^{\perp}_{1T,DY}$ (Collins)

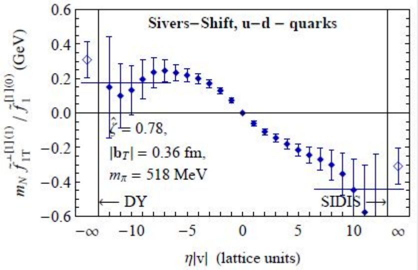


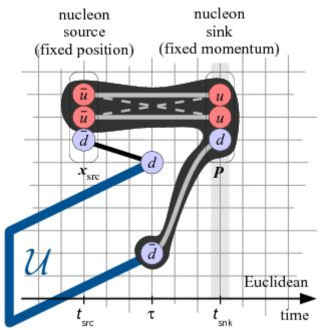


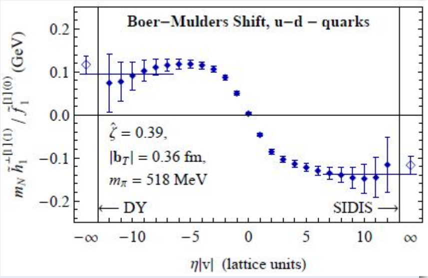




$f^{\perp}_{1T,SIDIS} = -f^{\perp}_{1T,DY}$ (Collins)







next: Orbital Angular Momentum

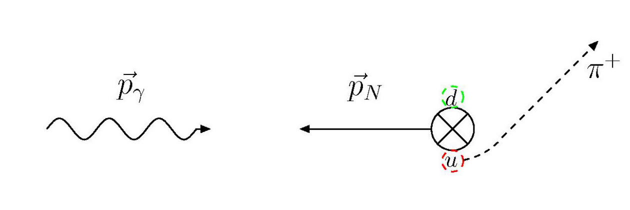
- nonforward matrix elements:
  - staple with long side in $\hat{z}$ direction
  - short side in $\hat{x}$ direction
  - (large) nucleon momentum in $\hat{z}$ direction
  - small momentum transfer in $\hat{y}$ direction

↪ generalized TMD F_{14} (Metz et al.)

- quark OAM
- renormalization same as $f_{1T}^{\perp}$

↪ study ratios...

Sivers $f_{1T}^{\perp}$ in semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by κ_u & κ_d
- attractive FSI deflects active quark towards the CoM

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ 'chromodynamic lensing'

$\Rightarrow$ $\kappa_p \, , \kappa_n \quad \longleftrightarrow \quad$ sign of SSA, d_2 !!!!!!!!

- confirmed by HERMES (and recent COMPASS) p data; consistent with vanishing isoscalar Sivers (COMPASS)