Double parton distribution functions of the nucleon and their evolution

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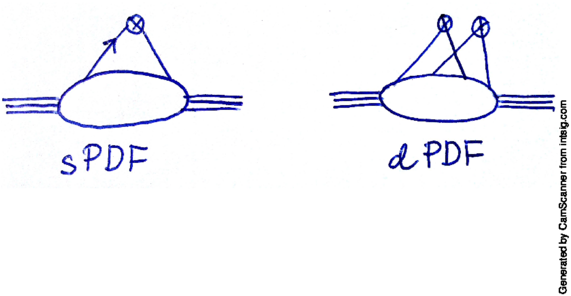
Motivation

- Old story, renewed interest (LHC) [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979 ..., reviews: Bartalani et al. 2011, Snigirev 2011]
- Multiple parton scattering, unitarity
- Very little explored [MIT bag: Chang, Manohar, Waalewijn 2013], constituent quark: Rinaldi, Scopetta, Vento 2013]
- Questions: factorization, interference of SPD and DPS [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013]
- Will be possible to see? - p+Pb enhancement [Blok et al. 2011, d'Enterria, Snigirev 2012]

New elements

- Valon model [Hwa, Zahir 1981, Hwa, Yang 2002] as a proper initial condition
- Numerical study of the dDGLAP evolution in the Mellin space and the parton correlations

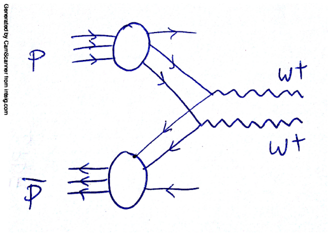
Probing two partons



(subleading twist, generically small)

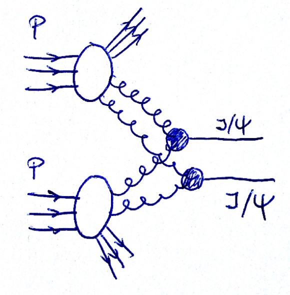
Master formula DPS production

$$\begin{aligned}\sigma^{DPS}_{hh'\to ab} \; = \; & \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx'_1 dx'_2 d^2b_1 d^2b_2 d^2b \times \\ & \Gamma^{ij}_h(x_1,x_2,b_1,b_2,Q_1^2,Q_2^2)\Gamma^{kl}_{h'}(x'_1,x'_2,b+b_1,b+b_2,Q_1^2,Q_2^2)\times \\ & \sigma^{ik}_h(x_1,x'_1,Q_1^2)\sigma^{jl}_{h'}(x_2,x'_2,Q_2^2)\end{aligned}$$



Master formula DPS production

$$
\begin{aligned}
\sigma^{DPS}_{hh' \to ab} \quad = \quad & \frac{m}{2} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2b_1 d^2b_2 d^2b \times \\
& \Gamma^{ij}_{h}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2) \Gamma^{kl}_{h'}(x'_1, x'_2, b + b_1, b + b_2, Q_1^2, Q_2^2) \times \\
& \sigma^{ik}_{h}(x_1, x'_1, Q_1^2) \sigma^{jl}_{h'}(x_2, x'_2, Q_2^2)
\end{aligned}
$$



Assumed factorization

$$\Gamma_h^{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2, Q_1^2, Q_2^2) f(b_1) f(b_2)$$

... or use the integrated dGPD's

$$D_h^{ij}(x_1, x_2, Q_1^2, Q_2^2) = \int d^2b_1 d^2b_2 \Gamma_h^{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2)$$

(transverse-longitudinal decoupling)

Experimental hopes

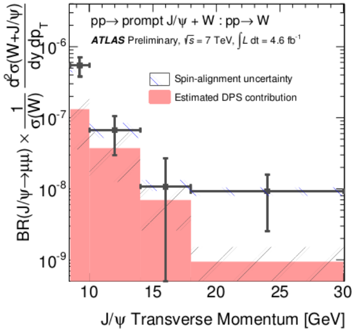


Figure 6: The inclusive (SPS+DPS) cross section ratio $dR^{\mathrm{incl}}_{J/\psi}/dp_{\mathrm{T}}$ is shown as a function of J/ψ transverse momentum. The shaded uncertainty corresponds to the variations due to the various spin-alignment scenarios. The DPS estimate is also shown for comparison, with the uncertainty again shown by a shaded region.

(sizable contribution from DPS)
Expected large enhancement for p+A [d'Enterria, Snigirev 2012, Treleani, Strikman, 2012]

Definition

Spin-averaged) dPDF [Collins, Sopper 1981, Diehl 2010, Diehl, Ostermeier, Schaeffer 2012] of the proton with momentum p

$$F_{q_1 q_2}(x_1, x_2, \vec{y}_\perp) = p^+ \int \frac{dz_1^- dz_2^-}{8\pi^2} \; e^{ix_1 z_1^- p^+ + ix_2 z_2^- p^+} \int dy^- \times$$
$$\left\langle p \right| \bar{q}_1(0) \gamma^+ q_1(z_2) \; \bar{q}_2(y) \, \gamma^+ q_2(y + z_1) \left| p \right\rangle \Big|_{z_i^+ = \vec{z}_{i\perp} = y^+ = 0}$$

$a^\pm = (a^0 \pm a^3)/\sqrt{2}$
$\vec{y}_\perp$ plays the role of the transverse distance between the two quarks

$$D_{j_1 j_2}(x_1, x_2) = \int d^2 y_\perp F_{j_1 j_2}(x_1, x_2, \vec{y}_\perp).$$

(similar definition for gluons, no link operators in LC gauge)

Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving based on the Mellin moments, similarly to the case of sPDF

$$M_j^n = \int_0^1 dx\, x^n D_j(x),$$
$$M_{j_1 j_2}^{n_1 n_2} = \int_0^1 dx_1 \int_0^1 dx_2 \theta(1-x_1-x_2) x_1^{n_1} x_2^{n_2} D_{j_1 j_2}(x_1, x_2)$$

The moments of the QCD splitting functions are

$$P_{i\to j}^n = \int_0^1 dx\, x^n P_{i\to j}(x),$$
$$P_{i\to j_1 j_2}^{n_1 n_2} = \int_0^1 dx\, x^{n_1} (1-x)^{n_2} P_{i\to j_1 j_2}(x),$$
$$\tilde{P}_{i\to j_1 j_2}^{n_1 n_2} = \delta_{j_1 j_2} P_{i\to j_1}^{n_1+n_2} - \delta_{i j_1} P_{j_1\to j_2}^{n_2} - \delta_{i j_2} P_{j_2\to j_1}^{n_1}$$

dDGLAP

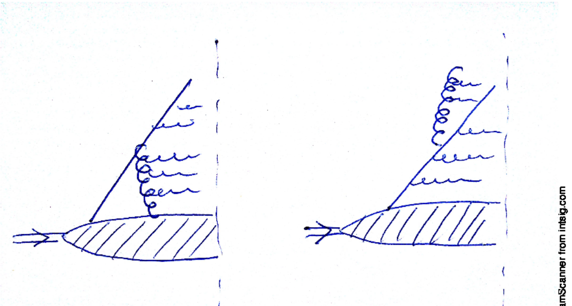
$$\frac{d}{dt} M^{n_1 n_2}_{j_1 j_2} = \sum_i P^{n_1}_{i \to j_1} M^{n_1 n_2}_{i j_2} + \sum_i P^{n_2}_{i \to j_2} M^{n_1 n_2}_{j_1 i} + \sum_i \left(P^{n_1 n_2}_{i \to j_1 j_2} + \tilde{P}^{n_1 n_2}_{i \to j_1 j_2} \right) M^{n_1 + n_2}_i$$

where

$$t = \frac{1}{2\pi\beta} \log\left[1 + \alpha_s(\mu)\beta \log(\Lambda_{\mathrm{QCD}}/\mu)\right], \ \beta = \frac{11 N_c - 2 N_f}{12\pi}$$

coupling

to sPDF's:



Evolution of the valence-valence distributions

Here we show only the evolution of the valence-valence distributions which leads to a technical simplification, as in this case there are no partons i decaying into a pair of valence quarks $(P_{i \to j_1 j_2} = 0)$ and the inhomogeneous term vanishes:

$$\frac{d}{dt} M^{n_1 n_2}_{j_1 j_2}(t) = \left(P^{n_1}_{j_1 \to j_1} + P^{n_2}_{j_2 \to j_2} \right) M^{n_1 n_2}_{j_1, j_2}(t)$$

For the sPDF the corresponding equation is

$$\frac{d}{dt} M^{n}_{j}(t) = P^{n}_{j \to j} M^{n}_{j}(t)$$

Solution

Due to the presence of correlations $M_{j_1 j_2}^{n_1 n_2}(t) \neq M_{j_1}^{n_1}(t) M_{j_2}^{n_2}(t)$ and the system is not separable. The solutions are

$$M_j^n(t) = e^{P_{j \to j}^n (t - t_0)} M_j^n(t_0)$$
$$M_{j_1 j_2}^{n_1 n_2}(t) = e^{(P_{j_1 \to j_1}^{n_1} + P_{j_2 \to j_2}^{n_2})(t - t_0)} M_{j_1 j_2}^{n_1 n_2}(t_0)$$

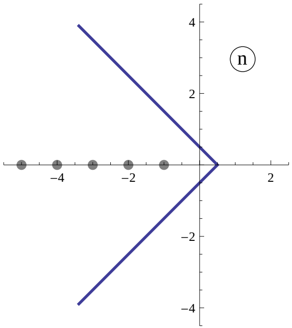
The inverse Mellin transform brings us to the evolved solution in the x-space, namely

$$D_j(x; t) = \int_C \frac{dn}{2\pi i} x^{-n-1} M_j^n(t)$$
$$D_{j_1 j_2}(x_1, x_2; t) = \int_C \frac{dn_1}{2\pi i} x_1^{-n_1 - 1} \int_{C'} \frac{dn_2}{2\pi i} x_2^{-n_2 - 1} M_{j_1, j_2}^{n_1, n_2}(t)$$

where n and n' are treated as complex variables and the contours C and C' are lying right to all singularities of M.

Practical method of solution

(as for the sPDF case)
The inverse Mellin transform is carried out numerically along a bent contour



Gaunt-Stirling sum rules

Conservation laws $\rightarrow$

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D^{j_1 j_2}(x_1, x_2; \mu) = (1 - x_2) D^{j_2}(x_2; \mu)$$

$$\sum_{j_{1v}} \int_0^{1-x_2} dx_1 D^{j_{1v} j_2}(x_1, x_2; \mu) = \begin{cases} N_{j_{1v}} D^{j_2}(x_2; \mu) & j_2 \neq j_1, j_2 \neq \bar{j}_1 \\ (N_{j_{1v}} - 1) D^{j_2}(x_2; \mu) & j_2 = j_1 \\ (N_{j_{1v}} + 1) D^{j_2}(x_2; \mu) & j_2 = \bar{j}_1 \end{cases}$$

- Preserved by the evolution
- Non-trivial to satisfy with the (guessed) initial condition
- Ansatz of the form $\sim D(x_1)D(x_2)(1 - x_1 - x_2)^n$

Valon model

Early multiparton models [Kuti, Weisskopf 1971] take $\Psi(x) \sim x^a$, $a = 1 - \alpha(0) \sim 0.5$ – only correlations from longitudinal momentum conservation

Valon [Hwa, Zahir 1980, Hwa, Yang 2002]:

$$F_{uud/p}(x_1, x_2, x_3) = A x_1^\alpha x_2^\alpha x_3^\beta \delta(1 - x_1 - x_2 - x_3)$$

Let $\phi(x) = |\psi(x)|^2$. The three-particle probability distribution incorporating the longitudinal momentum conservation is

$$D_3(x_1, x_2, x_3) = \phi(x_1)\phi(x_2)\phi(x_3)\delta(1 - x_1 - x_2 - x_3).$$

Its marginal projections define the dPDF and sPDF:

$$D_2(x_1, x_2) = \int_0^1 dx_3 D(x_1, x_2, x_3) = \phi(x_1)\phi(x_2)\phi(1 - x_1 - x_2)$$

$$D_1(x_1) = \int_0^1 D_2(x_1, x_2) = \int_0^1 \theta(1 - x_1 - x_2) dx_2 \phi(x_1)\phi(x_2)\phi(1 - x_1 - x_2)$$

GS sum rules satisfied by construction!

We take for simplicity $\alpha = \beta$, whence

$$\begin{aligned} D(x_1, x_2) &= \frac{\Gamma(3\alpha+3)}{\Gamma(\alpha+1)^3} x_1^\alpha x_2^\alpha (1 - x_1 - x_2)^\alpha \\ D(x_1) &= \frac{\Gamma(3\alpha+3)}{\Gamma(\alpha+1)\Gamma(2\alpha+2)} x_1^\alpha (1 - x_1)^{2\alpha+1} \end{aligned}$$

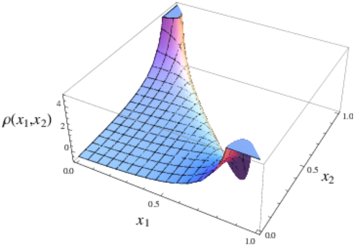
With $\alpha = 1$ the behavior of $D(x_1)$ at $x_1 \to 1$ conforms to the counting rules [Brodsky, Lepage 1980]

(evolution)

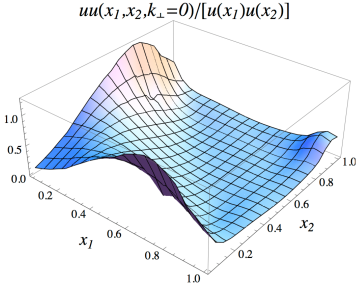
Correlation

$$\rho_{j_1 j_2}(x_1, x_2) = \frac{D_{j_1 j_2}(x_1,x_2)}{D_{j_1}(x_1) D_{j_2}(x_2)} - 1$$

valon

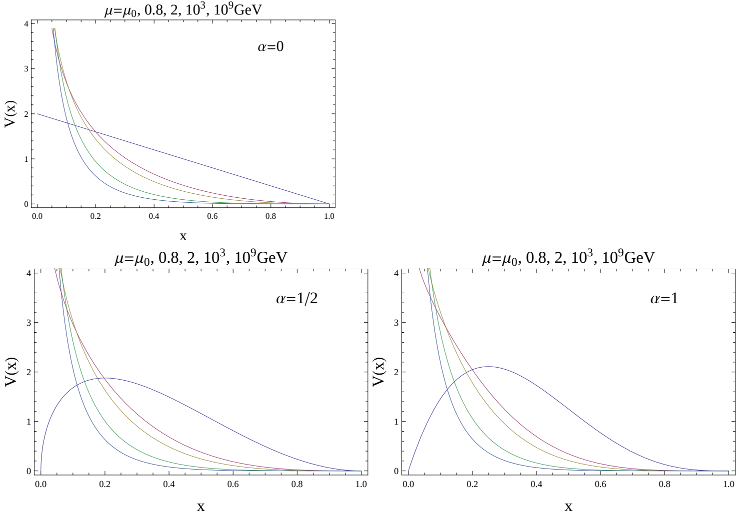


MIT bag [Chang, Manohar, Waalewijn 2013]

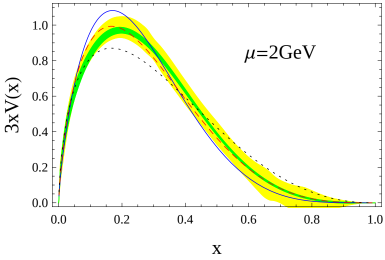


(the above are at the quark-model scale – no glue)

sPDF for valence, $V(x)$

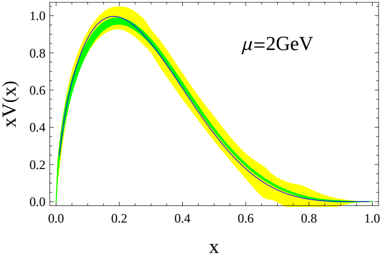


sPDF for valence vs data at $\mu = 2$ GeV



solid $\alpha = 1$, dashed $\alpha = 1/2$, dotted $\alpha = 0$
green NNPDF (no LHC), yellow NNPDF (collider)
At $\mu = 2$ GeV quarks carry 41.6% of the momentum (no gluons) $\rightarrow$ the initial scale is very low, $\mu_0 \sim 300$ MeV, similarly to a similar problem with the pion [Davidson, Arriola 1995, WB, Arriola, Golec-Biernat 2008]

$$\phi(x) = (1-x)^\alpha$$

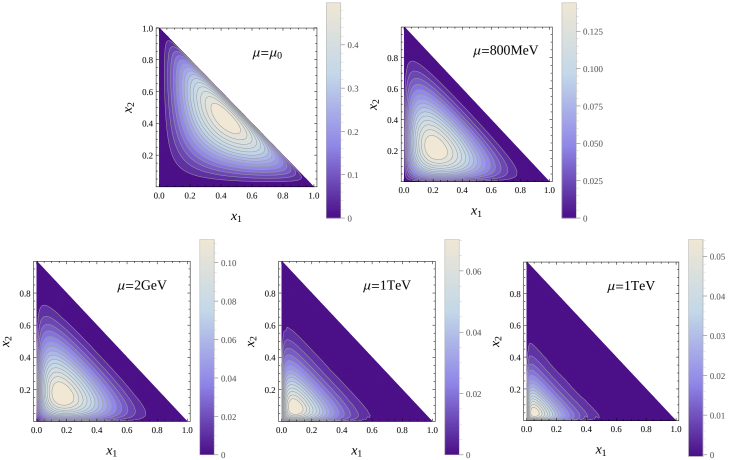


solid $\alpha = 2$,
green NNPDF (no LHC), yellow NNPDF (collider)
(can fit the data very well with models of this class)

dPDF for valence

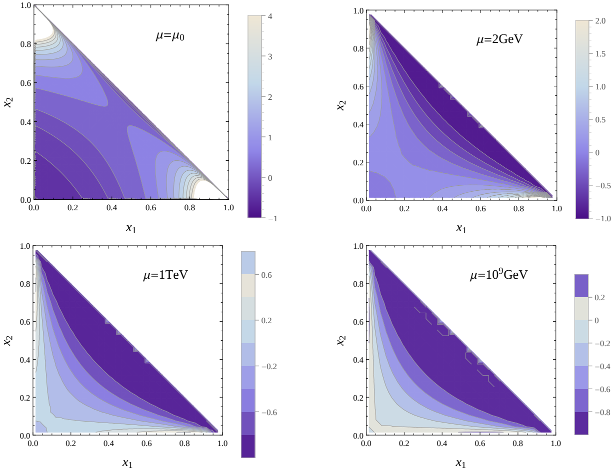
$x_1 x_2 V(x_1, x_2; \mu)$

Result of the numerical evolution to subsequent scales:

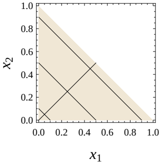


Correlation

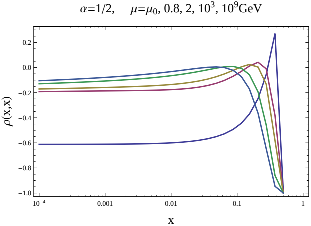
$$\rho_{j_1 j_2}(x_1, x_2) = \frac{D_{j_1 j_2}(x_1,x_2)}{D_{j_1}(x_1) D_{j_2}(x_2)} - 1$$

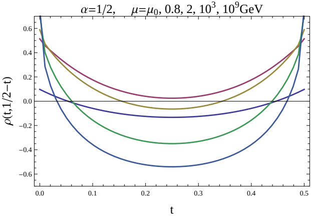


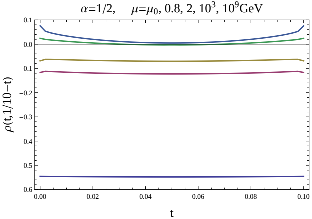
lack of factorization [Snigirev 2003, Korotkikh, Snigirev 2004]

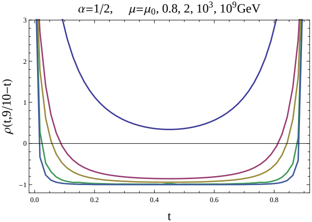


Sections of the 2D correlation plots









End-line behavior

[Snigirev 2010] shows that for $x_1 \to 1$, $x_2 \to 0$

$$D(x_1, x_2; t) \to H(t)(1 - x_1)^{k_{i,j_1,j_2}}(1 - x_1 - x_2)^{2C_F t + h_{i,j_1,j_2}}$$

– confirmed by our numerical analysis

- For $x_1 + x_2 \to 1$ one gets $\rho(x_1, x_2) \to -1$ implying that this region is much suppressed as compared to the independent parton approximation
- For $x_1 \sim x_2 \sim 0$ no correlations, i.e., the product ansatz works
- Along the axes the correlations are large and positive!

Conclusions

- Valon model offers a simple ansatz at the initial quark-model scale which is consistent with all formal requirements (support, GS sum rules, positivity) – grasps the essential features, longitudinal momentum conservation
- Method of solving the dDGLAP evolution in the Mellin space is practical and accurate (a few-lines of code!)
- Product ansatz good at low x_1, x_2, but correlations are large in other regions