Form Quarks & Gluons to Hadron Form Factors For Increasing Photon Virtualities

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Introduction

- Through SDEs, we can study hadron form factors from first principles in the continuum.

- SDE for QCD have been extensively applied to meson spectra and interactions below the masses ~ 1 GeV.

- They have been employed to calculate:

  the masses, charge radii and decays of mesons

P. Maris, C.D. Roberts, Phys. Rev. C56 3369 (1997).

P. Maris, P.C. Tandy, Phys. Rev. C62 055204 (2000).

D. Jarecke, P. Maris, P.C. Tandy, Phys. Rev. C67 035202 (2003).

Introduction

elastic pion and kaon form factors

P. Maris, P.C. Tandy, Phys. Rev. C62 055204 (2000).

pion and kaon valence quark-distribution functions

T. Nguyen, AB, C.D. Roberts, P.C. Tandy, Phys. Rev. C83062201 (2011).

nucleon form factors

D. Jarecke, P. Maris, P.C. Tandy, Phys. Rev. C67 035202 (2003).

G. Eichmann, et. al., Phys. Rev. C79 012202 (2009).
D. Wilson, L. Chang and C.D. Roberts, Phys. Rev. C85 025205 (2012).

review article

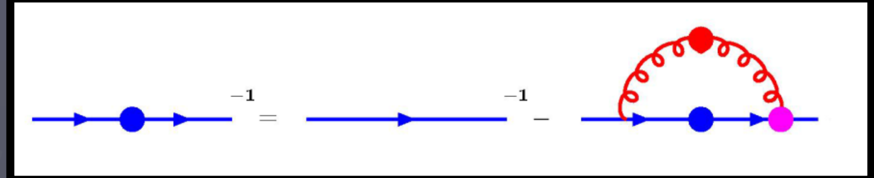
"Collective Perspective on advances in DSE QCD",AB , L. Chang, I.C. Cloet, B. El Bennich, Y. Liu, C.D. Roberts, P.C. Tandy, Commun. Theor. Phys. 58 79 (2012)

The Quark Propagator

The quark propagator:

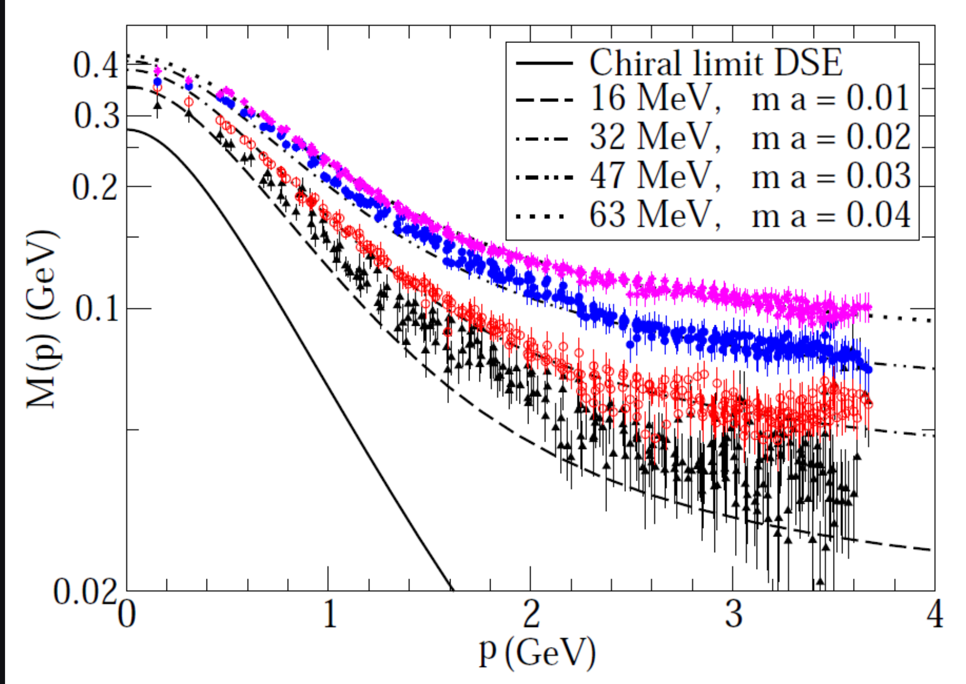
$$S(p^2,\mu^2) = i\,\gamma\cdot p\,A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{Z(p^2,\mu^2)}{i\,\gamma\cdot p + M(p^2)}$$

Schwinger-Dyson Equation for the quark propagator



Quark mass is a function of momentum, dropping as $1/p^2$ in the ultraviolet.

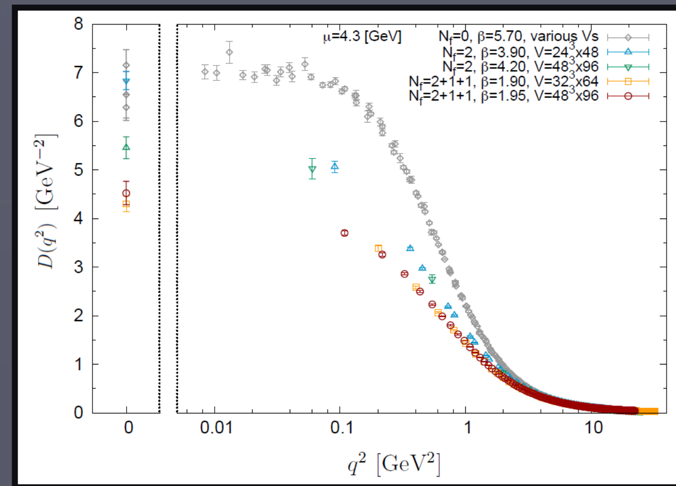
Higgs mechanism is almost irrelevant to the infrared enhancement of quark mass.



The Gluon propagator

The Gluon Propagator:

$$\Delta^{ab}_{\mu\nu}(q) \;=\; \delta^{ab}\, D(q^2)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$



Modern SDE and lattice results support decoupling solution for the gluon propagator.

A. Ayala, AB, D. Binosi, M. Cristoforetti, J. Rodríguez, Phys. Rev. D86 074512 (2012).
AB, A. Raya, J. Rodríguez, arXiv:1302.5829
(light flavor dependence of the quark mass)

Momentum dependent gluon mass is reminiscent of the momentum dependent quark mass function.
It is in accord with the refined GZ-picture.

$$D^{\rm RGZ}(q^2) \;=\; \frac{q^2 + M^2}{q^4 + q^2(m^2 + M^2) + 2g^2 N_c \gamma^2 + M^2 m^2}$$

The Gluon propagator

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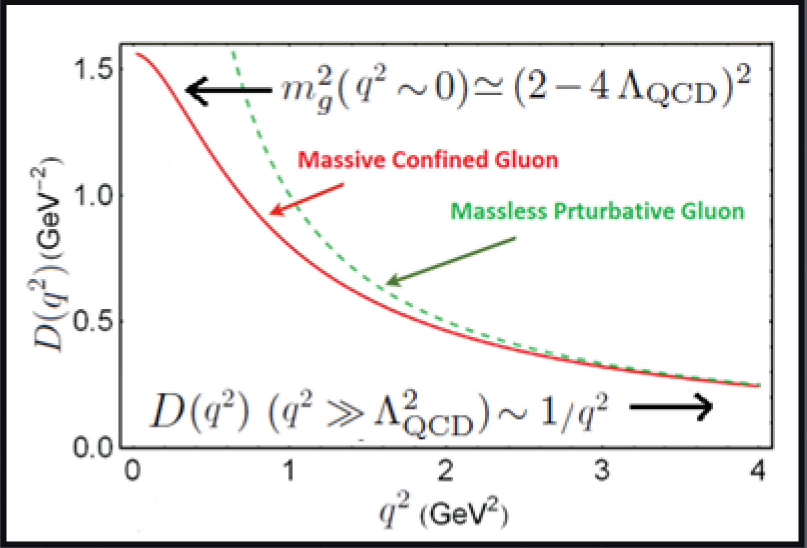

AB, C. Lei, I. Cloet, B. El Bennich, Y. Liu, C. Roberts, P. Tandy, Comm. Theor. Phys. 58 79-134 (2012)

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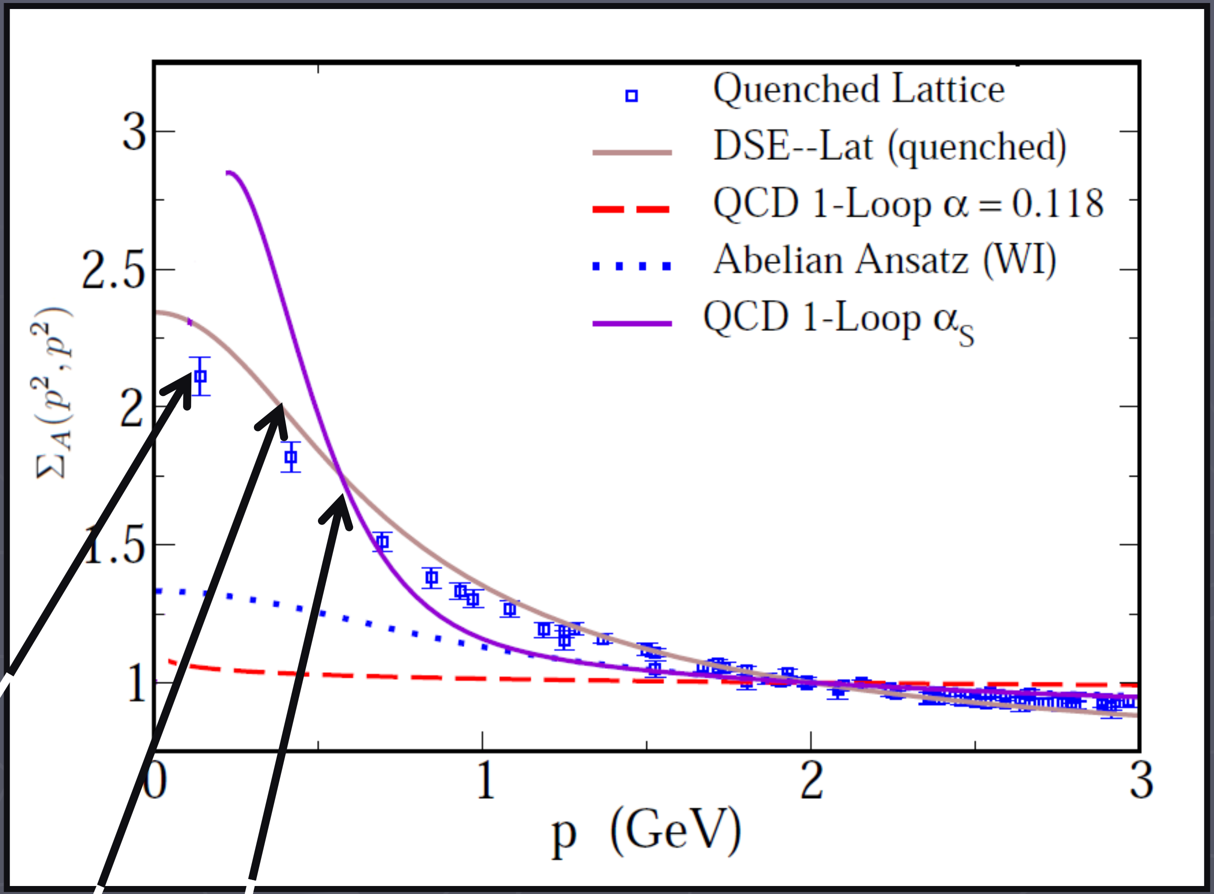
The Quark-Gluon Vertex

The Quark-Gluon Vertex:

One of the 12 form factors



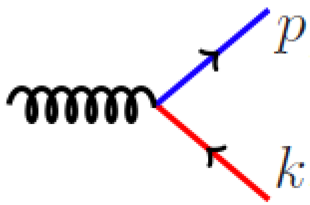
J. Skullerud, P. Bowman, A. Kizilersu, D. Leinweber, A. Williams, J. High Energy Phys. 04 047 (2003)

M. Bhagwat, M. Pichowsky, C. Roberts, P. Tandy, Phys. Rev. C68 015203 (2003).

AB, L. Gutiérrez, M. Tejeda, AIP Conf. Proc. 1026 262 (2008).

The Quark-Gluon Vertex

- The non perturbative **quark-gluon vertex** is a complicated object as compared to its perturbative counterpart.



$\gamma_\mu \quad k_\mu \quad p_\mu$

$1 \quad \gamma \cdot k \quad \gamma \cdot p \quad \gamma \cdot k \gamma \cdot p$

- **Quark gluon vertex** consists of 12 linearly independent Dirac structures.

- 5 of these 12 structures are generated dynamically in the chiral limit.

- Thus **DCSB** manifests itself not only in the quark propagator but also the **quark-gluon vertex**.

The Quark-Gluon Vertex

- The rainbow ladder truncation and the Ball-Chiu vertices describe π and the ρ mesons well but fail to do so for their parity partners σ and a_1.

MeV	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1020	1280
ρ	770	644	764	800	840
Mass splitting	455	115	121	220	440

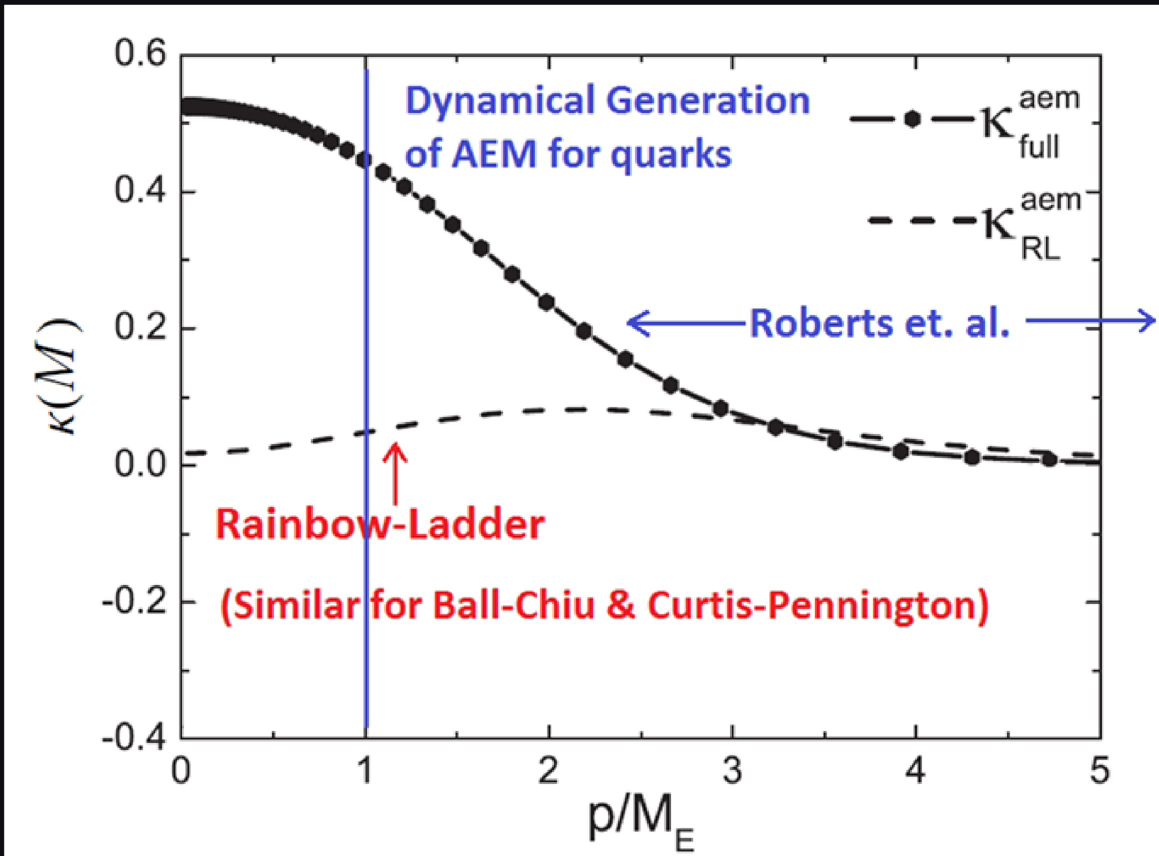
- Non perturbative details of the vertex explain the mass splitting of the parity partners through the generation of an anomalous magnetic moment distribution of quarks.

L. Chang, C.D. Roberts, Phys. Rev. Lett. 103 081601 (2009)

- The associated corrections cancel in the pseudoscalar and vector channels but add in the scalar and axial vector channels.

The Quark-Gluon Vertex

The vertex which successfully explains the mass splitting between ρ and a_1 mesons produces the following magnetic moment distribution.



MeV	Full Vertex
a_1	1230
ρ	745
Mass Difference	485

The Quark-Photon Vertex

Ball and Chiu construction: (Ward-Takahashi identity)

$$(k-p)_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

$$\Gamma_\mu(k,p) = \Gamma_\mu^{\mathrm{BC}}(k,p) + \Gamma_\mu^{\mathrm{T}}(k,p)\,, \quad (k-p)_\mu \Gamma_\mu^{\mathrm{T}}(k,p) = 0\,, \quad \Gamma_\mu^{\mathrm{T}}(p,p) = 0$$

The Ward identity is then invoked:

$$i\,\Gamma^\mu(p,p) \;=\; \frac{\partial}{\partial p_\mu} S^{-1}(p)$$

$$S(p) = \frac{1}{i\gamma\cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

$$i\Gamma_\mu^{\mathrm{BC}}(k,p) \;=\; i\Sigma_A(k^2,p^2)\,\gamma_\mu + 2\ell_\mu\left[i\gamma\cdot\ell\,\Delta_A(k^2,p^2) + \Delta_B(k^2,p^2)\right],$$

$$2\ell = k+p,\;\; \Sigma_\phi(k^2,p^2) = \frac{1}{2}[\phi(k^2)+\phi(p^2)]\,,\; \Delta_\phi(k^2,p^2) = \frac{\phi(k^2)-\phi(p^2)}{k^2-p^2}$$

The Quark-Photon Vertex

The transverse basis:

$$\Gamma^T_\mu(k,p) = \sum_{j=1}^{8} \tau^j(k^2,p^2,k\cdot p)\, T^j_\mu(k,p)$$

$$T^1_\mu(k,p) = i\left[p_\mu(k\cdot q) - k_\mu(p\cdot q)\right],\ q = k - p\,,$$

$$T^2_\mu(k,p) = -iT_{1\mu}(\gamma\cdot k + \gamma\cdot p)\,,$$

$$T^3_\mu(k,p) = q^2\gamma_\mu - q_\mu\gamma\cdot q =: q^2\gamma^{\rm T}_\mu\,,$$

$$T^4_\mu(k,p) = iT_{1\mu}p_\nu k_\rho\sigma_{\nu\rho}\,,$$

$$T^5_\mu(k,p) = \sigma_{\mu\nu}q_\nu\,,$$

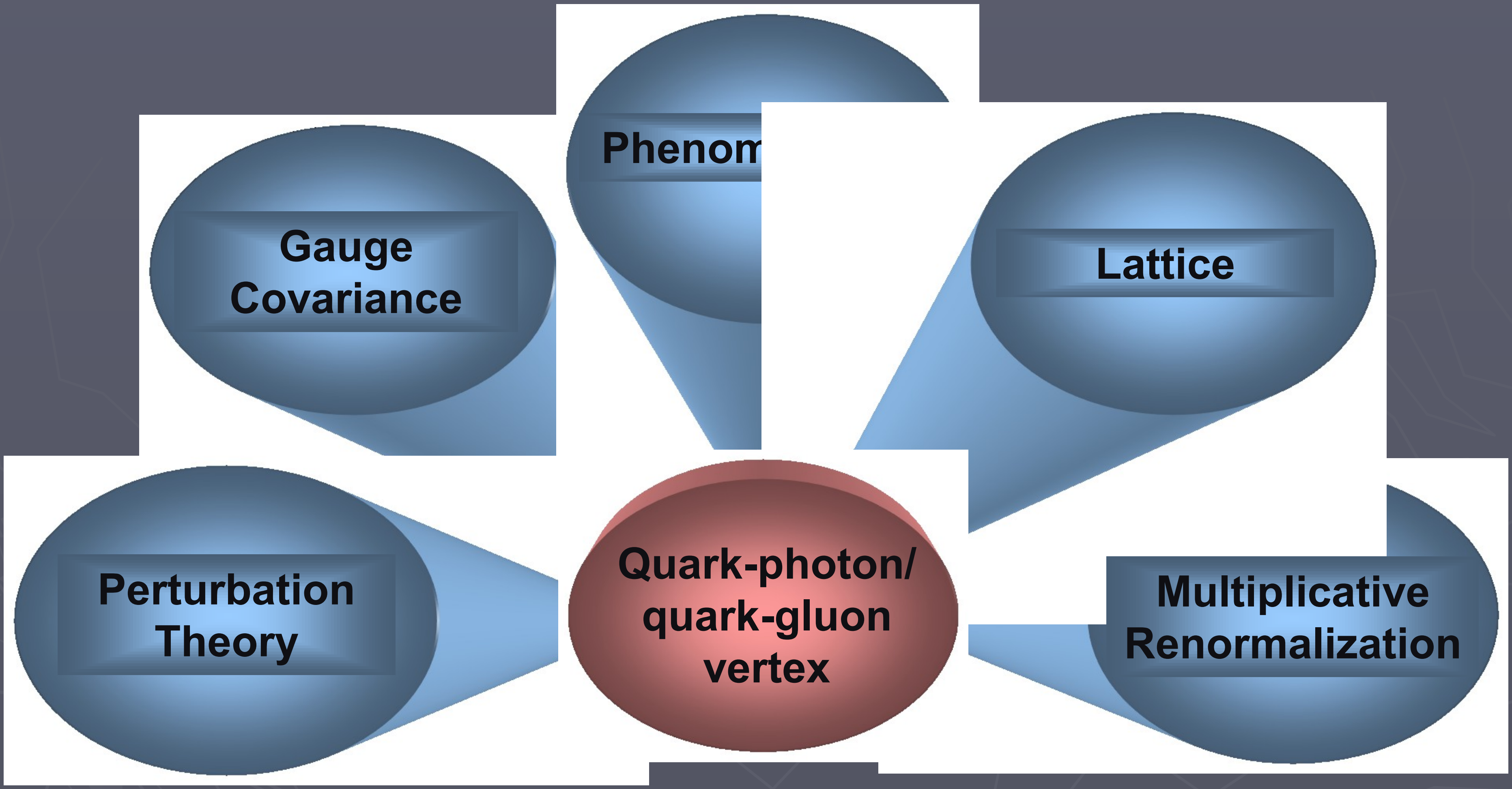
$$T^6_\mu(k,p) = -\gamma_\mu(k^2 - p^2) + (k+p)_\mu\gamma\cdot q\,,$$

$$T^7_\mu(k,p) = \frac{i}{2}(k^2-p^2)[\gamma_\mu(\gamma\cdot k + \gamma\cdot p) - (k+p)_\mu] + (k+p)_\mu p_\nu k_\rho\,\sigma_{\nu\rho}\,,$$

$$T^8_\mu(k,p) = k_\mu\gamma\cdot p - p_\mu\gamma\cdot k - i\gamma_\mu p_\nu k_\rho\sigma_{\nu\rho}$$

$$\sigma_{\mu\nu} = (i/2)[\gamma_\mu,\gamma_\nu]$$

The Quark-Photon Vertex



Quark-photon Vertex

The Quark-Photon Vertex

D.C. Curtis and M.R. Pennington Phys. Rev. D42 4165 (1990)

AB, M.R. Pennington Phys. Rev. D50 7679 (1994)

A. Kizilersu and M.R. Pennington Phys. Rev. D79 125020 (2009)

L. Chang, C.D. Roberts, Phys. Rev. Lett. 103 081601 (2009)

AB, C. Calcaneo, L. Gutiérrez, M. Tejeda, Phys. Rev. D83 033003 (2011)

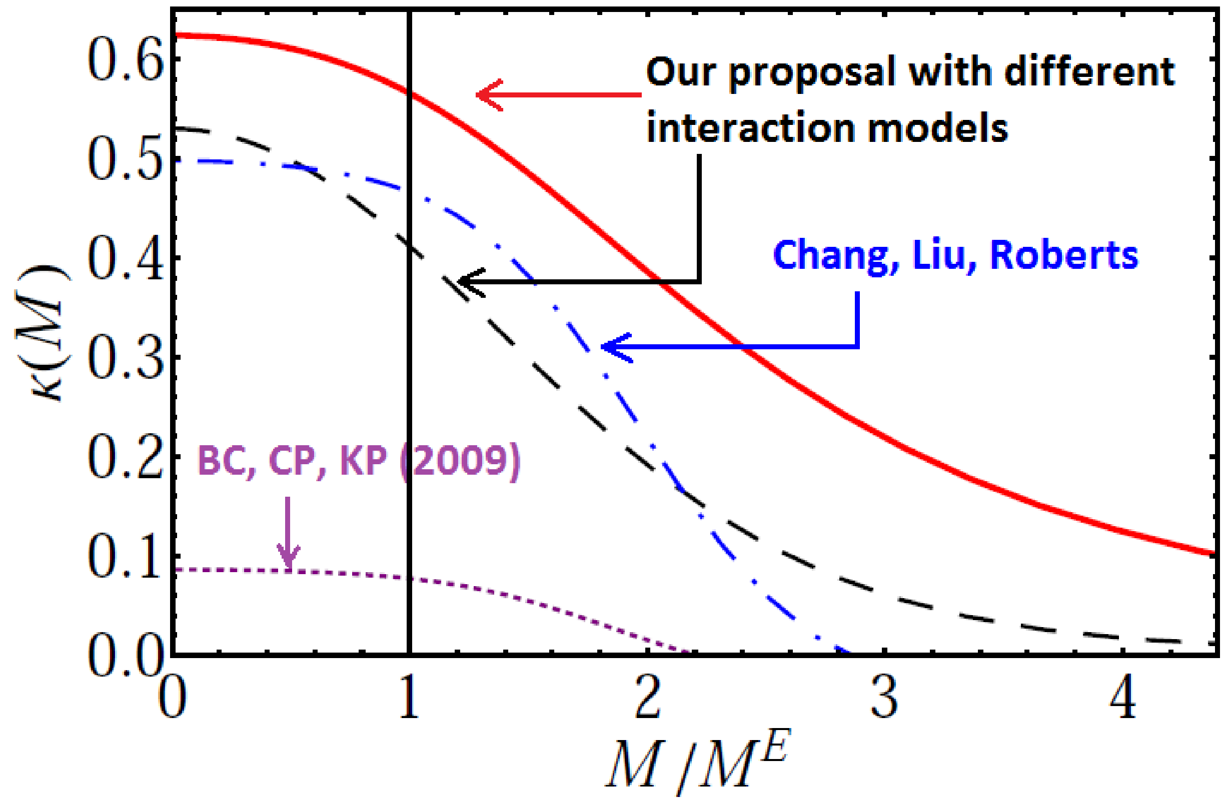
AB, R. Bermudez, L. Chang, C.D. Roberts, Phys. Rev. C85, 045205 (2012).

Significantly, this last ansatz contains nontrivial factors associated with those tensors whose appearance is solely driven by dynamical chiral symmetry breaking.

It yields gauge independent critical coupling in QED.

It also reproduces large anomalous magnetic moment distribution for quarks in the infrared.

The Quark-Photon Vertex

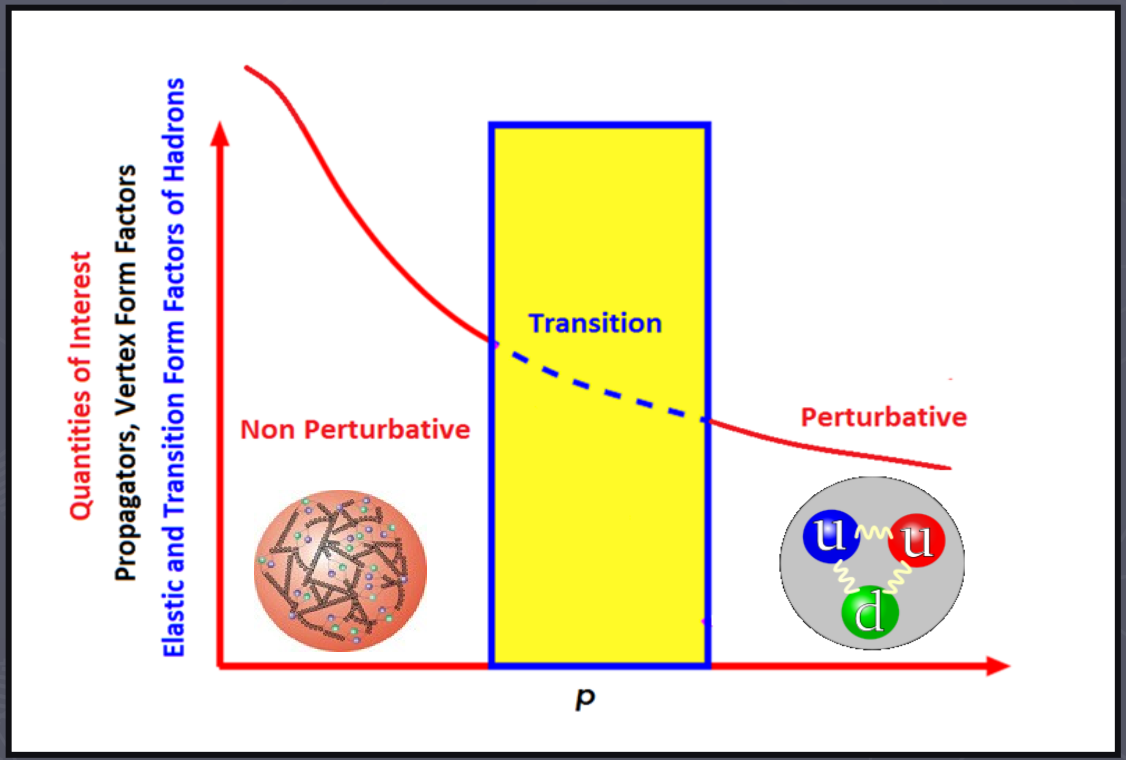


Dynamically Generated Anomalous Magnetic Moment Distribution for a Perturbatively Massless Fermion

AB, R. Bermudez, L. Chang, C.D. Roberts, Phys. Rev. C85, 045205 (2012).

Charting out the Q^2 Evolution

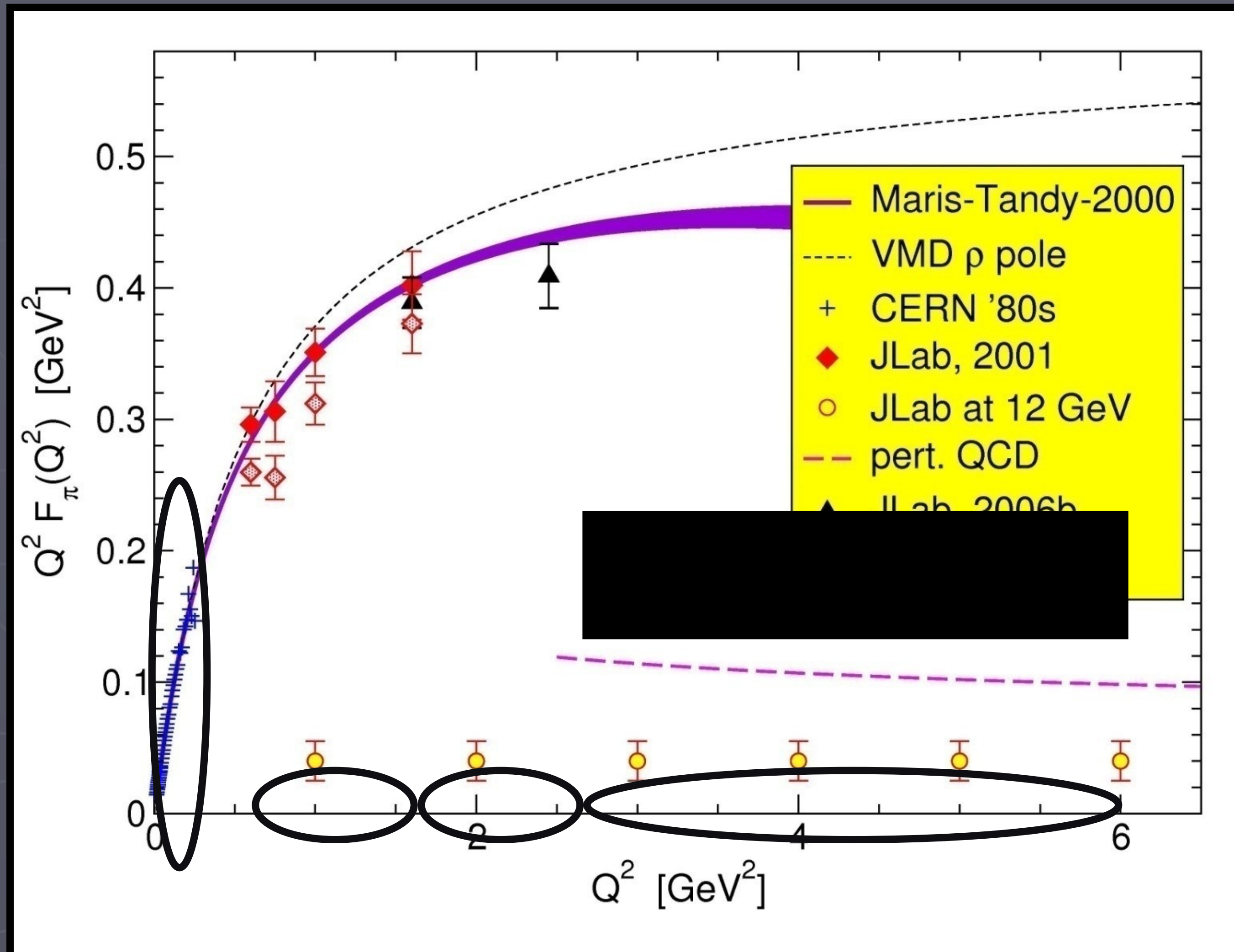
Schwinger-Dyson equations are the fundamental equations of QCD and combine its UV and IR behaviour.



Observing the transition of the hadron from a sea of quarks and gluons to the one with valence quarks alone is an experimental and theoretical challenge.

Charting out the Q^2 Evolution

Elastic Pion Form Factor



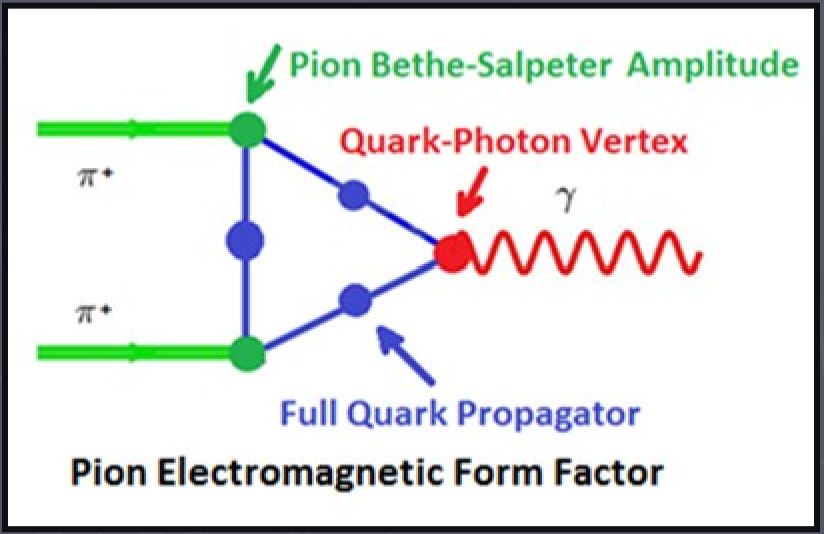
1980's 2001 2006

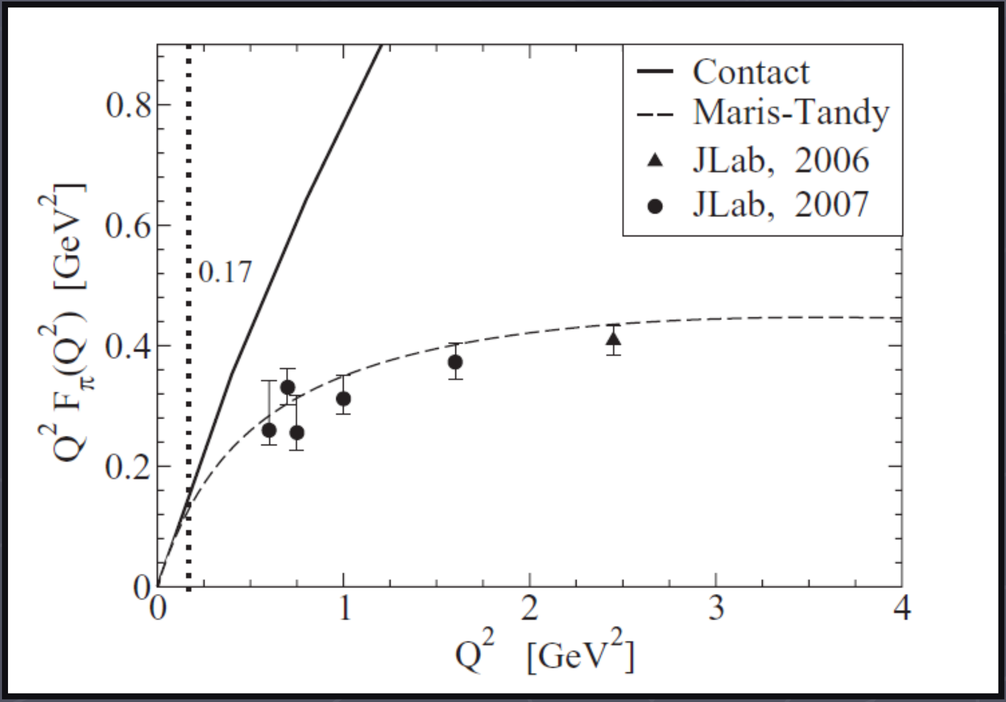
2015?

Charting out the Q^2 Evolution

Elastic Pion Form Factor:

The experiments would be able to distinguish between different patterns of chiral symmetry breaking with increasing photon virtualities:

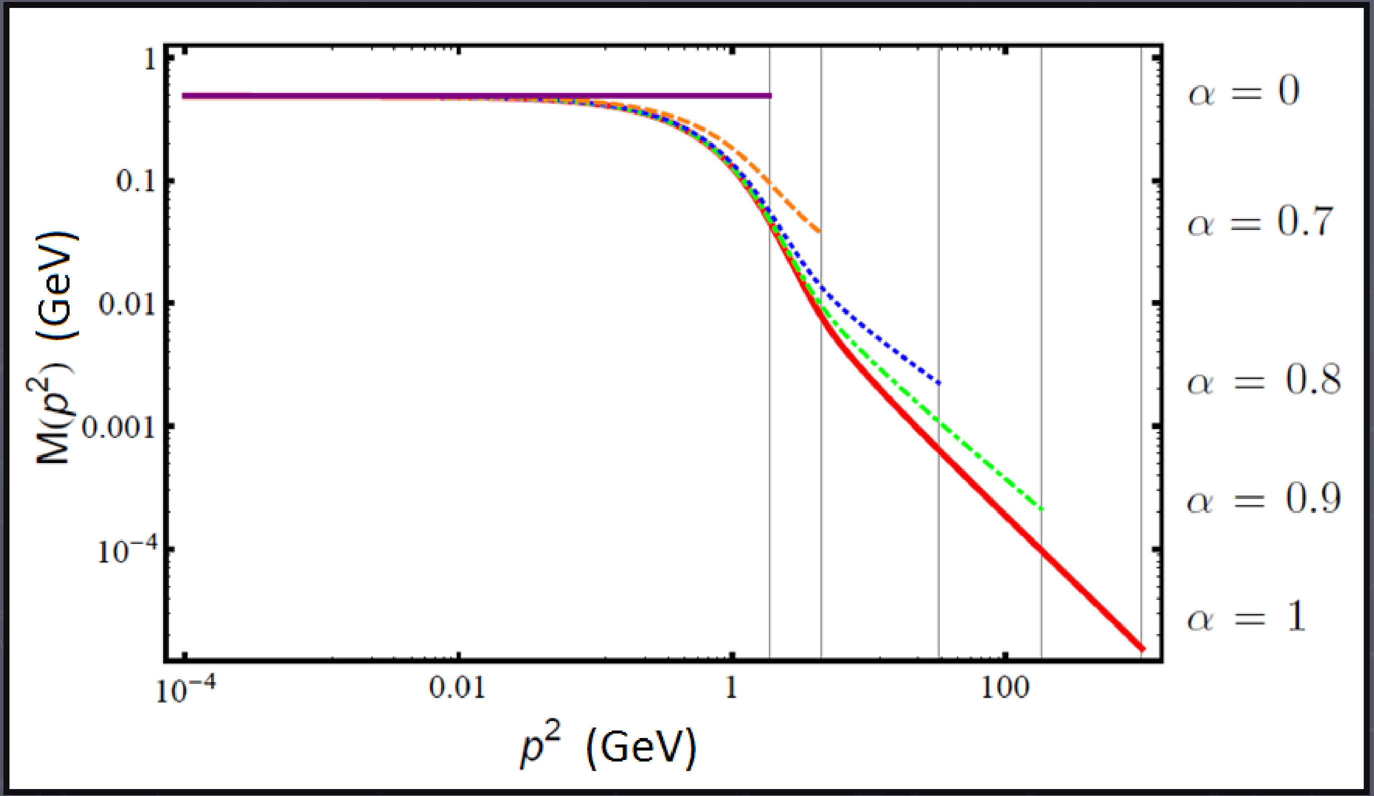




L. Gutiérrez, AB, I.C. Cloet, C.D. Roberts, Phys. Rev. C81 065202 (2010).

Charting out the Q^2 Evolution

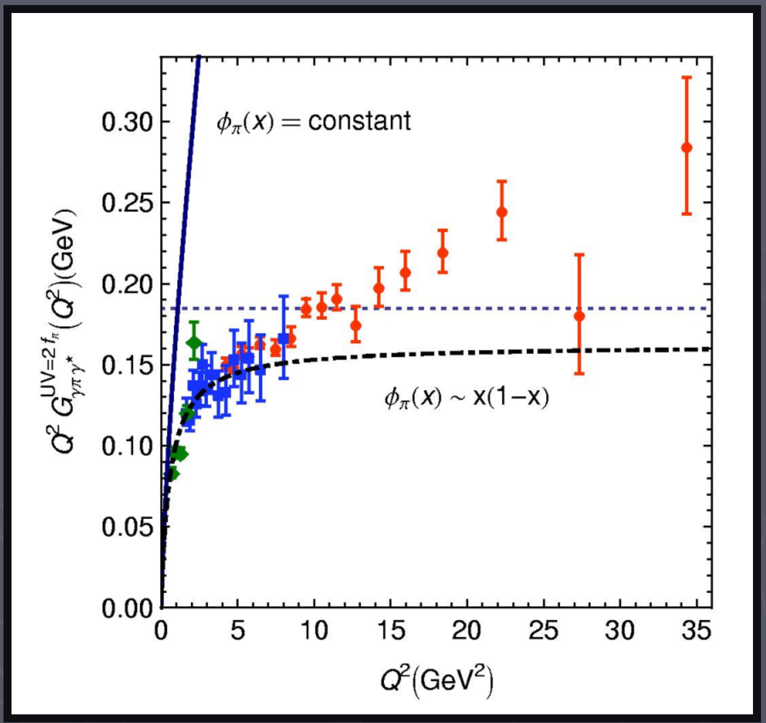
Future form factor measurements can lift the degeneracy between minute variations of the mass function.



F. Akram, AB, K. Raya, work in progress.

Charting out the Q^2 Evolution

The $\gamma^* \gamma \rightarrow \pi^0$ transition form factor:



H.L.L. Robertes, C.D. Roberts, AB, L.X. Gutiérrez and P.C. Tandy, Phys. Rev. C82, (065202:1-11) 2010.

CELLO H.J. Behrend et.al., Z. Phys C49 401 (1991). 0.7 – 2.2 GeV^2

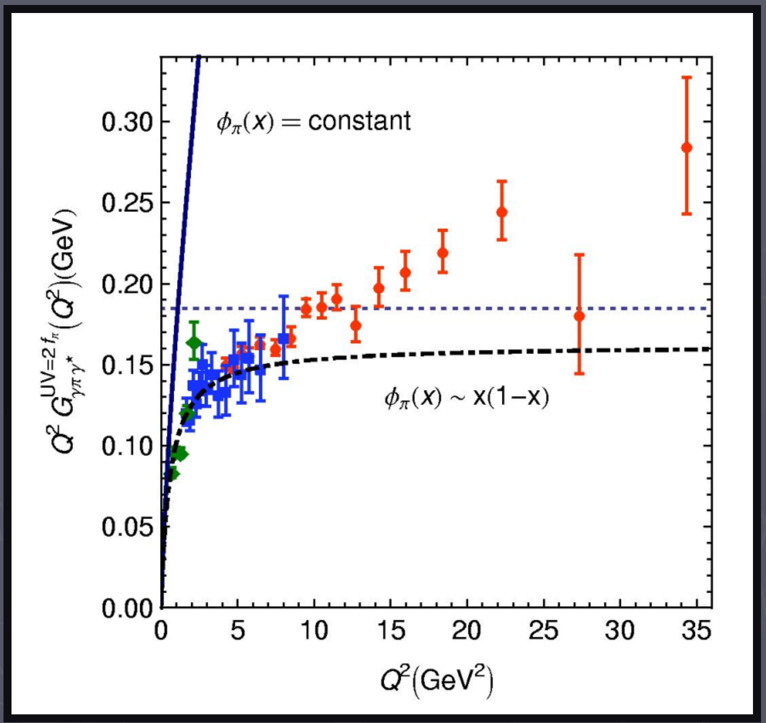
CLEO J. Gronberg et. al., Phys. Rev. D57 33 (1998). 1.7 – 8.0 GeV^2

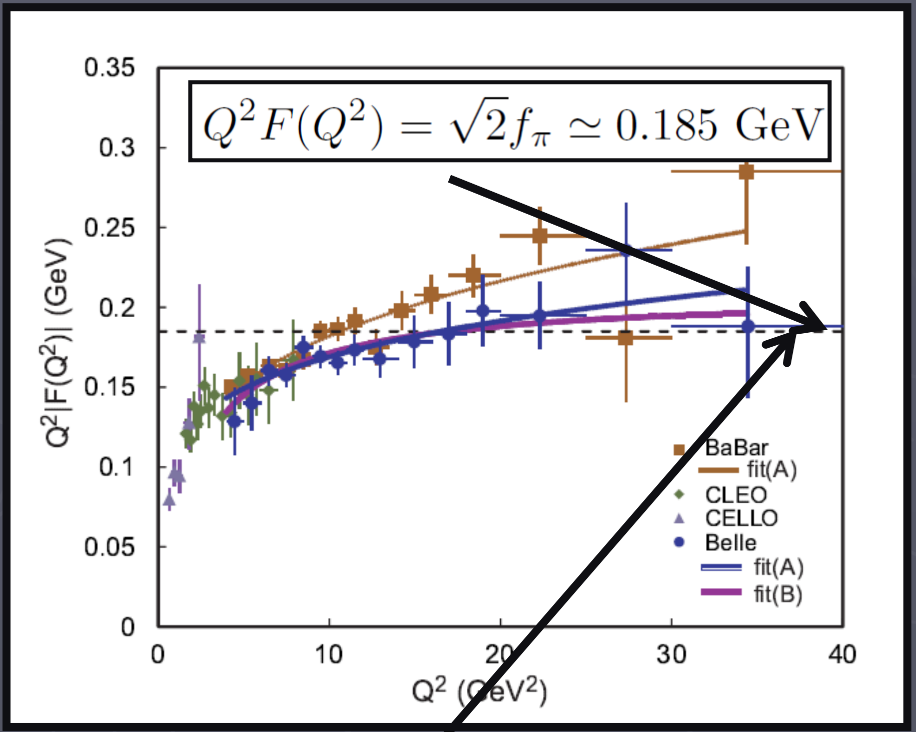
BaBar R. Aubert et. al., Phys. Rev. D80 052002 (2009). 4.0 – 40.0 GeV^2

Belle S. Uehara et. al., arXiv:1205.3249 [hep-ex] (2012). 4.0 – 40.0 GeV^2

Charting out the Q^2 Evolution

The $\gamma^*\gamma \rightarrow \pi^0$ transition form factor:





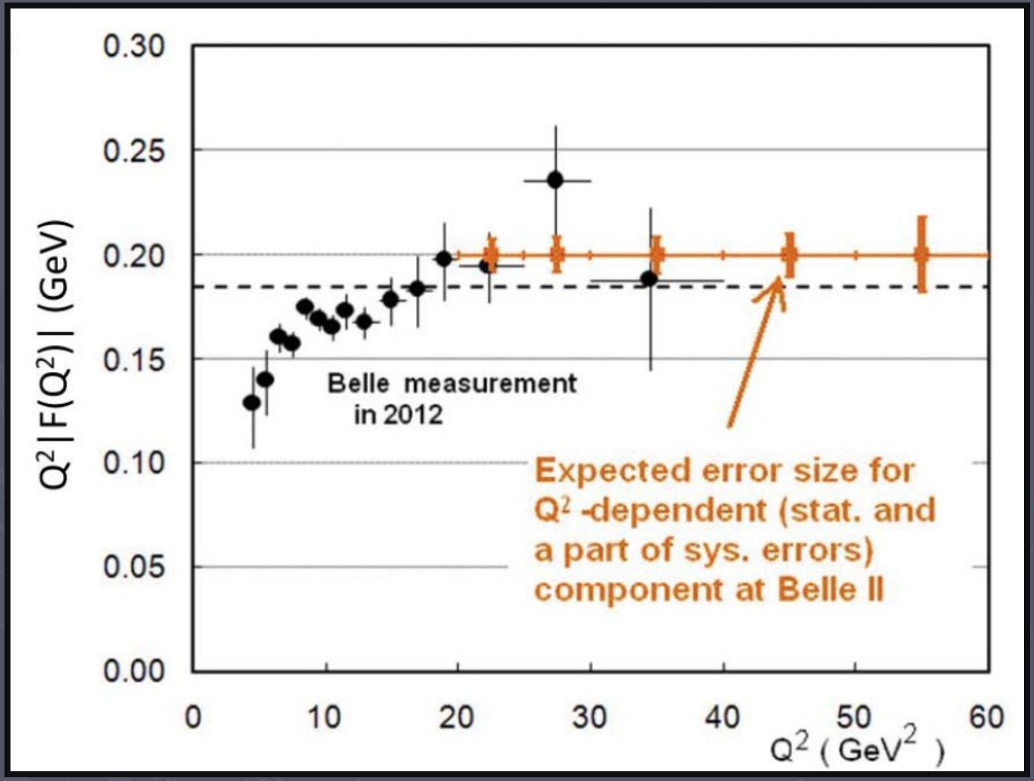
The leading twist pQDC calculation:

G.P. Lepage, and S.J. Brodsky, Phys. Rev. D22, 2157 (1980).

Charting out the Q^2 Evolution

The $\gamma^*\gamma \to \pi^0$ transition form factor:

- Belle II will have 40 times more luminosity.



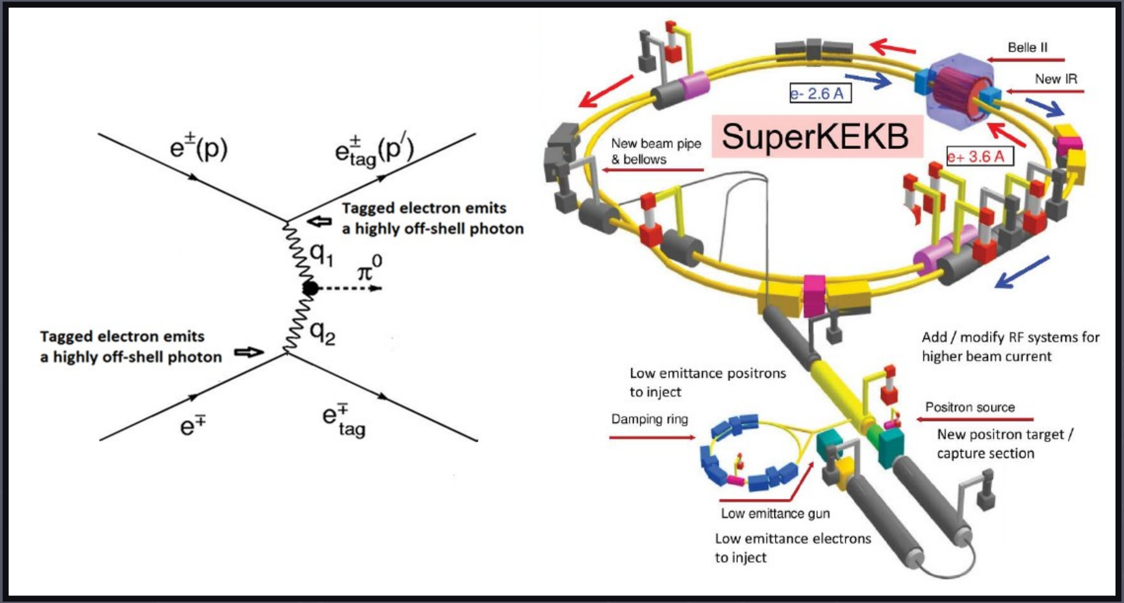
Vladimir Savinov:
5th Workshop of the APS Topical Group on Hadronic Physics

Precise measurements at large Q^2 will provide a stringent constraint on the pattern of chiral symmetry breaking.

Charting out the Q^2 Evolution

- Double tagging?

Vladimir Savinov



- As this process is independent of the PDA, probing the pattern of chiral symmetry breaking can be neater.
- At large Q^2, the form factor calculations require knowing mass function in complex space where singularities are present, making numerical integration impossible.
- How must one proceed?

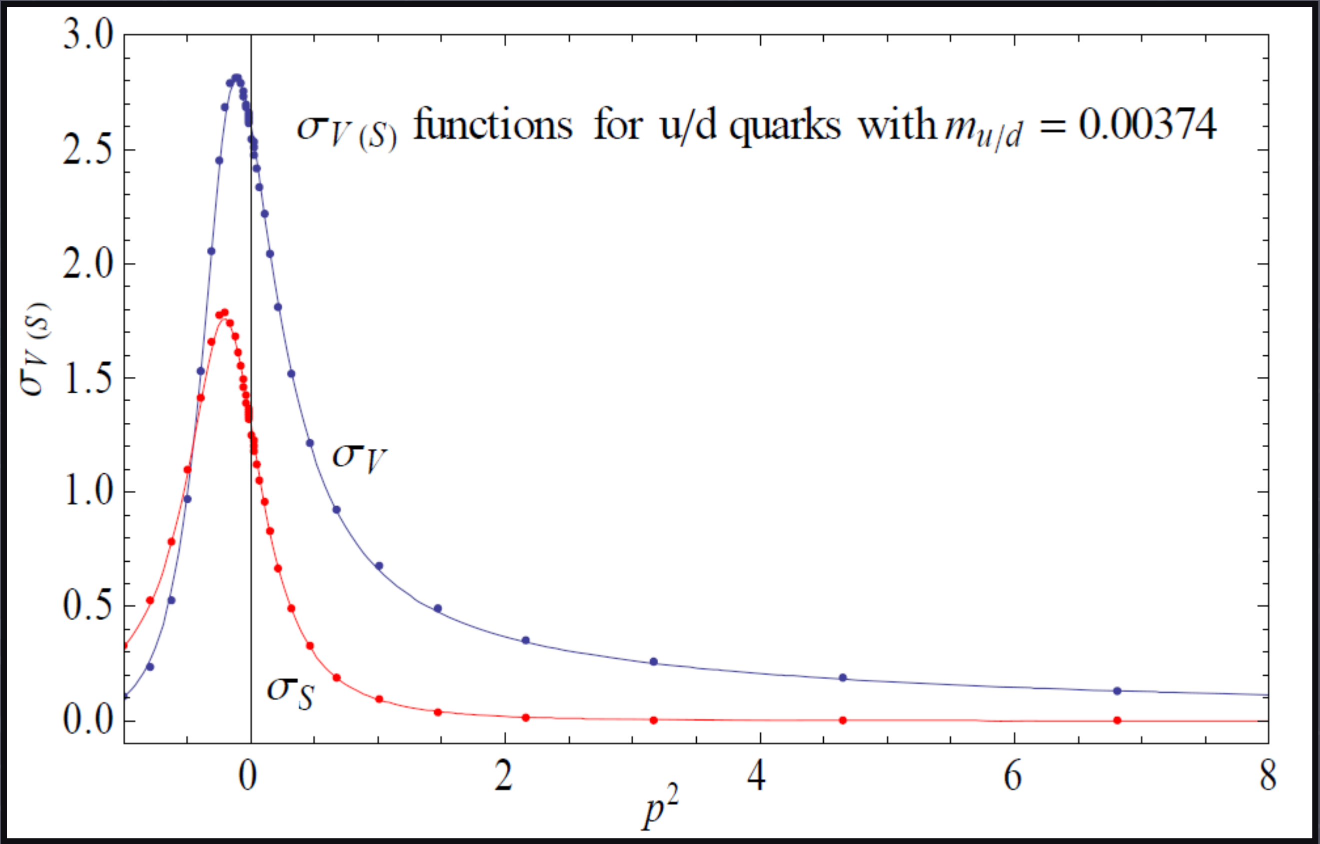
Charting out the Q^2 Evolution

- **Quark Propagator:**

$$S(p) = -i\gamma \cdot p\,\sigma_V(p^2) + \sigma_S(p^2)$$

$$\sigma_S(p^2) = \sum_{i=1}^{3}\left(\frac{z_i m_i}{p^2+m_i^2} + \frac{z_i^* m_i^*}{p^2+(m_i^*)^2}\right),$$

$$\sigma_V(p^2) = \sum_{i=1}^{3}\left(\frac{z_i}{p^2+m_i^2} + \frac{z_i^*}{p^2+(m_i^*)^2}\right),$$



also for s and c quarks

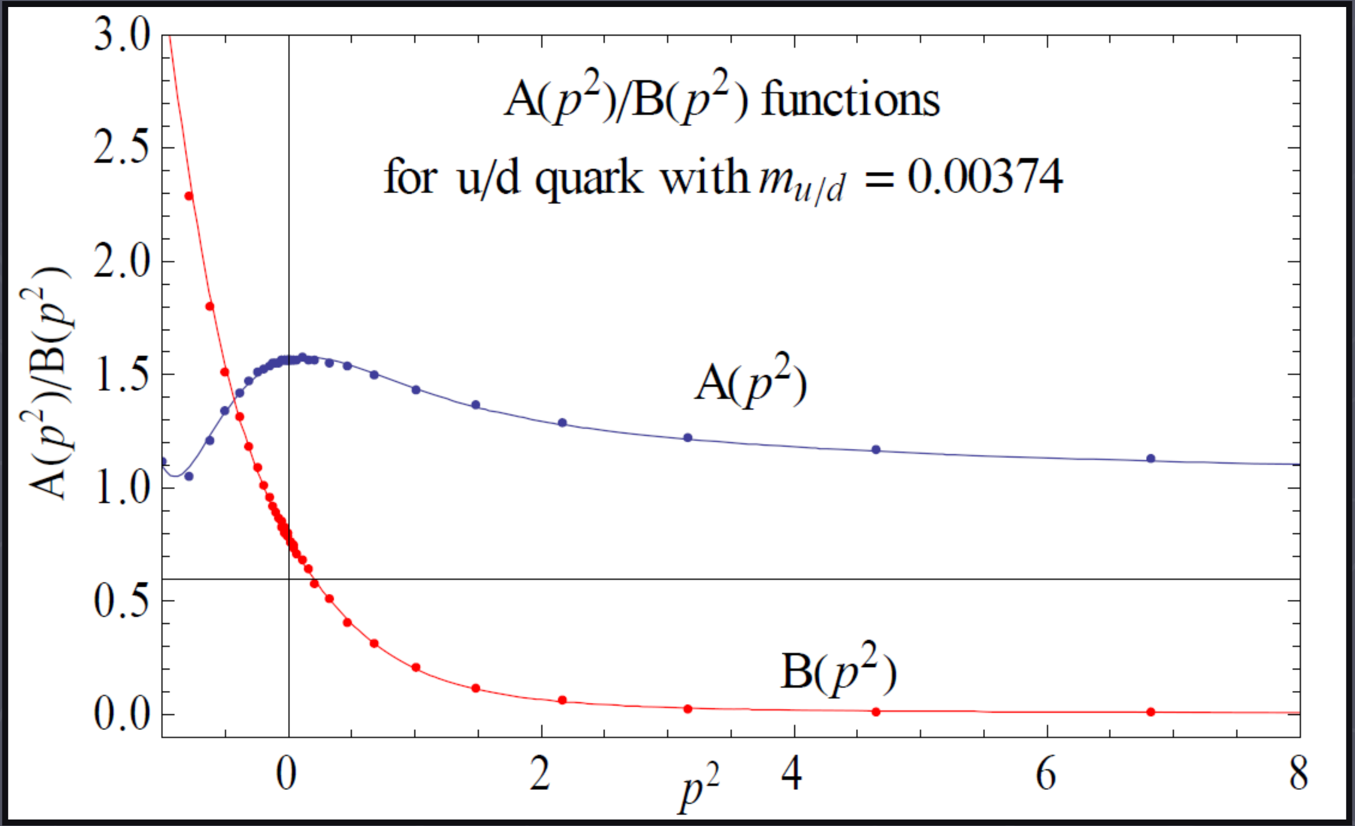
Charting out the Q^2 Evolution

- **Inverse Quark Propagator:**

$$S(p)^{-1} = i\gamma \cdot p\, A(p) + B(p)$$

$$A(p^2) = 1 + \sum_{i=1}^{3}\left(\frac{z_i m_i^2}{p^2 + m_i^2} + \frac{z_i^* m_i^{*2}}{p^2 + (m_i^*)^2}\right),$$

$$B(p^2) = m + \sum_{i=1}^{3}\left(\frac{z_i m_i^3}{p^2 + m_i^2} + \frac{z_i^* m_i^{*3}}{p^2 + (m_i^*)^2}\right),$$



also for s and c quarks

Charting out the Q^2 Evolution

- Fitting the BSA of the pion:

P. Tandy (Matalascañas 2012)

$$E_\pi(k^2, k \cdot P) = \frac{N}{k^2+\Lambda^2}$$

$$E_\pi(k^2, k \cdot P) = N\left\{\frac{1}{k^2+\alpha k\cdot P+\Lambda^2} + \frac{1}{k^2-\alpha k\cdot P+\Lambda^2}\right\}$$

$$E_\pi(k^2, k \cdot P) = N(1 + k^2/\lambda^2)\ \Sigma_i^2 \left\{\frac{1}{(k^2+\alpha_i k\cdot P+\Lambda_i^2)^2} + \frac{1}{(k^2-\alpha_i k\cdot P+\Lambda_i^2)^2}\right\}$$

Similarly $\mathbf{F}_\pi(..)$ Similarly $\mathbf{G}_\pi(..) = \frac{\mathbf{N}}{(\mathbf{k}^2 + \mathbf{\Lambda}^2)^2}$, etc

npQCD info is in the variables and constants that are not momenta---

Wick rotation is trivial as in pert thy.

These are essentially the Nakanishi repn with discrete weight functions $\rho(\alpha, \Lambda)$

- How well can one reproduce the BSA and the associated physical observables?

Charting out the Q^2 Evolution

- Fitting the BSA of the pion: (F. Akram, AB, …)

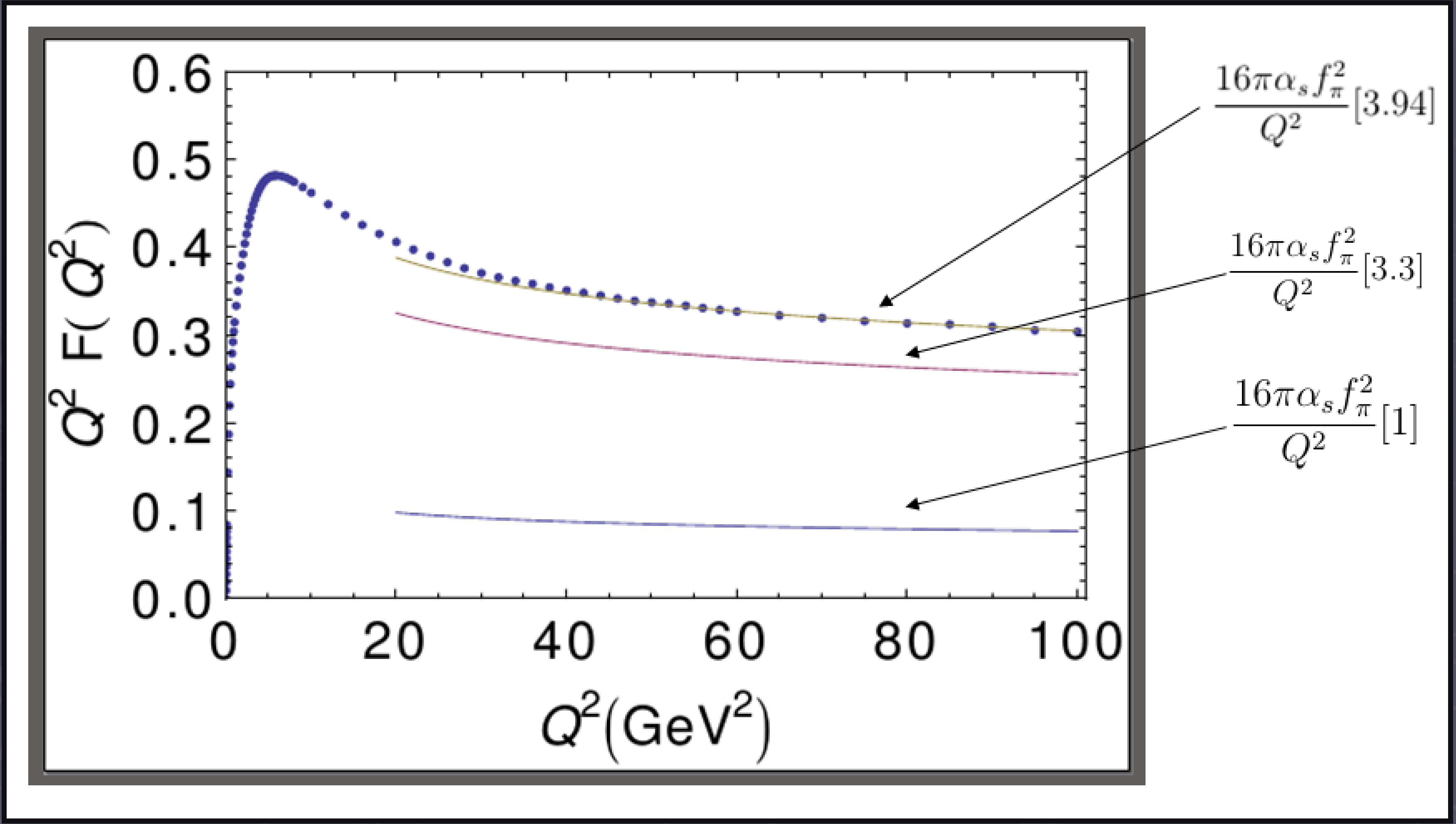
$$E(k^2,k\cdot P)=N\left(1+\frac{k^2}{\lambda_1^2}+\frac{k^4}{\lambda_2^4}+\frac{k^6}{\lambda_3^6}\right)\sum_{i=1,2}\left[\frac{1}{\left(k^2+\alpha_i k\cdot P+\Lambda_i^2\right)^4}+\frac{1}{\left(k^2-\alpha_i k\cdot P+\Lambda_i^2\right)^4}\right]\text{; and F}$$

$$G(k^2,k\cdot P)=N\left(1+\frac{k^2}{\lambda_1^2}+\frac{k^4}{\lambda_2^4}+\frac{k^6}{\lambda_3^6}\right)\sum_{i=1,2}\left[\frac{1}{\left(k^2+\alpha_i k\cdot P+\Lambda_i^2\right)^5}+\frac{1}{\left(k^2-\alpha_i k\cdot P+\Lambda_i^2\right)^5}\right]\text{; and H}$$

- Such form allows one to fit nicely the BSAs E,F,G,H as well as capture their asymptotic momentum dependence satisfactorily.
- The first few moments of Chebyshev polynomials are also rather well reproduced.
- These are being implemented in form factor calculations for arbitrarily high virtualities.

Charting out the Q^2 Evolution

- L. Chang – IV International Meeting on Non Perturbative Methods in Field Theory, Morelia, Mexico, 6-10 May, 2013.



Conclusions

Dynamical chiral symmetry breaking and the momentum dependence of the quark mass function in QCD have experimental signals which enable us to differentiate their predictions from alternative patterns of SB.

The Q^2 evolution of the hadronic form factors, their experimental evaluation and theoretical predictions are likely to provide better understanding of how DCSB and confinement work in QCD.

A systematic framework based upon the QCD equations of motion (Euclidean based SDE) and its symmetries can help chart out and comprehend the Q^2 evolution of these form factors.