A study of Compton form factors in scalar QED[1]

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[1]This work was done in collaboration with **Chueng Ji**

Outline

Motivation

Deeply-virtual Compton scattering (DVCS) has been proposed to determine the generalized-parton distributions (GPDs) of hadrons. It is commonly assumed that to allow for the extraction the GPDs, the experiments should be set-up in (approximately) collinear kinematics. Because such kinematics is not always possible to realize in concrete experiments, see e.g. JLab proposal E12-06-114, it is important to determine deviations that occur in a non-collinear kinematics.

We propose to first analyze the experimental data in terms of Lorentz-invariant amplitudes, Compton form factors (CFFs). By definition, the CFFs can be determined in any suitable kinematics. Once they are measured, it is the job of theorists to extract the GPDs.

Here, we devote special attention to the kinematics. Specifically, we analyze the DVCS limit where the virtuality of the incoming photon is large compared to the relevant mass scales. Moreover, we determine whether or not all CFFs can be extracted in collinear kinematics.

In virtual Compton scattering the physical amplitudes can be written as the contraction of a tensor operator with the photon polarization vectors.

It is important to use the most general form of that tensor operator consistent with EM gauge invariance.

Are there preferred reference frames for the extraction of CFFs from the data?

The quark-gluon structure of hadrons is supposed to manifest itself most transparently in processes where the hadrons are subjected to strongly virtual probes.

How about the scaling of the amplitudes with the virtuality Q?

Tensor Formulation

We write the physical amplitudes as contractions of a tensor with the polarization vectors of the photons:

$$A(h', h) = \epsilon^*(q'; h')_\mu T^{\mu\nu} \epsilon(q; h)_\nu .$$

This tensor must be transverse, i.e.,

$$q'_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} q_\nu = 0.$$

The tensor is written in terms of scalars (CFFs) and basis tensors.

In order to find the number of independent tensor structures we first identify the independent momenta.

From four-momentum conservation it follows that out of the external momenta occurring in the hadronic part of the amplitude, namely p, q, p', and q' one may choose 3 independent ones.

We keep q and q', to simplify a check of the transversity of the tensor. For the remaining one we choose the sum of the hadronic momenta, $\bar{P} = p' + p$. (This choice can also be motivated by perturbation theory.)

Our basis is $k_1 = \bar{P}$, $k_2 = q'$, $k_3 = q$.

The most general second-rank tensor expressed in terms of our basis is then:

$$T^{\mu\nu} = \mathcal{T}_0 \, g^{\mu\nu} + \sum_{i,j} \mathcal{T}_{ij} \, k_i{}^{\mu} k_j{}^{\nu}.$$

By contracting $T^{\mu\nu}$ with q'_{μ} and q_{ν}, which must give the result 0 for the physical tensor, one can determine the number of independent scalars $\mathcal{T}$.

As there are 10 $\mathcal{T}$s and the number of independent contractions is 5, there are 5 CFFs in the **effective** tensor[2].

As the 5 independent tensor structures can be chosen in an infinite number of ways, we look for a synthetic way to construct the effective tensor.

[2] This number was mentioned before by M. Perrottet, Lett. Nuovo Cim. **7**, 915 (1973) and R. Tarrach, Nuovo Cim. **28 A**, 409 (1975) and numerous more recent papers.

Following Tarrach, we find it useful to construct the tensor $T^{\mu\nu}$ by applying a two-sided projector $\tilde{g}^{\mu\nu}(q,q')$ to the most general second rank tensor expressed in terms of our basis:

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}, \quad t_{mn} = t_0\, g_{mn} + \sum_{i,j} t_{ij}\; k_{i\,m} k_{j\,n}.$$

The two-sided projector $\tilde{g}(q,q')$ is defined as follows:

$$\tilde{g}^{\mu\nu}(q,q') = g^{\mu\nu} - \frac{q^\mu\, q'^\nu}{q\cdot q'}.$$

This projector has the properties

$$\tilde{g}^{\mu m}\, g_{mn}\, \tilde{g}^{n\nu} = \tilde{g}^{\mu\nu}, \quad q'_\mu\, \tilde{g}^{\mu\nu} = 0, \quad \tilde{g}^{\mu\nu}\, q_\nu = 0.$$

Should we write $T^{\mu\nu}$ as the contraction

$$T^{\mu\nu} = \tilde{g}^{\mu m}(L,q')\, t_{mn}\, \tilde{g}^{n\nu}(q,R),$$

it would also be transverse in the same way as the $T^{\mu\nu}$ we constructed. Further contractions with $\tilde{g}^{\mu\nu}(q,q')$ would not change it.

The application of $\tilde{g}^{\mu m}$ and $\tilde{g}^{n\nu}$ removes the parts of t_{mn} that contain the left factor q_m or the right factor q'_n.

We define the following reduced momenta

$$\tilde{q}'^{\mu}_{\rm L} = \tilde{g}^{\mu m} q'_m = q'^{\mu} - \frac{q'^2}{q\cdot q'} q^{\mu}, \quad \tilde{q}^{\nu}_{\rm R} = q_n \, \tilde{g}^{n\nu} = q^{\nu} - \frac{q^2}{q\cdot q'} q'^{\nu}$$

$$\tilde{P}^{\mu}_{\rm L} = \tilde{g}^{\mu m} \bar{P}_m = P^{\mu} - \frac{P\cdot q'}{q\cdot q'} q^{\mu}, \quad \tilde{P}^{\nu}_{\rm R} = \bar{P}_n \, \tilde{g}^{n\nu} = P^{\nu} - \frac{q\cdot P}{q\cdot q'} q'^{\nu}.$$

and write for $T^{\mu\nu}$

$$T^{\mu\nu} = \mathcal{H}_0 \, \tilde{g}^{\mu\nu} + \mathcal{H}_1 \, \frac{\tilde{P}^{\mu}_{\rm L} \tilde{P}^{\nu}_{\rm R}}{Q^2} + \mathcal{H}_2 \, \frac{\tilde{P}^{\mu}_{\rm L} \tilde{q}^{\nu}_{\rm R}}{Q^2} + \mathcal{H}_3 \, \frac{\tilde{q}'^{\mu}_{\rm L} \tilde{P}^{\nu}_{\rm R}}{Q^2} + \mathcal{H}_4 \, \frac{\tilde{q}'^{\mu}_{\rm L} \tilde{q}^{\nu}_{\rm R}.}{Q^2}$$

The transverse tensors multiplying $\mathcal{H}_i$, i=1,...4, are divided by Q^2 to make them dimensionless

Contracting the tensor with $\epsilon^*_{\mu}(q')$ and $\epsilon_{\nu}(q)$ we find that all five pieces of the tensor contribute, if $q'^2 \neq 0$ and $q^2 \neq 0$.

The number of independent tensor structures is equal to the number of independent physical matrix elements consistent with parity conservation: $A(-h',-h) = (-1)^{h'-h} A(h',h),\ h',h = \pm 1, 0,$

$$A(1,1),\ A(1,0),\ A(1,-1),\ A(0,1),\ \text{and } A(0,0).$$

If either of the photons is real, some pieces of the tensor do not contribute to the physical amplitudes: the tensor is reduced to an **effective** tensor.

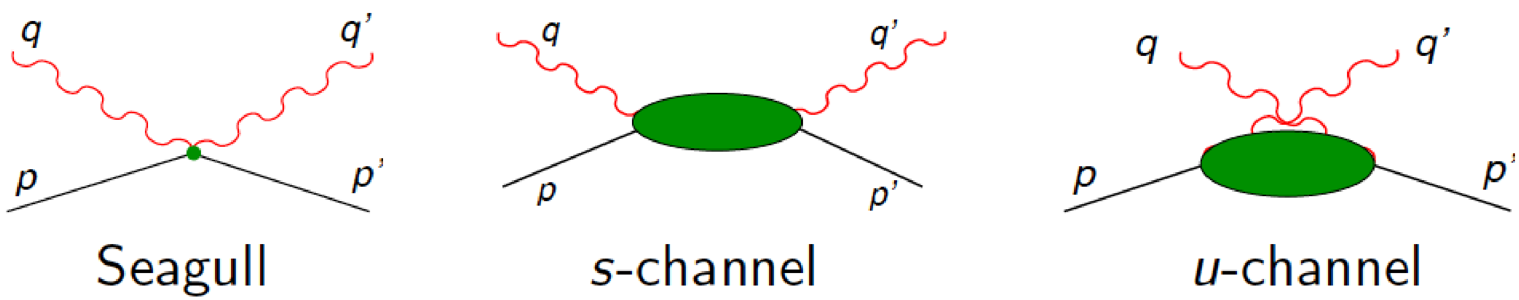
For instance, consider the case where one of the photons is real, say $q'^2 = 0$, the number of independent physical amplitudes reduces to three, say $A(1,1)$, $A(1,0)$, and $A(1,-1)$.

The vector $\tilde{q}'_{\rm L}$ reduces to q' which is orthogonal to $\epsilon(q')$ and thus the CFFs $\mathcal{H}_3$ and $\mathcal{H}_4$ do not contribute, reducing the full tensor $T^{\mu\nu}$ to an effective one with only three independent pieces.

Finally, if both photons are real, the number of active CFFs reduces to two, which equals the number of independent physical amplitudes $A(1,1)$ and $A(1,-1)$. The effective tensor has in this case the same form as the tree-level tensor.

Thus, the number of CFFs in the **effective** tensor equals the number of independent physical matrix elements.

Illustration: Tree-level DVCS



The tree-level DVCS amplitude corresponds to the CFFs

$$\mathcal{H}_0 = -2, \quad \mathcal{H}_1 = Q^2 \left(\frac{1}{s - M^2} + \frac{1}{u - M^2} \right).$$

Thus, only 2 out of 5 CFFs contribute. We note that $\mathcal{H}_1$ and $\mathcal{H}_0$ are of the same order at large Q.

The tree-level amplitude has the same number of CFFs whatever the kinematics. They are simple functions of the Mandelstam variables, but will be more complicated if one goes beyond the lowest order in perturbation theory (dynamical effect).

Kinematics

We shall in general work in the hadronic CMF. The momenta are

$$
\begin{aligned}
p^\mu &= (E_C, -q_C \sin\theta_C, 0, -q_C\cos\theta_C), \\
q^\mu &= (q_C^0, q_C \sin\theta_C, 0, q_C\cos\theta_C), \\
p'^\mu &= (E'_C, -q'_C \sin\theta'_C, 0, -q'_C\cos\theta'_C), \\
q'^\mu &= (q'_C, q'_C \sin\theta'_C, 0, q'_C\cos\theta'_C).
\end{aligned}
$$

with

$$
\begin{aligned}
q_C &= \frac{\sqrt{(M^2+Q^2-s)^2+4sQ^2}}{2\sqrt{s}}, \quad E_C = \frac{s+M^2+Q^2}{2\sqrt{s}}, \\
q_C^0 &= \frac{s-M^2-Q^2}{2\sqrt{s}}, \\
q'_C &= \frac{s-M^2}{2\sqrt{s}}, \quad E'_C = \frac{s+M^2}{2\sqrt{s}}.
\end{aligned}
$$

Superficially, the momenta scale as Q^2, but we can use the Bjorken variable x_{Bj} to relate the Mandelstam variable s to the mass M and Q^2.

Using the definition $x_{\mathrm{Bj}} = Q^2/(2p \cdot q)$ we find

$$x_{\mathrm{Bj}} = \frac{Q^2}{s + Q^2 - M^2} \leftrightarrow s = M^2 + \frac{1 - x_{\mathrm{Bj}}}{x_{\mathrm{Bj}}} Q^2.$$

Thus s is of order Q^2, which shows that all non-vanishing momentum components are of order Q.
We calculate the Mandelstam variables t and u for large Q:

$$t \to -\frac{1 - \cos\vartheta}{2x_{\mathrm{Bj}}} Q^2, \quad u \to -\frac{1 + \cos\vartheta}{2x_{\mathrm{Bj}}} Q^2.$$

The quantity $\vartheta = \theta'_{\mathrm{C}} - \theta_{\mathrm{C}}$ is the scattering angle in the CMF.

If $\vartheta \to 0$, t goes to zero up to corrections of $\mathcal{O}(M^2)$, thus t does not strictly vanish in the forward limit.

If the experimental set-up limits the scattering angle to values greater than ϑ_{lim}, t remains of order Q^2.

For large Q and small ϑ_{lim} one finds

$$t < -\frac{\vartheta_{\mathrm{lim}}^2}{4x_{\mathrm{Bj}}} Q^2.$$

In collinear kinematics, $\vartheta = 0$, and rotating the reference frame such that $\theta'_C = \theta_C = 0$, one finds for large Q the simplified expressions

$$\bar{P}^\mu = Q \frac{2 - x_{Bj}}{2\sqrt{x_{Bj}(1 - x_{Bj})}} (1, 0, 0, -1),$$
$$q'^\mu = Q \frac{1 - x_{Bj}}{2\sqrt{x_{Bj}(1 - x_{Bj})}} (1, 0, 0, 1),$$
$$q^\mu = Q \frac{1}{2\sqrt{x_{Bj}(1 - x_{Bj})}} (1 - 2x_{Bj}, 0, 0, 1).$$

The corrections to these expressions are of order M^2/Q.

One may check that $q' - q \propto \bar{P}$ in this limit and thus $t = (q' - q)^2 = 0$.

Coincidence Experiment

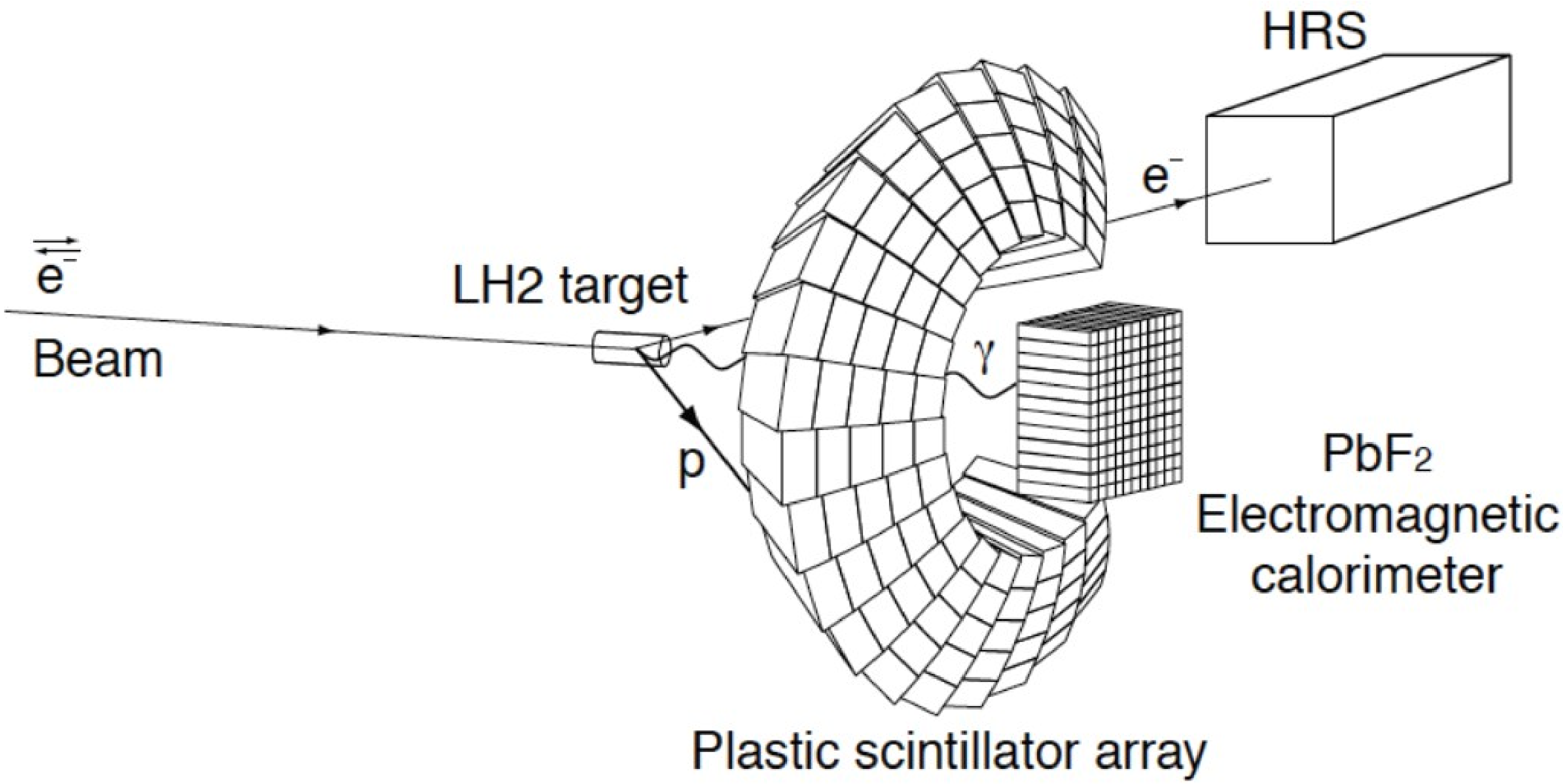


Figure 1.11: E00-110 schematic setup showing the three different detectors used to measured each of the particles in the final state. **Carlos Muñoz Camacho** thesis

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Block[{M = 0.938, Q = Sqrt[1.9], xBj = 0.36, Eb = 5.75, the = 19.3 Pi / 180},
 Graphics[Table[{{Red, Arrow[{{0, 0}, {qfT3mu[[2]], qfT3mu[[4]]}}]},
    {Blue, Arrow[{{0, 0}, {pfT3mu[[2]], pfT3mu[[4]]}}]},
    {Black, Arrow[{{0, 0}, {PT3mu[[2]], PT3mu[[4]]}}]}}, {thetaC, 1, Pi, Pi / 18}]]]
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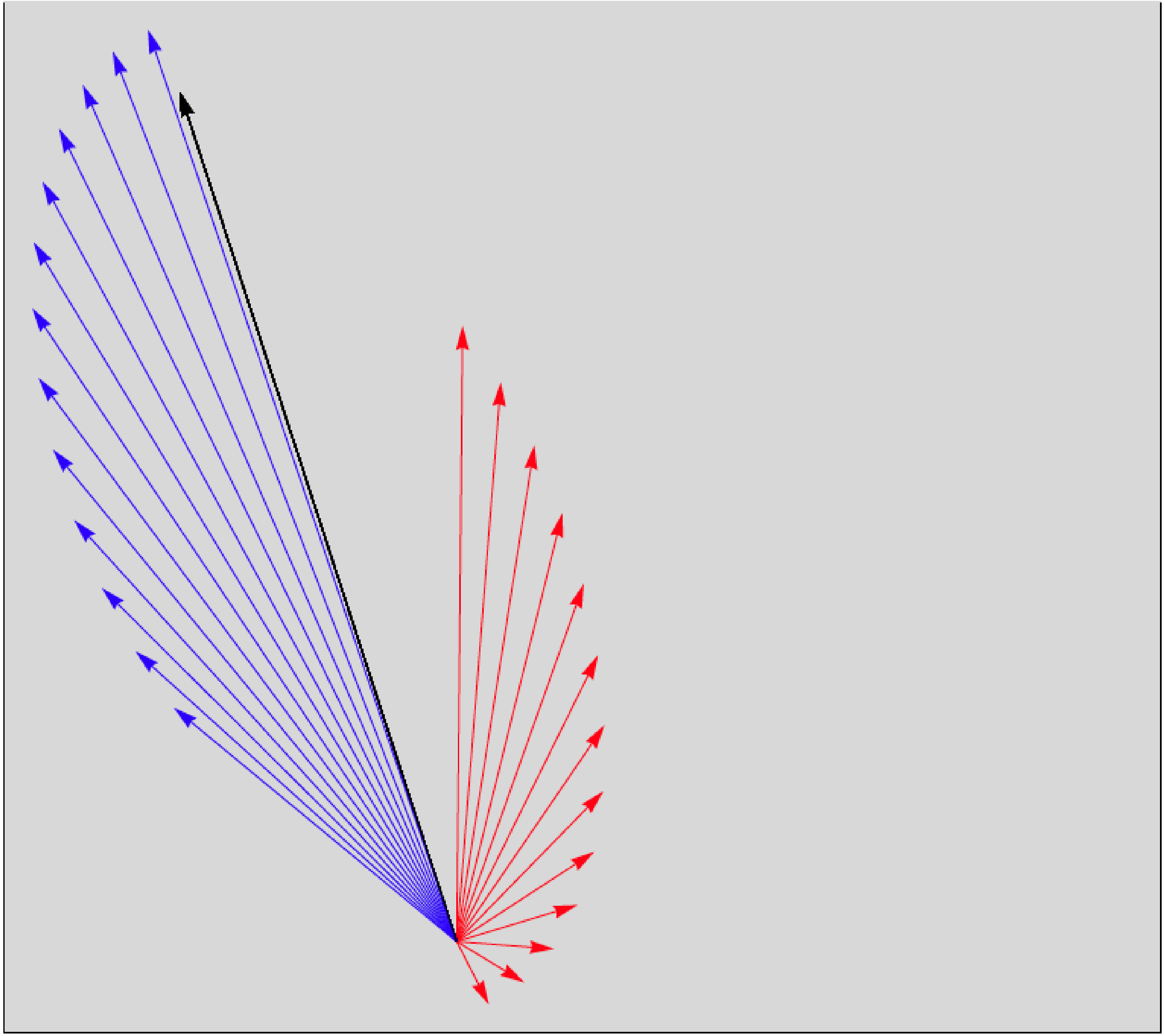


Reduced momenta

It is interesting to check the reduced momenta in the limits $Q \to \infty$ and $\vartheta_C \to 0$. First, we look at the projector:

$$\tilde{g} \to \frac{1}{x_{Bj}} \begin{pmatrix} 1 & 0 & 0 & 1 - 2x_{Bj} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 - 2x_{Bj} \end{pmatrix}$$

The reduced momenta that matter for DVCS, in this limit are:

$$\tilde{P}_L = \frac{Q}{2\sqrt{x_{Bj}(1 - x_{Bj})}} \frac{(1 - x_{Bj})(2 - x_{Bj})}{x_{Bj}} (1, 0, 0, 1),$$

$$\tilde{P}_R = \frac{Q}{2\sqrt{x_{Bj}(1 - x_{Bj})}} \frac{2 - x_{Bj}}{x_{Bj}} (1, 0, 0, 1 - 2x_{Bj}),$$

$$\tilde{q}_R = \frac{Q}{2\sqrt{x_{Bj}(1 - x_{Bj})}} (-1, 0, 0, -(1 - 2x_{Bj})).$$

The Compton tensor

We write the tensor $T^{\mu\nu}$ in the forward kinematics for the DVCS case where only three CFFs occur

$$T^{\mu\nu} = \begin{pmatrix} \frac{\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} & 0 & 0 & \frac{(1-2x_{\mathrm{Bj}})\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} \\ 0 & -\mathcal{H}_0 & 0 & 0 \\ 0 & 0 & -\mathcal{H}_0 & 0 \\ \frac{\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} & 0 & 0 & \frac{(1-2x_{\mathrm{Bj}})\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} \end{pmatrix}$$

where the compound CFF $\mathcal{H}_1'$ is defined by

$$\mathcal{H}_1' = 2x_{\mathrm{Bj}}^2\mathcal{H}_0 + (2 - x_{\mathrm{Bj}})^2\mathcal{H}_1 - x_{\mathrm{Bj}}(2 - x_{\mathrm{Bj}})\mathcal{H}_2.$$

Note that this result is effectively the same as the tree-level result, where only two CFFs occur.

Thus in the forward limit at large Q we cannot distinguish between the tree level tensor and the complete tensor. This a **kinematical effect**.

Polarization vectors

To calculate the amplitudes, we need the polarization vectors.

The polarization vectors of the incoming virtual photon in the CMF are

$$\begin{aligned}\epsilon^\mu(q', \pm 1) &= \frac{1}{\sqrt{2}}(0, \mp\cos\theta_\mathrm{C}, i, \pm\sin\theta_\mathrm{C}) \\ \epsilon^\mu(q', 0) &= \frac{1}{\sqrt{-Q^2}}(-q_\mathrm{C}, -q_\mathrm{C}^0\sin\theta_\mathrm{C}, 0, q_\mathrm{C}^0\cos\theta_\mathrm{C})\end{aligned}$$

The ones for the final state are obtained by replacing θ_C by θ'_C and dropping the one with helicity 0.

In the forward limit and $Q \to \infty$ we find

$$\epsilon^\mu(q, 0) = -\frac{Q}{2\sqrt{-Q^2}\sqrt{x_\mathrm{Bj}(1 - x_\mathrm{Bj})}}(1, 0, 0, 1 - 2x_\mathrm{Bj})$$

One may easily check that in this limit q and $\epsilon(q, h)$ are still orthogonal for all values of the helicity h.

Factorized amplitude

Recall the formulas

$$A(h', h) = \epsilon^*(q'; h')_\mu T^{\mu\nu} \epsilon(q; h)_\nu .$$

and

$$T^{\mu\nu} = \mathcal{H}_0 \, \tilde{g}^{\mu\nu} + \mathcal{H}_1 \, \frac{\tilde{P}_{\mathrm{L}}^{\mu} \tilde{P}_{\mathrm{R}}^{\nu}}{Q^2} + \mathcal{H}_2 \frac{\tilde{P}_{\mathrm{L}}^{\mu} \tilde{q}_{\mathrm{R}}^{\nu}}{Q^2}$$

which leads to the expression

$$\begin{aligned} A(h', h) &= \mathcal{H}_0 \, \epsilon^*(q'; h') \cdot \tilde{g} \cdot \epsilon(q; h) \\ &+ \mathcal{H}_1 \, [\epsilon^*(q'; h') \cdot \tilde{P}_{\mathrm{L}}] \, [\tilde{P}_{\mathrm{R}} \cdot \epsilon(q; h)] / Q^2 \\ &+ \mathcal{H}_2 \, [\epsilon^*(q'; h') \cdot \tilde{P}_{\mathrm{L}}] \, [\tilde{q}_{\mathrm{R}} \cdot \epsilon(q; h)] / Q^2 . \end{aligned}$$

This factorized form is useful to understand why in forward kinematics only $\mathcal{H}_0$ contributes:

$$A(1, 1) = A(-1, -1) = -\mathcal{H}_0 \quad \text{for} \quad \vartheta_{\mathrm{C}} \to 0,$$

All other matrix elements are proportional to $\sin\vartheta$. The reason is that in the limit $\vartheta \to 0$ the factors $\epsilon^*(q'; h') \cdot \tilde{P}_{\mathrm{L}}$ vanish for all h'.

In detail, we have the following expansions for small ϑ

$$\begin{aligned} A(1,1) &\to -\mathcal{H}_0 + \mathcal{O}(1-\cos\vartheta) \to -\mathcal{H}_0 + \mathcal{O}(\vartheta^2), \\ A(1,0) &\to \mathcal{O}(\sin\vartheta), \to \mathcal{O}(\vartheta), \\ A(1,-1) &\to \mathcal{O}\left(\sin^2\frac{\vartheta}{2}\right) \to \mathcal{O}(\vartheta^2). \end{aligned}$$

If we solve the amplitudes for the CFFs we find, besides the exact relation $A(1,1) + A(1,-1) = -\mathcal{H}_0$,

$$\mathcal{H}_1 \to -\frac{2x_{\rm Bj}^3}{1-x_{\rm Bj}}\frac{A(1,-1)}{\vartheta^2} + i\frac{\sqrt{2x_{\rm Bj}(1-x_{\rm Bj})}}{1-x_{\rm Bj}}\frac{A(1,0)}{\vartheta} - \frac{x_{\rm Bj}^2}{2}\mathcal{H}_0$$

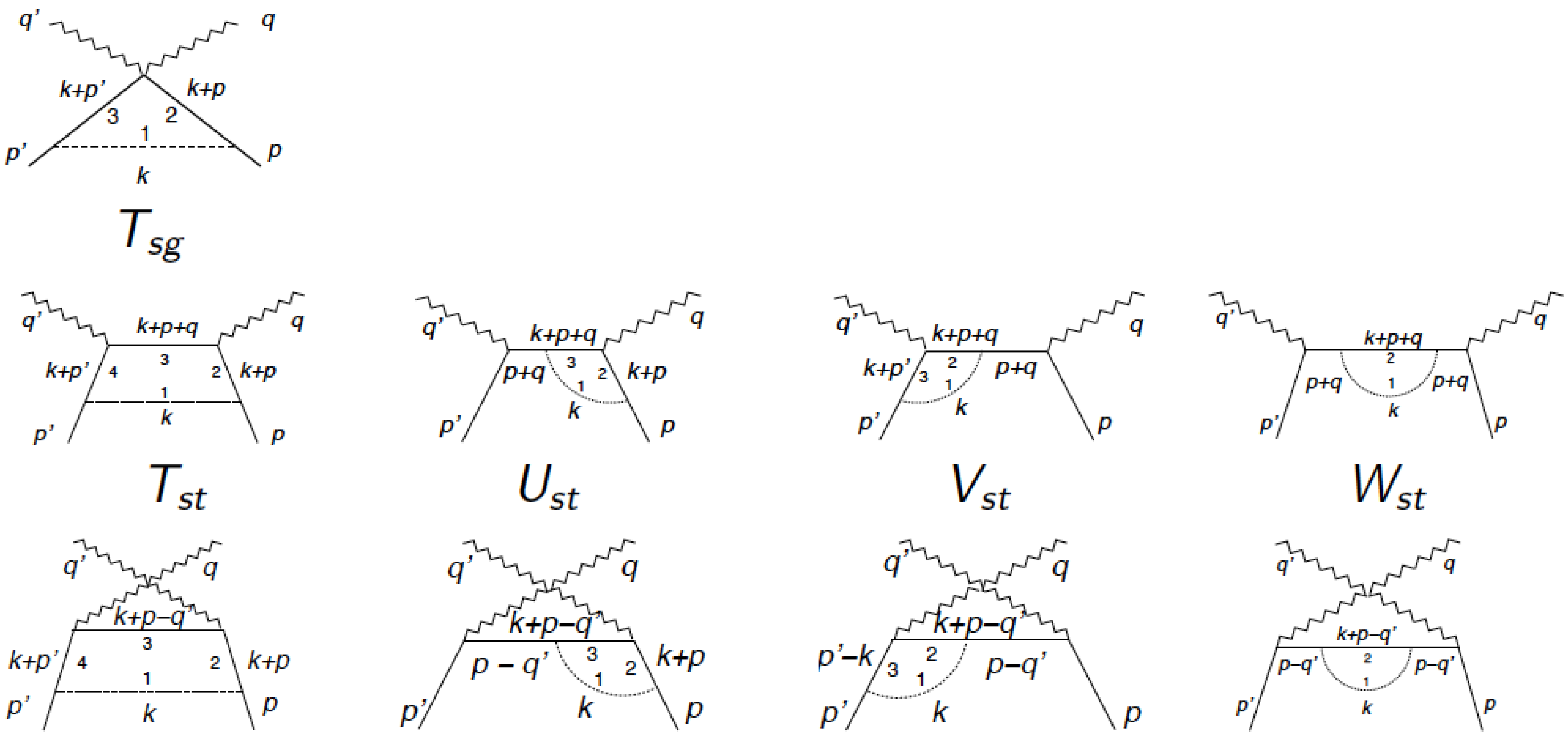
$$\mathcal{H}_2 \to -\frac{2x_{\rm Bj}^2(2-x_{\rm Bj})}{1-x_{\rm Bj}}\frac{A(1,-1)}{\vartheta^2} + i x_{\rm Bj}\sqrt{2x_{\rm Bj}(1-x_{\rm Bj})}\frac{A(1,0)}{\vartheta} + \frac{x_{\rm Bj}^2}{2}\mathcal{H}_0.$$

Because $A(1,-1)/\vartheta^2$ and $A(1,0)/\vartheta$ are finite for $\vartheta \to 0$, the limit $\vartheta \to 0$ exists, but it means that $\mathcal{H}_1$ and $\mathcal{H}_2$ must be determined from the angular dependence of the differential cross-section data.

Simple Model

We consider a simple model. A charged particle of mass M and charge e interacts with a neutral one of mass μ. The coupling constant is g.

It is known[3] that to second order in g the following diagrams must be included to guarantee EM gauge invariance:



T_{sg}

T_{st} U_{st} V_{st} W_{st}

T_{ut} U_{ut} V_{ut} W_{ut}

[3] CRJ and BLGB, Int. J. Mod. Phys. E **22**, 1330002 (2013)

Using the usual procedure involving Feynman parametrization of the integrals over the momenta and performing a shift to reduce the numerators to an even function of the integration variables, we find the contributions to the total second-order tensor $T^{\mu\nu}$.

Because the tree-level tensor $T^{\mu\nu}_{\text{tree}}$ is transverse by itself, we shall not discuss it.

The projector $\tilde{q}$ being idempotent, we know that a transverse tensor that can be written as

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}$$

does not change when the projector is applied again: $T^{\mu\nu} = \tilde{T}^{\mu\nu} \equiv \tilde{g}^{\mu m}\, T_{mn}\, \tilde{g}^{n\nu}$. Therefore, we may apply $\tilde{g}^{\mu\nu}$ to all parts of the second-rank tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{sg}} + T^{\mu\nu}_{\text{st}} + U^{\mu\nu}_{\text{st}} + V^{\mu\nu}_{\text{st}} + W^{\mu\nu}_{\text{st}} + T^{\mu\nu}_{\text{ut}} + U^{\mu\nu}_{\text{ut}} + V^{\mu\nu}_{\text{ut}} + W^{\mu\nu}_{\text{ut}}$$

individually, without changing their sum.

We shall now discuss the transverse parts of the individual tensors.

The simplest example is the seagull tensor, given by the integral

$$T_{\mathrm{sg}}^{\mu\nu} = \int_0^1 \mathrm{d}\alpha_2 \int_0^{1-\alpha_2} \mathrm{d}\alpha_3 \frac{\mathrm{d}k^4}{(2\pi)^4} \frac{-2g^{\mu\nu}}{(k^2 - M_{\mathrm{sg}}^2)^3}.$$

The αs are Feynman parameters and M_{sg} is the invariant mass function obtained following the usual procedure to the calculate the amplitude.

If $\tilde{g}^{\mu\nu}$ is applied, the tensor $g^{\mu\nu}$ changes to $\tilde{g}^{\mu\nu}$, thus $T_{\mathrm{sg}}^{\mu\nu}$ contributes only to the CFF $\mathcal{H}_0$.

Because in our model all invariant mass functions are definite functions of the Feynman parameters and Mandelstam variables in the physical domain, the momentum integrals are convergent after performing a Wick rotation and we can determine their scaling behaviour for large Q: all scale as $1/Q^2$.

The box diagrams $T_{\rm st}$ and $T_{\rm ut}$ turn out to contribute to all five CFFs. As an example consider $T_{\rm st}$:

$$\begin{aligned} T^{\mu\nu}_{\rm st} &= \int_0^1 d\alpha_2 \int_0^{1-\alpha_2} d\alpha_3 \int_0^{1-\alpha_2-\alpha_3} d\alpha_4 \, \frac{dk^4}{(2\pi)^4} \frac{N^{\mu\nu}_{T_{\rm st}}}{(k^2 - M^2_{\rm st})^4}, \\ N^{\mu\nu}_{T_{\rm st}} &= (2k + \bar{P} + q - 2\Delta_{T_{\rm st}})^\mu (2k + \bar{P} + q' - 2\Delta_{T_{\rm st}})^\nu \\ \Delta_{T_{\rm st}} &= \alpha_2 p + \alpha_3 (p+q) + \alpha_4 p'. \end{aligned}$$

After contracting $T^{\mu\nu}_{\rm st}$ with $\tilde{g}^{\mu\nu}$, one finds that $\tilde{T}^{\mu\nu}_{\rm st}$ contributes to all CFFs. In particular, one finds for the three CFFs occurring in DVCS:

$$\begin{aligned} \mathcal{H}_{0\,T_{\rm st}} &= \int [d\alpha] \frac{dk^4}{(2\pi)^4} \frac{k^2}{(k^2 - M^2_{T_{\rm st}})^4}, \\ \mathcal{H}_{1\,T_{\rm st}} &= \int [d\alpha] \frac{dk^4}{(2\pi)^4} \frac{(1-\alpha_2-\alpha_3-\alpha_4)^2 Q^2}{(k^2 - M^2_{T_{\rm st}})^4}, \\ \mathcal{H}_{2\,T_{\rm st}} &= \int [d\alpha] \frac{dk^4}{(2\pi)^4} \frac{(\alpha_2-\alpha_3-\alpha_4)(1-\alpha_2-\alpha_3-\alpha_4) Q^2}{(k^2 - M^2_{T_{\rm st}})^4}. \end{aligned}$$

The integral $\int [d\alpha]$ stands for the integral over three Feynman parameters. The factor Q^2 occurs owing to our convention for the CFFs. It guarantees that for $Q \to \infty$ all CFFs scale in the same way.

We summarize the results for the total tensor.

The CFFs relevant for DVCS are obtained after applying the projector $\tilde{g}^{\mu\nu}$ to the tensor $T^{\mu\nu}$.

By splitting the tensor in pieces that correspond to the nine diagrams, the projection also splits into nine parts. They do not all contribute in the same way to the three CFFS. Their contributions are

$$\begin{aligned}
\mathcal{H}_0 &= \mathcal{H}_{0\,T_{\rm sg}} + \mathcal{H}_{0\,T_{\rm st}} + \mathcal{H}_{0\,T_{\rm ut}}, \\
\mathcal{H}_1 &= \mathcal{H}_{1\,T_{\rm st}} + \mathcal{H}_{1\,U_{\rm st}} + \mathcal{H}_{1\,V_{\rm st}} + \mathcal{H}_{1\,W_{\rm st}} + \\
&\quad \mathcal{H}_{1\,T_{\rm ut}} + \mathcal{H}_{1\,U_{\rm ut}} + \mathcal{H}_{1\,V_{\rm ut}} + \mathcal{H}_{1\,W_{\rm ut}}, \\
\mathcal{H}_2 &= \mathcal{H}_{2\,T_{\rm st}} + \mathcal{H}_{2\,U_{\rm st}} + \mathcal{H}_{2\,T_{\rm ut}} + \mathcal{H}_{2\,V_{\rm ut}}.
\end{aligned}$$

In our convention for the CFFs, all of them have the same dimension, namely $1/Q^2$. This can be compared to the tree-level contributions, which are

$$\mathcal{H}_0 = -2, \quad \mathcal{H}_1 = Q^2\left(\frac{1}{s-M^2} + \frac{1}{u-M^2}\right),$$

which scale as Q^0.

Summary and conclusions

1. We have discussed the kinematical effects of taking the DVCS limit $Q \to \infty$ and the collinear limit $\vartheta \to 0$. These considerations are model-independent. They are relevant for DVCS on ^{4}He.
2. The question whether one can measure CFFs in experiments is answered in a model-independent way:

   only $\mathcal{H}_0$ is measured in strictly collinear kinematics.

   To measure the other two CFFs one must measure the angular dependence of the differential cross section.
3. To estimate the relative importance of the three CFFS, one should include the leptonic part and the Bethe-Heitler amplitude.
4. For illustration, we have discussed a toy model, inspired by a quark–di-quark model of a proton. In this model the so-called cat's ears diagrams do not exist.
5. To estimate the relative importance of the different pieces contributing to the CFFs, a numerical calculation would be necessary. Such an effort should better be made in a more realistic model.