LIGHT AND HEAVY PENTAQUARKS IN THE CONSTITUENT QUARK MODEL

Review and recent developements

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- **Dynamical calculations in the Flavour-spin models :**

  - **the light pentaquark antidecuplet**

  - **the charmed antisextet**

- **New generation of results for Θ^+**

- **Representation mixing**

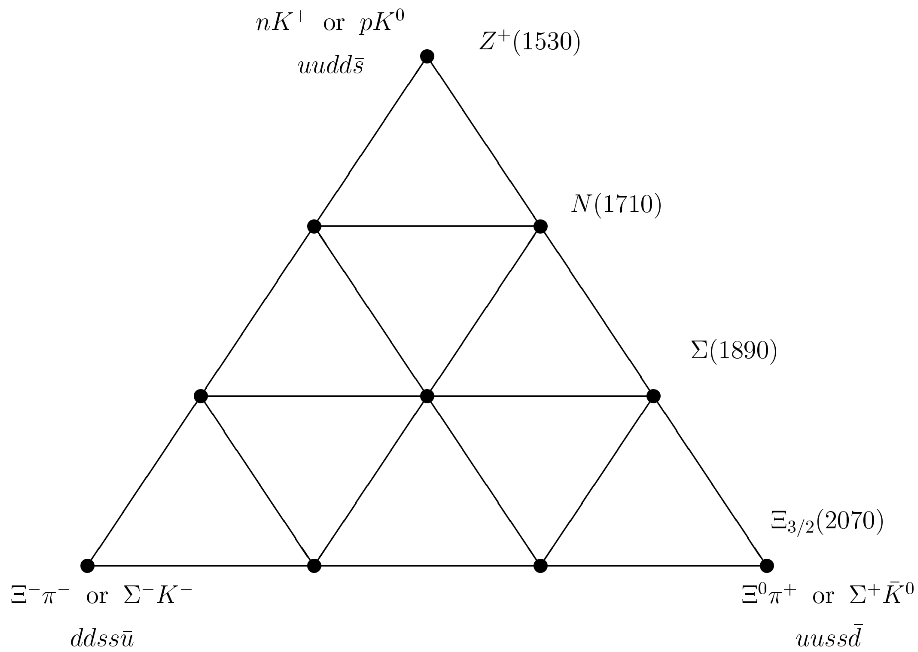
References:

F. S. PRC58 (1998) 111501

F. S. & D. O. Riska, PLB575 (2003) 242

F. S., PLB595 (2004) 269, PLB598 (2004) 295 (erratum)

$\textbf{D.Diakonov, V.Petrov, M.Polyakov}$, Z. Phys. A359(1997)305



Chiral soliton model: Θ^+ is a collective excitation of the mean chiral field $\rightarrow$ low mass and narrow width

PRESENT APPROACHES to the ANTIDECUPLET

- Spin and parity of $\Theta^+ \rightarrow$ polarization experiments
- Consistency with partial wave analysis
- Photoproduction cross sections on proton, neutron
- Chiral soliton model revisited, masses & widths
- Constituent quark models, masses & widths
- Octet-antidecuplet or higher representation mixing
- Group theoretical classification of

$q \times q \times q \times q \times \bar{q}$ states $\rightarrow$ mass formulae

- Heptaquarks or $K\pi N$ molecular picture
- Skyrme model (bound state or rigid rotator)
- Description of pentas in the instanton model
- QCD sum rules
- Pentaquarks in lattice calculations
- Magnetic moments of pentaquarks
- Θ^+ in relativistic heavy ion collisions

WHY CONSTITUENT QUARK MODELS ?

$\rightarrow$ **Describes a large number of observables**

$$8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8)_F + 1_F$$

THE HAMILTONIAN:

(1) spin-independent part (mass, kinetic, confinement)

(2) short-range hyperfine interaction

TWO STANDARD MODELS

Colour-Spin (CS)

$$V_{CS} = -\sum_{i<j}^{5} C_{ij}^{CS} \lambda_i^c \cdot \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Flavour-Spin (FS)

$$V_{FS} = -\sum_{i<j}^{4} C_{ij}^{FS} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$$

MAIN ISSUES IN ANY QUARK MODEL

- **The mass of Θ^+**
- **The strong decay width**
- **Spin and Parity**
- **Splitting between isomultiplets**
- **SU(3)- flavour representation mixing**

WHY THE FS MODEL ?

- Good description of baryon spectra,

- Previous experience in heavy pentaquarks,
  F. S. PRD58(1998)111501

- Symmetry consistent with the large N_c limit of QCD
  (Jenkins & Manohar, hep-ph/0402024)
  N. B. The irreducible representations at $N_c \to \infty$ are referred to as Skyrme representations

- Support from lattice calculations, F. Lee, PENTAQUARK04, SPring-8,
  see N. Mathur et al., PLB605(2005)137

CONSTITUENT QUARK MODELS:
POSITIVE PARITY SCENARIOS

N.B. Useful to look at the q^4 subsystem
I = 0, S = 0 compatible with $uudd\bar{s}$

- FS model (Pseudoscalar meson exchange) :
  Lowest state has positive parity, F. S. '98
  $[31]_{\rm O} \times [211]_{\rm C} \rightarrow [1111]_{\rm OC} \rightarrow [4]_{\rm FS}$ allowed

- CS model (One gluon exchange) :
  Positive parity lowest state is possible schematically (Jennings & Maltman'03), NOT true in realistic calculations
  $[31]_{\rm O} \times [22]_{\rm F} \rightarrow [211]_{\rm OF} \rightarrow [31]_{\rm CS}$ allowed

STABLE $\mathbf{uudd\bar{s}}$

F. S. & D. O. Riska, PLB575,242(2003)

1) qq **pairs, schematic Flavour-Spin interaction**

Glozman & Riska, PR268(1996)263

$$\mathbf{V}_\chi = -\,\mathbf{C}_\chi \sum_{\mathbf{i}<\mathbf{j}}^{4} \lambda_\mathbf{i}^\mathbf{F} \cdot \lambda_\mathbf{j}^\mathbf{F}\, \tilde{\sigma}_\mathbf{i} \cdot \tilde{\sigma}_\mathbf{j}$$

$$C_\chi \sim 30 \text{ MeV} \quad (\Delta - N \;\, splitting)$$

enough if the antiquark is heavy

2) $q\bar{s}$ pairs, schematic η-meson exchange

Pion decay $D_s^* \rightarrow D_s\pi^0$ requires $\pi^0 - \eta$ mixing
$\rightarrow \eta$ meson couples to s or $\bar{s}$ (Laehde and Riska, '02)

$$\mathbf{V}_\eta = \mathbf{V_0} \sum_{\mathbf{i}}^{\mathbf{4}} \tilde{\sigma}_\mathbf{i} \cdot \tilde{\sigma}_\mathbf{j}$$

$\mathrm{V_0 = 2C_\chi}$ lowers the binding energy by 350 MeV Conclusion:
$\mathrm{V_\chi + V_\eta}$ makes $\mathrm{uudd\bar{s}}$ J^P=1/2$^+$, I=0, S=+1 stable

THE ANTIDECUPLET. F. S. PLB595(2004)269 [hep-ph/0402044]

Basic assumptions:

- **Consider the lowest q^4 state**

$$|1\rangle \;=\; |[31]_O[211]_C\left[1^4\right]_{OC}\;;\;[22]_F[22]_S[4]_{FS}\rangle$$

- **Couple $\bar{q}$ to the q^4 state $\rightarrow$ 5-particle state**

- **qq interaction: Graz parametrization**

- **the $q\overline{q}$ interaction is spin dependent but flavour independent $\rightarrow$ assume a global shift of the spectrum**

REALISTIC FLAVOUR-SPIN HAMILTONIAN

Graz parametrization, Glozman, Papp & Plessas, PLB '96

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \frac{\vec{P}^2}{2M} + \sum_{i<j} V_{conf}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij})$$

$$V_{\text{conf}}(r_{ij}) = -\frac{3}{8}\lambda_i^c \cdot \lambda_j^c \, C \, r_{ij}$$

$$V_\chi(r_{ij}) = \left\{ \sum_{F=1}^{3} V_\pi(r_{ij}) \lambda_i^F \lambda_j^F + \sum_{F=4}^{7} V_K(r_{ij}) \lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

N. B. Parameters (m_i, meson masses, etc) fitted to baryon spectra
$\rightarrow$ good level order

Internal Jacobi coordinates

$$\vec{x} = \vec{r}_1 - \vec{r}_2 ,$$
$$\vec{y} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3}$$
$$\vec{z} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4)/\sqrt{6} ,$$
$$\vec{t} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5)/\sqrt{10}$$

Basis states of POSITIVE parity:

assume s^3p content for $[31]_O$

$$\psi_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|000\rangle\,\langle \vec{y}\,|000\rangle\,\langle \vec{z}\,|010\rangle$$

$$\psi_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|000\rangle\,\langle \vec{y}\,|010\rangle\,\langle \vec{z}\,|000\rangle$$

$$\psi_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|010\rangle\,\langle \vec{y}\,|000\rangle\,\langle \vec{z}\,|000\rangle$$

to be multiplied by $\langle \vec{t}\,|000\rangle$

Gaussian Ansatz

$$\psi_1^5 = \psi_0 \ z \ Y_{10} \ (\hat{z})$$

$$\psi_2^5 = \psi_0 \ y \ Y_{10} \ (\hat{y})$$

$$\psi_3^5 = \psi_0 \ x \ Y_{10} \ (\hat{x})$$

$$\psi_0 = [\frac{1}{48\pi^5\alpha\beta^3}]^{1/2} \exp \left[- \frac{1}{4\alpha^2} \left(x^2 + y^2 + z^2\right) - \frac{1}{4\beta^2} t^2 \right]$$

Variational parameters α , β

Expectation values (MeV) and total energy

$$\mathbf{E = \sum_{n=1}^{5} m_i + \langle T \rangle + \langle V_c \rangle + \langle V_\chi \rangle}$$

$q^4\bar{q}$	$\sum_{n=1}^{5} m_i$	$\langle T \rangle$	$\langle V_c \rangle$	$\langle V_\chi \rangle$	E	M	α(fm)	β(fm)
$uudd\bar{d}$	1700	1864	442	-2044	1962	1452	0.42	0.92
$udd\bar{s}$	1800	1848	461	-2059	2050	1540	0.42	1.01
$uuds\bar{d}$	1800	1535	461	-1563	2233	1723	0.45	0.92
$uuds\bar{s}$	1900	1634	440	-1663	2310	1800	0.44	0.87
$ddss\bar{u}$	1900	1418	464	-1310	2472	1962	0.46	0.92
$uuss\bar{s}$	2000	1410	452	-1310	2552	2042	0.46	0.87

The antidecuplet members with $Y=1$ and $Y=0$

$$M(N_{\overline{10}}) = \frac{1}{3}M(uudd\bar{d}) + \frac{2}{3}M(uuds\bar{s}) = 1684 \text{ MeV},$$
$$M(\Sigma_{\overline{10}}) = \frac{2}{3}M(uuds\bar{d}) + \frac{1}{3}M(uuss\bar{s}) = 1829 \text{ MeV}.$$

The octet members with $Y=1$ and $Y=0$

$$M(N_8) = \frac{2}{3}M(uudd\bar{d}) + \frac{1}{3}M(uuds\bar{s}) = 1568 \text{ MeV},$$
$$M(\Sigma_8) = \frac{1}{3}M(uuds\bar{d}) + \frac{2}{3}M(uuss\bar{s}) = 1936 \text{ MeV}.$$

THE ANTIDECUPLET MASS SPECTRUM (MeV) $P = +1$

Penta	Y, I, I_3	Present results	Carlson et al.
Θ^+	$2, 0, 0$	1540	1540
$N_{\overline{10}}$	$1, 1/2, 1/2$	1684	1665
$\Sigma_{\overline{10}}$	$0, 1, 1$	1829	1786
Ξ^{--}	$-1, 3/2, -3/2$	1962	1906

Carlson et al. PLB579(2004)52

NO kinetic energy, NO η'-meson exchange

NO symmetry breaking in η-meson exchange

NO orbital excitation in the radial matrix elements

$\rightarrow$ conflict with POSITIVE parity

N.B. Updated partial wave analysis gives 1680 MeV or 1730 MeV, Arndt et al., nucl-th/0312126

N.B. Smaller splitting in the CS model

V. Dmitrasinovic & F. S.

- **Totally antisymmetric q^4 wave fct., NO $\bar{3}$ S=0, I=0 diquarks**
- **Full CS interaction**
- **Broken SU(3) by the quark masses**

M = M_0 - 58 Y

M_0 is fixed by the Θ^+ mass $\rightarrow$ M(Ξ^{--}) $\simeq$ 1710 MeV

Representation mixing

$$8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8)_F + 1_F$$

Existing data require mixing $\rightarrow$ Find multiplets by fitting the mass and width $\rightarrow$ some mixing angle

Arbitrary number of quarks and antiquarks, model independent

Diakonov and Petrov, $J^P = 1/2^+$, PRD69(2004)

Pakvasa and Suzuki, PRD70(2004), $J^P = 1/2^+$, Ξ^{--} =1862
masses require large mixing and the widths small mixing

Hyodo and Hosaka, $J^P = 1/2^+, 1/2^-, 3/2^+, 3/2^-$ (hep-ph/0502093)
$\rightarrow$ consistent large mixing for $3/2^-$

Hyodo and Hosaka, hep-ph/0502093

Mass spectra and Θ^+ decay width (MeV) For $1/2^-$ the masses of Θ and Ξ are the input parameters, while for $1/2^+, 3/2^\pm$, the masses of Θ and N are the input parameters. Values in brackets are predictions and we show the candidates to be assigned for the states.

J^P	M_Θ	M_N	M_Σ	M_Ξ	Γ_Θ
	1540	[1647]	[1753]	1860	
$1/2^-$		N(1650)	Σ(1750)		$156.1\ ^{+90.8}_{-73.3}$
	1540	1710	[1880]	[2050]	
$1/2^+$			Σ(1880)	Ξ(2030)	$7.2\ ^{+15.3}_{-4.6}$
	1540	1720	[1900]	[2080]	
$3/2^+$			-	-	$10.6\ ^{+7.0}_{-5.0}$
	1540	1700	[1860]	[2020]	
$3/2^-$			-	Ξ(2030)	$1.3\ ^{+1.2}_{-0.9}$

Dynamical derivation in the FS model

$$\mathbf{V} = \begin{cases} \frac{2\sqrt{2}}{3}(\mathbf{m_s} - \mathbf{m_u}) + \frac{\sqrt{2}}{3}\ [\mathbf{S}(\mathbf{uuds\bar{s}}) - \mathbf{S}(\mathbf{uudd\bar{d}})] = \mathbf{166}\ \text{MeV} & \text{for N} \\ \frac{2\sqrt{2}}{3}(\mathbf{m_s} - \mathbf{m_u}) + \frac{\sqrt{2}}{3}\ [\mathbf{S}(\mathbf{uuss\bar{s}}) - \mathbf{S}(\mathbf{uuds\bar{d}})] = \mathbf{155}\ \text{MeV} & \text{for } \Sigma \end{cases}$$

$$\mathbf{S} = \langle \mathbf{T} \rangle + \langle \mathbf{V}_\chi \rangle$$

Physical states

$$|\mathbf{N}^*\rangle = |\mathbf{N_8}\rangle \cos\theta_\mathbf{N} - |\mathbf{N_{\overline{10}}}\rangle \sin\theta_\mathbf{N},$$
$$|\mathbf{N_5}\rangle = |\mathbf{N_8}\rangle \sin\theta_\mathbf{N} + |\mathbf{N_{\overline{10}}}\rangle \cos\theta_\mathbf{N}$$

The calculated mixing angle $\theta_N = 35.34^0$ $\rightarrow$

M(N$_5$) = 1801 MeV is 67 % antidecuplet

M(N*) = 1451 MeV is 67 % pentaquark octet

Also a q^3 **state N(1440) = 1495 MeV** $\rightarrow$

TWO resonances in the Roper region (P. Morsch et al. PRL '99)

N. B. Ideal mixing $\theta_N = 35.26^0$

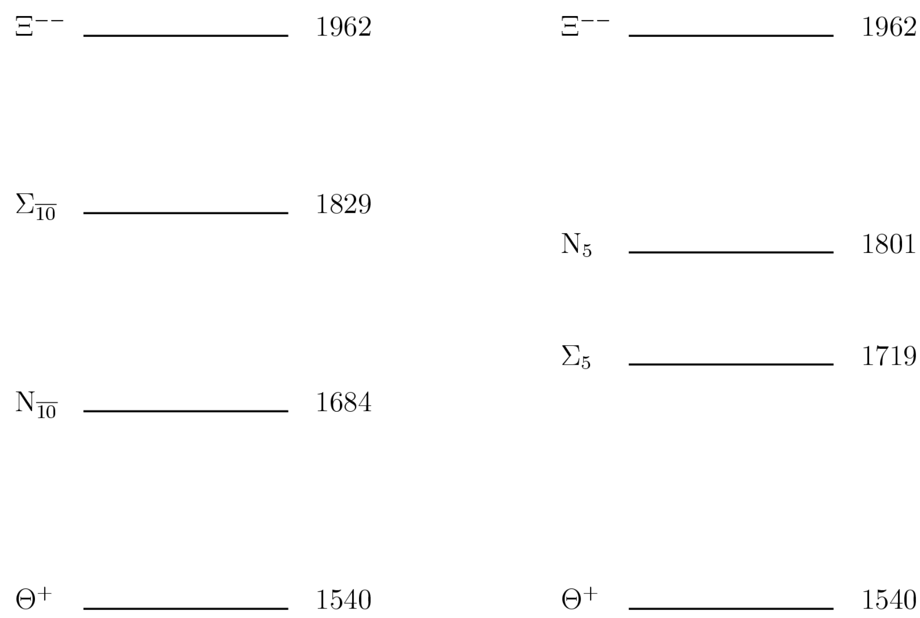
The calculated mixing angle $\theta_\Sigma = -35.48^0 \rightarrow$

M(Σ_5**) = 1719 MeV is 67 % pentaquark antidecuplet**

M(Σ^***) = 2046 MeV is 67 % pentaquark octet**

N. B. Ideal mixing $\theta_\Sigma = -35.26^0$

PENTAQUARK SPECTRUM in the FS MODEL



a) pure antidecuplet   b) mixed with the octet

ongoing searches for Ξ_5 and N_5 at BaBar (SLAC) ?

RECENT CONSTITUENT QUARK MODEL RESULTS

Takeuchi & Shimizu, hep-ph/0411016

FS model - semirelativistic version - Graz

The lowest state is $J^P = 1/2^+$

CS model - nonrelativistic one gluon exchange

1)The lowest resonant state is $J^P = 3/2^-$

2)The $J^P = 1/2^-$ **states is lower, but in the continuum**

Takeuchi & Shimizu, hep-ph/0411016

Five parameter sets.

Each parameter set is denoted by Rπ, or Rgπ, etc., according to the kinetic energy term and the qq interaction. Each meson mass is taken to be the observed one, and $m_\sigma = 675$ MeV.

Model ID	qq int.	m_u MeV	m_s MeV	α_s	Λ_g [fm^{-1}]	$\frac{g_8^2}{4\pi}$	$(\frac{g_0}{g_8})^2$	Λ_0 [fm^{-1}]	κ	a_{conf} [MeV/fm]	V_0 MeV
Rπ†	$\pi\ \sigma\ \eta$	313	530	0	-	0.69	0	1.81	0.92	170	-378.3
Rgπ	OGE $\pi\ \sigma\ \eta\ \eta'$	340	560	0.35	3	0.69	1	1.81	0.92	172.4	-381.7
Ngπ‡	OGE $\pi\ \sigma\ \eta$	313	550	0.35	5	0.592	0	2.87	0.81	172.4	-453
Ng	OGE	313	680	1.72	3	0	-	-	-	172.4	-345.5
Graz§	$\pi\ \eta\ \eta'$	340	500	0	-	0.67	1.34	2.87	0.81	172.4	-416

Takeuchi & Shimizu, hep-ph/0411016

Masses of meson, baryon, and pentaquark, P(TJ^P), in MeV.

	N	Δ	K*	NK*	P($0\frac{1}{2}^-$) m_P	P($0\frac{1}{2}^-$) m'_P	P($0\frac{3}{2}^-$) m_P	P($0\frac{3}{2}^-$) m'_P	P($1\frac{1}{2}^-$) m_P	P($1\frac{1}{2}^-$) m'_P	P($0\frac{1}{2}^+$) m_P	P($0\frac{1}{2}^+$) m'_P
Rπ	941	1261	979	1921	2109	2054	2141	2083	2143	2083	2165	2045
Rgπ	938	1232	931	1869	1985	1947	2064	2018	2078	2021	2129	2006
Ngπ	936	1232	814	1846	2029	1996	2106	2066	2106	2061	2321	2144
Ng	938	1232	814	1846	1966	1971	2153	2144	2170	2145	2345	2197
Graz	937	1239	927	1864	2231	2160	2240	2168	2251	2173	2248	2120
Exp.†	939	1232	892	1831			1540					

THE DECAY WIDTH

Nakano et al., LEPS Collab. $\Gamma \leq 25$ MeV
Scattering of K^+ on nuclei $\Gamma \leq 1$ MeV

Options to reduce the width in constituent quark models
$\rightarrow$ diquark correlations (JW scenario)

Melikhov, Simula, Stech, hep-ph/0405037
a) size parameter similar to the nucleon $\rightarrow \Gamma \simeq 150$ MeV
b) asymmetric "peanut" structure $\rightarrow \Gamma(\alpha_{\rho\theta}, \alpha_{\lambda\theta}) \simeq 1$ MeV

$$\Psi_{\Theta} = N_\theta \exp(-\frac{\rho_\theta^2}{2\alpha_{\rho\theta}^2} - \frac{2\lambda_\theta^2}{3\alpha_{\lambda\theta}^2} - \frac{(r_{23}^2 + r_{45}^2)}{2\alpha_D^2})$$

no dynamics

Hosaka, Oka, Shinozaki, hep-ph/0409102

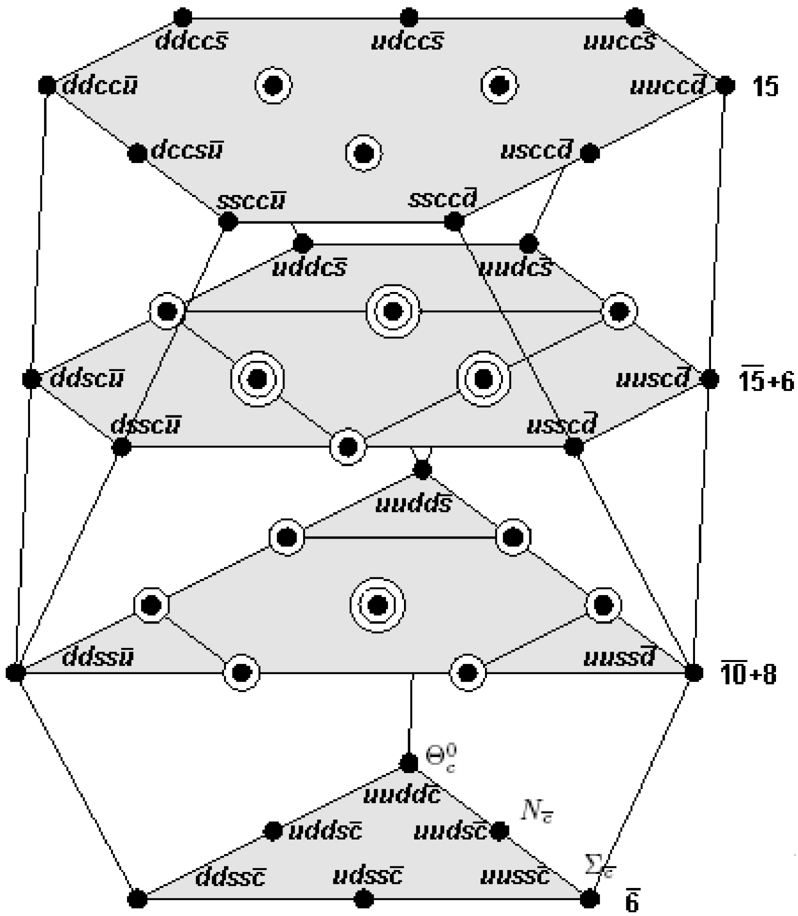
The decay width (MeV)

$$\mathcal{L}_{int} = -ig\overline{\psi}\gamma_5\Phi\psi; \quad g = \frac{g_{\pi NN}}{5} = 2.6$$

$\langle r^2\rangle^{1/2}$	α^2	$\Gamma(J^P = 1/2^-)$	$\Gamma(J^P = 1/2^+)$ FS	$\Gamma(J^P = 1/2^+)$ CS	$\Gamma(J^P = 1/2^+)$ JW
0.7 fm	3 fm^{-2}	890	63	32	11

CHARMED PENTAQUARKS in the $\overline{60}$-plet of SU(4)

Bin Wu & Bo-Qiang Ma, hep-ph/0402244



MASSES (MeV) of the antisextet charmed pentaquarks $P = +1$

Penta	I	Content	FS model FS 1998	$D - D - \bar{c}$ JW 2003	$D - T$ KL 2003	Lattice CH 2004
Θ_c^0	0	u u d d $\bar{c}$	2902	2710	2985 ± 50	2977 ± 109
N_c^0	1/2	u u d s $\bar{c}$	3161			3180 ± 70
Ξ_c^0	1	u u s s $\bar{c}$	3403			3650 ± 95

FS 1988: F.S. PRD58(1998)111501

JW 2003: Jaffe & Wilczek PRL 91 (2003) 232003

KL 2003: Karliner & Lipkin, PL B575 (2003) 249

CH 2004: Chiu & Hsieh, hep-ph/0404007

N.B. **N + D = 2808 MeV**

CONSTITUENT QUARK MODELS

* One can accommodate Θ^+ and partners
* absolute mass is not determined

* Ξ^{--} - Θ^+ mass difference is model dependent
* Coupling $\overline{\mathbf{10}}_\mathbf{F}$ & $\mathbf{8}_\mathbf{F}$ due to SU$_F$(3) breaking $\rightarrow$ IDEAL mixing

* 2 states: q^3 and $q^4\bar{q}$ in the Roper mass region $\rightarrow$ higher Fock components for baryon q^3 states

* small width: $\langle\Theta|KN\rangle$ flavour-spin or orbital space

CONCLUSIONS

- **New look of baryon spectroscopy**

  Difficult problems: Roper resonance, $\Lambda(1405)$, strong decays

- **Understand the role of chiral symmetry breaking**

  Negative parity baryons ?