# Review and recent developments in the chiral soliton model approach to the antidecuplet and pentaquarks

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Kraków, Poland

# Soliton models are quark models

Constituent QM:

$$H_{CM} \sim - \sum_{i<j=1,\cdots,5} \vec{\sigma}_i \cdot \vec{\sigma}_j \; F_i^a F_j^a \frac{f_{CM}(\vec{r}_{ij})}{m_i m_j}$$

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all qu. # are determined by quarks

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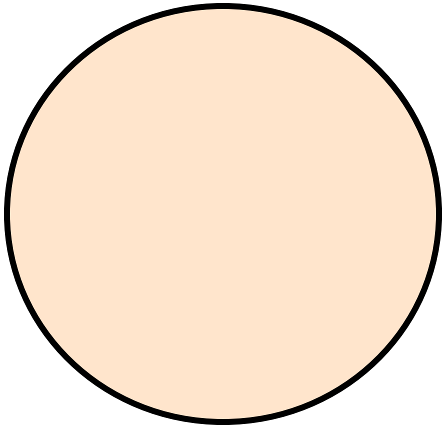

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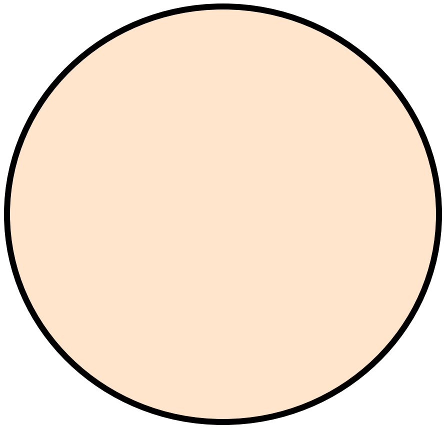

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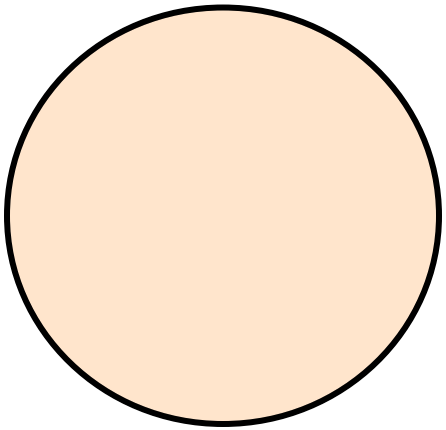

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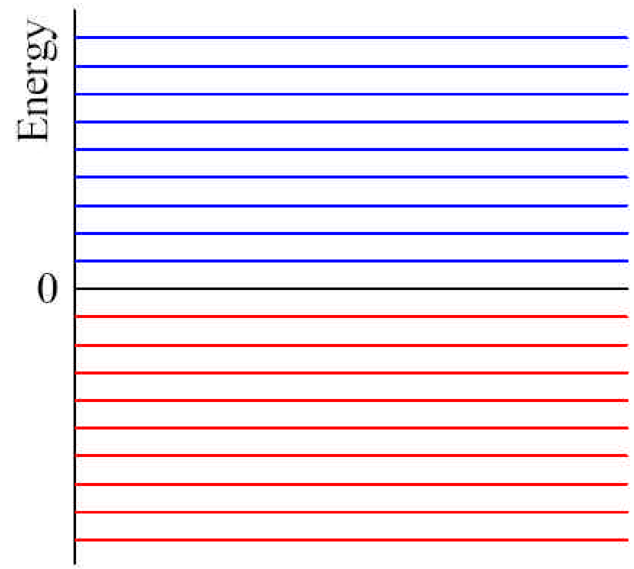


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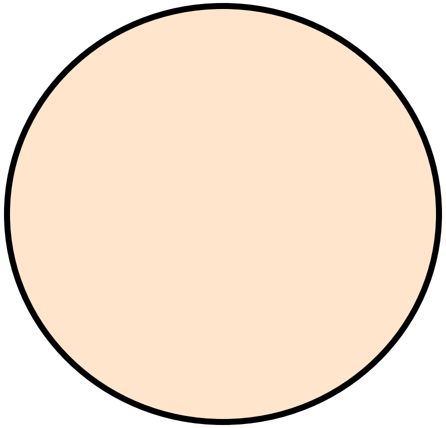

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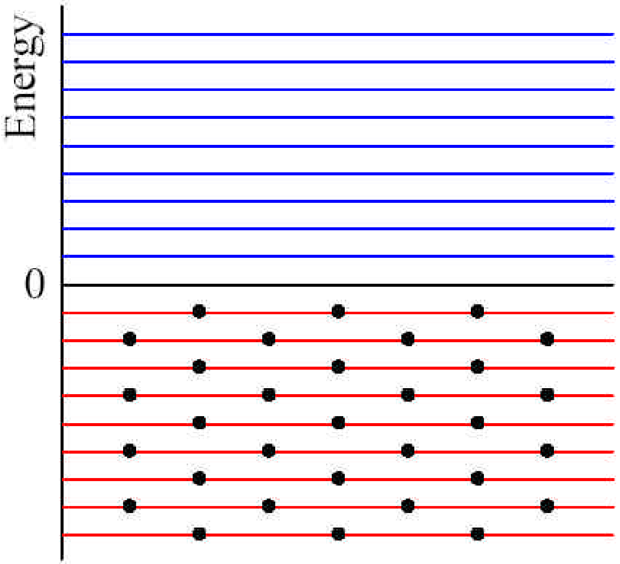

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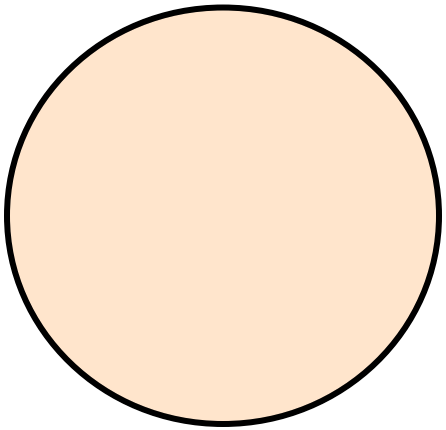

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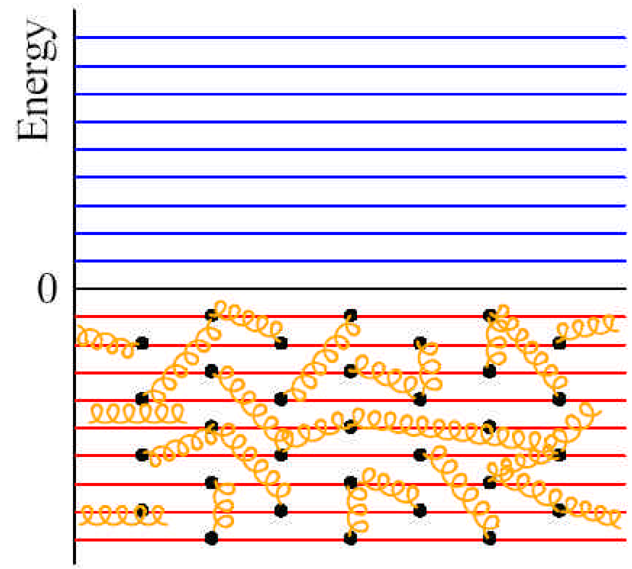

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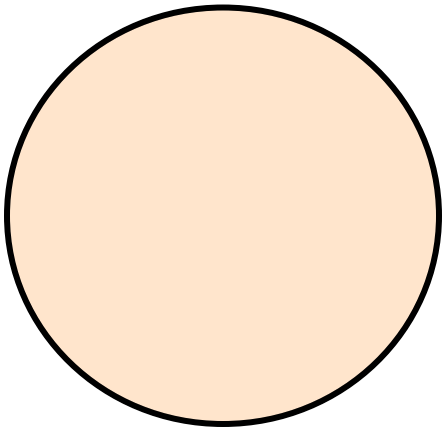

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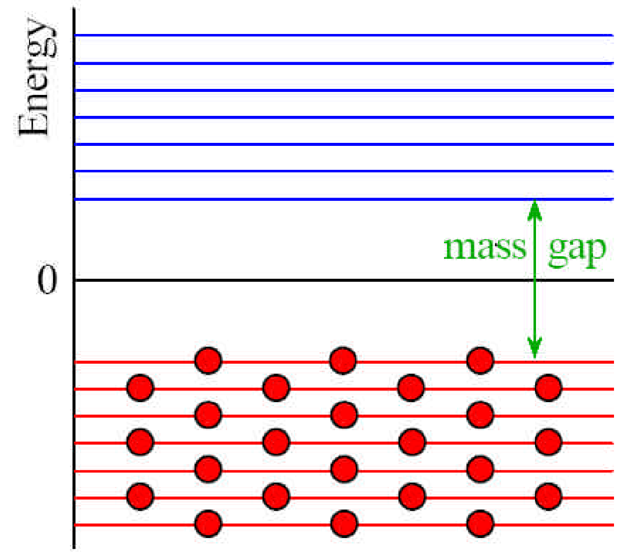


chiral symmetry breaking

chirally inv. manyquark int.

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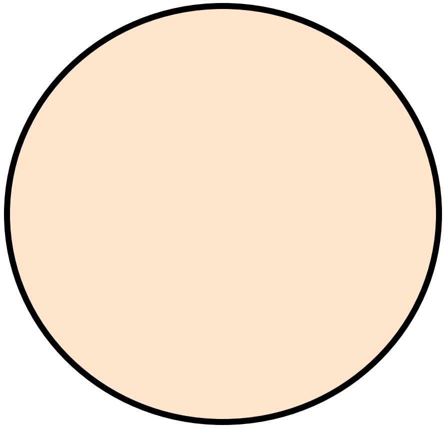

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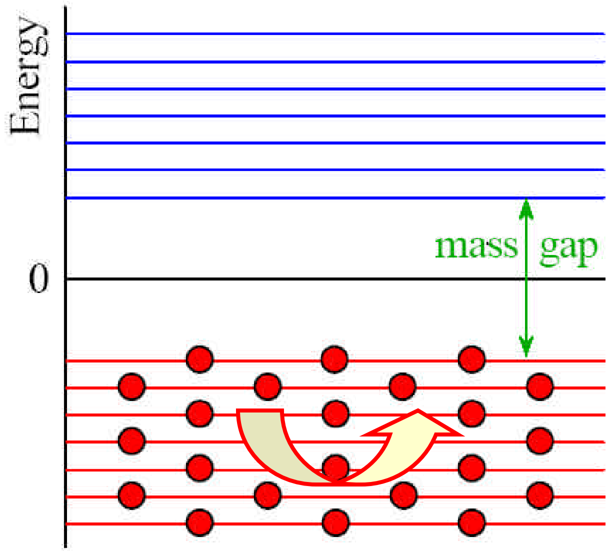


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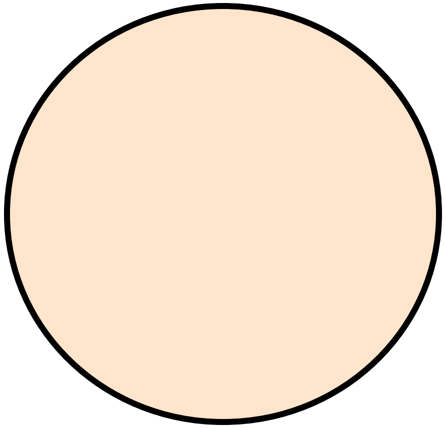

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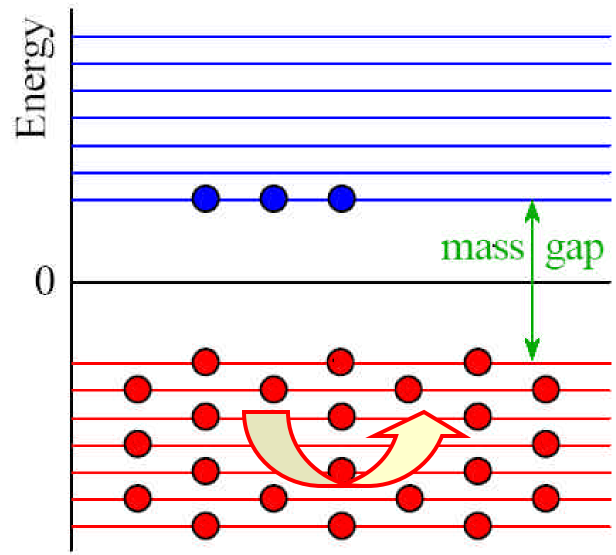


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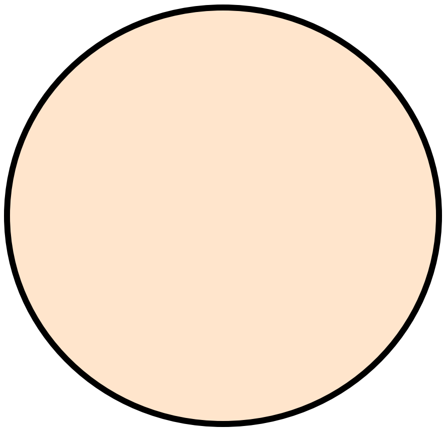

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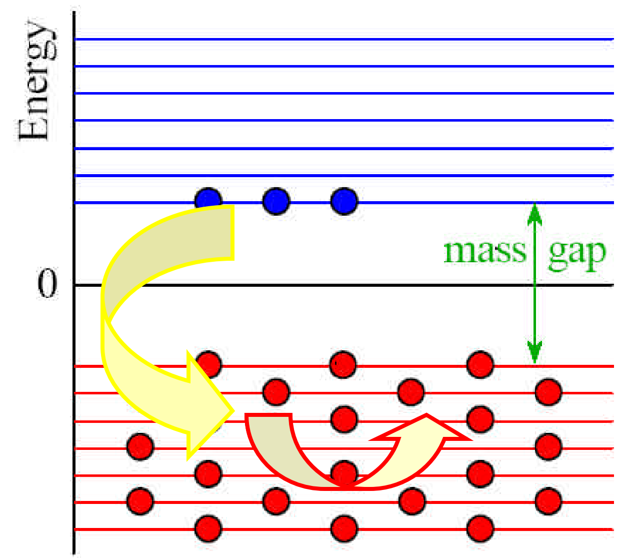


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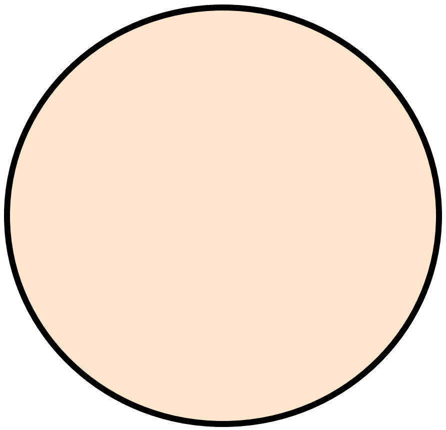

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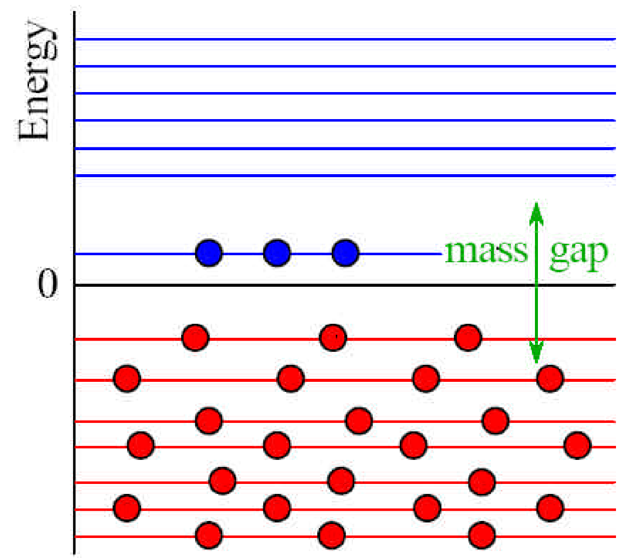


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no quantum numbers except B

rotation generates flavor and spin

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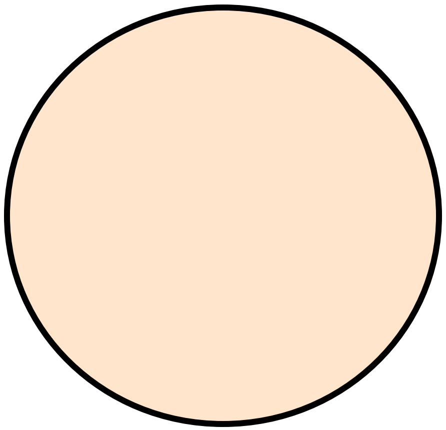

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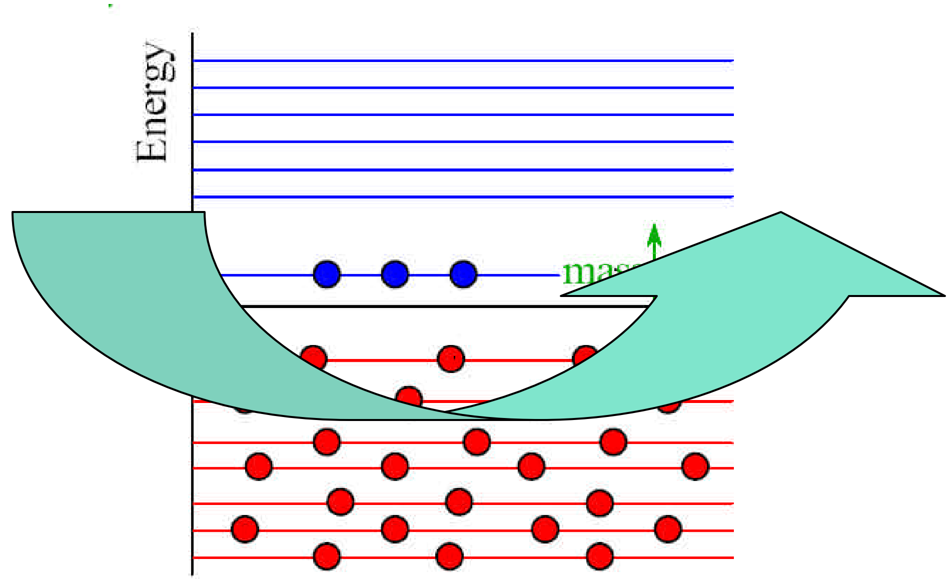


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# Soliton models are quark models

$$\bar{q}\ \left[i\partial\!\!\!/ - M \exp(i\,\gamma_5\,\pi^A \lambda^A / F_\pi)\right]\ q$$

is invariant, because one can absorb chiral rotation into the redefined pseudoscalar meson fields $\pi^A$

Note that $\pi = f(q, \bar{q}) \rightarrow$ quarks do interact

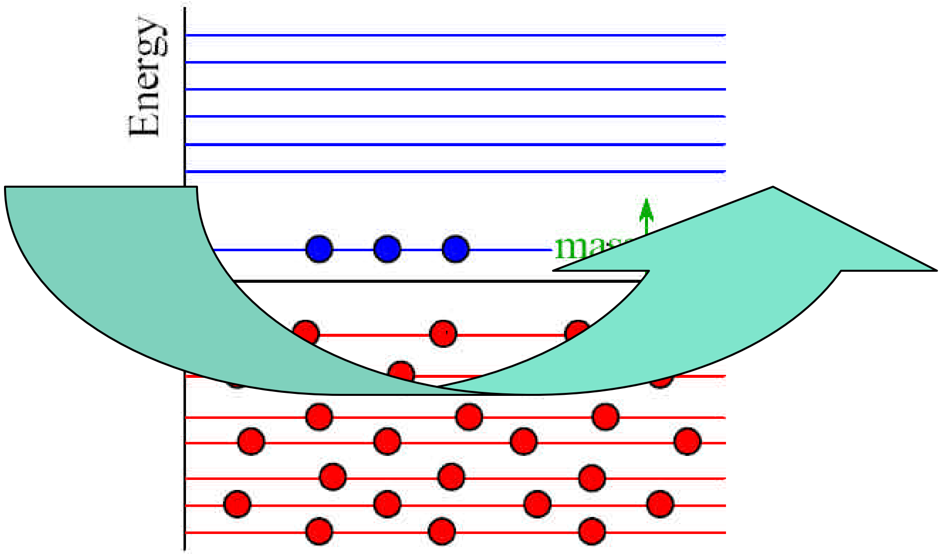
Chiral symmetry is spontaneously broken: $< \pi^A > = 0$

SKYRMION:

Integrating quarks one is left with dynamical GB field

Soliton in this model is stabilized by specific term in Lagrangian

## Soliton Models:

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# Soliton models are quark models but some are not

constituent quark mass ~ 350 MeV

$$\mathcal{Z} = \int D\pi^A \int D\psi^\dagger D\psi \, \exp \int d^4x \, \psi^\dagger(x) \left[ i\partial\!\!\!/ + iMU^{\gamma_5}(x) \right] \psi(x)$$

pions

fermions

$$U = \exp i \frac{\pi^A \lambda^A}{F_\pi}$$

integrate out quarks

# "Skyrme" Model

$$\mathcal{L} = \frac{F_\pi^2}{4} \mathrm{Tr}\left( \partial_\mu U^\dagger \partial^\mu U \right) + \frac{1}{32e^2} \mathrm{Tr}\left( \left[ \partial_\mu U \, U^\dagger, \partial_\nu U \, U^\dagger \right]^2 \right) + \Gamma_{\mathrm{WZ}} + \ldots .$$

only massless pion fields, kinetic term + interaction terms

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only massless pion fields, kinetic term + interaction terms

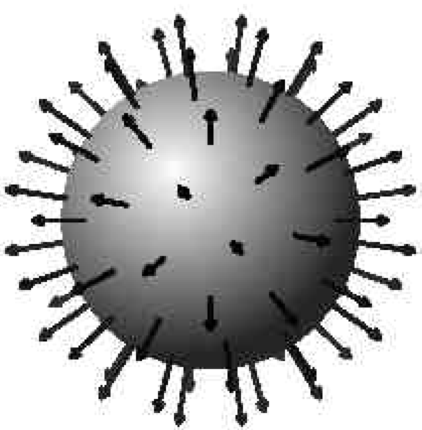
Soliton in the Skyrme model is stabilized by the Sk. term

# Variational approach to the soliton

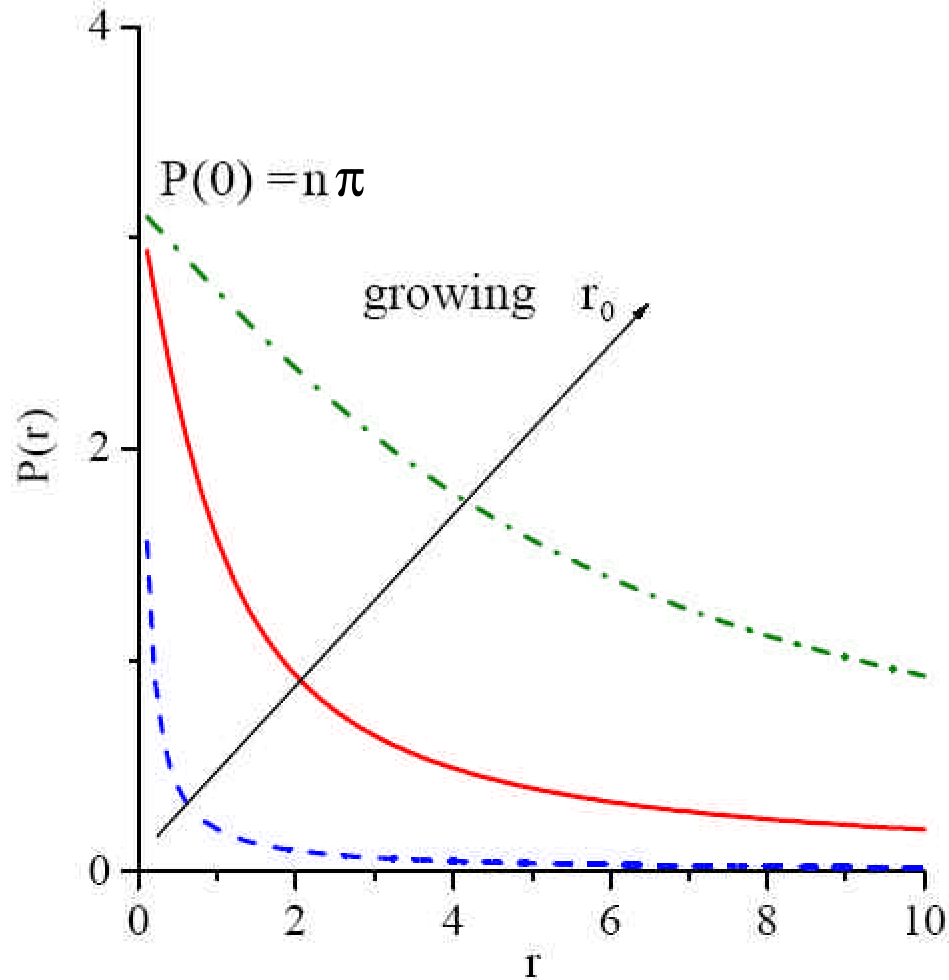
$$U = \exp i \frac{\pi^A \lambda^A}{F_\pi}$$

hedgehog Ansatz: $$\pi^a(\mathbf{x}) = \frac{n^a}{r} P(r)$$

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}\,P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

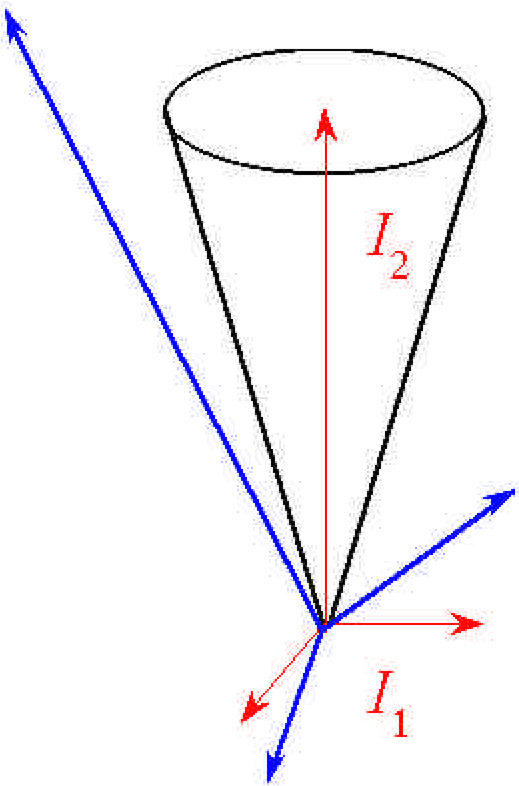


hedgehog



Collective quantiztion proceeds in both cases identically → symmetric top

$$L_0 = -M_{\rm cl} + \frac{I_1}{2}\sum_{a=1}^{3}\Omega_a^2 + \frac{I_2}{2}\sum_{a=4}^{7}\Omega_a^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$



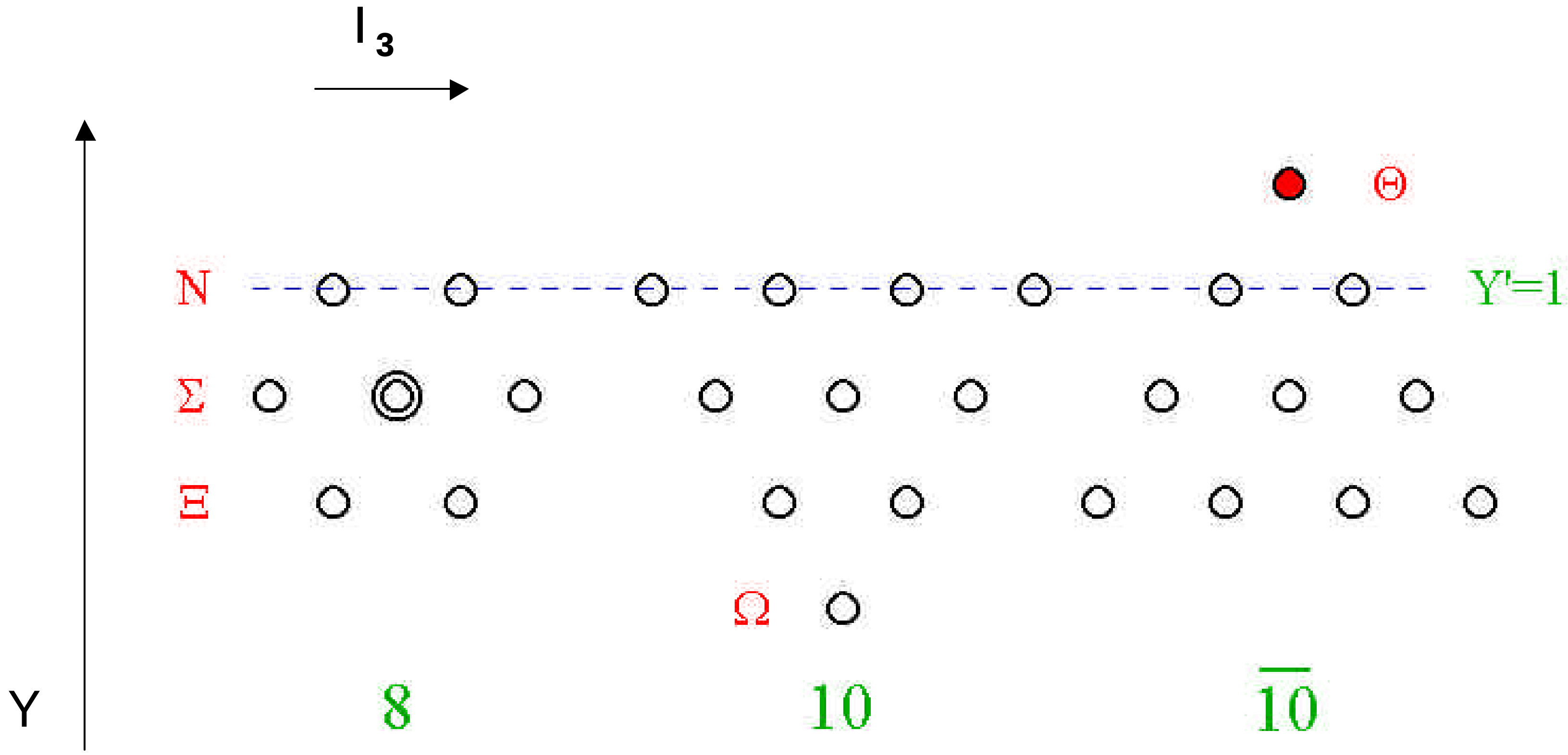

only the coefficients are given by different expressions

There is no kinetic term for 8-th angular velocity
® conjugated momentum is constant and produces constraint:

$$\pi_8 = N_c/2\sqrt{3}$$

# Wave functions and allowed states

$$|R, B, S\rangle = \sqrt{R}\, D^{(\mathcal{R})\,*}_{\underbrace{Y I I_3}_{B}\ \underbrace{Y' J J_3}_{S}}(A) \qquad\qquad J_3 = -S_3$$



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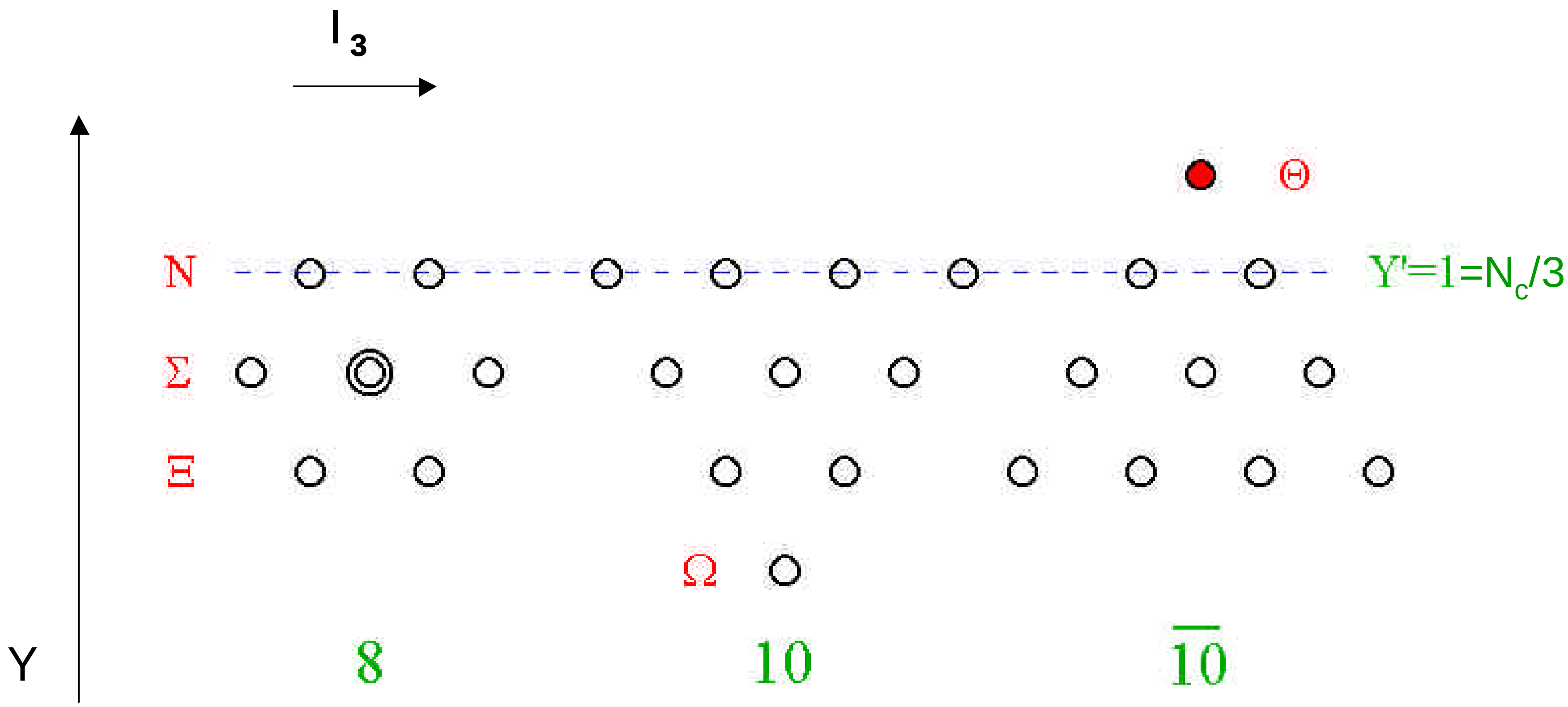

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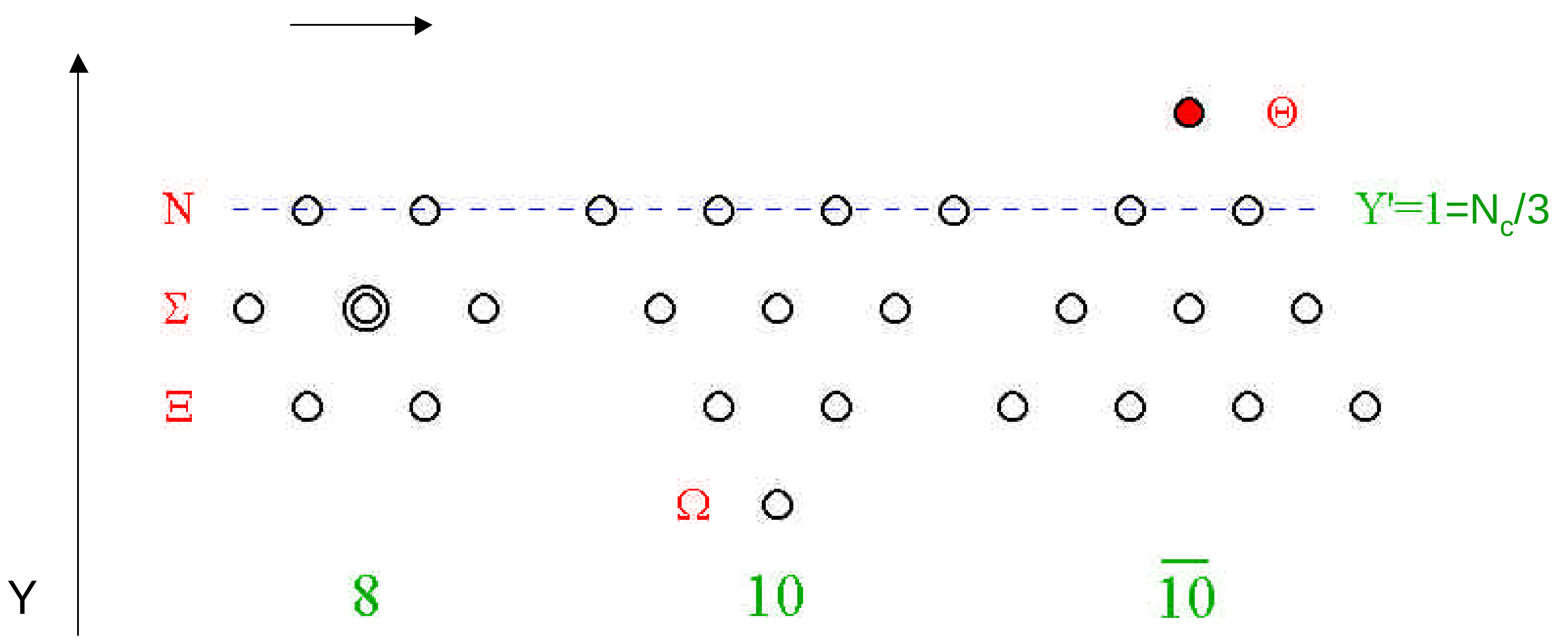
how far we can go? ®



O(1) corrections
to $M_{cl}$ do not allow
for absolute mass predictions

# Mass formula

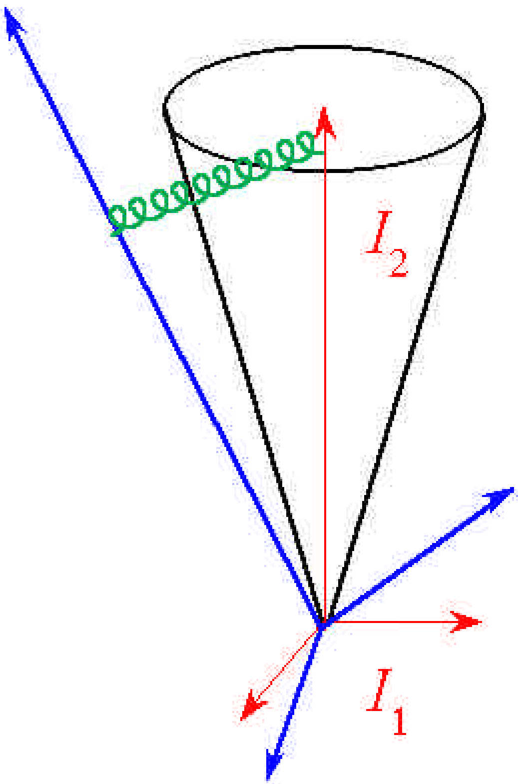
$$H_0 = M_{\rm cl} + \frac{1}{2I_1} S(S+1) + \frac{1}{2I_2}\left( C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12}\right)$$

octet-decuplet
splitting

known

?

exotic-nonexotic splittings




$$\sigma = \frac{1}{3}\frac{\Sigma_{\pi N}}{\overline{m}}$$

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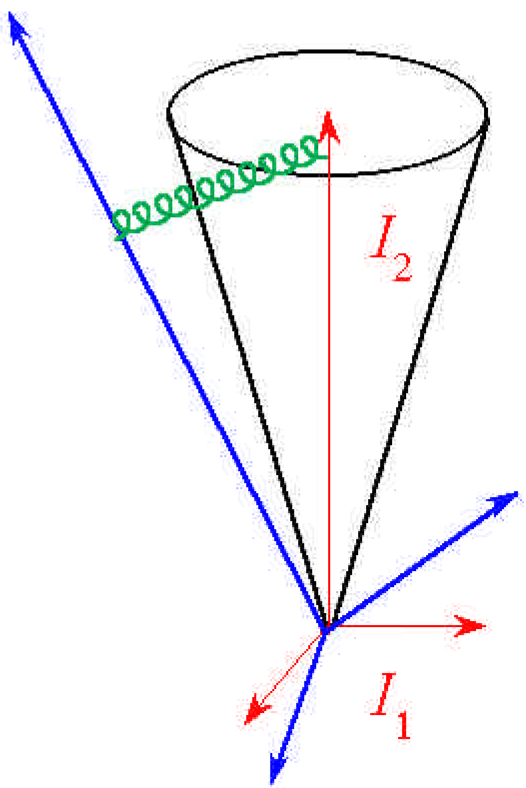


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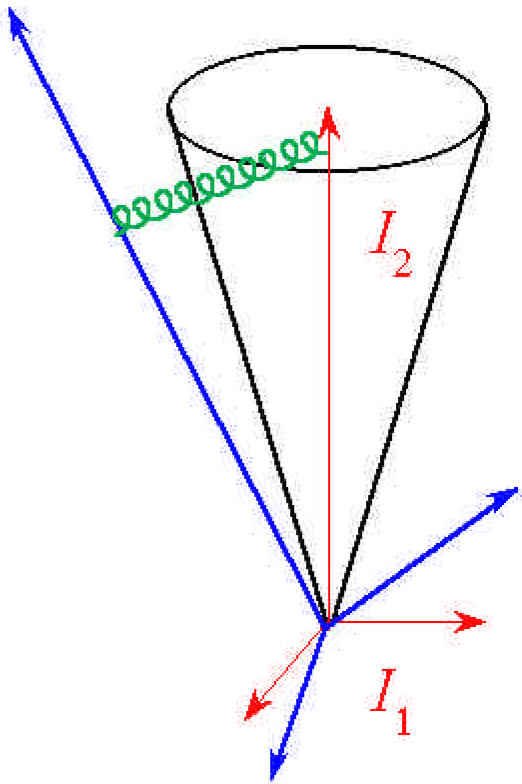


first order perturbation
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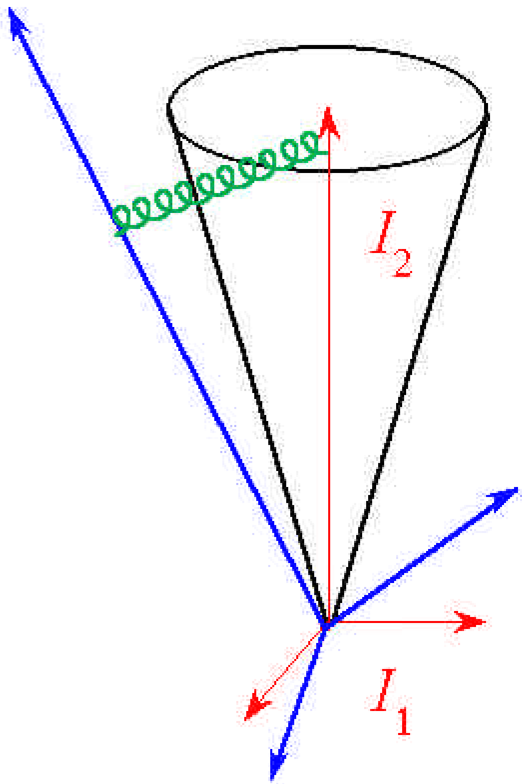


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$$H' = m_{\rm cl} - \alpha D^{(8)}_{88}(A) = m_s \underbrace{\sigma}_{\sigma = \frac{1}{3}\frac{\Sigma_{\pi N}}{\overline{m}}} - m_s \sigma D^{(8)}_{88}(A)$$

# Early predictions:

soliton models: positive parity

Biedenharn, Dothan (1984):

$D_{\overline{10}-8}$ ~ 600 MeV from Skyrme model

MP (1987):

$M_Q$= 1535 MeV from Skyrme model

in model independent approach, second order

Diakonov, Petrov, Polyakov (1997):

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Monopolar Harmonics in $SU_f(3)$ as Eigenstates of the Skyrme-Witten Model for Baryons*

L. C. Biedenharn

and

Yossef Dothan**

Physics Department, Duke University
Durham, NC 27706 USA



*To Professor Yuval Ne'eman on the occasion of his Sixtieth Birthday*

Thus the first state violating the three quark rule is a $(\overline{10},\frac{1}{2})$, which--using numerical values[4] in the Hamiltonian--yields an excitation energy $\approx$ 600 Mev above the $(8,\frac{1}{2})$. Since the theory is a low energy effective theory we believe that this gives an aposteriori excitation energy limit on the validity. Otherwise stated this means that when baryons are probed with momentum transfers of the order of 600 MeV one starts to feel their compositness.

## Footnotes and References


1) E. Witten, Nucl. Phys. B223(1982) 422.

2) T.H.R. Skyrme, Proc. Roy. Soc. A260 (1961) 127.

3) E. Guadagnini, Nucl. Phys., B236, (1984), 35.
L.C. Biedenharn, Y. Dothan and A. Stern, Phys. Lett. 146D (1983) 289.

4) L.C. Biedenharn, J.D. Louck, Encl. for Math. and Appl., Vol. 9: "The Racah-Wigner Algebra in Quantum Theory", Additon-Wesley (Reading, MA) 1981.

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4) L.C. Biedenharn, J.D. Louck, Encl. for Math. and Appl., Vol. 9: "The Racah-Wigner Algebra in Quantum Theory", Additon-Wesley (Reading, MA) 1981.

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Footnotes and References

146 B (1984) 289

1) E. Witten, Nucl. Phys. B223(1982) 422.

2) T.H.R. Skyrme, Proc. Roy. Soc. A260 (1961) 127.

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In the printed version:

*From SU(3) to Gravity*

Festschrift in honor of Yuval Ne'eman

Eds. E. Gotsman, G. Tauber



ordering for $B = 1$ is: $(8, 1/2); (10, 3/2); (\overline{10}, 1/2); \ldots$ Thus the first state violating the three quark rule is a $(\overline{10}, 1/2)$, which — using numerical values[3] in the Hamiltonian — yields an excitation energy $\approx 600$ Mev above the $(8, 1/2)$. Since the theory is a


1. Witten, E. *Nucl. Phys.* **B223,** 422 (1982).
2. Skyrme, T.H.R. *Proc. Roy. Soc.* **A260, 127** (1961).
3. Guadagnini, G. *Nucl. Phys.* **B236,** 35 (1984).
   Biedenharn, L.C., Dothan, Y. and Stern, A. *Phys. Lett.* **146D,** 289 (1984).
4. Biedenharn, L.C., Louck, J.D. *Encl. for Math. and Appl.*, vol.9 "The Racah-Wigner Algebra in Quantum Theory," (Reading, MA, Addison-Wesley, 1981, (see Topic 2: Monopolar Harmonics, p. 201 ff).

# BARYONS AS QUARKS IN A SKYRMION BUBBLE

L.C. BIEDENHARN [1], Y. DOTHAN [2] and A. STERN
*Center for Particle Theory, Physics Department, University of Texas at Austin, Austin, TX 78712, USA*

Received 4 June 1984
Revised manuscript received 24 July 1984

$$E_{\mathrm{qu}}^{\mathrm{SU(3)}} = E_0 + (2F_\pi^2 R^3)^{-1} [(p^2 + 3p + q^2 - pq - \tfrac{9}{4}B^2)/3C_{\mathrm{SU(3)}} + J(J+1)(C_{\mathrm{rot}}^{-1} - C_{\mathrm{SU(3)}}^{-1})] , \tag{24}$$

with the wave section having the form of an $(\mathrm{SU(3)})_\mathrm{f} \times (\mathrm{SU(2)})_{\mathrm{spin}}$ monopolar harmonic [21]:

$$\phi(A) = D^{[pqo]^*}{}_{I,I_3,Y;J,J_3,B}(\phi_1, \ldots, \phi_7, \phi_8 = \pm\phi_4) . \tag{25}$$

The quantum numbers are: $(\mathrm{SU(3)})_\mathrm{f}$ irrep labels $[pqo]$; isospin $I, I_3$; hypercharge $Y$; spin $J, J_3$; baryon number $B = B_\mathrm{U}$.

The additional moment of inertia is

$$C_{\mathrm{SU(3)}} = \tfrac{1}{2}\pi \int_0^\infty \mathrm{e}^{3s} [1 - \cos\theta(s)]\, \mathrm{d}s \simeq 12.93 . \tag{26}$$

$$E_{\mathrm{qu}}^{\mathrm{SU}(3)} = E_0$$

$$+ (2F_\pi^2 R^3)^{-1} [(p^2 + 3p + q^2 - pq - \tfrac{9}{4}B^2)/3C_{\mathrm{SU}(3)}$$

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$$\Delta_{\overline{10}-8} = 330 \quad \mathrm{MeV}$$

+ 3q

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$$\Delta_{\overline{10}-8} = 590 \quad \mathrm{MeV}$$

# Early predictions:

soliton models: positive parity

Biedenharn, Dothan (1984):

$\Delta_{\overline{10}-8} \sim$ 600 MeV from Skyrme model

MP (1987):

$M_\Theta$= 1535 MeV from Skyrme model
in model independent approach, second order

Diakonov, Petrov, Polyakov (1997):

$\chi$QM - model independent approach,
$1/N_c$ corrections
$M_\Theta$= 1530 MeV, $\Gamma_\Theta$< 15 MeV

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Small mass is natural in chiral models,
small width is much less trivial

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Small mass is natural in chiral models,
small width is much less trivial

# cQM breaking hamiltonian

calculate next-to-leading contributions to H'

$$H'_{\chi\mathrm{QM}} = -\alpha' D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i} S_i$$

$O(N_c)+O(1)$ $O(1)$ $O(1)$ all $O(m_s)$

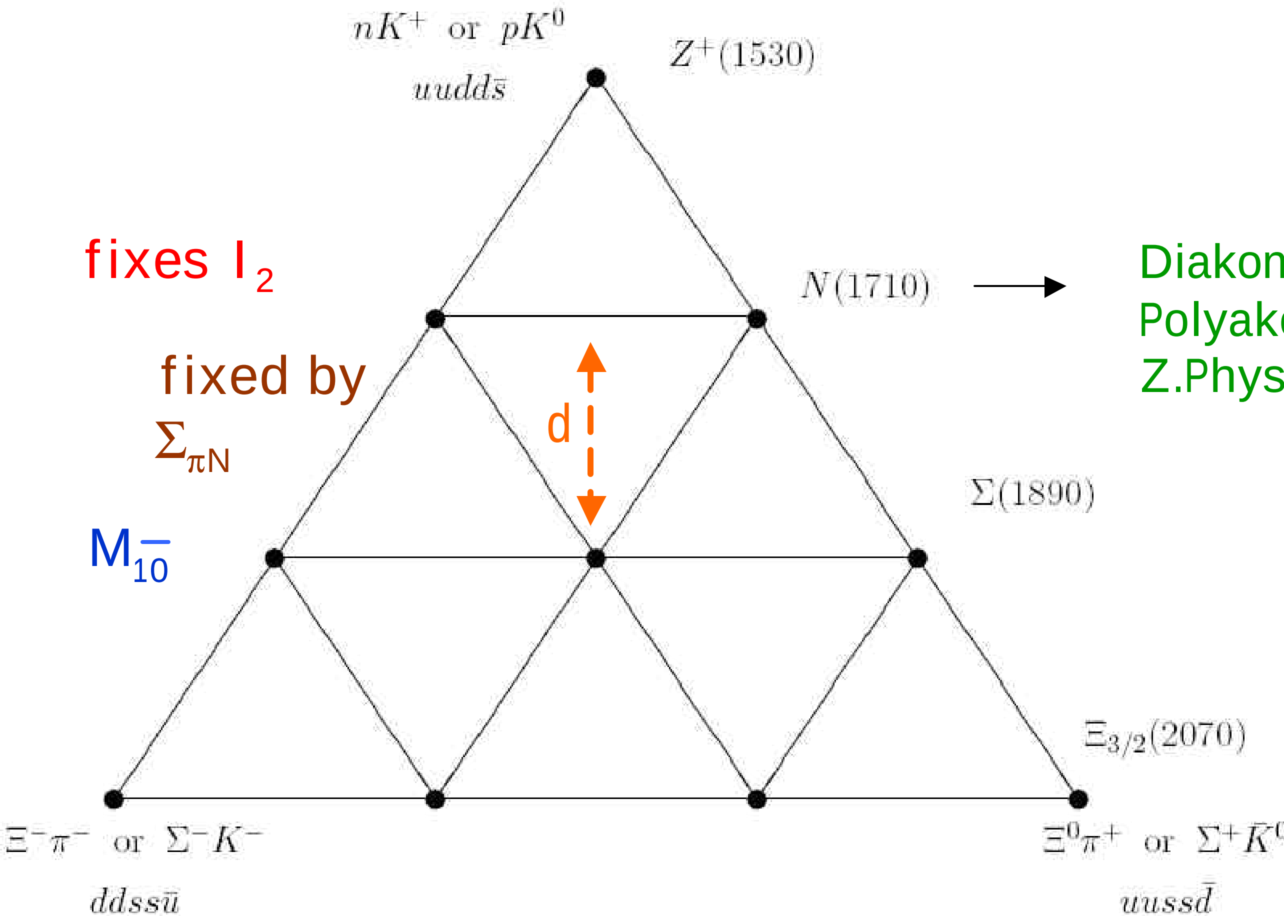
Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

* no handle on $I_2$
* only 2 linear combinations of parameters
  a', b and g enter nonexotic splittings
* splittings in 10 $\neq$ $\overline{10}$ because third combination enters

$$\boxed{\begin{array}{c} \alpha' + \frac{3}{2}\gamma \\ \beta - \frac{1}{2}\gamma \end{array}}$$

# Antidecuplet in cQM

richer H': splittings in 10 $\neq$ $\overline{10}$ , still no handle on $I_2$




Diakonov, Petrov
Polyakov
Z.Phys A359 (97)

$$M_{\overline{10}} = 1890,\ \delta_{\overline{10}} = 180,\ M_{Z^+} = 1530\ \mathrm{MeV}$$

# Antidecuplet in cQM

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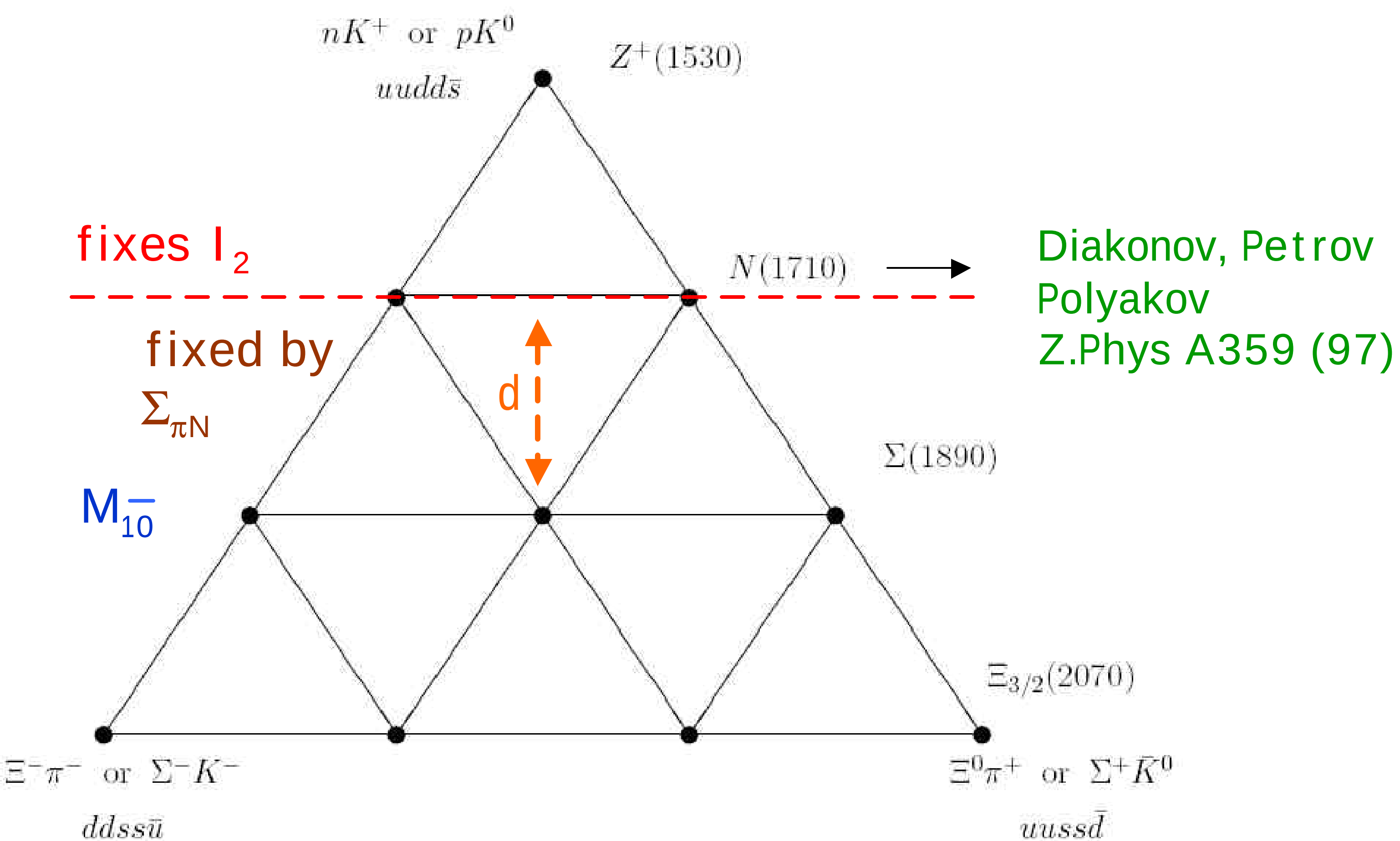


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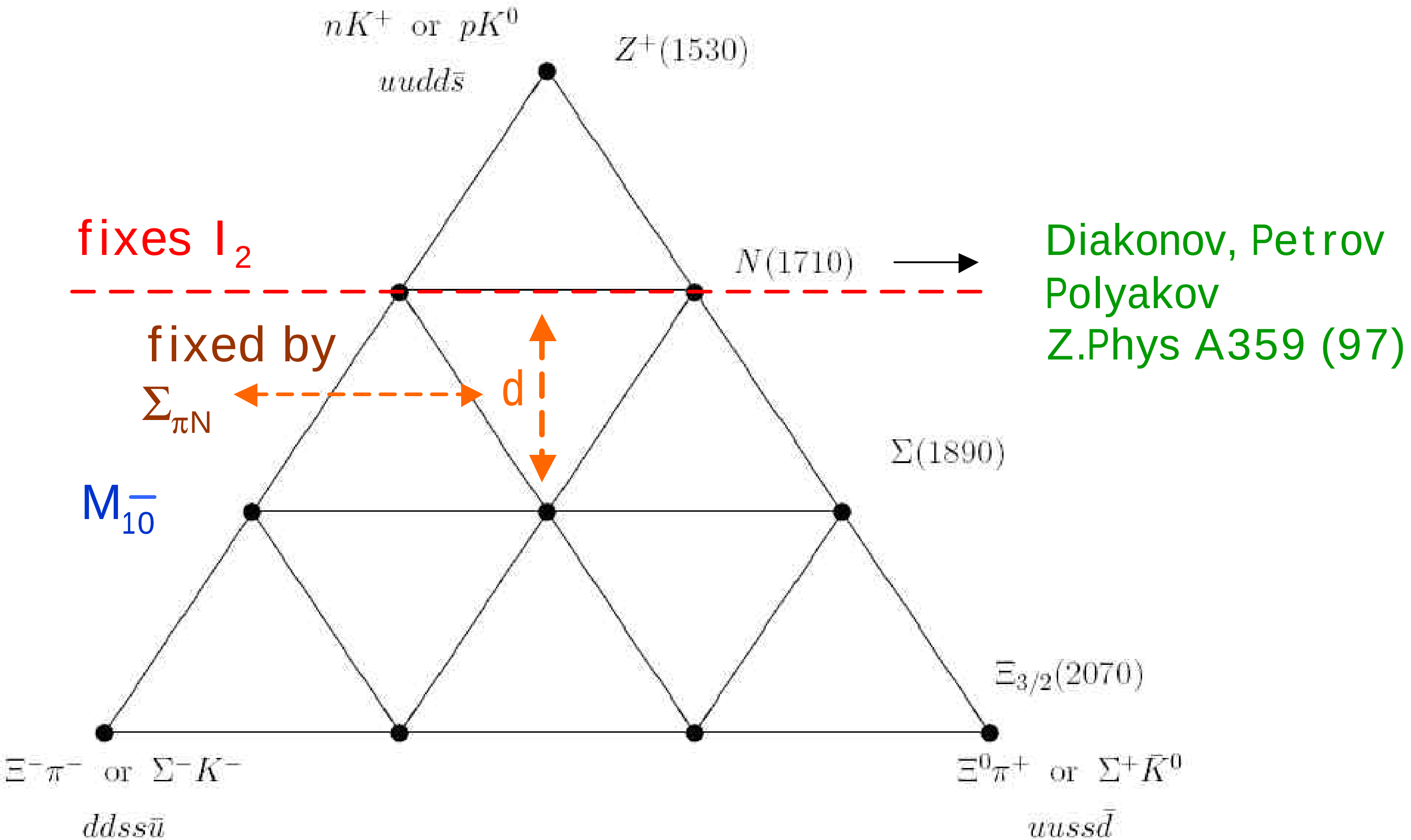


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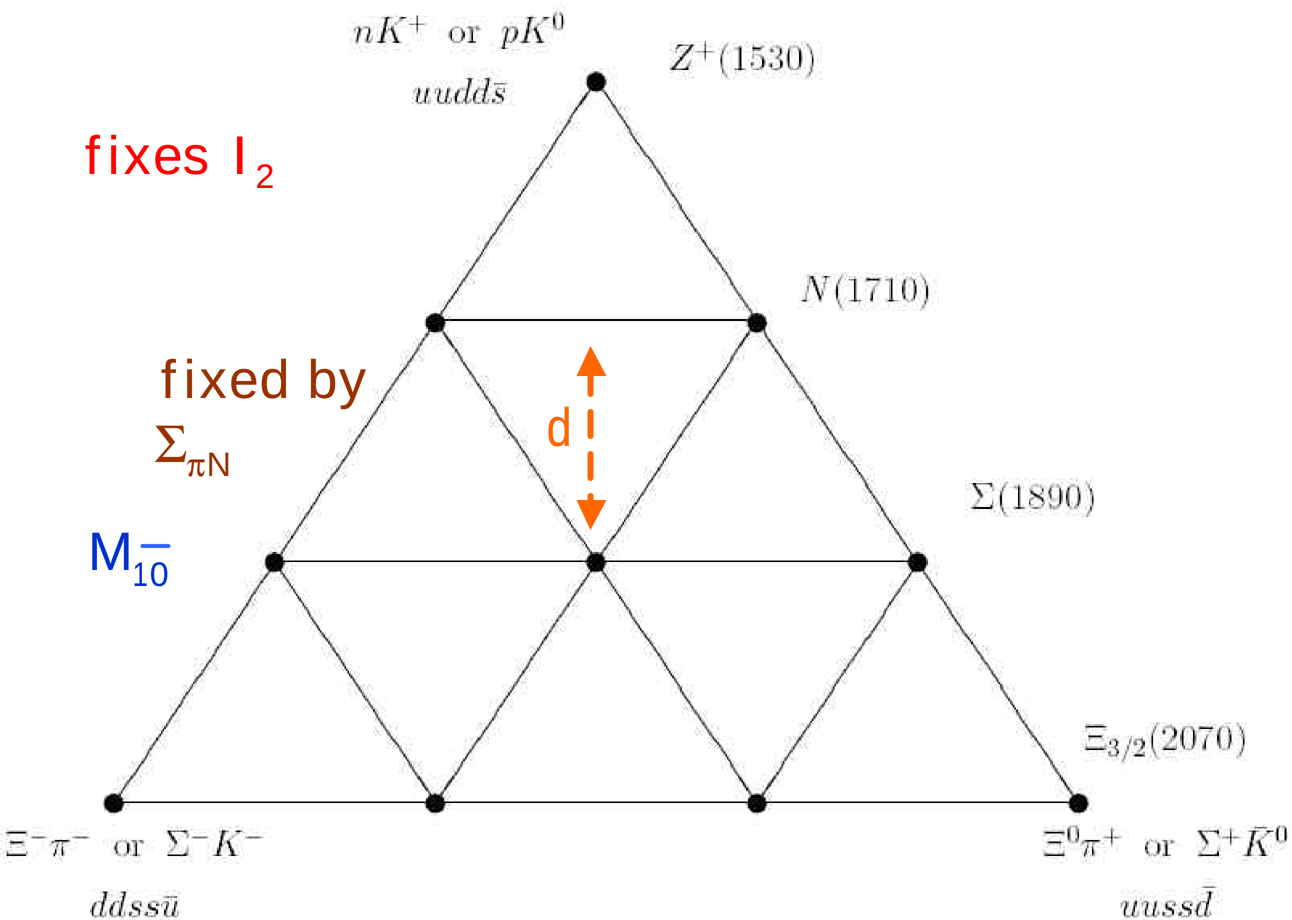
**TODAY:**



# Antidecuplet in cQM

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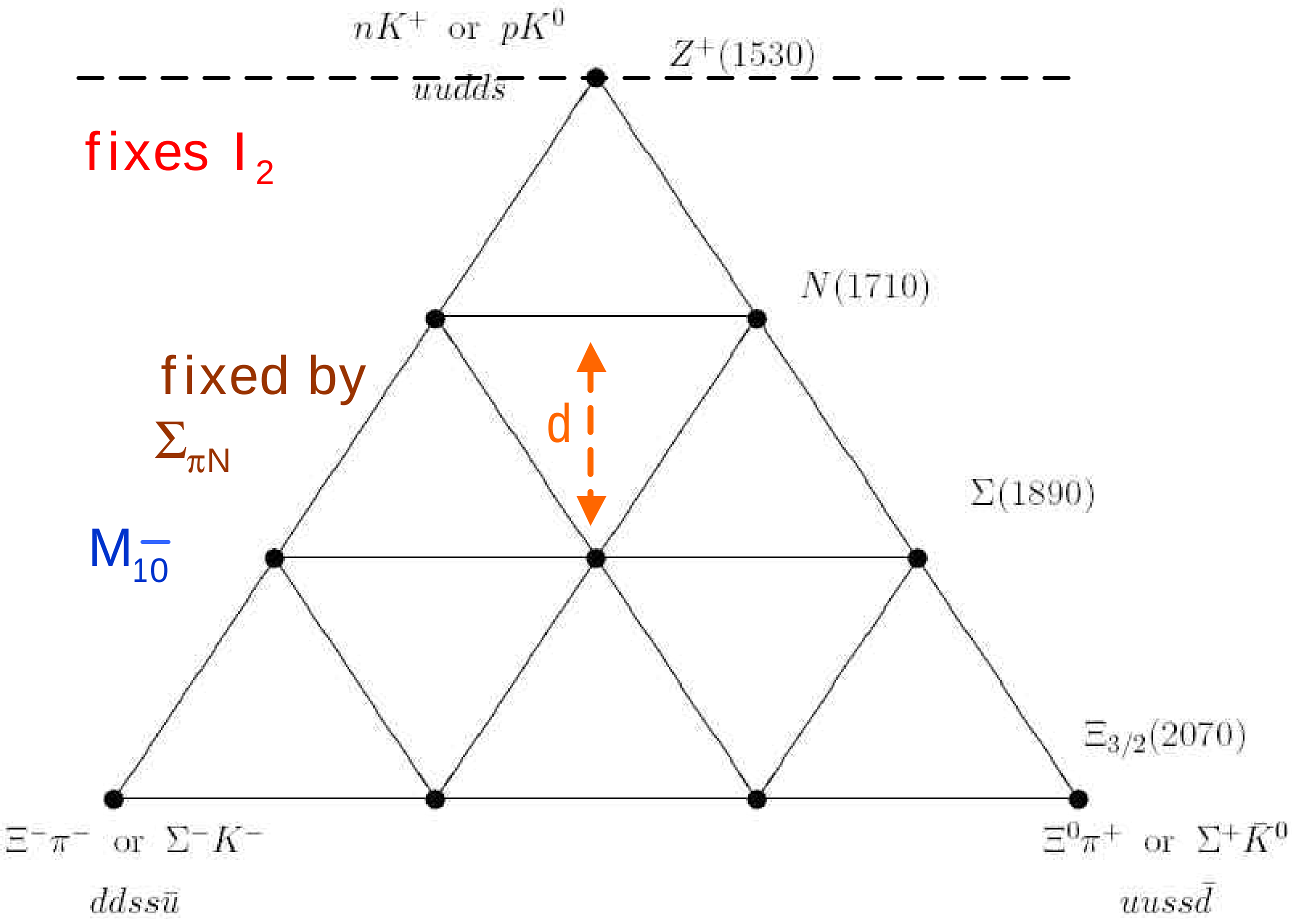

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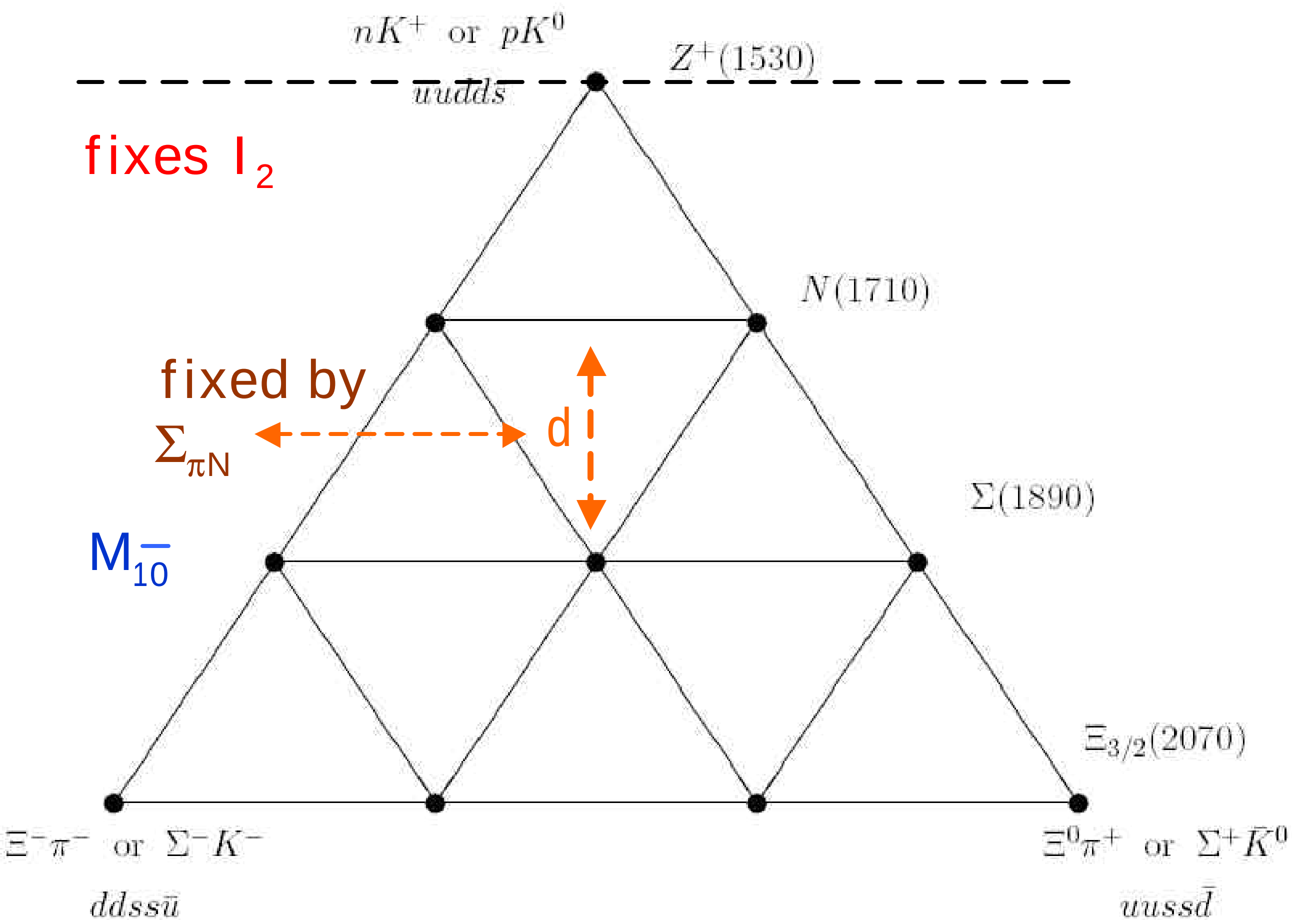
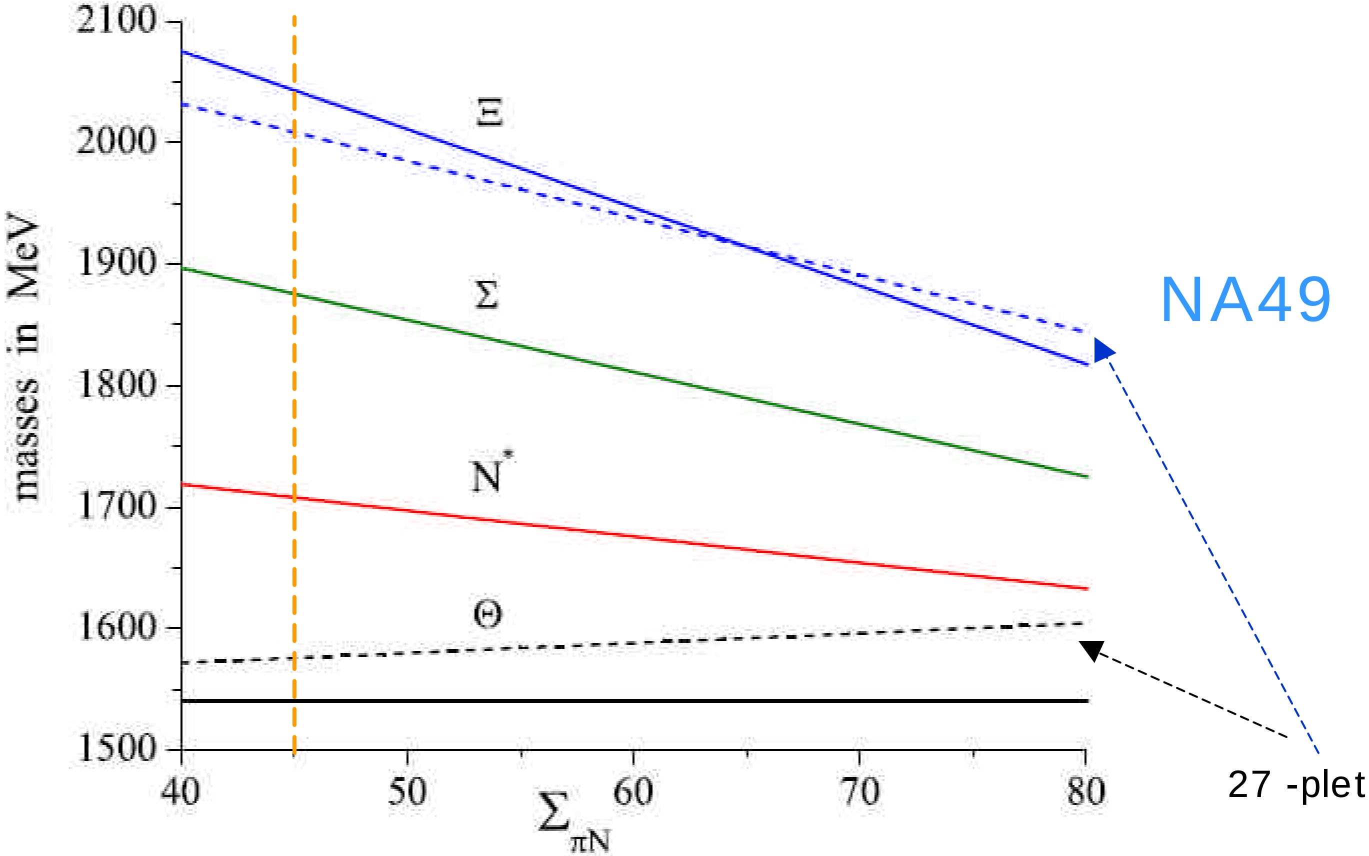


# Freedom in cQM




M.M.Pavan, I.I.Strakovsky, R.L.Workman, R.A.Arndt, PiN Newslett. **16** (2002) 110
T.Inoue, V.E. Lyubovitskij, T.Gutsche, A.Faessler, arXiv:hep-ph/0311275

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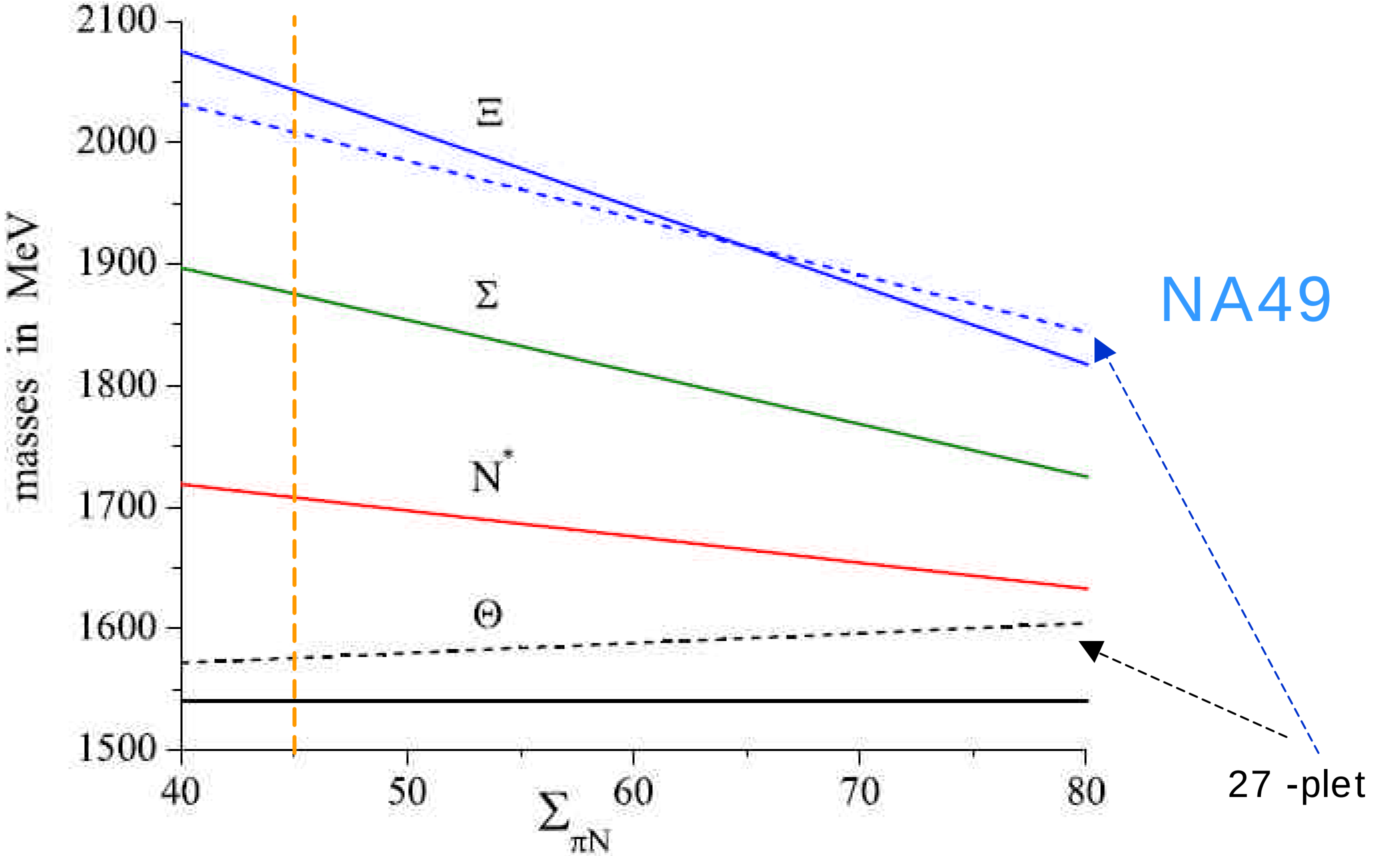


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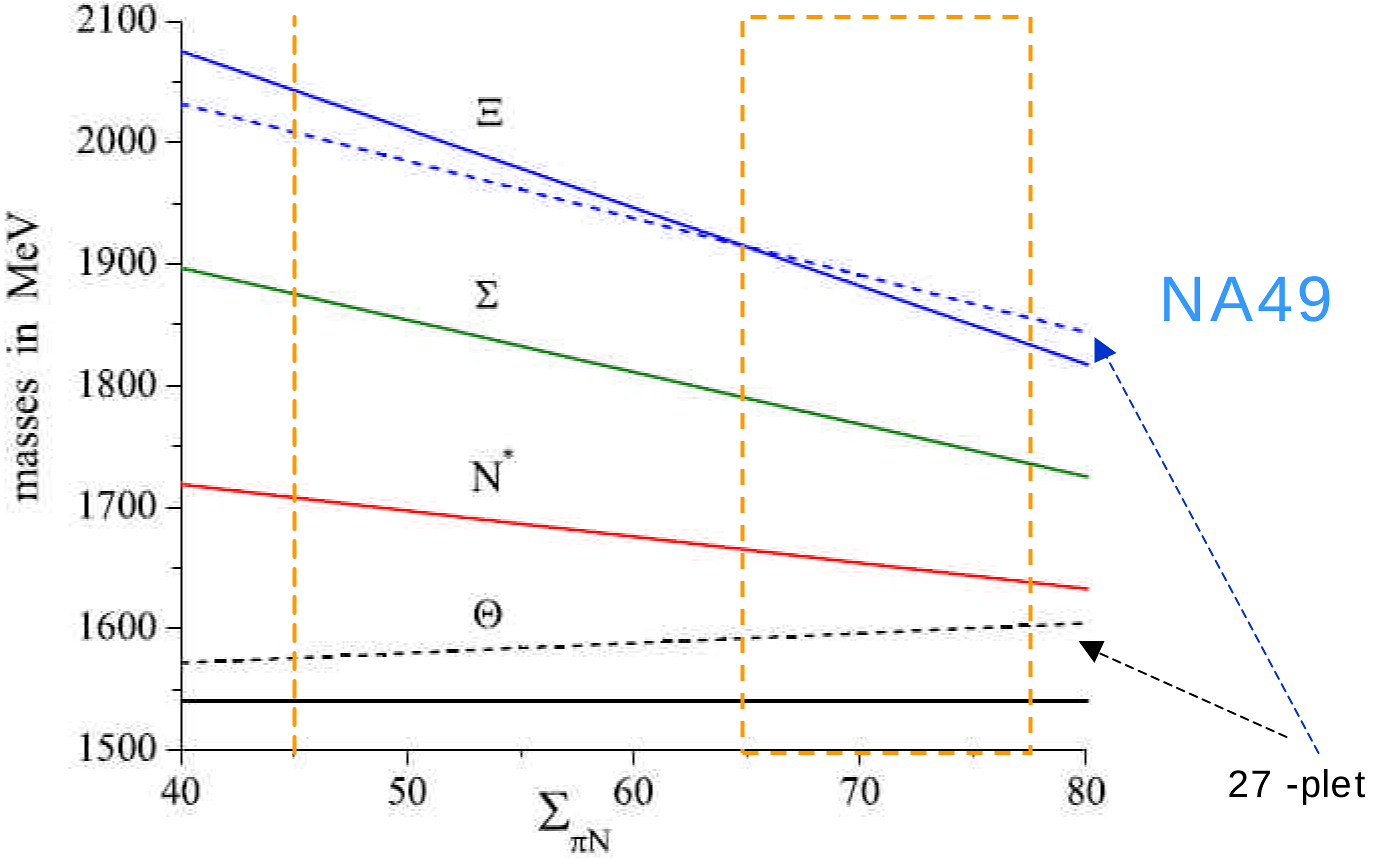


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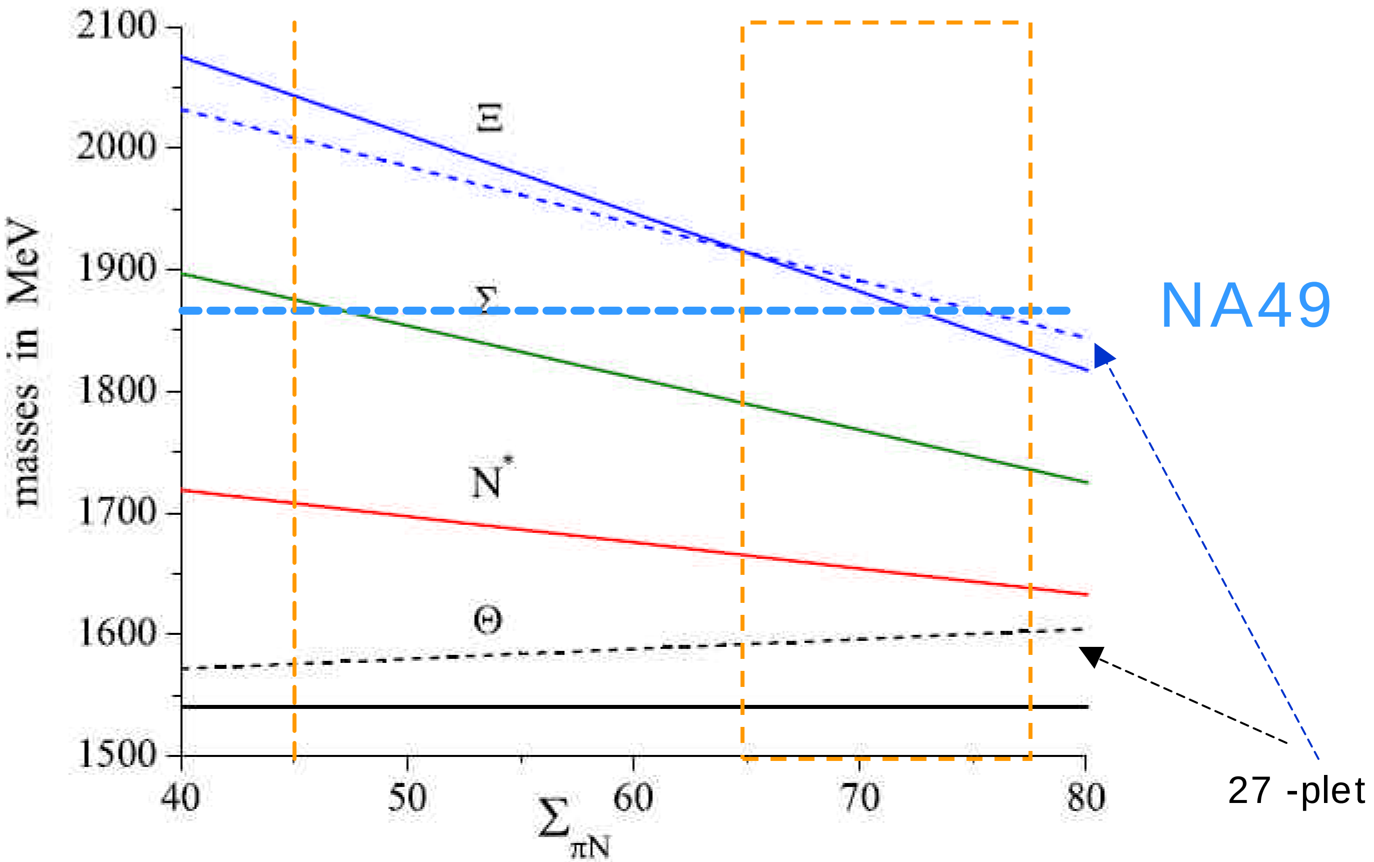


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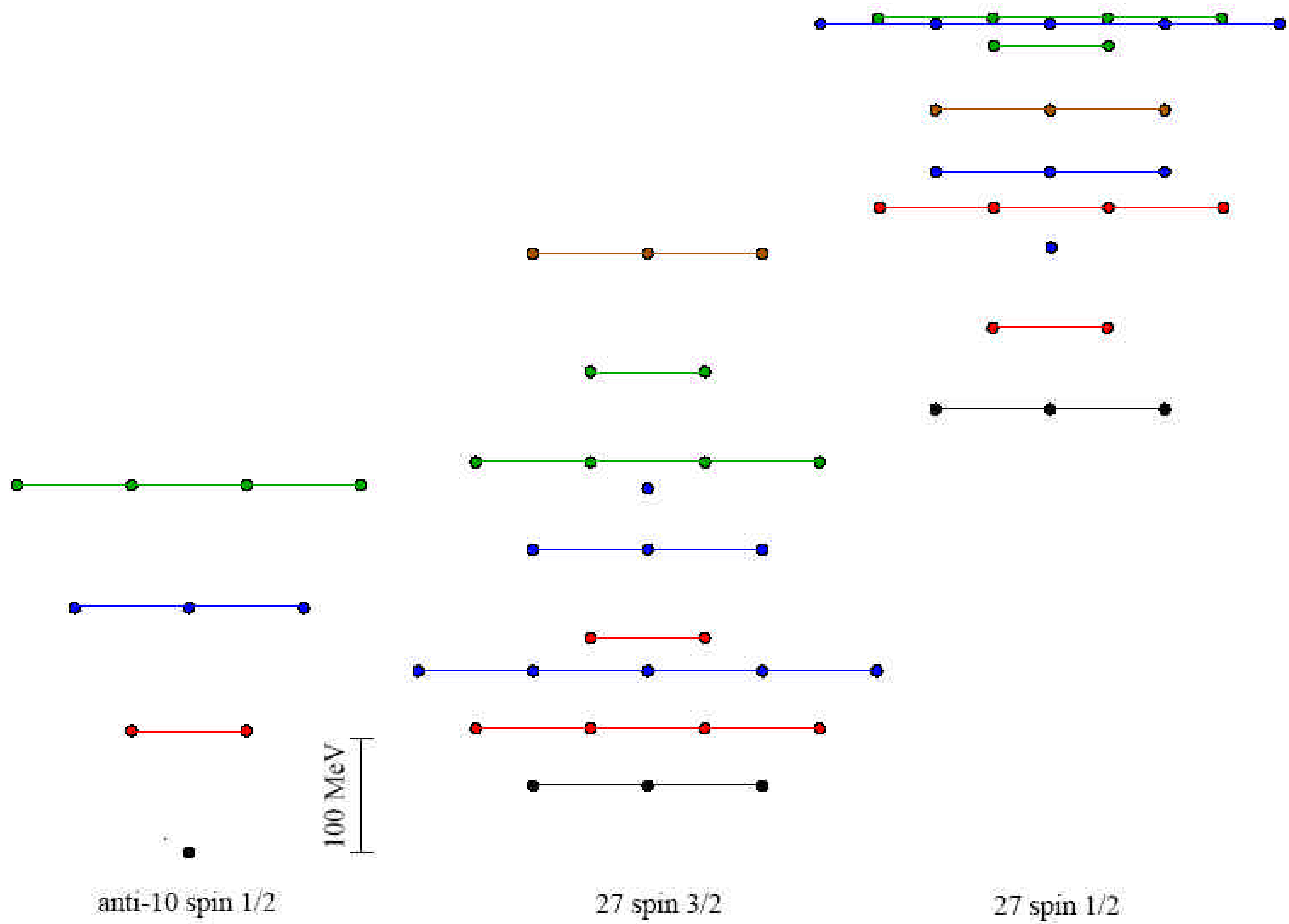
100 MeV
anti-10 spin 1/2
27 spin 3/2
27 spin 1/2

# Width

$$L_{int} = g\, \partial^i \varphi^\alpha\, \hat{V}_{\alpha i}$$

G.S. Adkins, C.R. Nappi, E. Witten, Nucl. Phys. B228 (1983) 552

operator V has the same structure as axial current

$$\hat{V}_{\alpha i} = \left( A_0 - \frac{A_1}{I_1} \right) D_{\alpha i} - 2\frac{B}{I_2} d_{ibc} D_{\alpha b} \hat{J}_c - \frac{2}{\sqrt{3}} \frac{C}{I_1} D_{\alpha 8} \hat{J}_i$$

Diakonov, Petrov, Polyakov, Z.Phys **A359** (97) 305

Weigel, Eur.Phys.J.**A2** (98) 391, hep-ph/0006619

$$\hat{O}_\alpha = -i \frac{3}{2M_B} \times \left[ G_0 D_{\alpha i} - G_1 d_{ibc} D_{\alpha b} \hat{J}_c - G_2 D_{\alpha 8} \hat{J}_i \right] \times p_i$$

In practice we calculate only matrix elements of
❧ and plug them into the QM formula dor the width

# Width in the soliton model

SU(3) relations

D $$\Gamma^{(10)}_{B_1\to B_2\varphi} = \frac{G^2_{10}}{8\pi M_1 M_2} \times C^{(10)}(B_1, B_2, \varphi) \times p^3_\varphi$$

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Decuplet decay: $$G_{10} = G_0 + \frac{1}{2}G_1$$

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In NRQM limit:

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In NRQM limit: $G_{\overline{10}} = 0$

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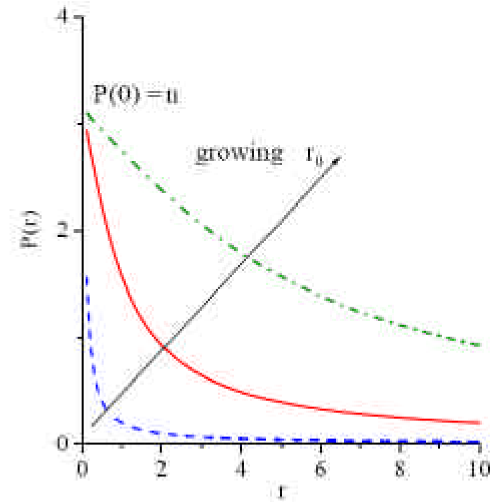
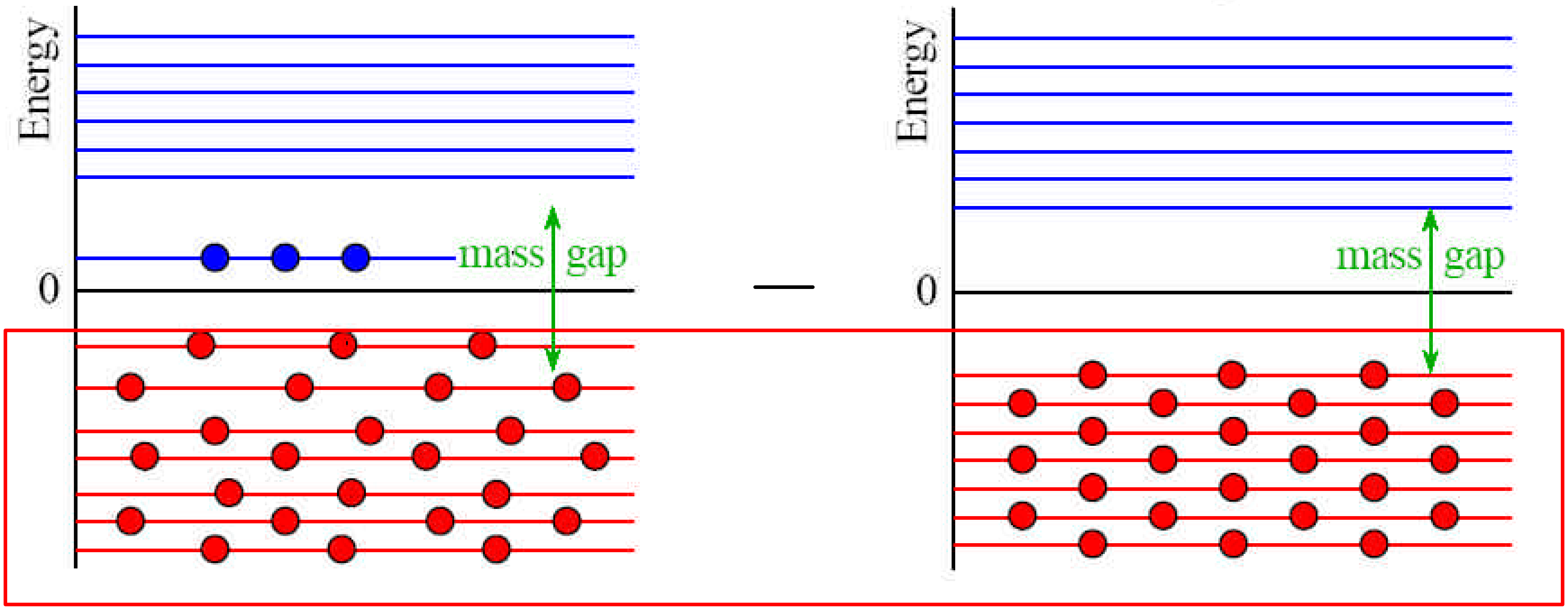
# NRQM Limit

Diakonov, Petrov, Polyakov, Z.Phys **A359** (97) 305

MP, A.Blotz K.Goeke, Phys.Lett.**B354**:415-422,1995

energy is calculated
with respect to the vacuum:



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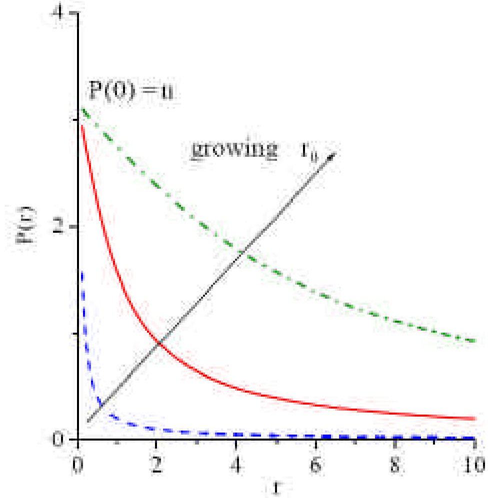


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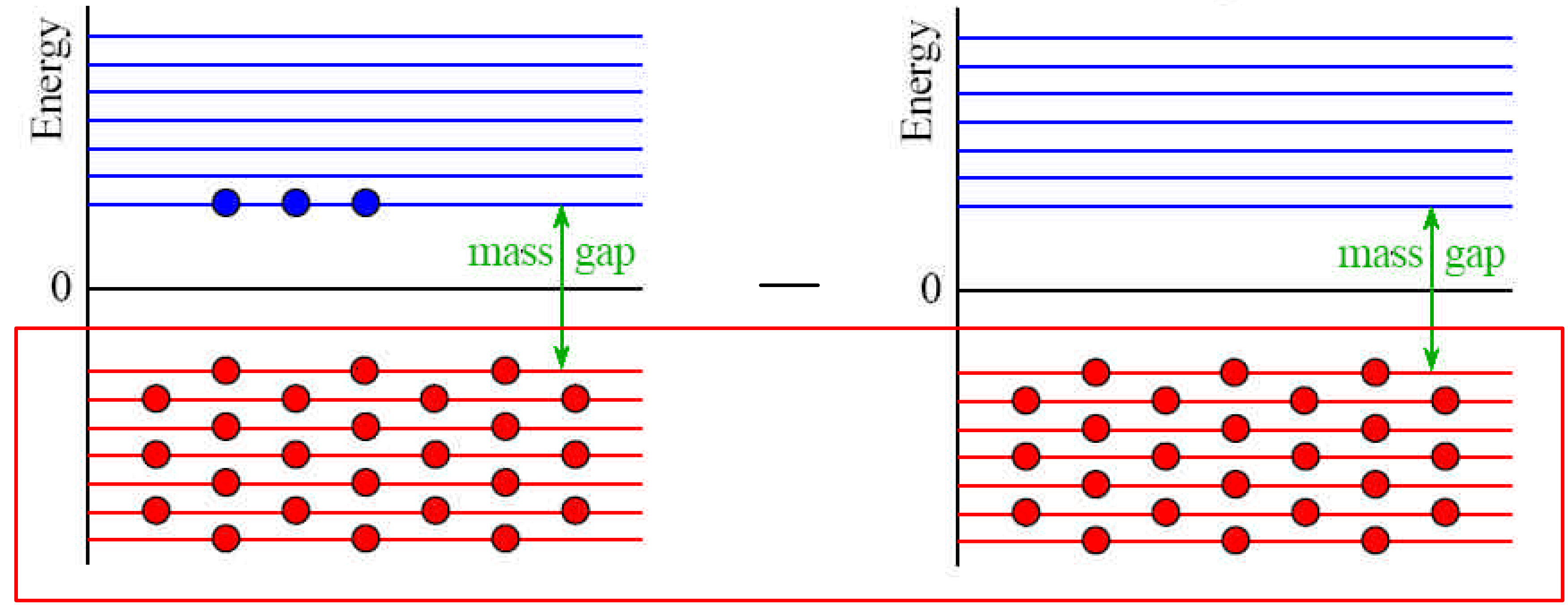
with respect to the vacuum:



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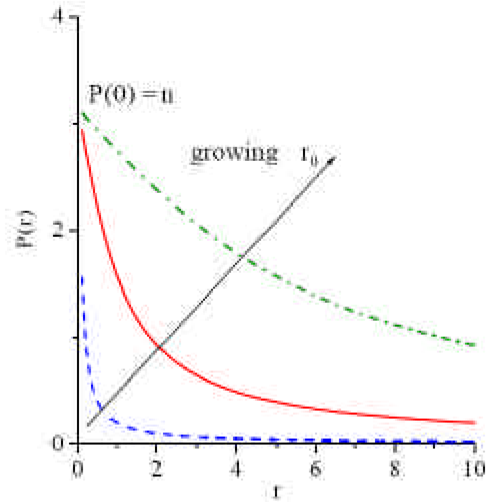


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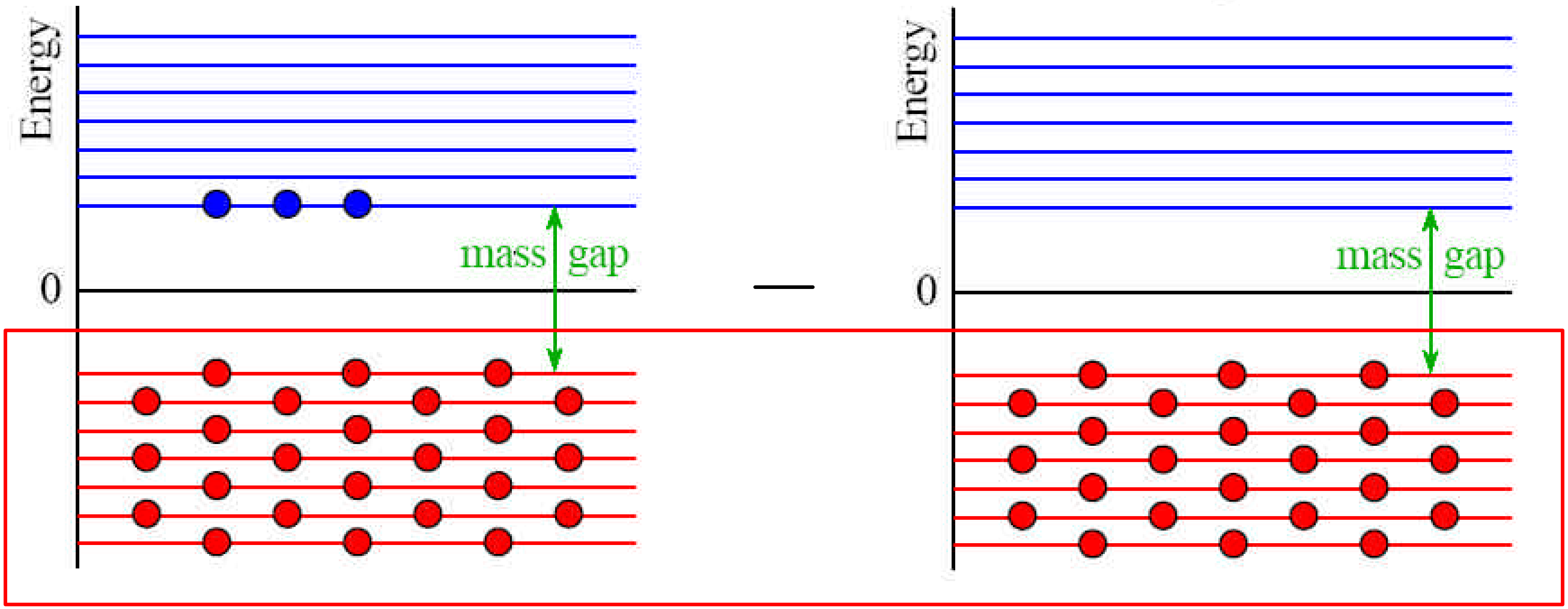


in the NRQM limit only valence level contributes

# Width

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

$$\hat{O}_\alpha = -i\frac{3}{2M_B} \times \left[G_0 D_{\alpha i} - G_1 d_{ibc} D_{\alpha b} \hat{J}_c - G_2 D_{\alpha 8} \hat{J}_i\right] \times p_i$$

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Antidecuplet decay: $G_{\overline{10}} = G_0 - G_1 - \frac{1}{2} G_2$

In small soliton limit: $G_{\overline{10}} \cancel{\neq} 0$

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< 15 MeV

# Width

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

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In small soliton limit:
In reality: $G_{\overline{10}} \neq 0$

$$\Gamma^{(10)}_{B_1\to B_2\varphi} = \frac{G_{10}^2}{8\pi M_1 M_2} \times C^{(10)}(B_1, B_2, \varphi) \times p_\varphi^3$$

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Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

$$\hat{O}_\alpha = -i\frac{3}{2M_B}\times\left[G_0 D_{\alpha i} - G_1 d_{ibc} D_{\alpha b}\hat{J}_c - G_2 D_{\alpha 8}\hat{J}_i\right]\times p_i$$

$\sqrt{N_c}\ \times$ [ $O(N_c)$ + $O(1)$ $\quad$ $O(1)$ $\quad$ $O(1)$ ]

In small soliton limit: $G_{\overline{10}} = 0$

Decuplet decay: $G_{10} = G_0 + \frac{1}{2}G_1$

Antidecuplet decay: $G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2$

# Width

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

$$\hat{O}_\alpha = -i\frac{3}{2M_B} \times \left[ G_0 D_{\alpha i} - G_1 d_{ibc} D_{\alpha b} \hat{J}_c - G_2 D_{\alpha 8} \hat{J}_i \right] \times p_i$$

$\sqrt{N_c}\ \times$ [ $O(N_c)$ + $O(1)$ $\quad$ $O(1)$ $\quad$ $O(1)$ ]

Is this cancellation consistent with large $N_c$ counting?

MP Phys.Lett.**B583**:96-102,2004

In small soliton limit: $\quad G_{\overline{10}} = 0$

Decuplet decay: $\quad G_{10} = G_0 + \frac{1}{2} G_1$

Antidecuplet decay: $\quad G_{\overline{10}} = G_0 - G_1 - \frac{1}{2} G_2$

Three sources of $N_c$ factors:

- QCD: corrections to the effective lagrangian
- parametric $M,\ I_{1,2} \sim N_c$
- quantum and combinatorial $Y' = N_c/3$

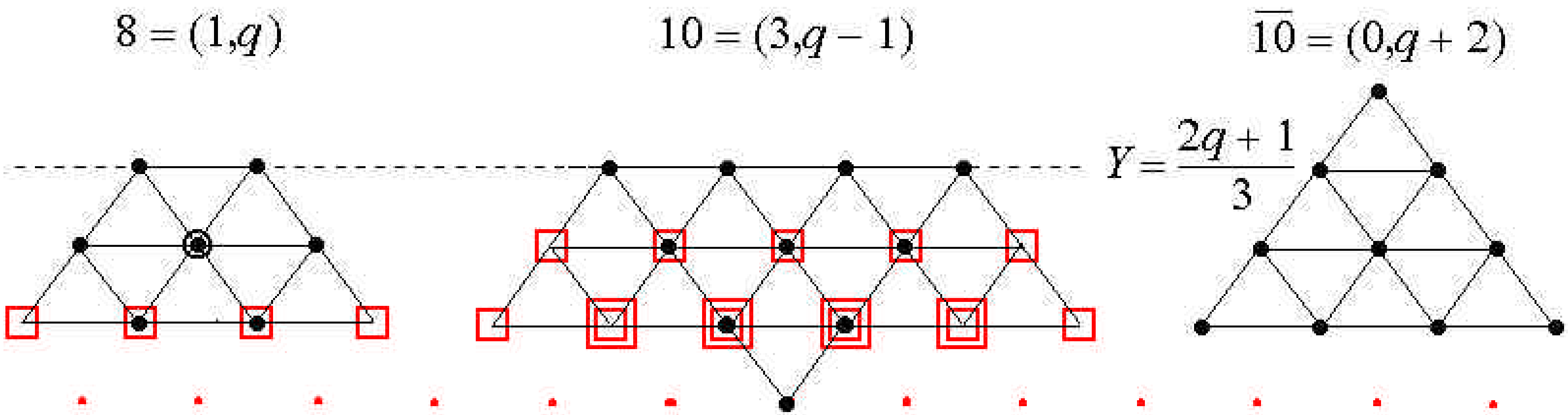
SU(3) C-G's for arbitrary $N_c$

# Wave functions and allowed states

$$\psi^{(\mathcal{R})}_{BS} = \psi_{(\mathcal{R},B)(\mathcal{R}^*,S)} = (-)^{Q_S}\sqrt{\dim(\mathcal{R})}\,D^{(\mathcal{R})*}_{BJ}$$

G. Karl, J. Patera, S. Perantonis, Phys. Lett **172B** (1986) 49,
J. Bijnens, H. Sonoda, M. Wise, Can. J. Phys. **64** (1986) 1,
Z. Dulinski, M. Praszalowicz, Acta Phys.Pol. **B18** (1988) 1157.

$$q = \frac{N_c - 1}{2}$$



# Width

$$G_{10} = G_0 + \frac{1}{2} G_1$$

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2} G_2$$

$$G_{10} = G_0 + \frac{1}{2} G_1$$

$$G_{\overline{10}} = G_0 - \frac{N_c + 1}{4} G_1 - \frac{1}{2} G_2$$

# Width

$$G_{10} = G_0 + \frac{1}{2} G_1$$

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MP Phys.Lett.**B583**:96-102,2004

in small soliton limit cancellation takes place separately in each order in $N_c$

# Width

$\Delta \to N\pi$

$\longrightarrow$ 1/5

$$\Gamma_\Delta \sim \frac{3}{(M_N + M_\Delta)^2} \; C^{(10)}(B_1, B_2, \varphi)\, G_{10}^2 \times p_\pi^3$$

$\Theta^+ \to NK$

$$\Gamma_{\Theta^+} \sim \frac{3}{(M_N + M_\Theta)^2} \, C^{(\overline{10})}(B_1, B_2, \varphi)\, G_{\overline{10}}^2 \times p_K^3$$

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# Width

$$\Delta \to N\pi$$

$$\Gamma_\Delta \sim \frac{3}{(M_N + M_\Delta)^2} C^{(10)}(B_1, B_2, \varphi) \cdot G_{10}^2 \times p_\pi^3$$

O(1/N$_c$$^2$) O(N$_c$$^3$)

$$\Theta^+ \to NK$$

$$\Gamma_{\Theta^+} \sim \frac{3}{(M_N + M_\Theta)^2} C^{(\overline{10})}(B_1, B_2, \varphi)\, G_{\overline{10}}^2 \times p_K^3$$

O(1/N$_c$$^2$) O(N$_c$$^3$)

# Width

$\Delta \to N\pi$

1/5

$$\Gamma_\Delta \sim \frac{3}{(M_N+M_\Delta)^2}\frac{(N_c-1)(N_c+5)}{2(N_c+1)(N_c+7)}G_{10}^2 \times \boxed{p_\pi^3}$$

O(1/N$_c$$^2$) O(1) O(N$_c$$^3$)

$\Theta^+ \to NK$

$$\Gamma_{\Theta^+} \sim \frac{3}{(M_N+M_\Theta)^2}\boxed{\frac{3(N_c+1)}{(N_c+3)(N_c+7)}}G_{10}^2 \times p_K^3$$

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O(1/N$_c$$^2$) O(1/N$_c$) O(N$_c$$^3$)

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1/5

O(1/N$_c$$^2$) O(1) O(N$_c$$^3$)

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$$\Gamma_{\Theta^+} \sim \frac{3}{(M_N+M_\Theta)^2}\boxed{\frac{3(N_c+1)}{(N_c+3)(N_c+7)}}G_{10}^2 \times p_K^3$$

O(1/N$_c$$^2$) O(1/N$_c$) O(N$_c$$^3$)

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1/5

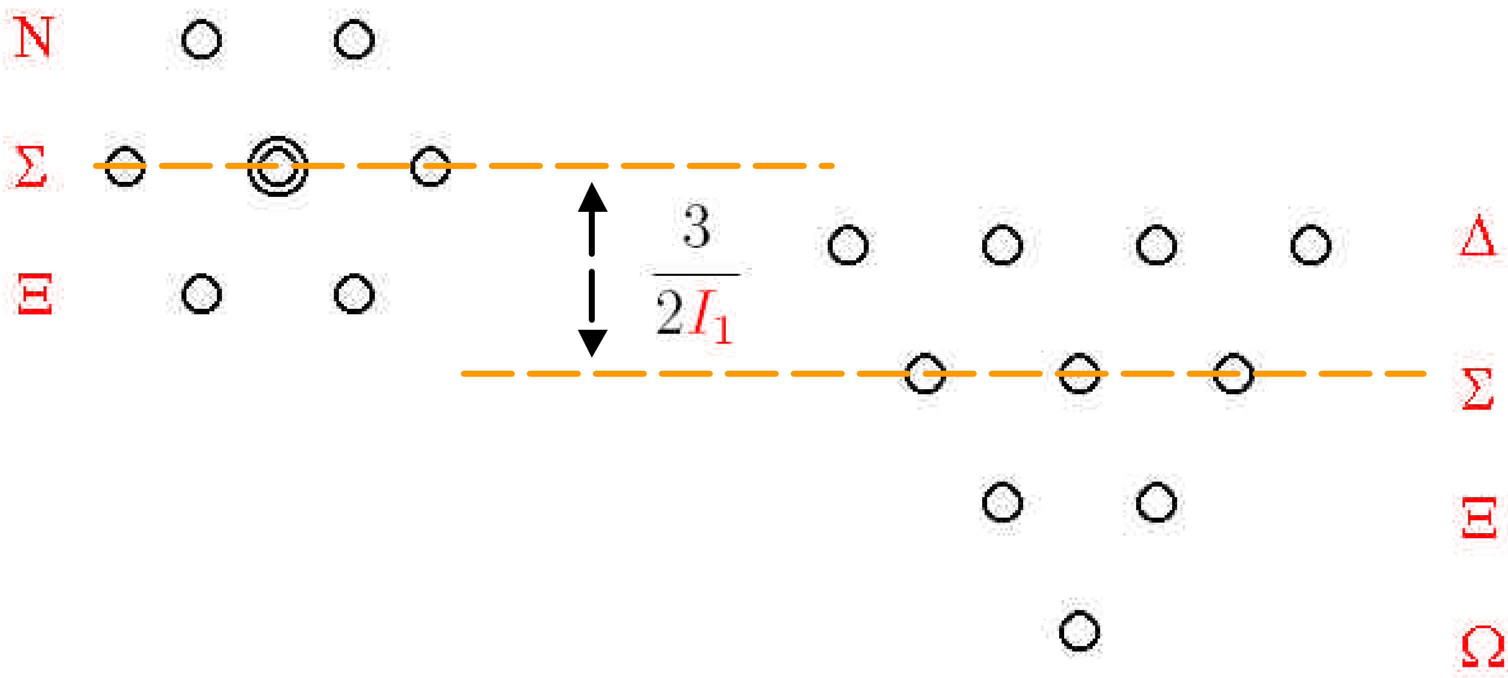
O(1/$N_c^2$) O(1) O($N_c^3$)

$\Theta^+ \to NK$

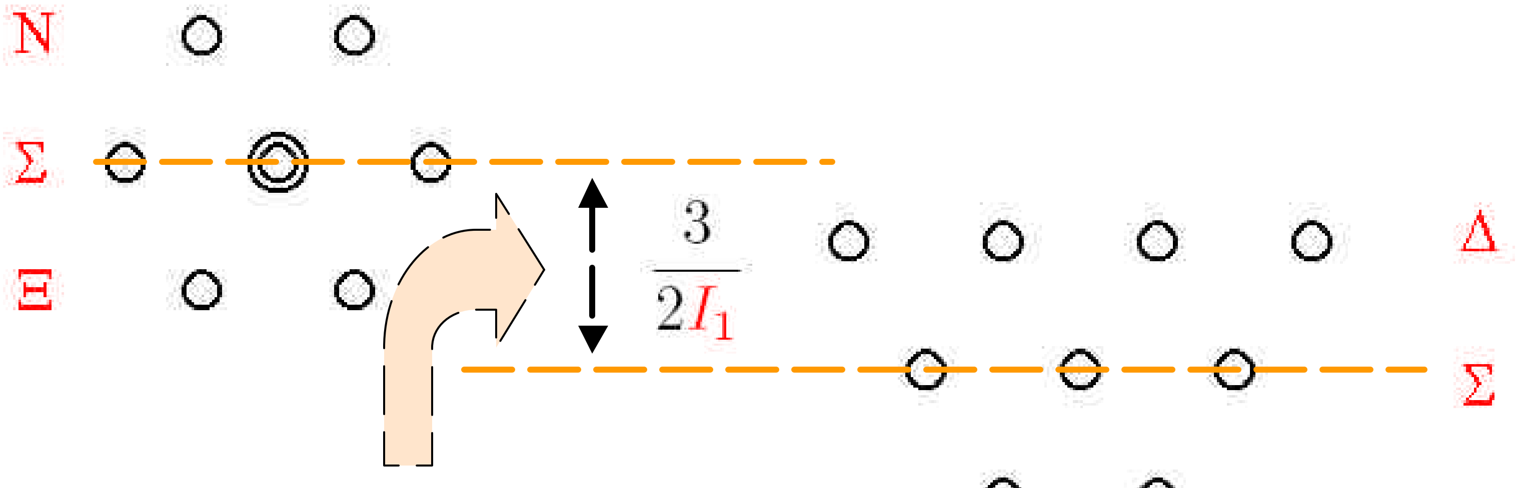
$$\Gamma_{\Theta^+} \sim \frac{3}{(M_N + M_\Theta)^2} \boxed{\frac{3(N_c+1)}{(N_c+3)(N_c+7)}} G_{10}^2 \times \boxed{p_K^3}$$

O(1/$N_c^2$) O(1/$N_c$) O($N_c^3$)

# Mass splittings for large $N_c$



# Mass splittings for large $N_c$




$$\Delta - N = \frac{3}{2I_1} \xrightarrow[N_c \to \infty]{} 0$$

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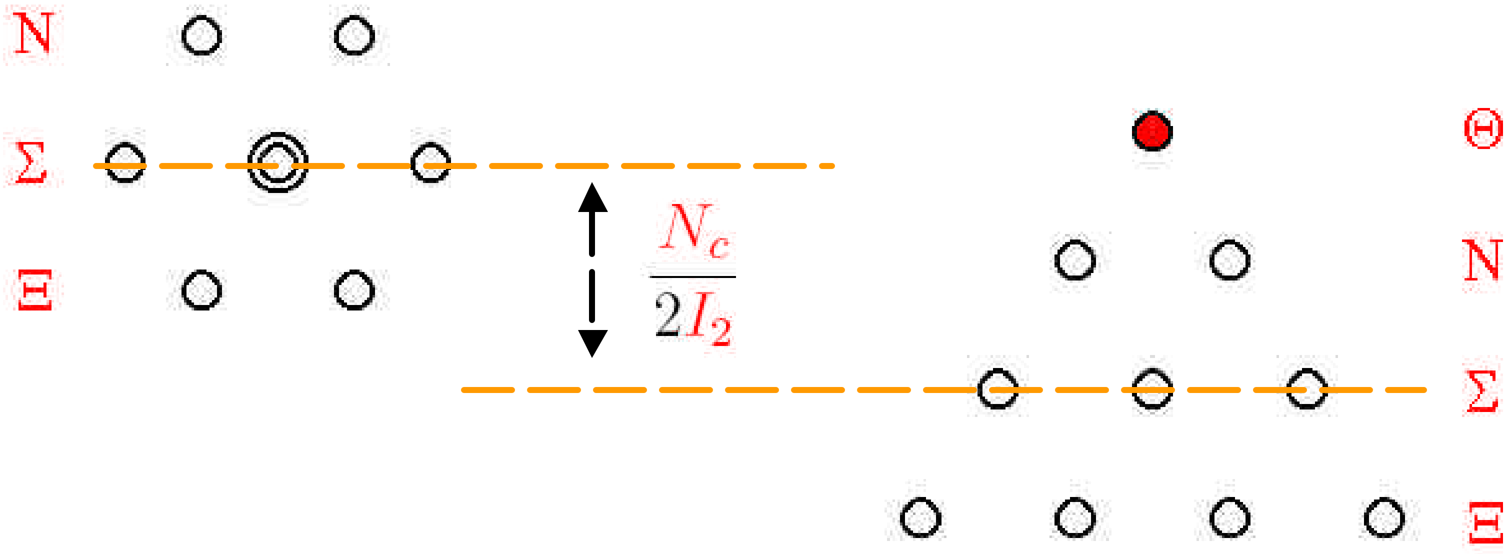
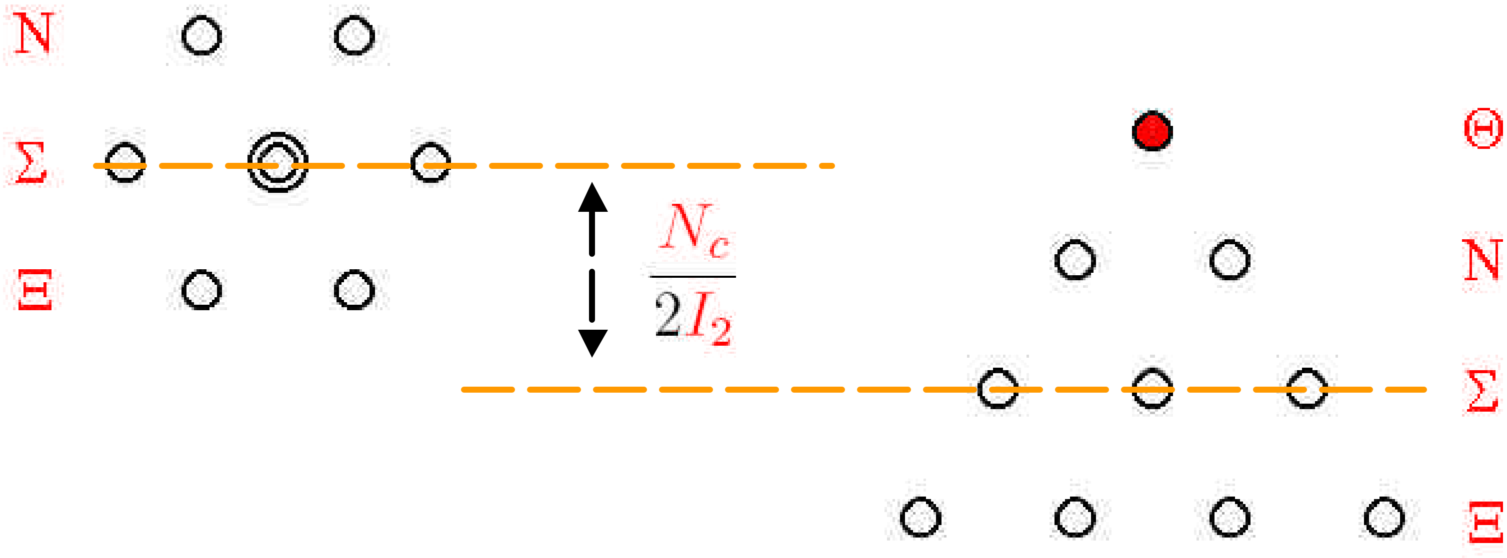



$$\overline{M}_{\overline{10}} - \overline{M}_8 = 601$$

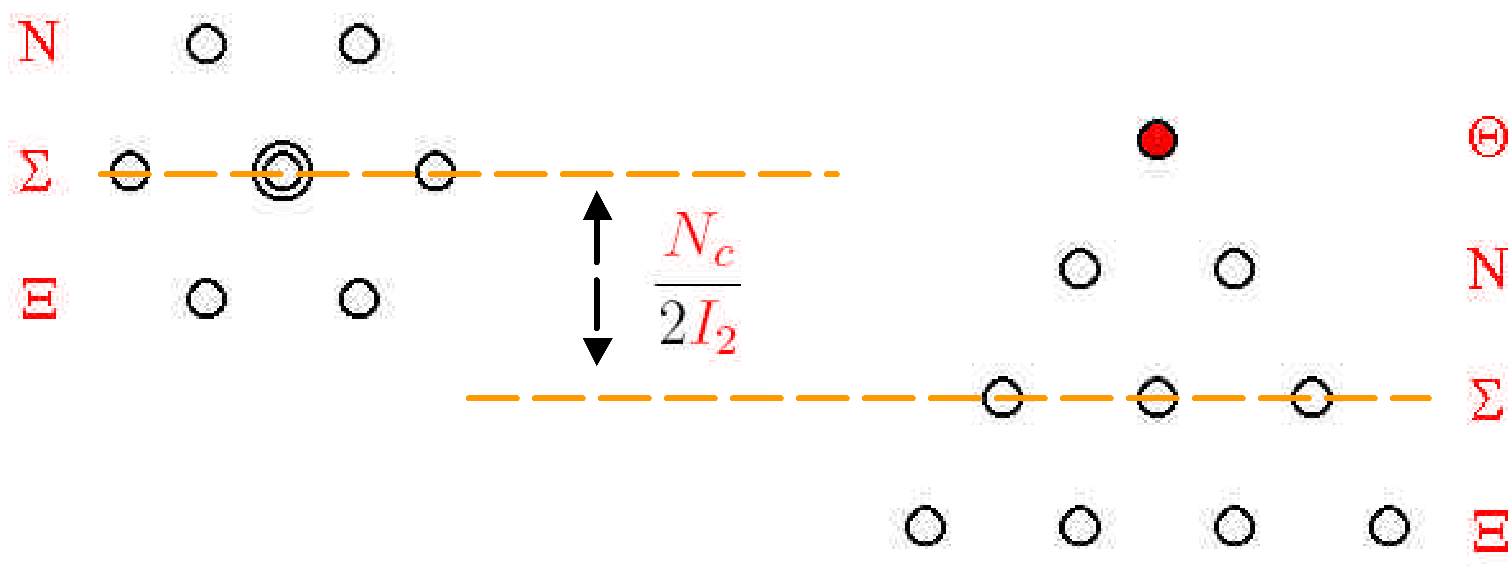
# Mass splittings for large $N_c$




$$\Delta - N \sim \mathcal{O}(\frac{1}{N_c})$$

$$\Theta - N \sim \mathcal{O}(1) \qquad \overline{M}_{\overline{10}} - \overline{M}_8 = 601$$

# Mass splittings for large $N_c$




$$\overline{M}_{10} - \overline{M}_8 = 234 \quad \Delta - N \sim \mathcal{O}(\frac{1}{N_c})$$

$$\Theta - N \sim \mathcal{O}(1) \qquad \overline{M}_{\overline{10}} - \overline{M}_8 = 601$$

# Width

$$\Gamma_\Delta \sim \mathcal{O}(N_c) \times p^3$$
$$\Gamma_\Theta \sim \mathcal{O}(1) \times p^3$$

where $p$ for $1 \longrightarrow 2$ decay reads

$$p = \frac{\sqrt{M_1^2 - (M_2 + m)^2}\sqrt{M_1^2 - (M_2 - m)^2}}{2M_1}$$

$$\Delta \to \pi N \quad p \sim \mathcal{O}(\frac{1}{N_c}) \qquad \Gamma_\Delta \sim \mathcal{O}(\frac{1}{N_c^2})$$
$$\Theta \to KN \quad p \sim \mathcal{O}(1) \qquad \Gamma_\Theta \sim \mathcal{O}(1)$$

$p_\pi = 225, \qquad p_K = 268$

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chiral limit:

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in Nature:

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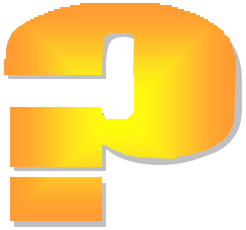
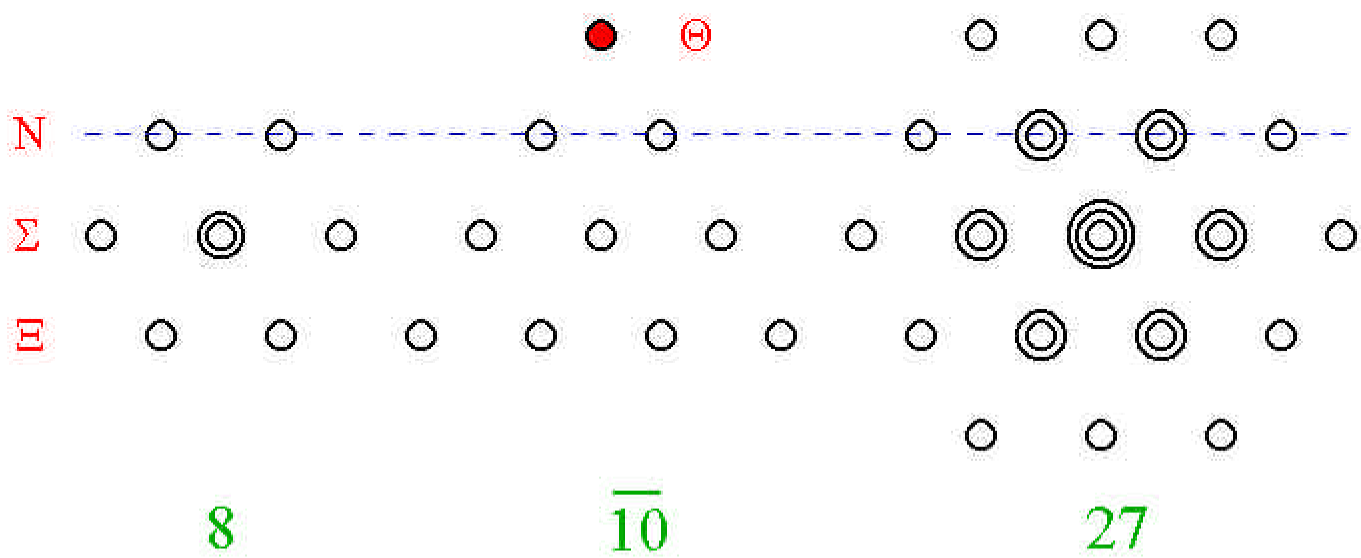


# Width in the soliton model - mixing effects

Once $G_{\overline{10}}$ is small, even moderate admixtures of other representations with nonsuppressed transitions modify the width




For Q only the admixture in the final state matters

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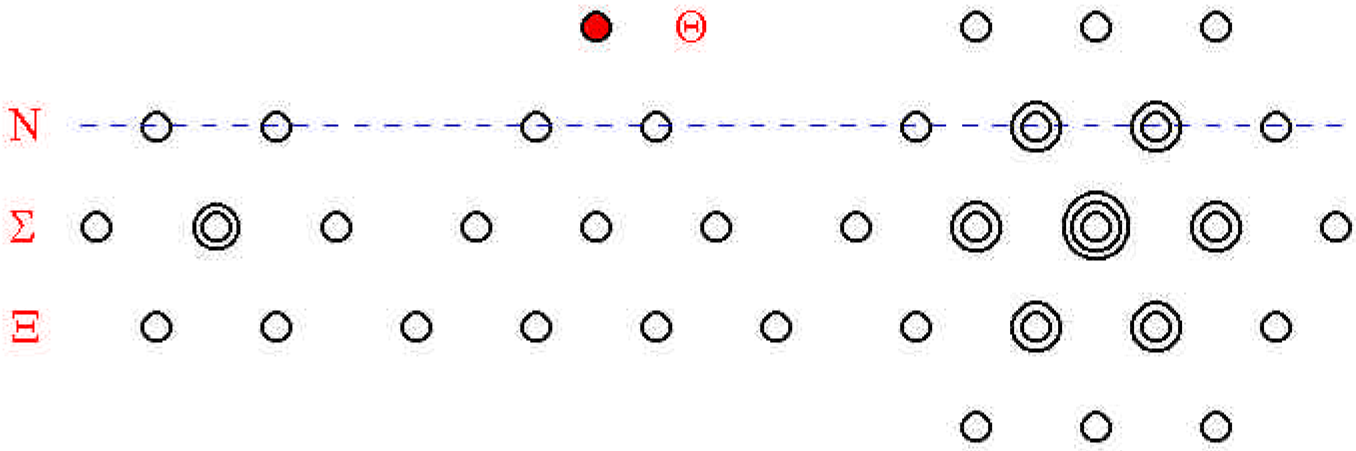


$$|B_8\rangle = |8_{1/2}, B\rangle + c^B_{\overline{10}}|\overline{10}_{1/2}, B\rangle + c^B_{27}|27_{1/2}, B\rangle$$

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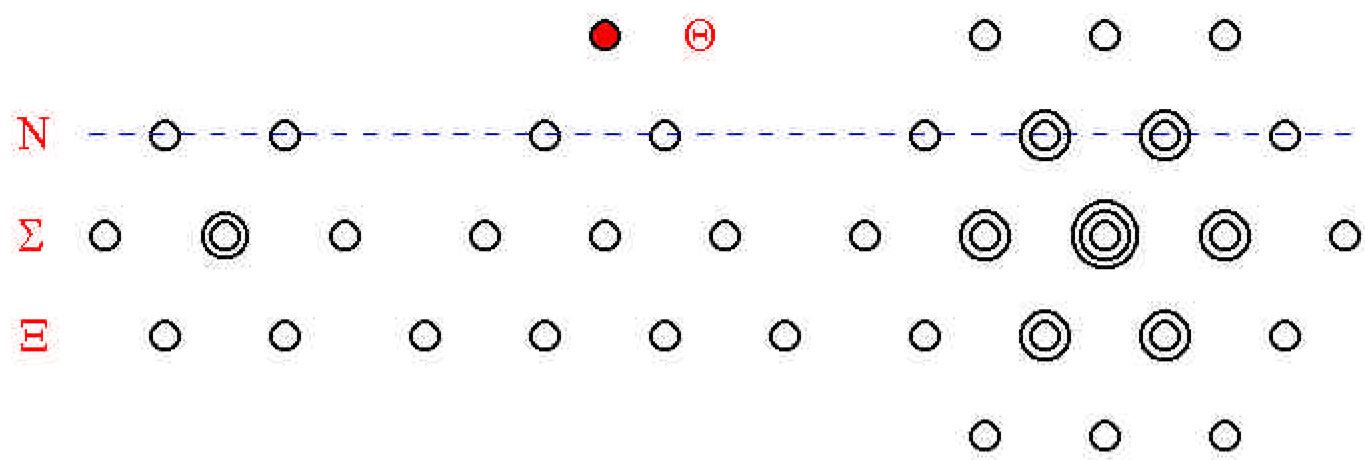


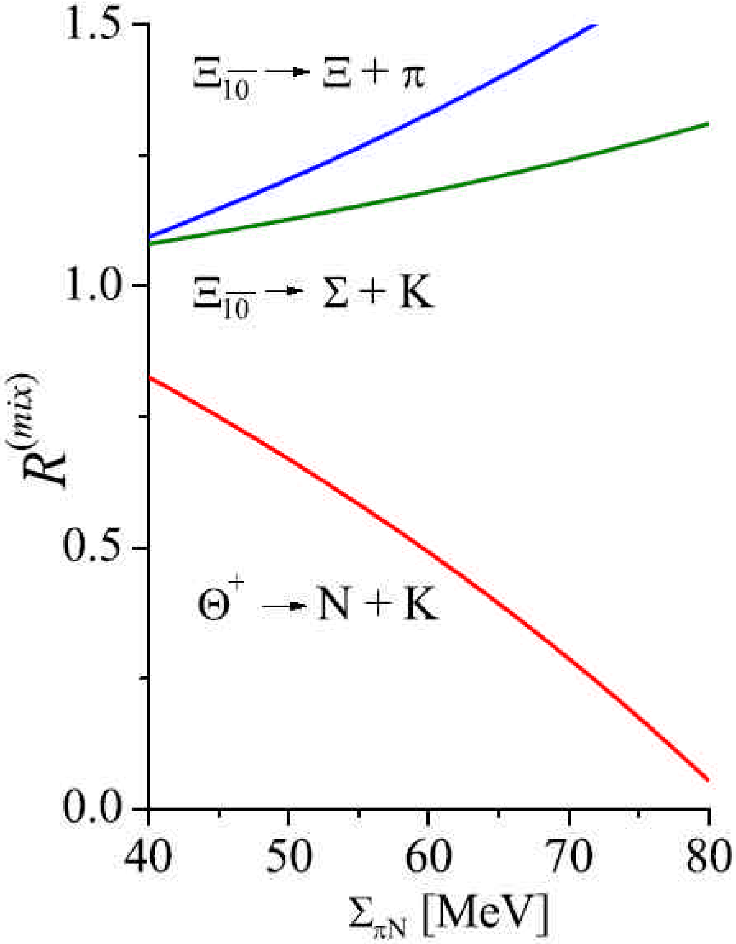

$$|B_8\rangle = \left|8_{1/2}, B\right\rangle + c^{B}_{\overline{10}} \left|\overline{10}_{1/2}, B\right\rangle + c^{B}_{27} \left|27_{1/2}, B\right\rangle$$

For Q only the admixture in the final state matters

$$\Gamma_{B_1 \to B_2 \varphi} = \frac{G^2_{\overline{10}}}{8\pi M_1 M_2} \times C^{(\overline{10})}(B_1, B_2, \varphi) \times p_\varphi^3 \times R^{(mix)}$$

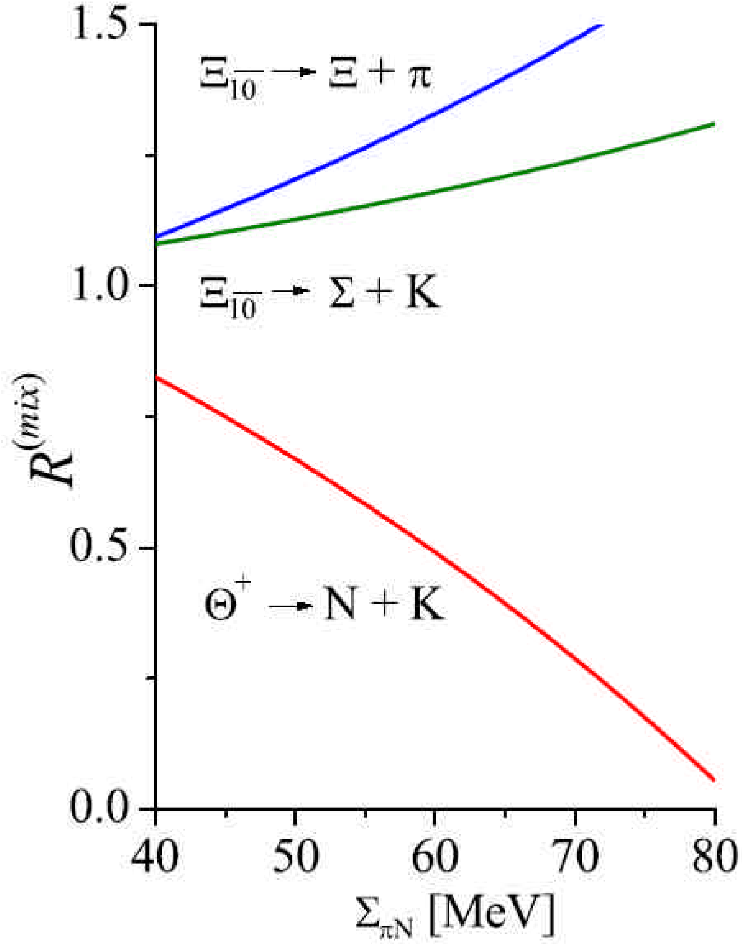
# Width in the soliton model - mixing effects

modification factor



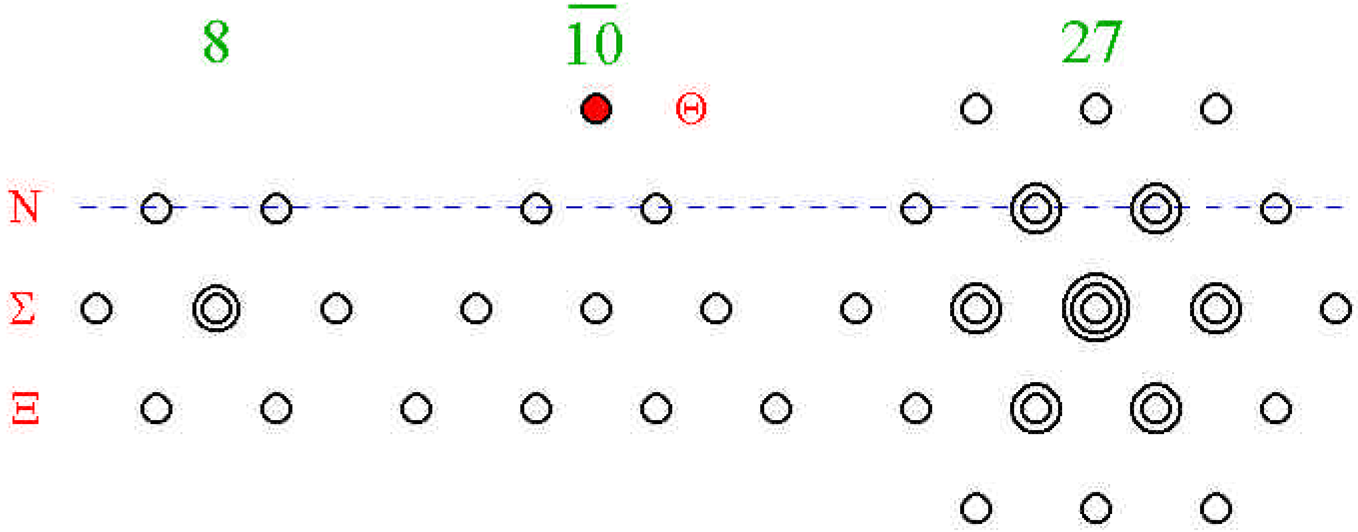
# Width in the soliton model - mixing effects

modification factor →

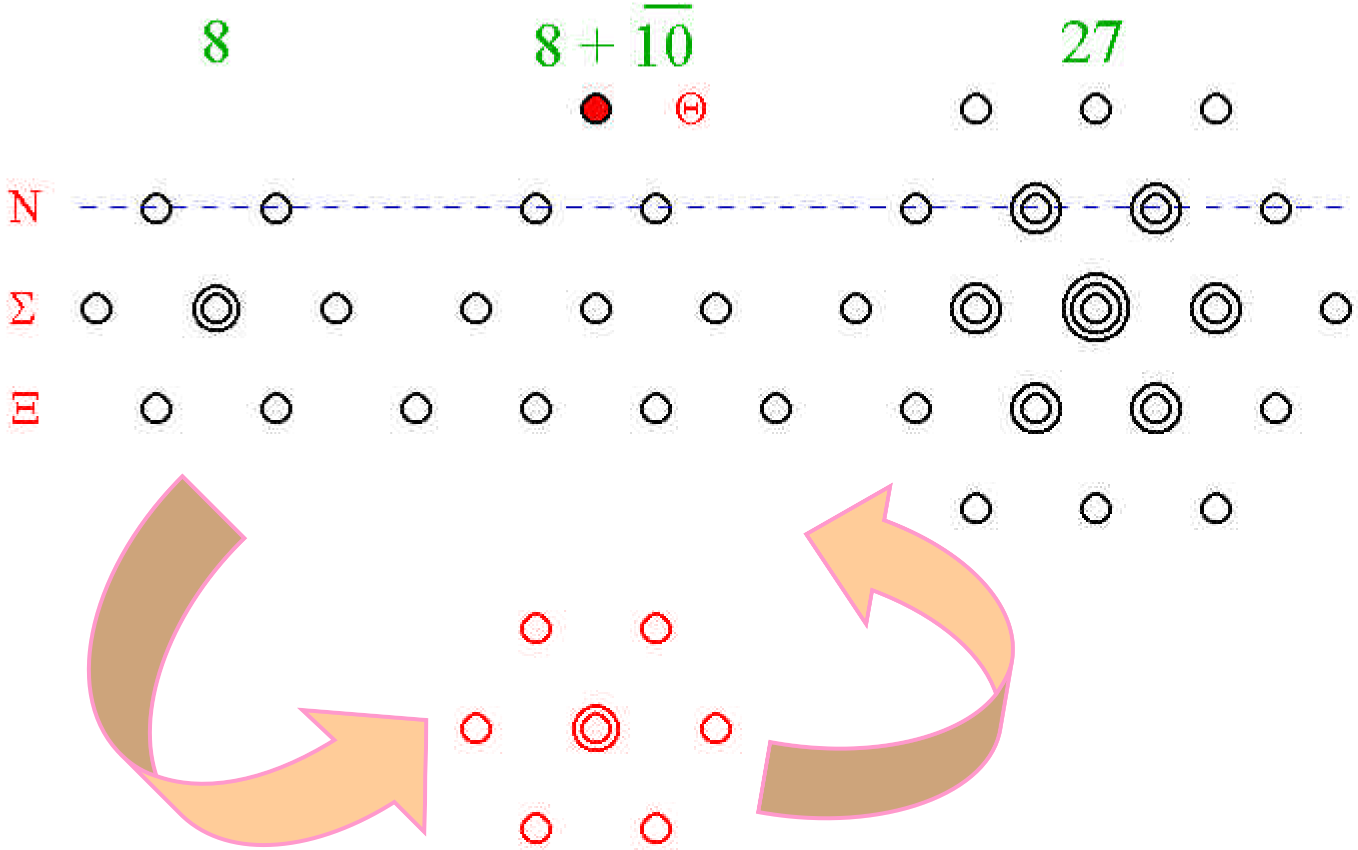



Warning: SU(3) relations for $G$'s will not hold

# Mixing



# Mixing



# $m_s$ corrections to the operator

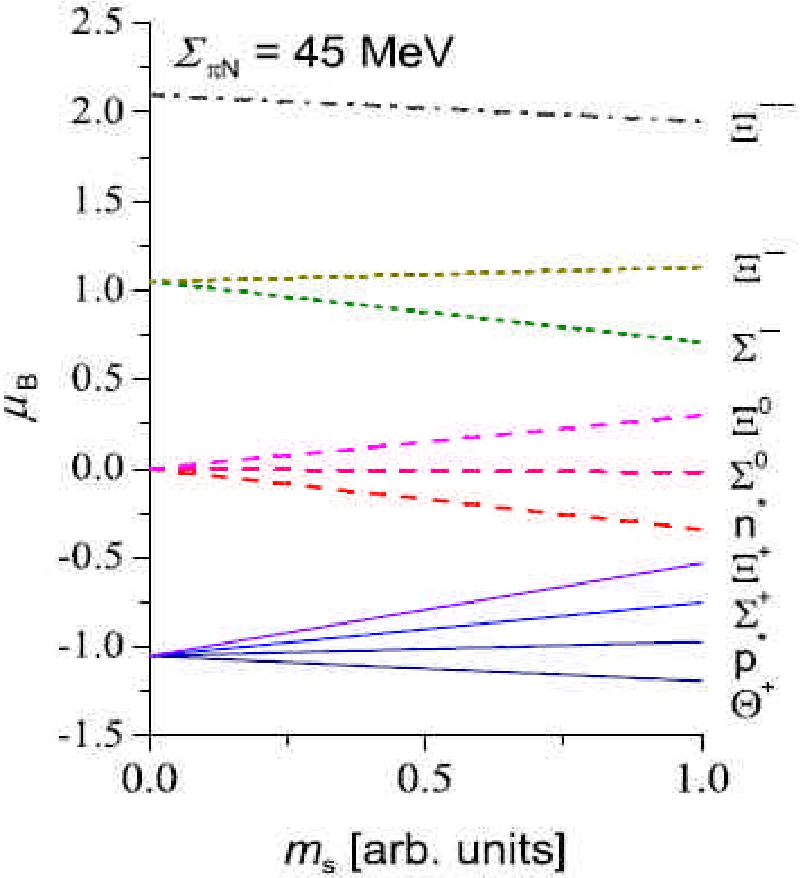
## Example: magnetic moments

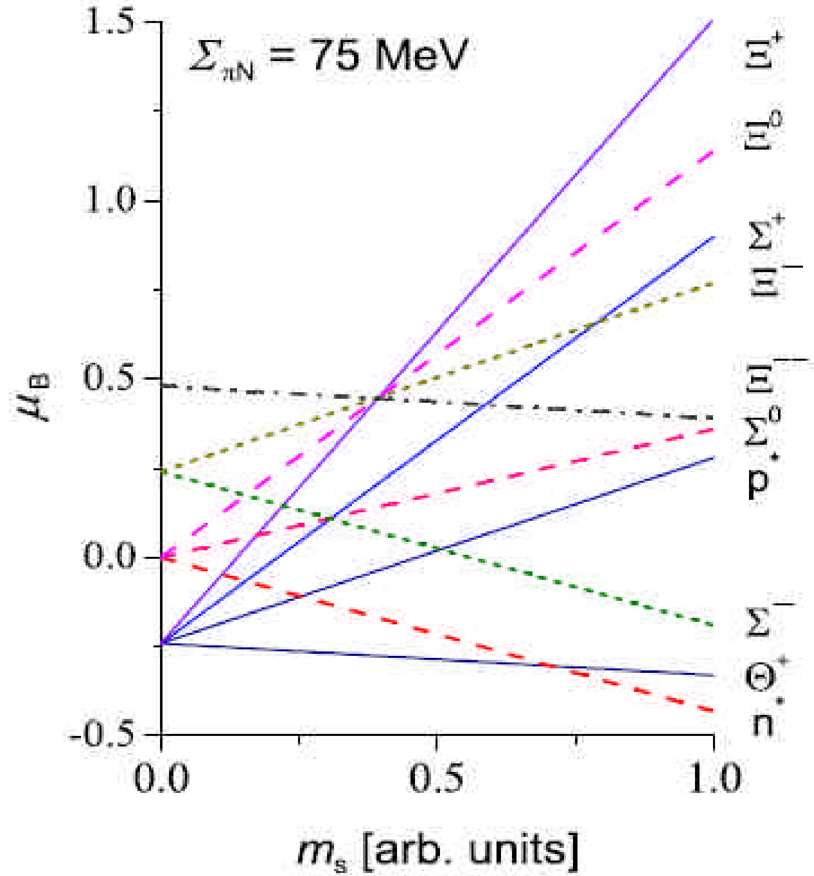
$$\hat{\mu}^{(0)} = w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3,$$

$$\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{8q} + w_5 \left( D^{(8)}_{Q3} D^{(8)}_{88} + D^{(8)}_{Q8} D^{(8)}_{83} \right) + w_6 \left( D^{(8)}_{Q3} D^{(8)}_{88} - D^{(8)}_{Q8} D^{(8)}_{83} \right)$$

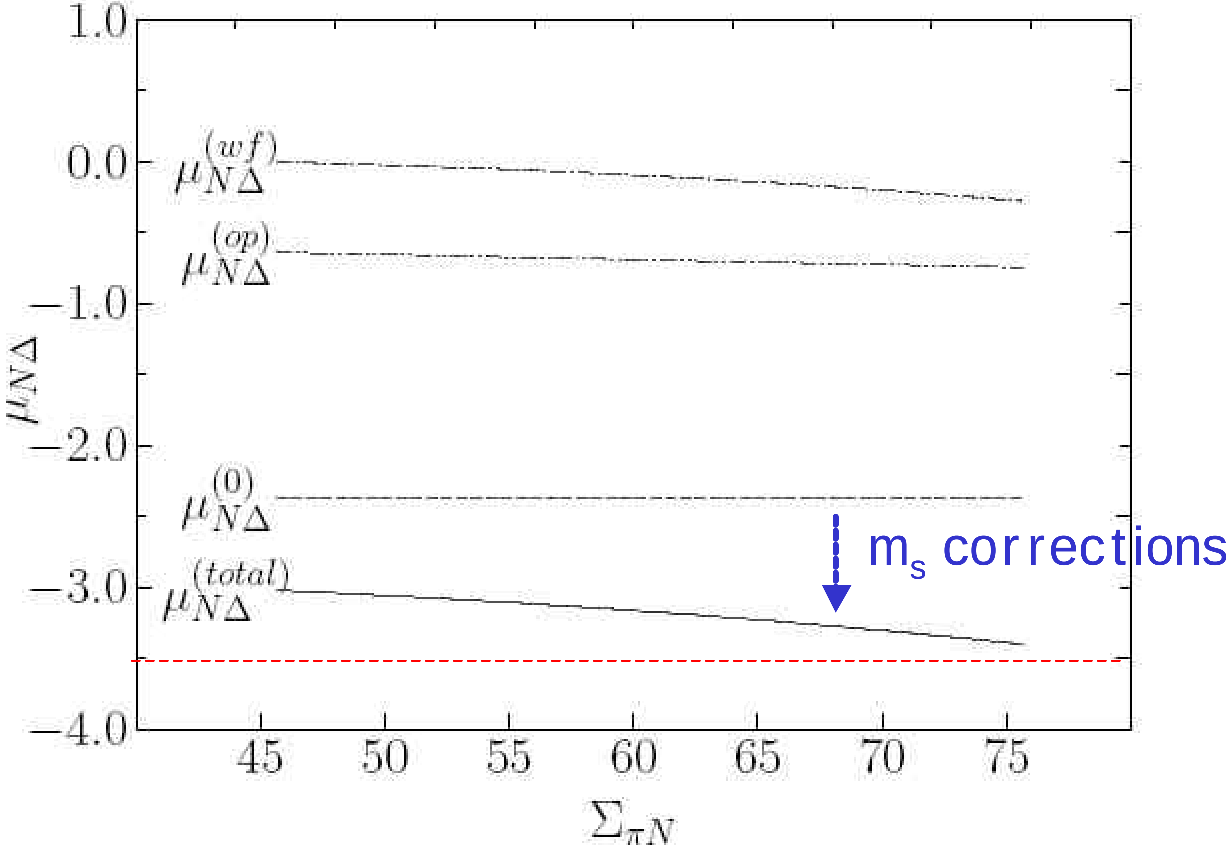
# $m_s$ corrections to the operator

## Example: magnetic moments

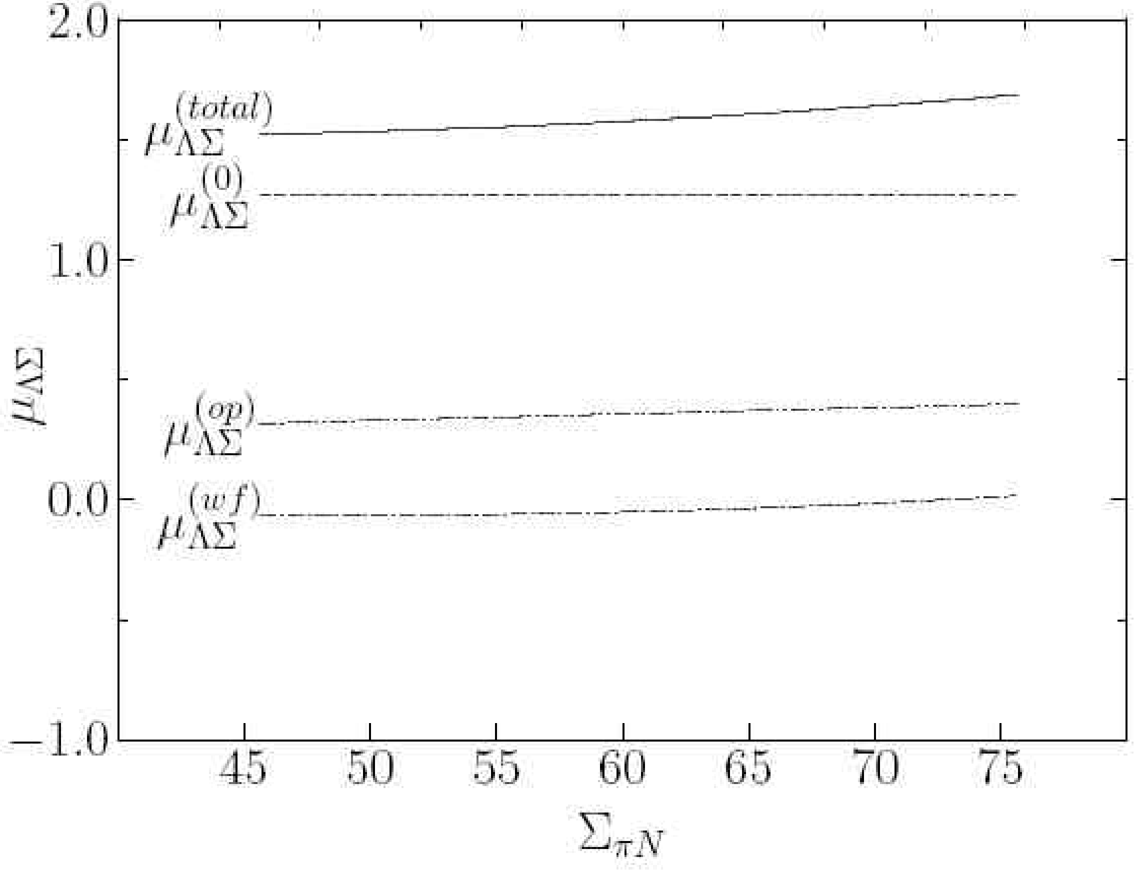


# Magnetic transitions



# Magnetic transitions



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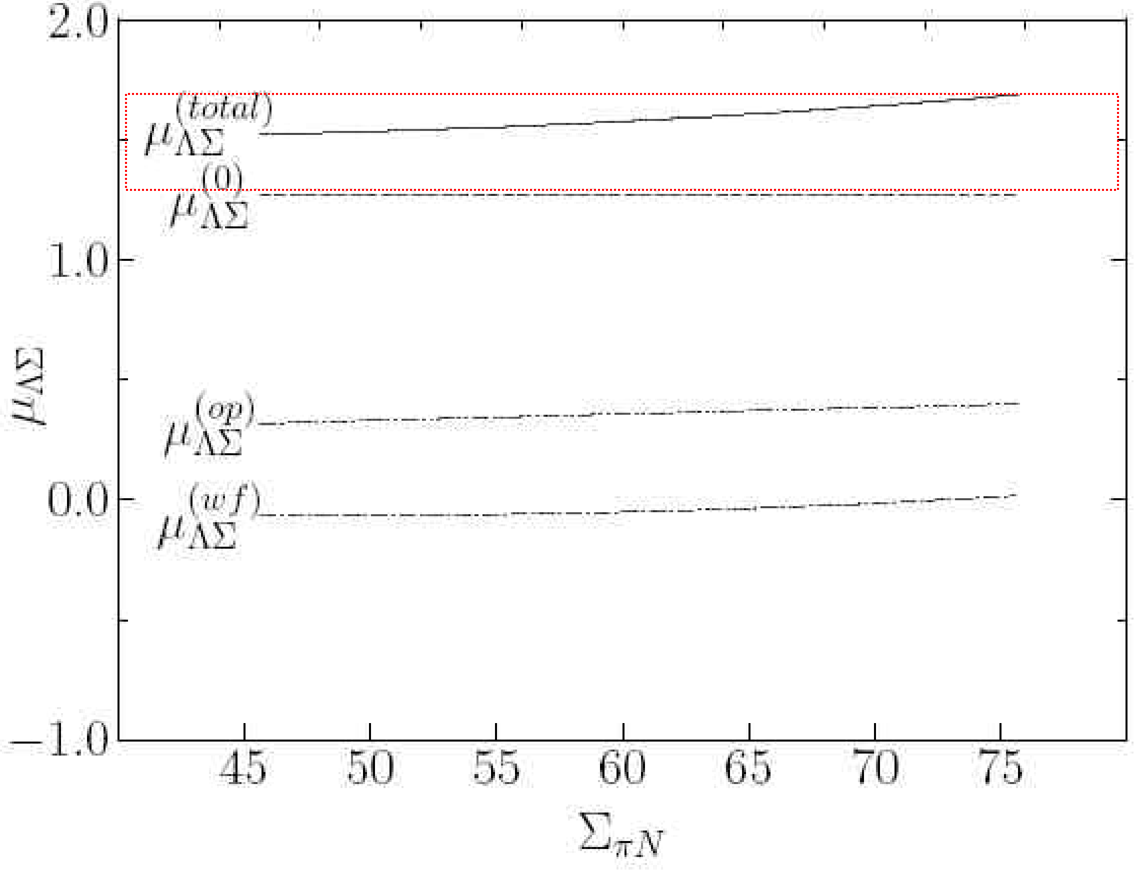
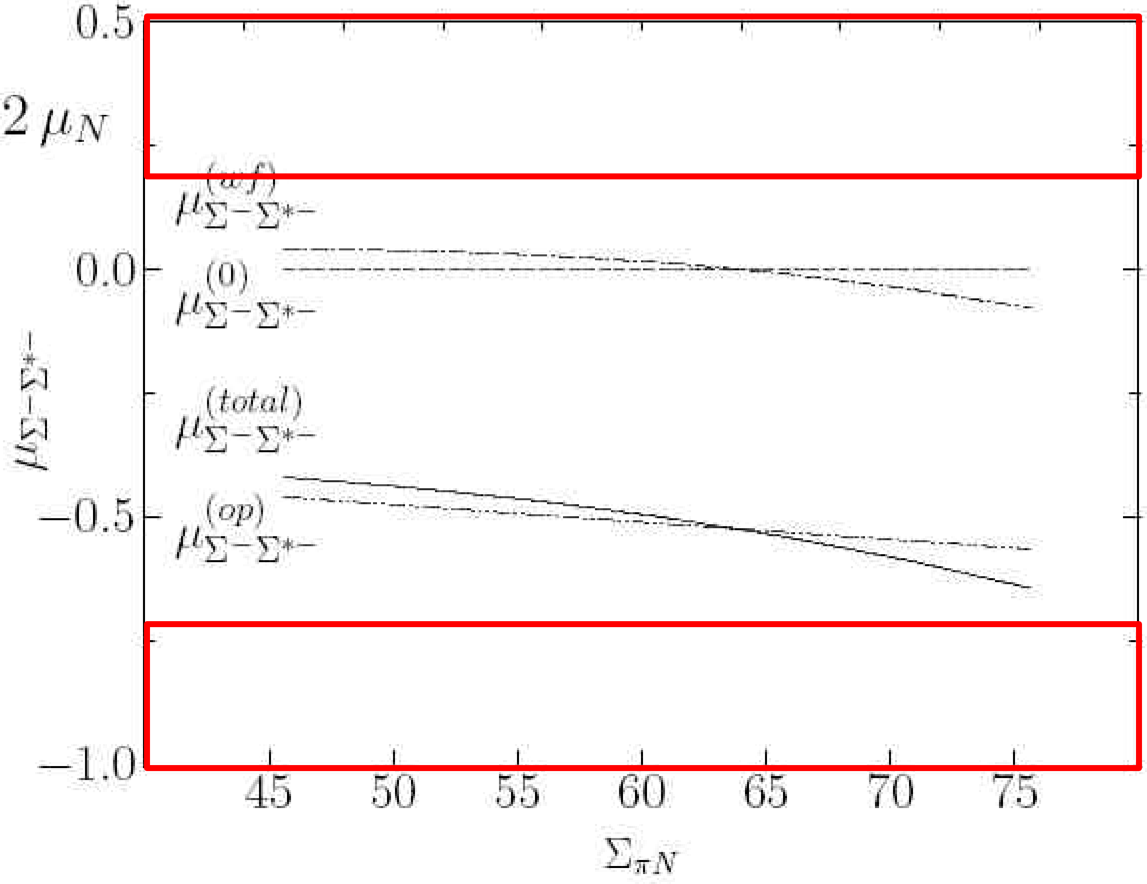


# Magnetic transitions

SELEX

$$|\mu_{\Sigma^- \Sigma^{*-}}| < 0.72\,\mu_N$$

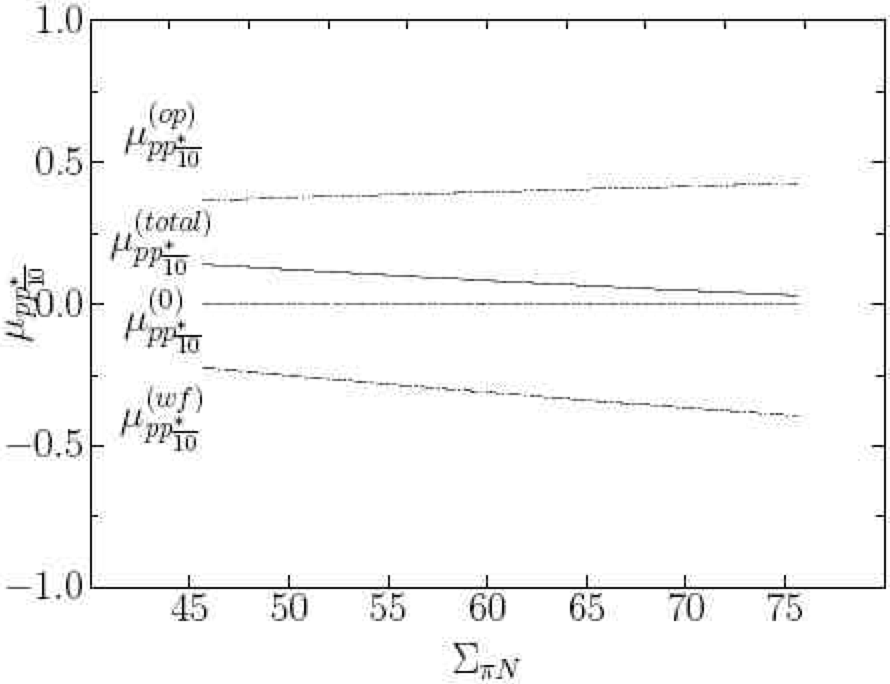



$$\mu^{(0)}_{\Sigma\Sigma^*} = -\frac{1}{6\sqrt{5}}(Q+1)(w_1 - \frac{1}{2}w_2)$$

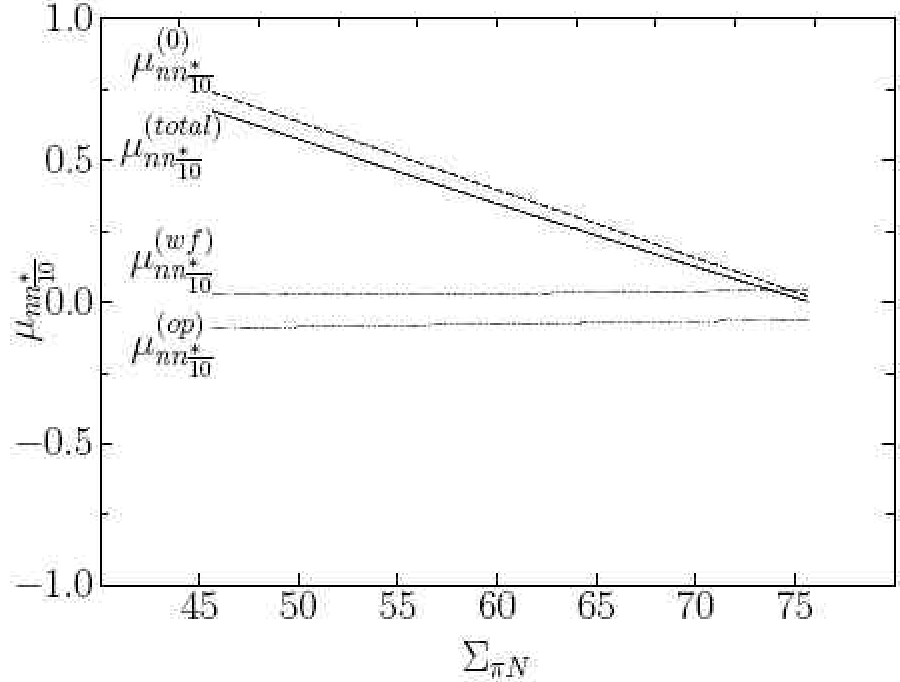
# Magnetic transitions

GRAAL

$p^*_{\overline{10}} \longrightarrow p\eta$



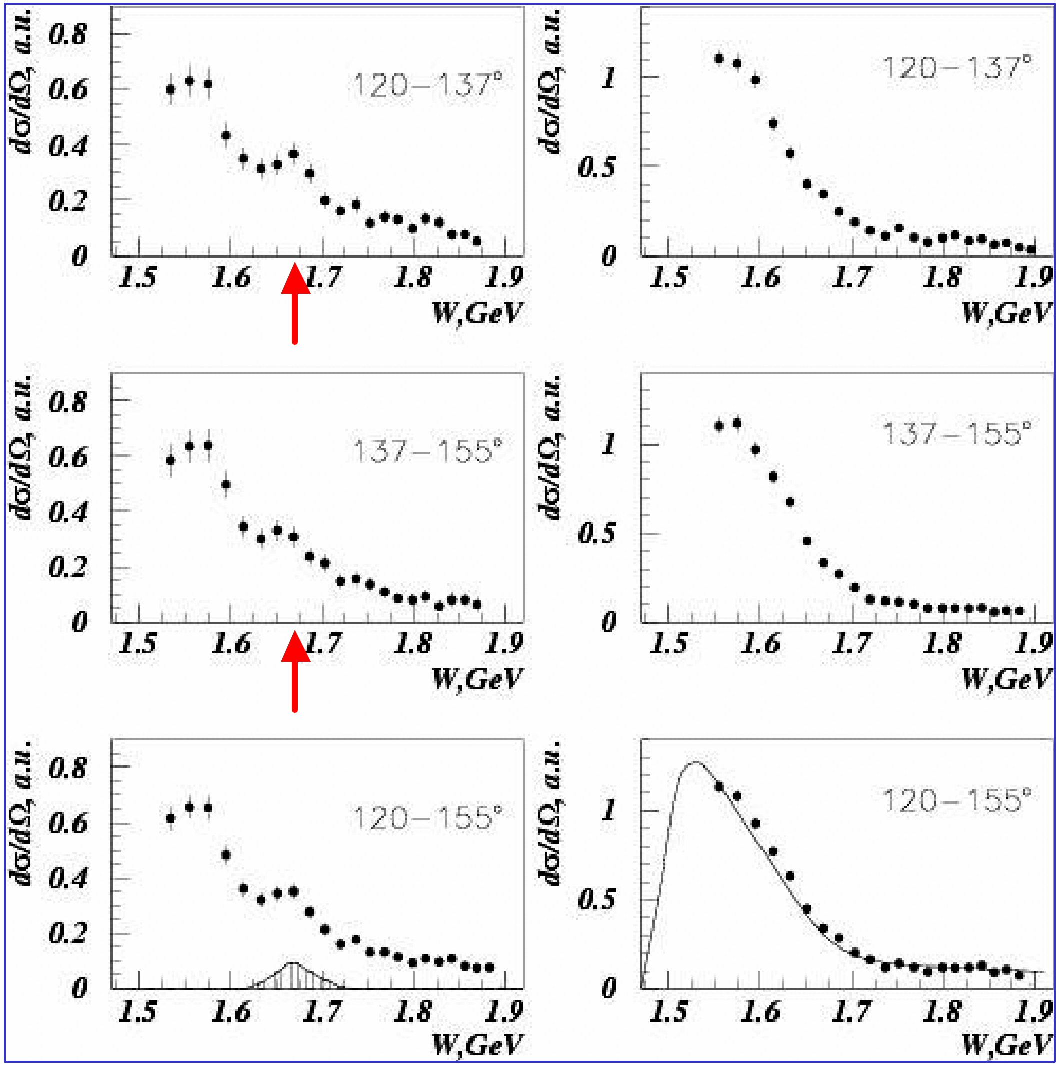

$n^*_{\overline{10}} \longrightarrow n\eta$




$$\mu^{(0)}_{NN^*_{\overline{10}}} = -\frac{1}{6\sqrt{5}}(Q-1)(w_1 + w_2 + \frac{1}{2}w_3)$$

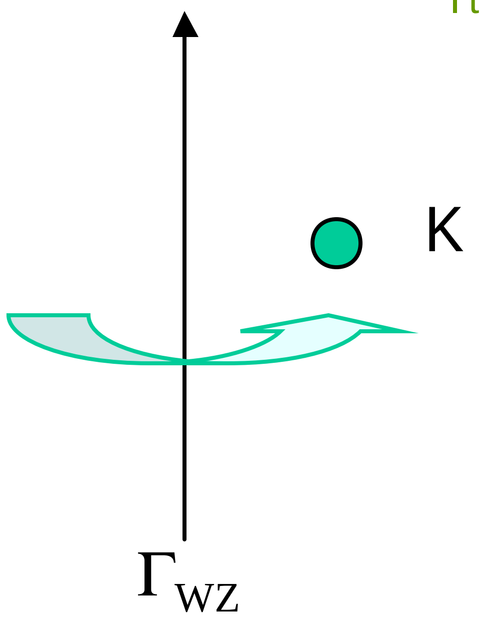
# Magnetic transitions

$n^*_{\overline{10}} \rightarrow n\eta$ GRAAL $p^*_{\overline{10}} \rightarrow p\eta$



# Matching with the bound state approach

Callan, Klebanov Nucl.Phys.**B262**:365,1985
Nadeau, Nowak, Rho, VentoPhys.Rev.Lett.**57**:2127-2130,1986
Callan, Klebanov , Hornbostel, Phys.Lett**.B202**:269,1988
Itzhaki, Klebanov, Quyang, Rastelli, Nucl.Phys.**B684**:264-280,2004




$K^-$ is bound
$K^+$ is not bound and has
no smooth limit to rigid rotator

# Summary

1. Soliton models are descendants of QCD, in comparison to quark models different approximations are made
2. Soliton models are used to dedscribe many different properties, not only spectra
3. Phenomenologically both small mass and small width of $10^*$ are in soliton models natural
4. If $\Theta$ exists there must be other states both exotic and cryptoexotic
5. Mixing of $10^*$ with other irreps. is important because primary transitions $10^* \rightarrow 8$ is very small
6. Large $N_c$ limit and chiral limit are very subtle, especially for the width
7. Bound state approach (?)