The Exotic Sector of the Skyrme Model

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Reported observations of exotic baryons have sparked substantial controversy among both theorists and experimenters!

At present there is no theoretical model of Θ^+ that is generally accepted. All suffer from major assumptions and limitations, which may or may not be experimentally relevant.

The big questions are

- do exotics exist?
- if so, what are their properties?
- if not, why not?

Because of the assumptions involved in constructing any model these questions are hard to answer definitively. However we can ask what properties of exotics seem to hold generically and which may be special.

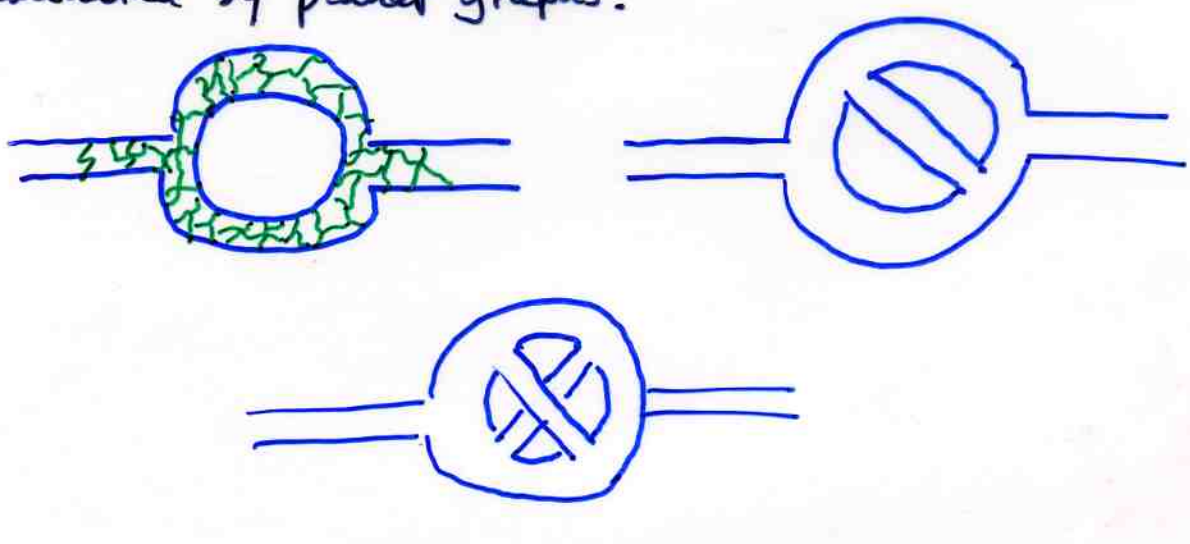
It is also worthwhile to distinguish between features "generic" in a theorist's sense of depending only on general properties and features "generic" in the sense of being robust on variation of parameters around physical values.

Main points of the talk

- physics of exotic vs. nonexotic <u>very different</u> - captured by Skyrme
- existence is <u>not</u> generic (but not excluded)
- p-wave is favored (generic in the 2nd sense)

Philosophy of $1/N$ expansion

QCD is of course $SU(N)$ gauge theory with $N=3$, but we obtain substantial insight by formally taking $N \to \infty$. In this limit the usual Feynman diagrams of perturbative field theory may be organized into a topological expansion, dominated by planar graphs.



The sensible way to take the large-N limit is to hold $g^2 N$ fixed.

$$g \sim \frac{1}{\sqrt{N}}$$

Some simple conclusions

- Meson-meson scattering is suppressed $\sim g^2 \sim \frac{1}{N}$

- Baryon masses are $\mathcal{O}(N)$, including all interactions
  $m_B \sim N \sim 1/g^2 \Rightarrow$ solitons ?

Skyrme Model

- Theory of baryons realized as solitons of a nonlinear chiral Lagrangian.
- Consistent with large-N_c reasoning

- $$\mathcal{L}_{Sk} = \frac{f_\pi^2}{16} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{1}{32e^2} \mathrm{Tr}\left([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2\right)$$

$$+ \text{ mass terms}$$

U = SU(3) matrix

$e \sim 5.5$

$f_\pi \sim 186$ MeV (expt, but we will vary it)

- Soliton solutions are possible because of the 4-derivative term; otherwise Derrick's theorem says you could collapse the soliton with no energy cost. The solitons are of hedgehog form:

$$U_0 = \begin{pmatrix} e^{i\vec{\tau}\cdot\hat{r}F(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

$\vec{\tau}$ = Pauli SU(2) matrices

- There is an additional term called the Wess-Zumino term which is crucial and which I will describe later.

SU(2) Flavor, Rigid Rotator

The hedgehog soliton $U_0 = \exp\left(i\vec{\tau}\cdot\hat{r}F(r)\right)$ solves the equations of motion, but so do isospin-rotated hedgehogs:

$$U_0 \longrightarrow A U_0 A^{-1}$$

These SU(2) rotations may be ~~[illegible]~~ promoted to dynamical rotations

$$U(t) = A(t) U_0 A^{-1}(t)$$

and canonically quantized (exactly like a rigid top.)
The resulting Hamiltonian:

$$H = M_{cl} + \frac{1}{2\Omega} J(J+1)$$

Numerically $\quad M_{cl} \sim 36.5 \dfrac{f_\pi}{e}$

$$\Omega \sim \frac{107}{e^3 f_\pi}$$

Fitting N, Δ masses gives $e = 5.45$, $f_\pi = 129$ MeV.

Constraint $\underline{I = J}$ on allowed wavefunctions!

Many physical quantities are computable (20-30% accuracy.)

Mass splittings $\sim \dfrac{1}{\Omega} \sim \dfrac{1}{N_c}$

SU(3) Flavor in the Skyrme Model

Flavor SU(3) physics is affected dramatically by the Wess-Zumino term. It can be written as

$$S_{WZ} = -\frac{iN_c}{240\pi^2}\int d^5x\, \varepsilon^{\mu\nu\alpha\beta\gamma}\, \mathrm{Tr}\, \partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \partial_\gamma U U^\dagger$$

vanishes for SU(2).

Nice properties:

- includes "naive" parity-violating interactions like $K^+K^- \to 3\pi$
- when gauged it generates the ABJ anomaly
- makes the Skyrmion a fermion for N_c odd

Suppose you tried to repeat the SU(2) RR analysis generalized to SU(3). You find

- a constraint on allowed multiplets from WZ.

  must contain a state with $Y = N/3$ $(I=J, S=0)$
- mass formula

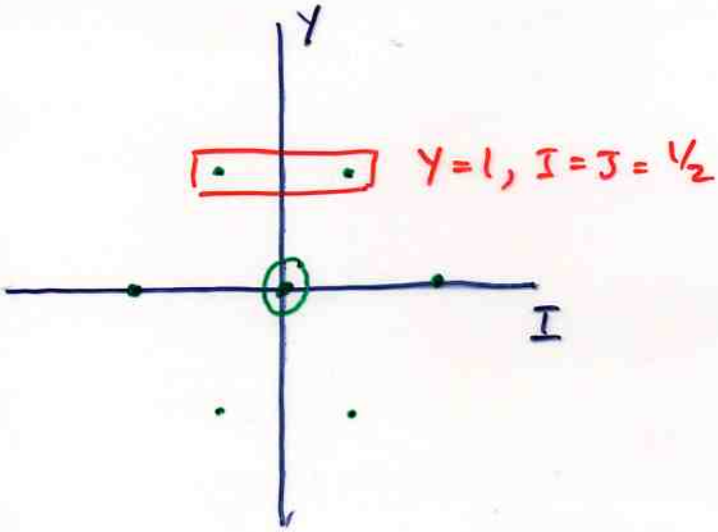
  for SU(3) rep (p,q)

$$M^{(p,q)} = \frac{1}{2\Omega}J(J+1) + \frac{1}{2\Phi}\left(C^{(p,q)} - J(J+1) - N^2/12\right)$$

$$C^{(p,q)} = \frac{1}{3}\left[p^2+q^2+3p+3q+pq\right]$$

Quarks are still massless. SU(2) splittings within multiplet.

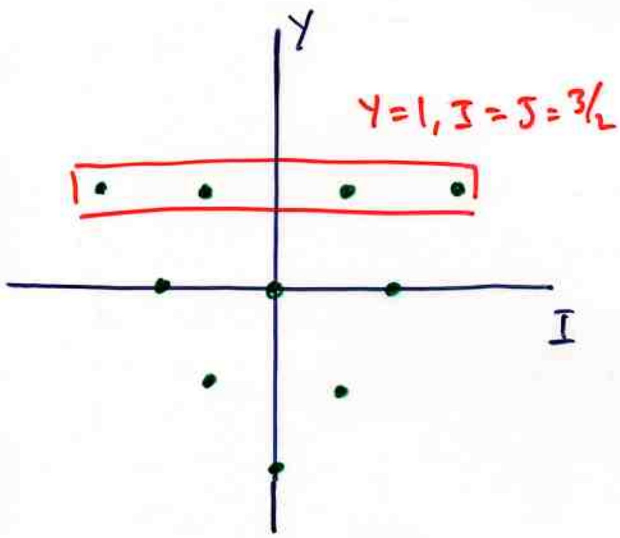
Allowed $SU(3)_f$ multiplets



(1,1) octet

$\left(1, \frac{N_c-1}{2}\right)$ at large N_c

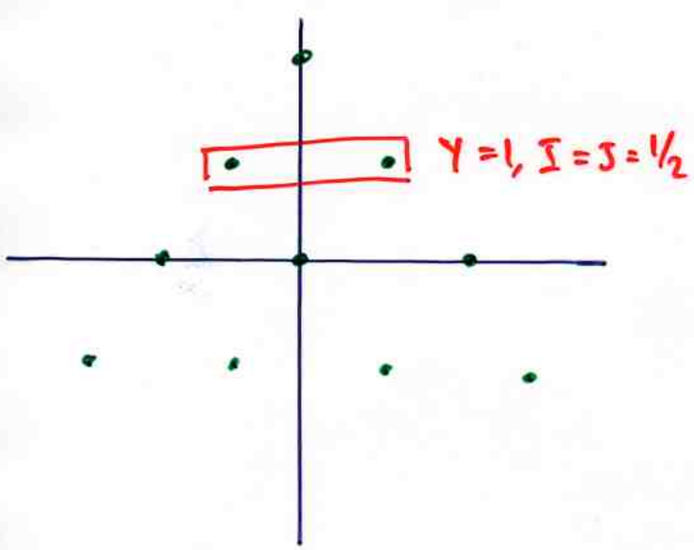
$$M = M_{cl} + \frac{N_c}{4\Phi} + \frac{3}{8\Omega}$$



(3,0) decuplet

$\left(3, \frac{N_c-3}{2}\right)$ at large N_c

$$M = M_{cl} + \frac{N_c}{4\Phi} + \frac{15}{8\Omega}$$

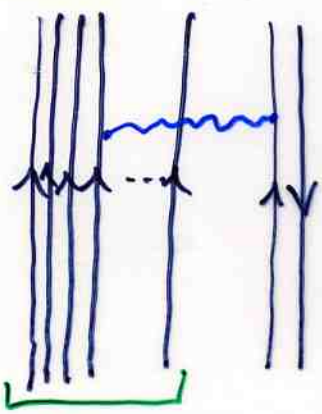


(0,3) antidecuplet

$\left(0, \frac{N_c+3}{2}\right)$ at large N_c

$$M = M_{cl} + \frac{N_c}{2\Phi} + \frac{3}{8\Omega} + \frac{3}{4\Phi}$$

The $\mathcal{O}(1)$ splitting is completely natural.



N_c

Line counting:

$$g^2 N_c \sim 1$$

More precisely, expand the collective coordinate Lagrangian to 2nd order in K and $\mathcal{O}(N_c^0)$. $\Omega \sim \Phi \sim N$.

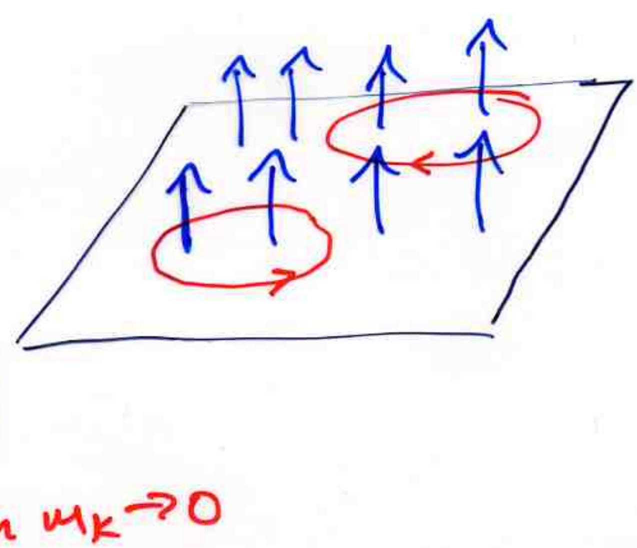
$$L \sim 2\Phi \dot{k}^\dagger \dot{k} + \frac{iN}{4}\left(k^\dagger \dot{k} - \dot{k}^\dagger k\right) - \frac{1}{2}\Gamma m_K^2 k^\dagger k$$

Compute equations of motion

$$4\Phi \ddot{k} - iN\dot{k} + \Gamma m_K^2 k = 0$$

$$k \sim e^{-i\omega t}$$

$$4\Phi\omega^2 + N\omega - m_K^2 \Gamma = 0$$



$$\Rightarrow \quad \omega = \frac{N}{8\Phi}\left(1 \pm \sqrt{1 + \frac{16 m_K^2 \Gamma \Phi}{N^2}}\right)$$

$$\rightarrow 0,\ \frac{N}{4\Phi} \sim \mathcal{O}(1) \text{ when } m_K \rightarrow 0$$

Splitting analogous to magnetic Landau levels. B-field comes from WZ term.

The "Bound State" Approach

Large kaonic fluctuations in the exotic sector make collective coordinate quantization dangerous. A more precise approach is to consider <u>all</u> fluctuations of the soliton (to quadratic order) and quantize them. (Callan & Klebanov)

- Recover "good" results in non-exotic sector
- <u>improvements</u> for non-exotics when SU(2) hyperfine splittings are included
- exotic feels "repulsive" force due to WZ term.

The $\mathcal{O}(1)$ corrections drastically modify the spectrum!

With

$$U = \sqrt{U_0}\, U_K \sqrt{U_0}$$

$$U_K = \exp\left(\frac{2i}{f_\pi}\sum_{a=4}^{7}\lambda_a K^a\right)$$

set

$$K(\vec{x},t) = \frac{1}{\sqrt{2}}\begin{pmatrix} K_4 - iK_5 \\ K_6 - iK_7 \end{pmatrix} = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \underbrace{k(r,t)}_{\text{kaon profile}}\, \underbrace{Y_{TLT_z}}_{SU(2)_I \text{ \& angular mtm eigenfunction}}$$

Find $(ef_\pi = 1)$

$$L = 4\pi\int dr\, r^2\Big[f(r)\dot{k}^\dagger\dot{k} + \overbrace{i\lambda(r)(k^\dagger\dot{k} - \dot{k}^\dagger k)}^{WZ} - h(r)\partial_r k^\dagger \partial_r k$$
$$- k^\dagger k\left(m_K^2 + V_{eff}(r,T,L)\right)\Big]$$

$\vec{T} = \vec{I} + \vec{L}$

$f(r)$, $\lambda(r)$, $h(r)$, $V_{eff}(r)$ are complicated functions of $F(r)$. Compute numerically.

The quadratic Lagrangian gives <u>linear</u> eqns of motion which you can solve numerically. Decompose into eigenmodes:

$$k(r,t) = \sum_{n>0} \underset{\uparrow}{\tilde{k}_n(r)} e^{i\tilde{\omega}_n t} b_n^\dagger + k_n(r) e^{-i\omega_n t} \underset{\uparrow}{a_n}$$

creates $\bar{K}$ annihilates K

$$\left.\begin{aligned} \left(f(r)\omega_n^2 \boxed{+ 2\lambda(r)\omega_n} + \theta\right) k_n &= 0 \\ \left(f(r)\tilde{\omega}_n^2 \boxed{- 2\lambda(r)\tilde{\omega}_n} + \theta\right) \tilde{k}_n &= 0 \end{aligned}\right]$$ equations of motion

WZ contributes with opposite sign.

$$\theta = \frac{1}{r^2}\frac{d}{dr} r^2 h(r) \frac{d}{dr} - m_K^2 - V_{eff}$$

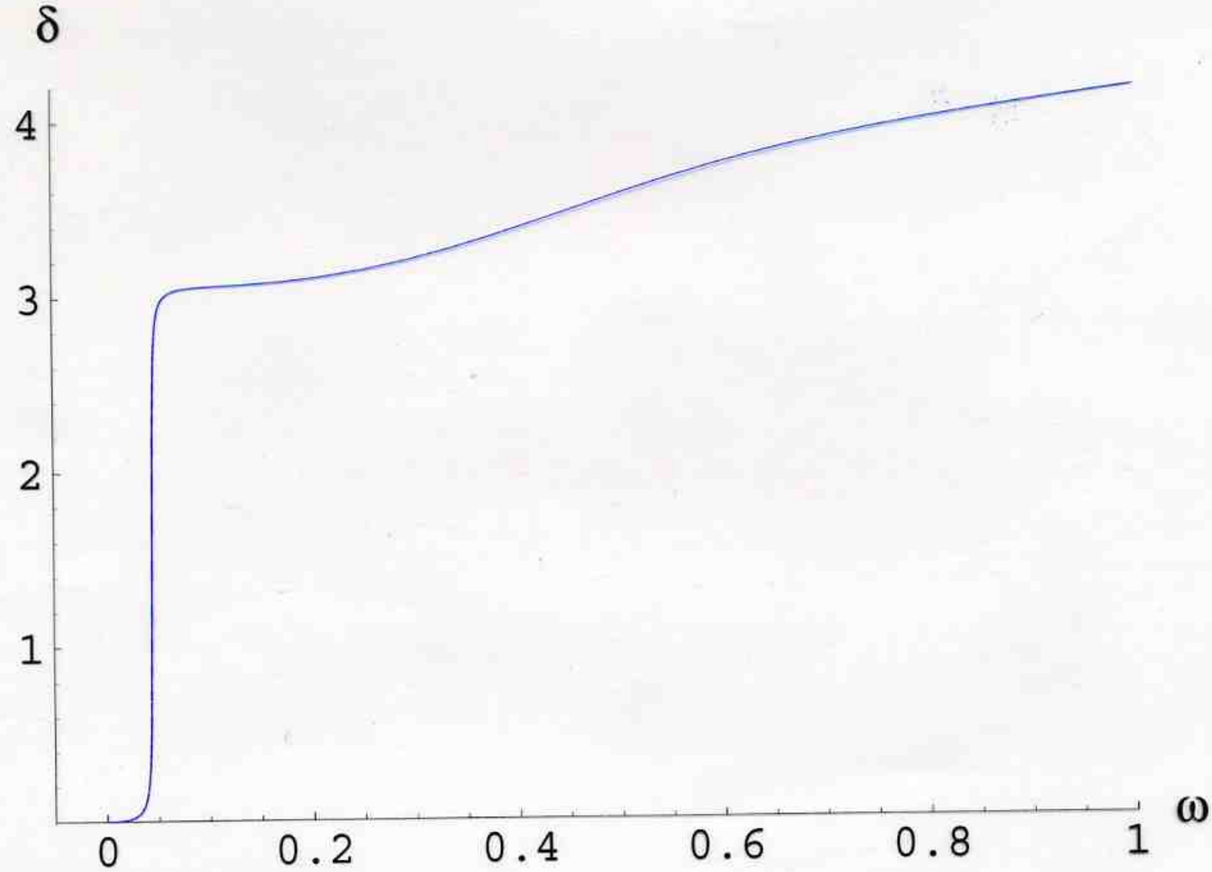
In the bound state approach, the input parameters are e, f_π, f_K, masses. With this input, everything is <u>calculated</u>. In particular, m_K is treated <u>exactly</u>, not in perturbation theory.

Particle	J	I	L	Mass (exp)	Mass (simple fit)
Λ	$\frac{1}{2}$	0	1	1115	1048
Σ	$\frac{1}{2}$	1	1	1190	1122
Σ	$\frac{3}{2}$	1	1	1385	1303
Λ	$\frac{1}{2}$	0	0	1405	1281

$$f_\pi = f_K = 129 \text{ MeV}, \quad e = 5.45, \quad m_\pi = 0.$$

Naturally reproduce $\Lambda(1405)$. All states are robust.

d-wave resonance



Phase shift as a function of energy in the L=2, T=3/2, S=-1 channel. The energy ω is measured in units of ef_π (with the kaon mass subtracted, so that ω=0 at threshold) and the phase shift δ is measured in radians. Here e=5.45 and f_π= 129 MeV.

Particle	J	I	L	Mass(exp)	Mass(th)
$\Lambda(D_{03})$	3/2	0	2	1520	1462
$\Sigma(D_{13})$	3/2	1	2	1670	1613
$\Sigma(D_{15})$	5/2	1	2	1775	1723

$M(1, 5/2) - M(0, 3/2) = 255$ (expt) $= 261$ (th)

$M(1, 3/2) - M(0, 3/2) = 150$ (expt) $= 151$ (th)

ratio ≈ 1.70 (expt) ≈ 1.73 (th)

Bound State & Exotic vs. Nonexotic in the Chiral Limit

For $m_\pi = m_K = 0$, look in the $I = 1/2$, $L = 1$ partial wave. The equations are analytically tractable!

In the non-exotic sector you find a bound state <u>exactly</u> at threshold. This is consistent: uds, udd, etc. should be degenerate in the limit of exact SU(3). Moreover, SU(2) rotator (hyperfine) energies agree exactly with the RR.

$$k(r) \sim \sin\left(\frac{F(r)}{2}\right)$$

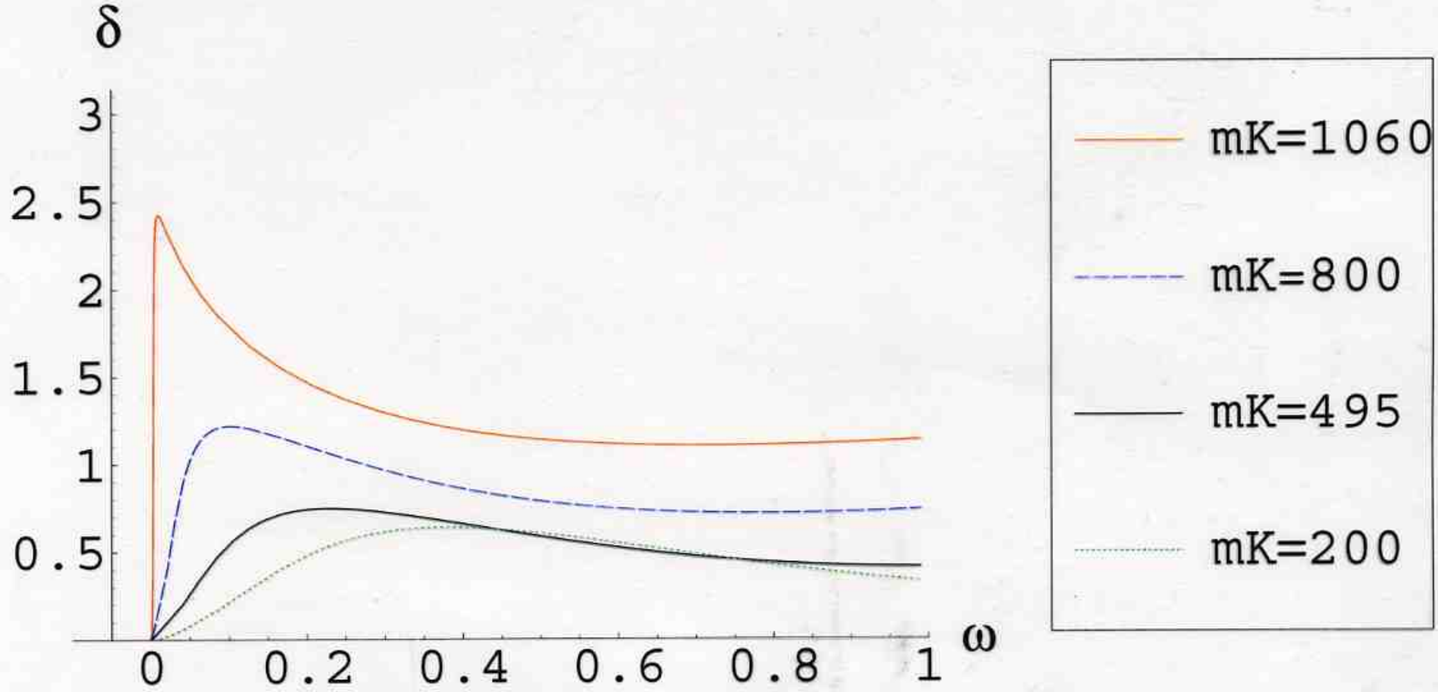
As you turn on m_K, the kaon field becomes more tightly bound to the skyrmion and the threshold state becomes a true bound state.

The exotic sector is completely different.

The equations are the same for $\omega = 0$, so there is still a bound state at threshold. However, <u>any</u> finite m_K destabilizes the solution. The WZ term contributes a repulsive force between the skyrmion and the kaon field.

Thus the threshold state is an <u>artifact</u> of exact SU(3). Collective coordinate quantization done naively differs dramatically from the full fluctuation treatment.

S=+1 Resonance?



Phase shifts as a function of energy in the L=1, T=1/2, S=+1 channel, for various values of the kaon mass mK. The energy ω is measured in units of ef_π (e=5.45, f_π=129 MeV) and the phase shift δ is measured in radians. ω=0 corresponds to the K-N threshold.

Play similar games varying the π mass, strength of WZ term, varying decay constants. All require adjustment of parameters far away from physical parameters to get a stable state.

It is certainly possible that adding terms to the Lagrangian leads to a stable Θ^+ (see Min, Park, Rho) but it is not a robust feature of the Skyrme model.

Charmed pentaquarks may be robust.

Why you might question everything I've told you

1. The Skyrme Model is an uncontrolled approximation. No separation of scales justifies truncating the chiral Lagrangian.

   This is true. However, the Skyrme model captures enough physics to be a reasonable toy model. Any predictions it makes which are model-dependent are suspect, but generic predictions may be reliable, as well as "model-independent" relations in the $1/N_c$ sense.

2. $1/N_c$ expansion has no intrinsic phenomenological relevance.

   Partly true.

   - $1/N$ is a sensible perturbative expansion, and nothing else is available!
   - "Model-independent" relations in which subleading $1/N$ corrections cancel are satisfied to high accuracy.
   - $\frac{e^2}{4\pi} = \frac{1}{137} \Rightarrow e = 1/3.3$ (Witten)

3. Perturbation theory in m_s won Gell-Mann the Nobel Prize. Why can't it work for me?

   - Even before you include $SU(3)$ breaking the exotic sector is split from the non-exotic sector by $O(1)$ effects. Physical intuition from non-exotic physics may be misleading!
   - Binding is invisible from PT.

4. What about the Roper resonance? In soliton models we often identify N(1440) with a radial excitation of the soliton. In the simple Skyrme model this state is unbound. If you raise m_π (past the physical value) the state is bound (Breit & Nappi, others) and one can obtain a resonance by adding terms to the chiral Lagrangian. Could similar effects bind Θ^+?

The short answer is yes, such a mechanism is possible. However, it falls in the category of non-generic predictions. Moreover, one would not expect a small width ($\Gamma_{N(1440)} \sim 250$ MeV.)