Mixing and decays of the antidecuplet in the context of approximate SU(3) symmetry

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Motivation

- Approximate flavor SU(3) symmetry of strong interactions allows to group all hadrons into certain multiplets. Only singlets, octets ($\mathbf{8}$) and decuplets ($\mathbf{10}$) were believed to be realized in Nature.

  The discoveries of the Θ^+ and Ξ^{--}, if confirmed, mean the existence of a new physical multiplet: the antidecuplet ($\overline{10}$).

- Approximate SU(3) symmetry works surprisingly well: Mass splittings and partial decay widths of all baryon multiplets (singlets, octets, decuplets) are described and predicted with good accuracy: Gell-Mann and Ne'eman 1964; Kokkedee 1969;
  Samios, Goldberg, Meadows 1974.

- Approximate SU(3) suits best for establishing in a model-independent way the overall structure of a given SU(3) multiplet: Mass splittings, necessity of mixing with other multiplets due to SU(3) breaking, correlations between partial decay widths.

- This is exactly what one needs for the antidecuplet: a reliable overall picture of $\overline{10}$ and its mixing with other multiplets and a way to sort out the present experimental info on the $\overline{10}$ decays.

- Since SU(3) is broken, states from different multiplets with the same spin and parity can mix. Because of the small width of Θ^+, even small mixing dramatically affects predictions for the $\overline{10}$ decays.

  At the same time, small mixing with $\overline{10}$ affects very little non-exotic multiplets. One can use the results of SU(3) analysis of the non-exotic multiplets (three octets in our case) in the SU(3) analysis of $\overline{10}$ decays.

- After the SU(3) picture of $\overline{10}$ is established using the scarce experimental info on $\overline{10}$ decays, one can make model-independent predictions for unmeasured decays and assess available models of the $\overline{10}$ mixing.

Antidecuplet mixing with three octets

We consider the scenario that $\overline{10}$ mixes with three $J^P = 1/2^+$ octets:
the ground-state octet,
the octet containing $N(1440)$ $\Lambda(1600)$, $\Sigma(1660)$ and $\Xi(1690)$,
the octet containing $N(1710)$, $\Lambda(1800)$, $\Sigma(1880)$ and $\Xi(1950)$.

The mixing takes place through the $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ and the corresponding N and Σ octet states:

$$\begin{pmatrix} |N_1^{\rm phys}\rangle \\ |N_2^{\rm phys}\rangle \\ |N_3^{\rm phys}\rangle \\ |N_{\overline{10}}^{\rm phys}\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \sin\theta_1 \\ 0 & 1 & 0 & \sin\theta_2 \\ 0 & 0 & 1 & \sin\theta_3 \\ -\sin\theta_1 & -\sin\theta_2 & -\sin\theta_3 & 1 \end{pmatrix} \begin{pmatrix} |N_1\rangle \\ |N_2\rangle \\ |N_3\rangle \\ |N_{\overline{10}}\rangle \end{pmatrix} .$$

- We assume that θ_i mixing angles are small, $\theta_i = \mathcal{O}(\epsilon)$, where ϵ is a small parameter of SU(3) breaking. We systematically neglect $\mathcal{O}(\epsilon^2)$ terms.

- The $|N_1\rangle$, $|N_2\rangle$ and $|N_3\rangle$ states can mix among themselves, i.e. they can belong to several different octets. Using the χ^2 fit to the measured decays, we find that $|N_2\rangle$ and $|N_3\rangle$ states are slightly mixed and that the $|N_1\rangle$ is unmixed.

  After this is taken into account, it is sufficient to consider only the mixing of each individual $|N_i^{\rm phys}\rangle$ with $|N_{\overline{10}}^{\rm phys}\rangle$.

- The mixing angles θ_i and θ_i^{Σ} are related,

$$\sin\theta_i \left(N_i^{\rm phys} - N_{\overline{10}}^{\rm phys}\right) = \sin\theta_i^{\Sigma} \left(\Sigma_i^{\rm phys} - \Sigma_{\overline{10}}^{\rm phys}\right) \, ,$$

  which becomes $\theta_i = \theta_i^{\Sigma}$ ignoring $\mathcal{O}(\epsilon^2)$ terms.

- Gell-Mann–Okubo mass formulas, which describe the mass splitting inside SU(3) multiplets, are not sensitive to small mixing

$$N_i^{\rm phys} \equiv \langle N_i^{\rm phys}|\hat{M}|N_i^{\rm phys}\rangle = N_i + \sin^2\theta_i N_{\overline{10}} = N_i + \mathcal{O}(\epsilon^2)\,,$$

It is not legitimate to estimate the mixing angles from the Gell-Mann–Okubo mass formula.

Instead, one has to consider decays which contain $\mathcal{O}(\epsilon)$ terms.

- We assume that SU(3) symmetry is violated by non-equal masses inside a given multiplet and mixing among multiplets and that SU(3) is exact in decay vertices $\rightarrow$ two universal SU(3) coupling constants for $\overline{10}$ and three mixing angles.

General expressions for $\overline{10}$ couplings: Γ_{Θ^+} and $G_{\overline{10}}$

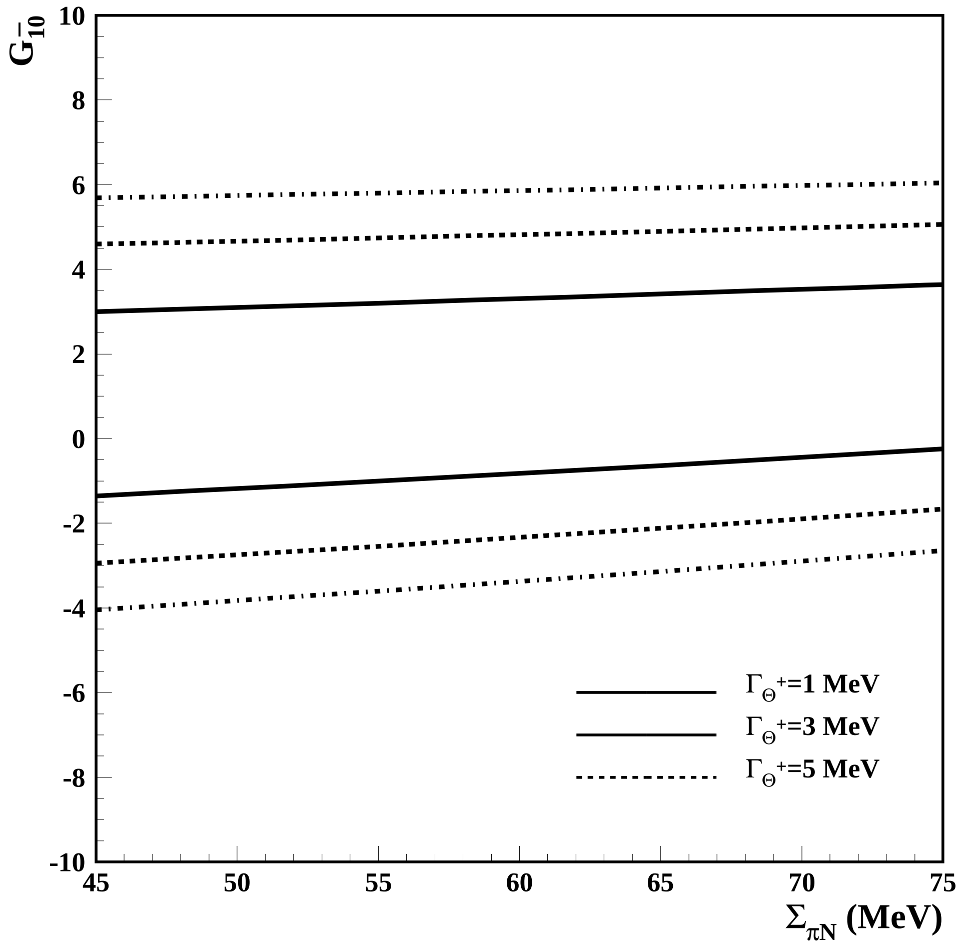
In our analysis, Γ_{Θ^+} and $\Sigma_{\pi\, N}$ are external parameters, which are varied in the following intervals: $1 \leq \Gamma_{\Theta^+} \leq 5$ MeV; $45 \leq \Sigma_{\pi\, N} \leq 75$ MeV.

$\Sigma_{\pi\, N}$ determines the θ_1 mixing angle with the ground state octet;
Γ_{Θ^+} determines the $G_{\overline{10}}$ and $H_{\overline{10}}$ $\left(H_{\overline{10}} = 2\, G_{\overline{10}} - 18\right)$ coupling constants through the $\Theta^+ N\, K$ coupling

Praszalowicz, hep-ph/0402038; R.A. Arndt *et al.*, PRC 69 (2004) 035208

$$g_{\Theta^+ N\, K} = \frac{1}{\sqrt{5}} \left(G_{\overline{10}} + \sin\theta_1 H_{\overline{10}} \frac{\sqrt{5}}{4} \right)$$

At given Γ_{Θ^+} and $\Sigma_{\pi\, N}$, the $\overline{10}$ decays are parameterized in terms of the θ_2 and θ_3 mixing angles, which are free parameters.



$G_{\overline{10}}^{-}$
10
8
6
4
2
0
-2
-4
-6
-8
-10
45
50
55
60
65
70
75
$\Sigma_{\pi N}$ (MeV)
Γ_{Θ^+}=1 MeV
Γ_{Θ^+}=3 MeV
Γ_{Θ^+}=5 MeV

General expressions for $N_{\overline{10}}$ couplings

$$g_{N_{\overline{10}}N\,\pi}=\frac{1}{2\sqrt{5}}\left(G_{\overline{10}}+\sin\theta_1\left(H_{\overline{10}}\frac{\sqrt{5}}{4}-G_8\frac{7}{\sqrt{5}}\right)-\sum_{i=2,3}\sin\theta_i\,g_{N_iN\,\pi}\right)\,,$$

$$g_{N_{\overline{10}}N\,\eta}=\frac{1}{2\sqrt{5}}\left(-G_{\overline{10}}+\sin\theta_1\left(H_{\overline{10}}\frac{\sqrt{5}}{4}-G_8\frac{1}{\sqrt{5}}\right)+\sum_{i=2,3}\sin\theta_i\,g_{N_iN\,\eta}\right)\,,$$

$$g_{N_{\overline{10}}\Lambda\,K}=\frac{1}{2\sqrt{5}}\left(G_{\overline{10}}+\sin\theta_1 G_8\frac{4}{\sqrt{5}}+\sum_{i=2,3}\sin\theta_i\,g_{N_i\Lambda\,K}\right)\,,$$

$$g_{N_{\overline{10}}\Delta\,\pi}=\frac{2}{\sqrt{5}}\left(\sin\theta_1 G_8+\sum_{i=2,3}\sin\theta_i\,g_{N_i\Delta\,\pi}\right)$$

- The $g_{N_iB\,P}$ coupling constants are determined by the χ^2 fit to the measured decays of the octets; $g_{N_iB\,P}>G_{\overline{10}}$.

- Important correlation: Mixing with the octets can decrease $g_{N_{\overline{10}}N\,\pi}$ and simultaneously increase $g_{N_{\overline{10}}N\,\eta}$.

- The $N_{\overline{10}}\Delta\,\pi$ decay is possible only due to mixing.

What is presently known about $N_{\overline{10}}$?

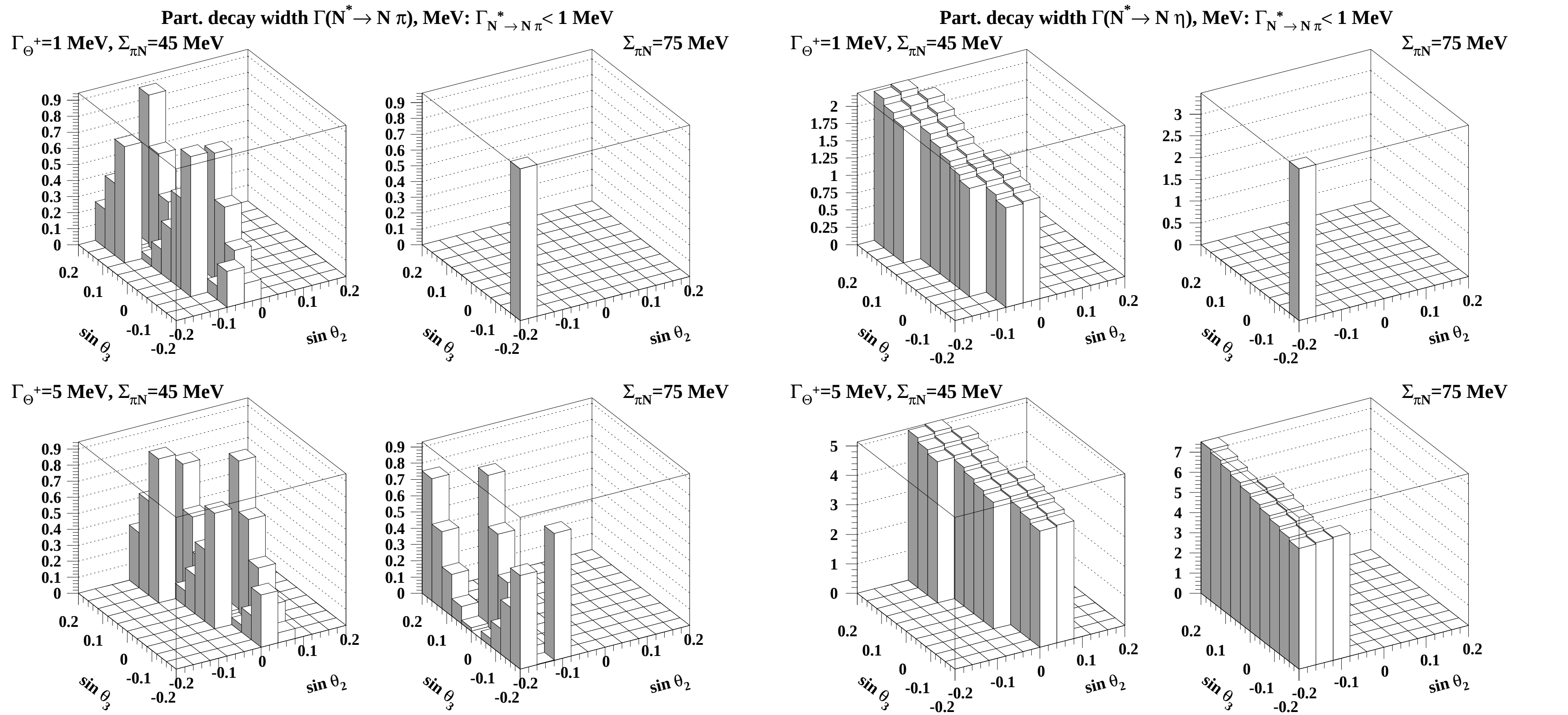
- The PWA analysis of R.A. Arndt *et al.*, PRC 69 (2004) 035208 gives two candidate states with masses 1680 MeV and 1730 MeV. Both states should have $\Gamma_{N_{\overline{10}} \to N\,\pi} \leq 0.5$ MeV.

- GRAAL observes a narrow nucleon resonance near 1670 MeV in the reaction $\gamma\, n \to n\, \eta$ and no resonance in $\gamma\, p \to p\, \eta$ V. Kuznetsov for the GRAAL Collab., hep-ex/0409032. Interpretation: $\Gamma_{N_{\overline{10}} \to N\,\eta}$ should not be too small.

  The much stronger photocoupling to the neutron than to the proton is a benchmark property of the $N_{\overline{10}}$ state, as predicted in the chiral quark soliton model Polyakov and A. Rathke, Eur. Phys. J. A **18**, 691 (2003)

- STAR observes a narrow peak at 1734 MeV and only a weak indication of a narrow peak at 1693 MeV in the $\Lambda\, K_S$ invariant mass S. Kabana for the STAR Collab., hep-ex/0406032. Interpretation: $\Gamma_{N_{\overline{10}} \to \Lambda\, K}$ is possibly suppressed.

Identifying $N_{\overline{10}}$ with the GRAAL's $N(1670)$, we find that this picture of $N_{\overline{10}}$ decays can be realized by suitable choice of θ_i. Imposing the $\Gamma_{N_{\overline{10}} \to N\,\pi} \leq 1$ MeV cut, we find unsuppressed $\Gamma_{N_{\overline{10}} \to N\,\eta}$ and somewhat suppressed $\Gamma_{N_{\overline{10}} \to \Lambda\,K}$ (anyway suppressed by the phase space).



General expressions for $\Sigma_{\overline{10}}$ couplings

$$g_{\Sigma_{\overline{10}}\Lambda\,\pi}=\frac{1}{2\sqrt{5}}\left(G_{\overline{10}}-\sin\theta_1^{\Sigma}G_8\frac{3}{\sqrt{5}}-\sum_{i=2,3}\sin\theta_i^{\Sigma}\,g_{\Sigma_i\Lambda\,\pi}\right)\,,$$

$$g_{\Sigma_{\overline{10}}\Sigma\,\pi}=\frac{1}{\sqrt{30}}\left(G_{\overline{10}}+\sin\theta_1\left(H_{\overline{10}}\frac{\sqrt{5}}{2}-G_8\sqrt{5}\right)-\sum_{i=2,3}\sin\theta_i^{\Sigma}\,g_{\Sigma_i\Sigma\,\pi}\right)\,,$$

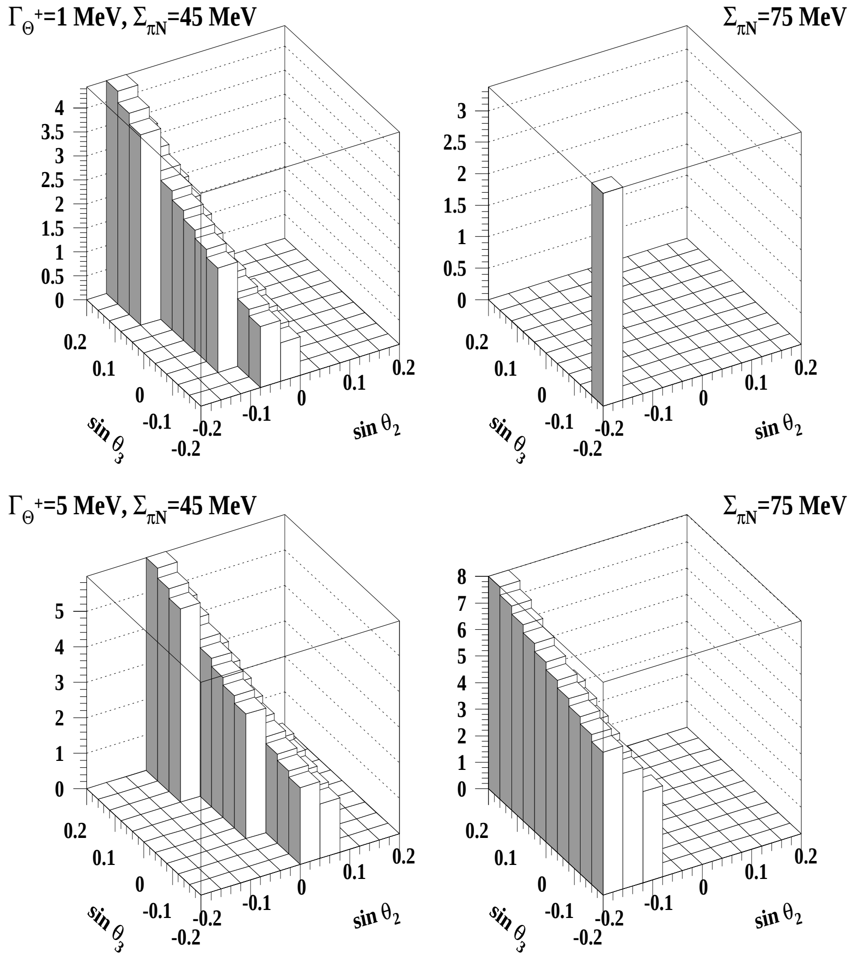
$$g_{\Sigma_{\overline{10}}N\,\overline{K}}=\frac{1}{\sqrt{30}}\left(-G_{\overline{10}}+\sin\theta_1 H_{\overline{10}}\frac{\sqrt{5}}{2}+\sin\theta_1^{\Sigma}G_8\frac{4}{\sqrt{20}}+\sum_{i=2,3}\sin\theta_i^{\Sigma}\,g_{\Sigma_i N\,\overline{K}}\right)\,,$$

$$g_{\Sigma_{\overline{10}}\Sigma_{10}\,\pi}=\frac{\sqrt{30}}{15}\left(G_8\sin\theta_1+\sum_{i=2,3}\sin\theta_i^{\Sigma}\,g_{\Sigma_i\Sigma_{10}\,\pi}\right)$$

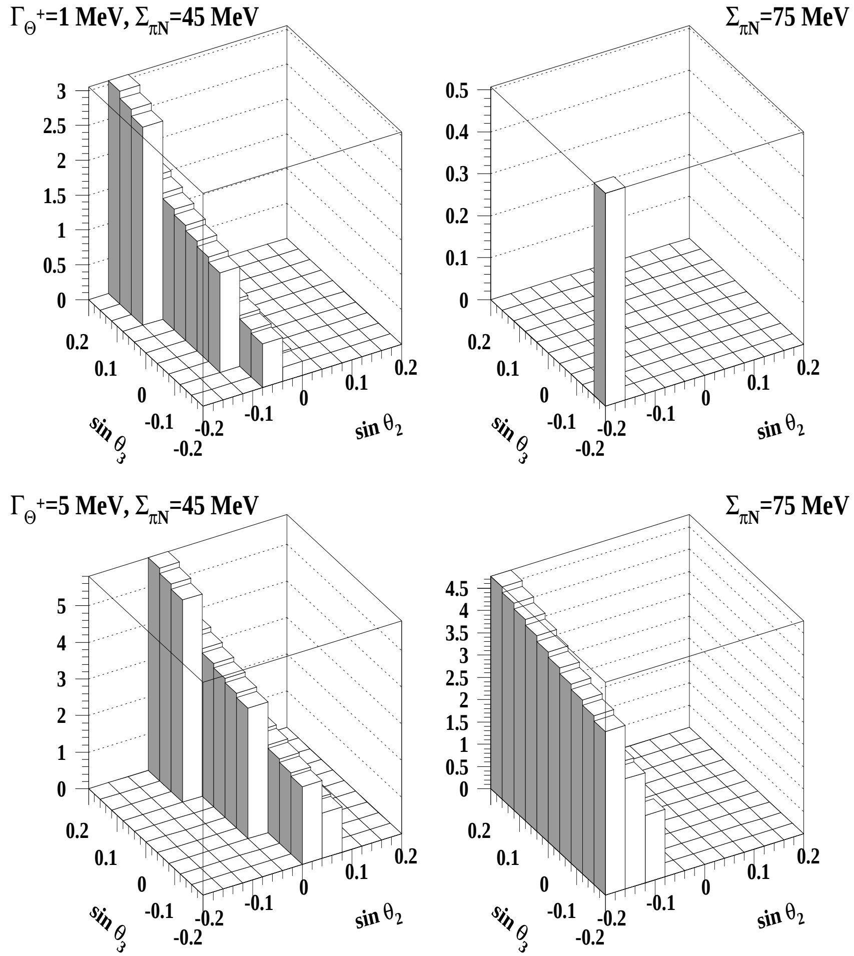
There are no distinct correlations among the partial decay widths when $\theta_{2,3}$ are free.

Imposing the $\Gamma_{N_{\overline{10}}\to N\,\pi}\leq 1$ MeV cut, one can have e.g. $\Gamma_{\Sigma_{\overline{10}}N\,\overline{K}}>\Gamma_{\Sigma_{\overline{10}}\Lambda\,\pi},\Gamma_{\Sigma_{\overline{10}}\Sigma\,\pi}\to\Sigma_{\overline{10}}$ looks like a narrow $\Sigma(1770)$.

Part. decay width $\Gamma(\Sigma^* \to NK)$, MeV: $\Gamma_{N^* \to N\pi} < 1$ MeV



Part. decay width $\Gamma(\Sigma^* \to \Lambda\pi)$, MeV: $\Gamma_{N^* \to N\pi} < 1$ MeV



What is presently known about $\Sigma_{\overline{10}}$?

- This is the least known member of $\overline{10}$. We use $m_{\Sigma_{\overline{10}}} = 1765$ MeV (equally spaced between the $N(1670)$ and $\Xi^{--}(1862)$).

- There are four experiments, where the Θ^+ was observed as a peak in the $p\,K_S$ invariant mass. Since $\Sigma_{\overline{10}}$ decays in the same final state ($N\,\overline{K}$), the four experiments give direct information on the $\Sigma_{\overline{10}} \to N\,\overline{K}$ decay!

- The analysis of A.E. Asratyan, A.G. Dolgolenko and M.A. Kubantsev, Phys. At. Nucl. **67**, 682 (2004) reveals peaks the $1650 < M_{p\,K_S} < 1850$ MeV mass range.

- The analysis of SVD Collab., A. Aleev *et al.*, hep-ex/0401024 contains at least two prominent peaks in the 1700-1800 MeV mass range, before the cuts.

- HERMES and ZEUS $p\,K_S$ invariant mass spectra extend only up to 1.7 MeV.

A possibly narrow $\Sigma_{\overline{10}}(1765)$ should be searched for in the available data!

Discussion

In order to assess the theoretical uncertainty of our predictions, we add an additional mixing of the antidecuplet with a 27-plet J. Ellis, M. Karliner, M. Praszalowicz, JHEP 0405 (2004) 002.

We arrive at two scenarios. In the first case, the picture of the $N_{\overline{10}}$ decays does not change. The correlations between the $\Sigma_{\overline{10}}$ decay widths do change $\rightarrow$ impossible to identify $\Sigma_{\overline{10}}$ with $\Sigma(1770)$.

In the second case, the picture of $N_{\overline{10}}$ decays is only marginally compatible with experiment. The pattern of $\Sigma_{\overline{10}}$ decays is similar to $\Sigma(1770)$. Both $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ states are very narrow.

We find that the scenario of large (ideal) mixing is incompatible with the experimental data Jaffe and Wiczek, PRL 91 (2003) 042003; D.K. Hong, hep-ph/0412132. Identifying $N_{\overline{10}}$ with $N(1710)$, which is ideally mixed with the Roper $N(1440)$, the χ^2 fit fails to simultaneously accommodate $\Gamma_{\Theta^+} \leq 10$ MeV and a large $\Gamma_{N(1440)\rightarrow N\,\pi}$. An acceptably low χ^2 can be only found assuming very small mixing, $|\theta_N| \approx 4^0$.

Conclusions

We considered mixing of the antidecuplet with three $J^P = 1/2^+$ octets in the framework of approximate flavor SU(3) symmetry and in the limit of small mixing angles.

We presented general expressions for the $\overline{10}$ partial decay widths as functions of the three mixing angles and Γ_{Θ^+}.

Identifying $N_{\overline{10}}$ with the $N(1670)$ observed by GRAAL, we arrive at the following picture of $\overline{10}$ decays: Θ^+ could be as narrow as 1 MeV; the $N_{\overline{10}} \to N\,\eta$ decay is sizable, while the $N_{\overline{10}} \to N\,\pi$ decay is suppressed and the $N_{\overline{10}} \to \Lambda\,K$ decay is possibly suppressed.

Constraining the mixing angles by the $N_{\overline{10}}$ decays, we make predictions for the $\Sigma_{\overline{10}}$ decays. $\Sigma_{\overline{10}}(1765)$ could be searched for in the available data on $K_S\,p$ invariant mass spectrum, which already revealed the Θ^+ peak.

It is important to experimentally verify the decay properties of $\Sigma(1770)$ because its mass and $J^P = 1/2^+$ make it an attractive candidate for $\Sigma_{\overline{10}}$.