Pentaquark hadrons from lattice QCD

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1. Lattice QCD and hadron spectroscopy

2. Separation of scattering states

3. Results

4. Conclusions

Lattice can identify low-lying hadronic states, starting from QCD with systematically controllable approximations.

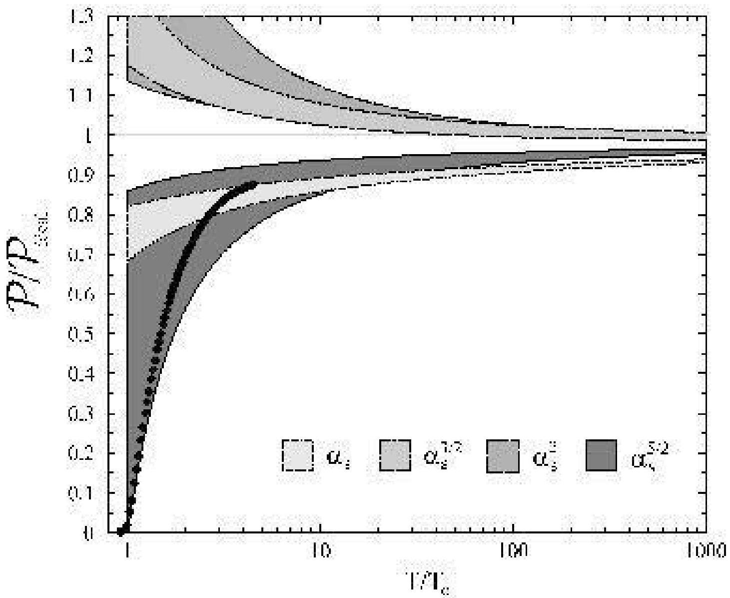
- Does QCD predict exotic states?
- If yes, what are the experimentally unkwnon quantum numbers?
- First opportunity for the lattice to predict new particles.

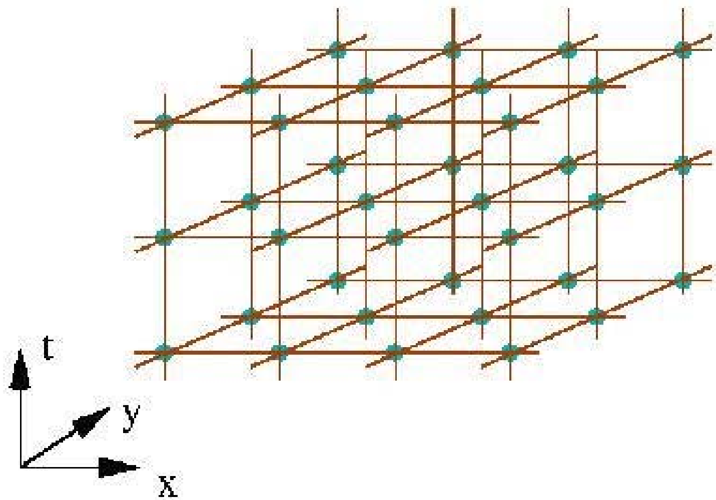
This talk

- Why is it not so easy?
- How to interpret the results?
- Available (still inconclusive) results.

Review: F. Csikor, Z. Fodor, S.D. Katz and TGK, hep-lat/0407033.

systematic approach: SPACE-TIME LATTICE





quantummechanics with path integral: add exp(iS) for each path

quantum field theory: add exp(iS) for each field configuration

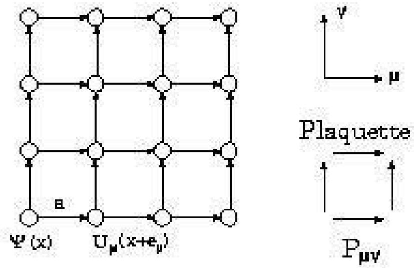
Euclidean space ($t=i\tau$): exp(-S) sum of Boltzmann factors

formally a four-dimensional statistical system

10^{10} dimensional integral $\approx$ 1000 billion operations per second

$\Longrightarrow$ CPU power of a supercomputer

notation: lattice action of QCD and Monte-Carlo techniques



$$\mathcal{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}(D_\mu\gamma^\mu + m)\psi$$

anti-commuting $\psi(x)$ quark fields live on the sites
gluon fields, $A^a_\mu(x)$ are used as links and plaquettes

$$U(x,y) = \exp\left(ig_s \int_x^y dx'^\mu\, A^a_\mu(x')\lambda_a/2\right)$$

$$P_{\mu\nu}(n) = U_\mu(n)U_\nu(n+e_\mu)U^\dagger_\mu(n+e_\nu)U^\dagger_\nu(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = 6/g_s^2 \cdot \textstyle\sum_{n,\mu,\nu}\left[1 - \mathrm{Re}(P_{\mu\nu}(n))\right]$$

a$\longrightarrow$0 limit reproduces the continuum value upto O(a^2)

differencing scheme for quarks:

$$\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) \rightarrow \bar{\psi}_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$
$$\bar{\psi}(x)\gamma^\mu D_\mu\psi(x) \rightarrow \bar{\psi}_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \ldots$$

chemical potential acts: $\mu a\bar{\psi}_x\gamma_4\psi_x$
fourth component of an imaginary(!), constant vector potential

fermionic part as a bilinear expression: $S_f = \bar{\psi}_n M_{nm}\psi_m$

Euclidean partition function gives Boltzman weights

$$Z = \int \prod_{n,\mu}[dU_\mu(x)][d\bar{\psi}_n][d\psi_n]e^{-S_g - S_f} = \int \prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\,\mathrm{det}(M[U])$$

Metropolis step:

$$P(U \rightarrow U') = \min\left[1, \exp(-\Delta S_g)\,\mathrm{det}(M[U'])/\,\mathrm{det}(M[U])\right]$$

for μ=0 the determinant is real, for $\mu\neq$0 it is complex
$\Rightarrow$ no probability interpretation, no Monte-Carlo method

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a "particle" at time 0 and the same "particle" at time t
$\Rightarrow$ compute the Euclidean correlation function of a composite op. $\mathscr{O}$:

$$\langle 0|\mathscr{O}(t)\mathscr{O}^\dagger(0)|0\rangle$$

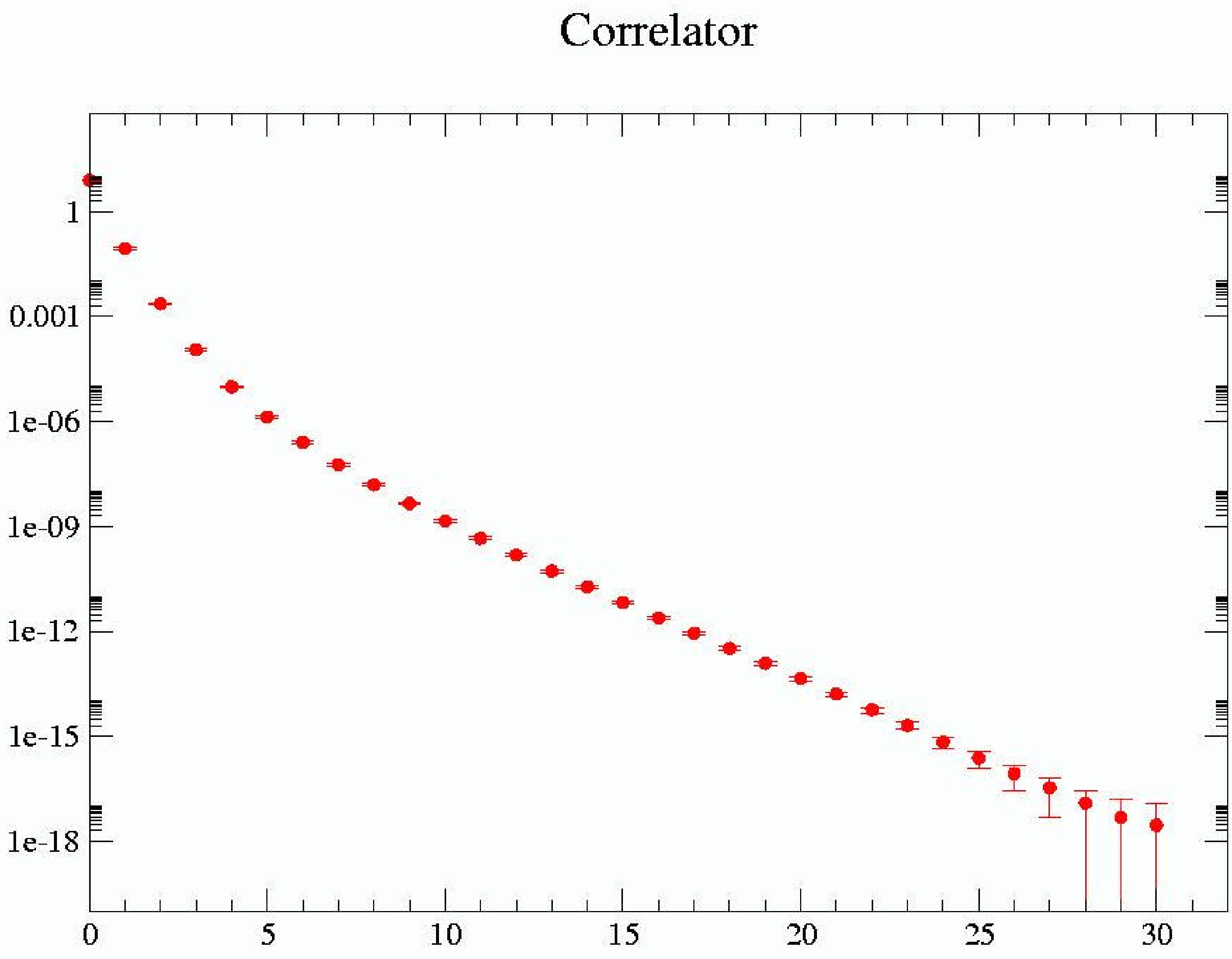
insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0|\mathrm{e}^{-Ht}\ \mathscr{O}(0)\ \mathrm{e}^{Ht}|i\rangle\langle i|\mathscr{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathscr{O}^\dagger(0)|i\rangle|^2\ \mathrm{e}^{-(E_i-E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

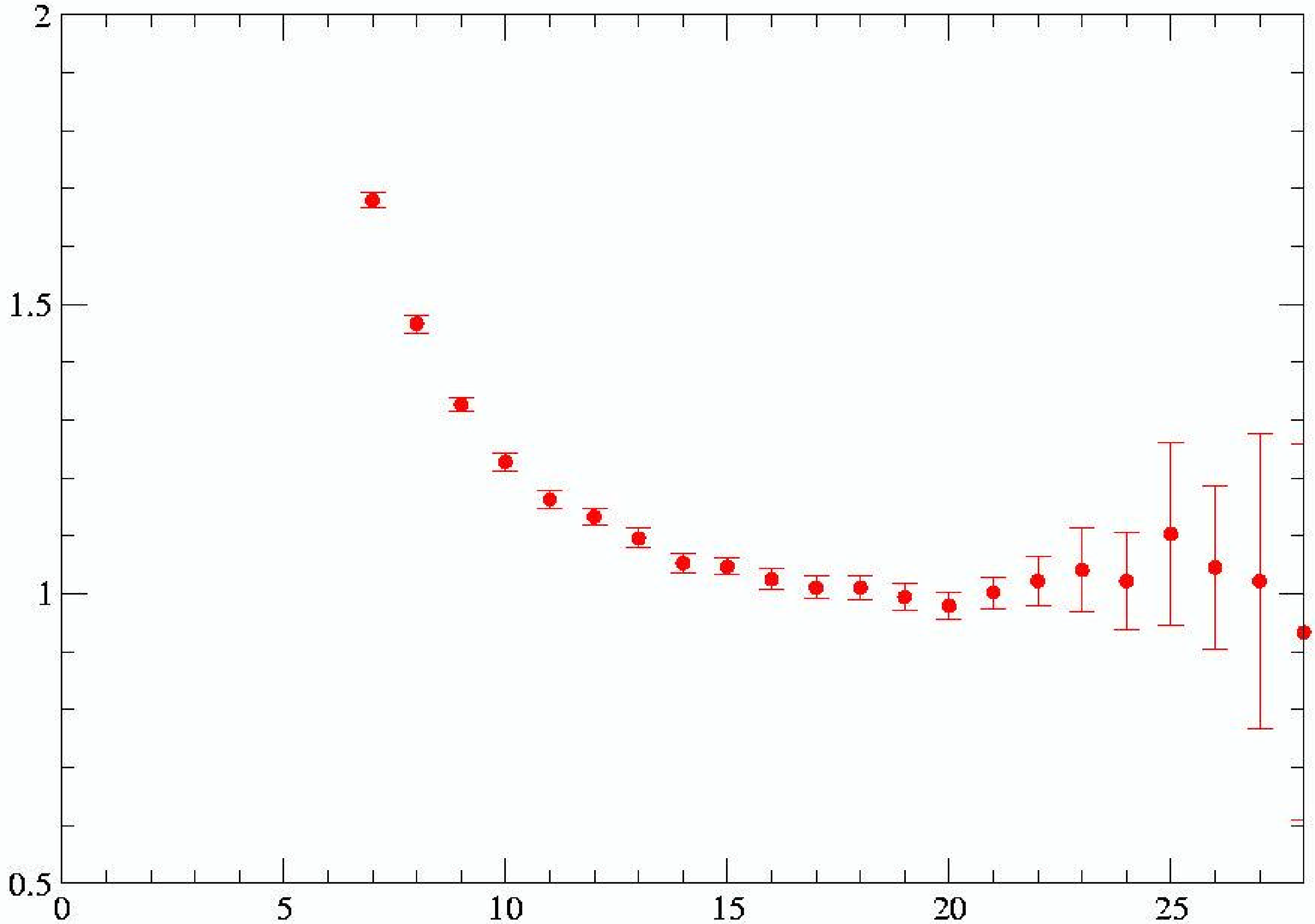
and $$\mathscr{O}(t) = \mathrm{e}^{-Ht}\ \mathscr{O}(0)\ \mathrm{e}^{Ht}.$$

t large $\Rightarrow$ Lightest states (created by $\mathscr{O}$) dominate.
$\Rightarrow$ Exponential fits give E_i's

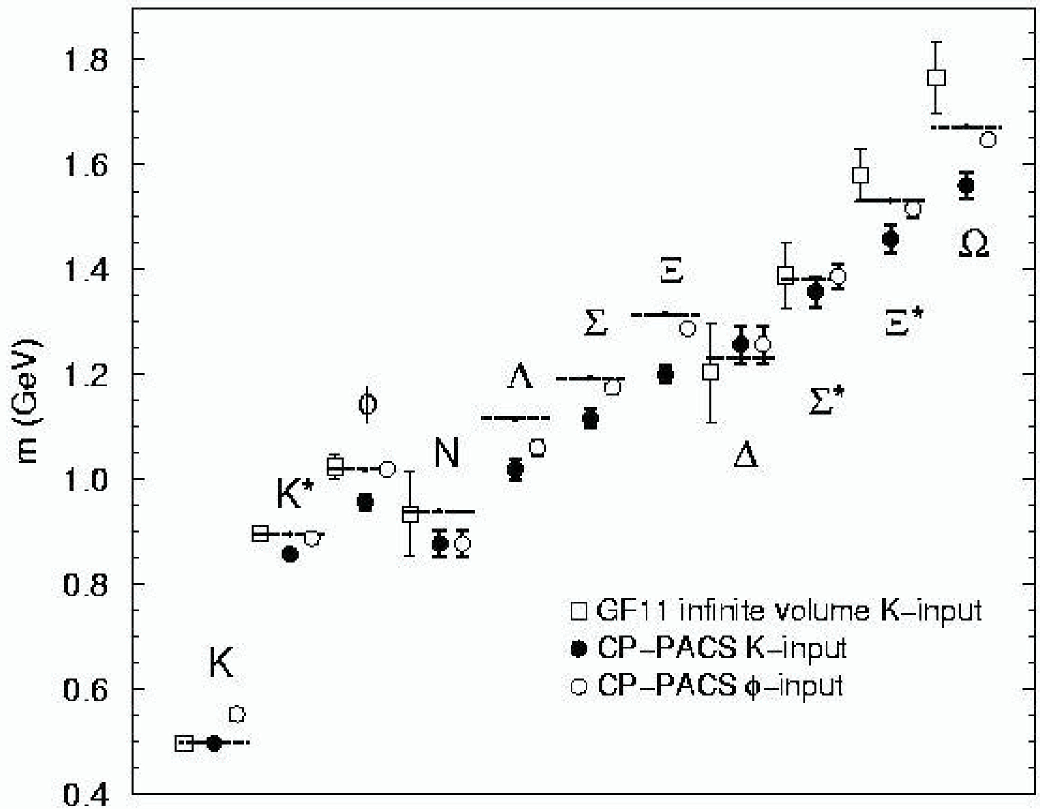


Correlator
1
0.001
1e-06
1e-09
1e-12
1e-15
1e-18
0
5
10
15
20
25
30

Effective mass



hadronic spectrum: no more no less than the detected one



lattice results show confinement

no sign of free quarks or gluons

• unfortunatley Monte-Carlo techniques work only in vacuum, since 20 years no results at finite density (chemical potential)

• Peter Blau (USA Sociologist 1964) "exchange theory".

extremely successful Paradigm (history, law, sociology etc.)

dominant binary code (everywhere):

maximizing benefits

high performance computing: number of operations (Tflop).

we optimize the price/performance ratio

does not matter, whether you are rich or poor

"supercomputer" based on PC-s for lattice gauge theory

PC-s are very cost effective: the market is huge

Linux: free operating system, compilers, add as little as possible

Communication hardware and driver

- PC-cluster development: local field theory

optimal price/performance, uses the PC-market: $1/Mflops

Computer Physics Communication 134 (2001) 139, [hep-lat/9912059]

standard PC-s in a 3 dimensional mesh (periodic)

next neighbours reached by a self-made communication card

lesson: PC industry is faster, we have to use them

Computer Physics Communication 152 (2003) 121, [hep-lat/0202030]

1. driving force: video-games $\Rightarrow$ rotation SO(3) group

locally isomorf to SU(2) group $\Rightarrow$ action can be calculated

2. internet music/video download: gigabit ethernet (switch)

cross twisted cables: connect 2 PC-s; 4 cards: 2 dim. mesh

Budapest architecture, several copies,

Wuppertal: (HBFG) $\approx$ 1.5 million Euro

>1000 CPU, largest research cluster on the continent

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How to choose $\mathcal{O}$?

Has to have quantum numbers of the desired state for $\langle i|\mathcal{O}(0)|0\rangle \neq 0$.

- Internal quantum numbers, spin (later)
- Total momentum
  Projection to zero total momentum: $\sum_{\vec{x}} \mathcal{O}(0,\vec{x})$
  - Single particle state $\Rightarrow E_1 = m$
  - 2-particle "continuum" $\Rightarrow E_1 = m + M$,
    $E_2 = (m^2 + p^2)^{1/2} + (M^2 + p^2)^{1/2}$, relative momentum can be $p_i = 2\pi k_i / L$.
- Large overlap with desired state $\Rightarrow$ smaller statistical errors.
- Small overlap with competing states. (Nucleon + Kaon)
- Essential to build in knowledge about the wave function!

If you see nothing, either there is nothing or you chose the wrong operator.
Lattice calculations cannot disprove the existence of a state!

Spatial wave function (why can't we do arbitrary w.f.'s?)

How is the correlator $C(t)$ computed?

Simple example: pion $\mathcal{O} = \bar{u}\gamma_5 d$

$$\langle 0|\bar{u}(x)\gamma_5 d(x) \quad \bar{d}(0)\gamma_5 u(0)|0\rangle$$

Use Wick's theorem to decompose into sums of products of the form

$$\langle 0|d_\alpha(x)\bar{d}_\beta(0)|0\rangle = (D+m)^{-1}_{x\alpha,0\beta}$$

General wave function:

$$\mathcal{O}(0) = \int d^3x \int d^3y \, f(\vec{x},\vec{y}) \, \bar{u}(\vec{x},0)\gamma_5 d(\vec{y},0)$$

For arbitrary wave fn., quark propagators $D^{-1}(x\alpha, y\beta)$ (from any point to any other point) are needed.

$\Rightarrow$ Order 10^{13} matrix elements i.e. $\approx$ 100 Tbytes to be stored

Solution: We store only $\Sigma_{\vec{y}\beta}(D+m)^{-1}_{\vec{x},0,\alpha;\vec{y},0,\beta}\psi_\beta(\vec{y})$ for some ψ's

Consequences: $\mathcal{O}$ can be built only as

- a product of 1-quark wave functions
- each 1-quark wave function requires "only" 12 Dirac op. inversions
- Iterative sparse matrix techniques can be used
- Typical lattice spectroscopy uses only one source $\psi(x)$, all wave functions are built out of this.
- Extended "source" wave function $\Longrightarrow$ smaller overlap with higher states.
- Same propagator can be used for u and d quark.

Spin, flavour, colour structure

Index summations exponentially more expensive with the number of quarks

- 3q baryons: summation negligible compared to inversion

$\Rightarrow$ • Simplest 5q operators: summation is order 50%

Pentaquark operators used so far on the lattice:

- All with one type of quark wave function
- $\mathcal{O}$ is a product of 1-particle states
- A few different spin, flavour, colour structures

Spin, flavour, colour (continued)

- $\mathcal{O}_{I=0/1} = \varepsilon_{abc}\,[u_a^T C\gamma_5 d_b]\,\{u_e\,\bar{s}_e i\gamma_5 d_c \mp (u \leftrightarrow d)\}$
  also with Nucleon×Kaon colour structure (Csikor et al., Liu et al.)

- Diquark-diquark antiquark
  $$\mathcal{O}_{I=0} = \varepsilon_{adg}\,[\varepsilon_{abc}\,u_b^T C\gamma_5 d_c]\;[\varepsilon_{def}\,u_e^T C\gamma_5 d_f]\;C\bar{s}_g^T$$
  symmetric in (ad) $\Rightarrow$ cannot be spatially symmetric
  $\Rightarrow$ non-trivial spatial wave-function needed (No lattice results so far.)

- To avoid spatially antisymmetric wave-function.:
  $$\mathcal{O}_{I=0} = \varepsilon_{adg}\,[\varepsilon_{abc}\,u_b^T C d_c]\;[\varepsilon_{def}\,u_e^T C\gamma_5 d_f]\;C\bar{s}_g^T$$
  (Sasaki, Chiu et al., Csikor et al.)

Parity assignment

- $\mathscr{O}$ does not create parity eigenstates, but

$$P\mathscr{O}P^{-1} = \pm\gamma_0\mathscr{O},$$

  - $\pm$ is the internal parity of $\mathscr{O}$
  - For non-pointlike operators, can be more complicated.

- Parity projection: $\frac{1}{2}(\mathscr{O} \pm P\mathscr{O}P^{-1})$

- Compute correlators separately in both channels.

- Identify the state we are looking for by its energy.

Separating two-particle states and 5q state

The Θ^+ is above threshold $\Rightarrow$ embedded in 2-particle NK continuum.
finite V $\Rightarrow$ all states are discrete.

In realistic lattice volumes:

- Level structure:
  - $N \times K,\ p = \sqrt{2} * 2\pi/L$ + −
  - $N \times K,\ p = 1 * 2\pi/L$ + −
  - m_{Θ^+} ???
  - $N \times K,\ p = 0$ −

- Lowest state in **−** parity channel: $m_N + m_K$ (S-wave)
- Lowest state in **+** parity channel: Θ^+ or $(m_N^2 + p^2)^{1/2} + (m_K^2 + p^2)^{1/2}$
  ($p = 2\pi/L \approx 400$ MeV, P-wave)

Pentaquark signal can be safely confirmed only after all nearby 2-particle states have been clearly identified

How to tell 2-particle state from 1-particle state?

Changing the volume

- Is there a volume dependence of the mass consistent with

$$(m_N^2 + p^2)^{1/2} + (m_K^2 + p^2)^{1/2}$$

$$p = \frac{2\pi}{L}$$

- Does the amplitude $|\langle 1|\mathcal{O}^\dagger(0)|0\rangle|^2$ go down with the volume?

How to identify several low-lying states?

- Fit with sum of exponentials: (Sasaki)
  $C_1 \mathrm{e}^{-m_1 t} + C_2 \mathrm{e}^{-m_2 t} + ...$
  Does not really work...

- Use cross correlators (Csikor et al.)

$$\mathscr{O}|0\rangle \approx A_1|1\rangle + A_2|2\rangle + ...,$$

$$\mathscr{O}'|0\rangle \approx A_1'|1\rangle + A_2'|2\rangle + ...$$

  - Mix two operators $\mathscr{O}\cos\phi + \mathscr{O}'\sin\phi$
  - Choose ϕ to cancel lowest or 2nd lowest state
    (Eg. minimise (maximise) effective mass w.r.t. α)
  - Can be generalised to more operators
  - Expensive! Have to compute $\langle\mathscr{O}\mathscr{O}\rangle$, $\langle\mathscr{O}\mathscr{O}'\rangle$, $\langle\mathscr{O}'\mathscr{O}'\rangle$.

Sources of errors

- Statistical errors.
- Extrapolation in quark mass and lattice spacing.
  Mass ratios do not vary much.
- Quenching
  (neglecting fermion determinant i.e. omitting quark loops).
  Stable hadron mass ratios OK within few %.
- Finite volume (different types of volume dependences)
  Tricky, but can be used to disentangle level structure.
- Contamination from other nearby states
  Cross correlator technique, choice of volume.
- Choice of operator No good recipe.

Extrapolations

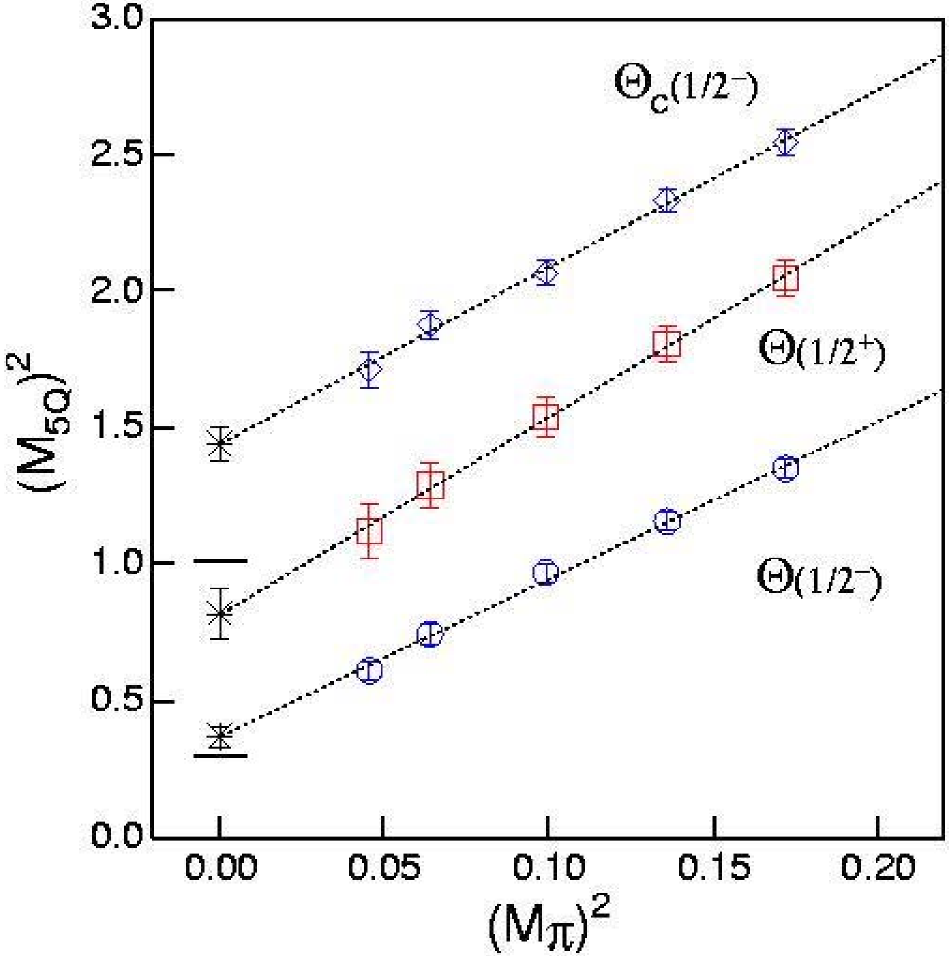
Any lattice calculation involves two extrapolations

- Chiral extrapolation:
  Simulation at physical ud quark masses too expensive
  $\Rightarrow$ simulate heavier quarks and extrapolate to $m_\pi = 135$ MeV.

- Continuum extrapolation:
  lattice spacing $\rightarrow$ 0; different actions have different cut-off effects

	action	a (fm)	smallest m_π (MeV)
Csikor et al.	Wilson	0.17-0.09	420
Sasaki	Wilson	0.07	650
Mathur et al.	chiral	0.20	180
Chiu & Hsieh	chiral	0.09	400
Alexandrou et al.	Wilson	0.12	400
Ishii et al.	improved Wilson	0.15	850
Takahashi et al.	Wilson	0.17	500

Chiral extrapolation of 5q states (Sasaki)



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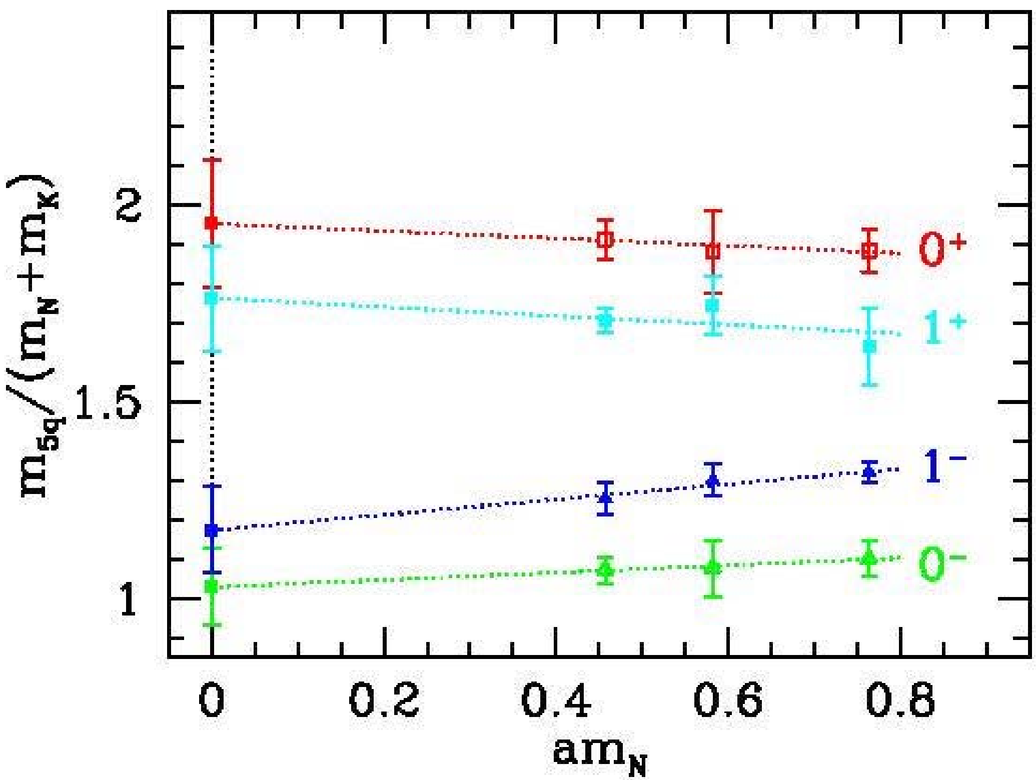
Continuum extrapolation of 5q states (Csikor et al.)



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Results and interpretation

Lowest state in $I^P = 0^-$ channel, mass in other parity channel much higher.

- Csikor et al. & Sasaki
  Consistent: $I^P = 0^-$, however NK scattering state still not ruled out.
- Mathur et al.
  identify only NK scattering states by volume dependence of amplitudes
  $\Rightarrow$ explain why only 2-particle states: use Nucleon$\times$Kaon operator
- Alexandrou et al.
  potential supports diquark-diquark-antiquark and not NK picture
  signal in the negative parity chanel (do not see scattering state)
- Takahashi et al.
  p=0 scattering and a pentaquark (positive parity is noisy)
- Ishii et al.
  lowest state is NK scattering (twisted boundary conditions)

Lowest state seen is in $I^P = 0^+$ channel,
mass in other parity channel much higher.

- Chiu & Hsieh
  $I^P = 0^+$??? they claim, due to better chiral action
  Not very likely:
  - In all studies huge difference between two parity channels
  - No precedent for such a discrepancy
    between chiral and Wilson action
  - 0^- has mass of 1433(72) MeV and 0^+ has mass of 1562(121) MeV
  - $\approx$ 1-σ: consistent with each other and with the scattering

  Might be misidentified parity $\Rightarrow$ Independent cross-check needed

Warning!
Nobody sees the lowest expected scattering state in both parity channels.

Perspectives

Seven independent lattice studies so far.

F. Csikor, Z. Fodor, S.D. Katz and T.G. Kovács, JHEP 0311 (2003) 070.

S. Sasaki, PRL **93** (2004) 152001.

N. Mathur et al. PRD **70** (2004) 074508.

Ting-Wai Chiu and Tung-Han Hsieh, hep-ph/0403020, hep-ph/0404007

C. Alexandrou, G. Koutsou, A. Tsapalis, Phys. Rev. D71 (2005) 014504

N. Ishii, T. Doi, H. Iida, M. Oka, F. Okiharua and H. Suganuma, hep-lat/0408030.

T. Takahashi et al., hep-lat/0410021,hep-lat/0410025

- To be done
  - Resolve discrepancy of Chiu & Hsieh
  - Systematically map out low-lying level structure (including the expected 2-particle states).
  - Technically:
    other operators with non-trivial spatial wave function and finite volume analysis.