Pentaquarks and Large Nc QCD:

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The Good, The Bad and The Ugly

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The Good, The Bad and The Ugly

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- Large Nc and the existence of exotics
- Large Nc and multiplets of exotics

Apologies

The Latin is in due deference to the Council of Trento

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Mea Maxima Culpa

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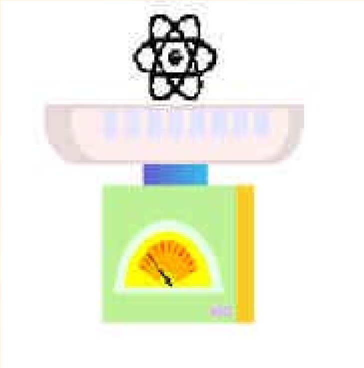
- Scientifi
  - Nc=3 i ot always large e blem

- Issue arises because 1/Nc expansion assumes all coefficients are natural. If multiple scales in the problem arising from physics other large Nc things get *ugly*.
  - Eg. bound states nuclear physics
  - Nuclear scales are radically smaller than typical hadronic scales for essentially unknown reason---reasons that have nothing to do with Nc.



- *Eg. Binding energy per nucleon is formally of order N_c^{1} and is 16 MeV in nuclear matter; The N-Δ mass splitting is N_c^{-1} and is 300 MeV .*
- ***Clearly large scales and small scales from large Nc are mixed; there is no clean scale separation based on large Nc.***
- ***On the other hand 1/Nc works quite beautifully in describing the properties of the Δ . Prediction of $g_{\pi N\Delta}$***
- ***In practice, the utility of the approach in making even semi-quantitative predictions depends on the quantity being studied.***

Why do large Nc?

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- Only known "practical" nonperturbative approach is lattice QCD.
  - The question arises about how practical.
  - My old joke was that waiting for reliable lattice results was like being a character in a Samuel Beckett play:
  - Situation has clearly improved: the era of reliable lattice computation is dawning (particularly for heavy quark systems)
  - Computation of resonant states, however, remain a very hard problem.

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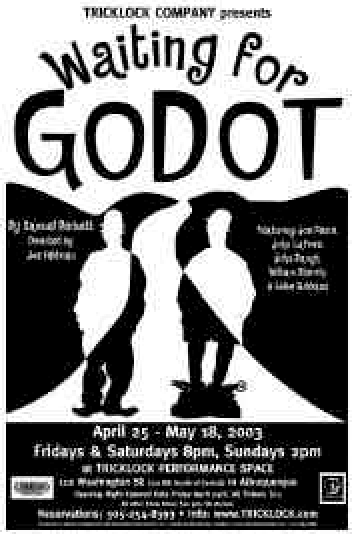

- Most theory about pentaquarks is based on modeling
  - chiral solitons
  - quark models
  - QCD sum rules
- Generic Problem: the models are all to some extent *ad hoc*. Are predictions of models related in any real way to the predictions of QCD?
- Large Nc enables one to make model independent predicts---albeit about a fiction large Nc world. *If* 1/Nc corrections are small it is useful for the real world.

- **In practice, the method *is* powerful as a (semi-)quantitative tool for some quantities.**
- **For these quantities the virtues of a model-independent approach are quite evident.**
- **However, the method may be of little practical use if there are scales in the problem arising from physics other than 1/Nc**

A Motto for 1/Nc practitioners

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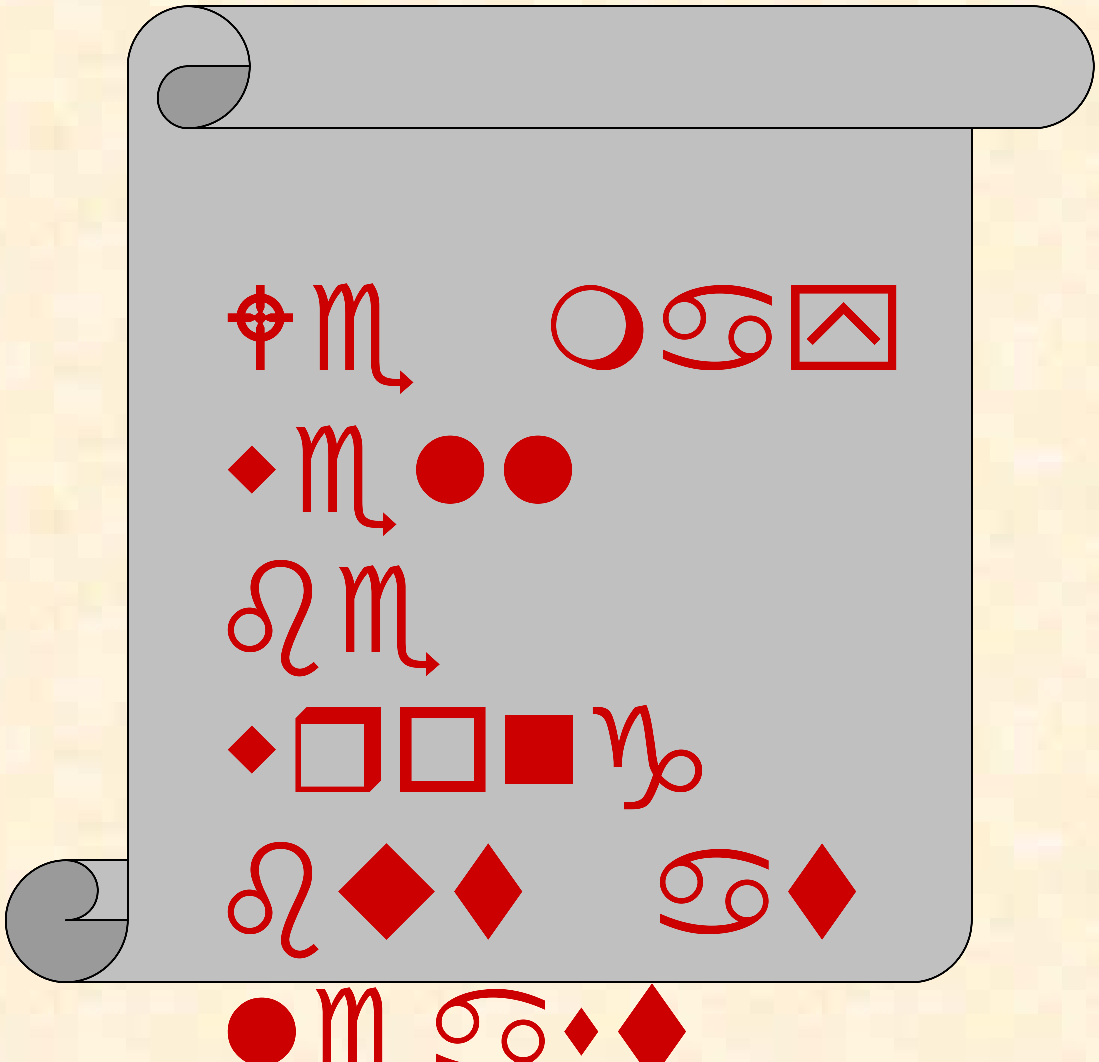
A Motto for 1/Nc practitioners



- Even in circumstances where the large Nc world is rather different from the real world approach still may be useful for qualitative reasons
  - One can use known behavior at large Nc to illustrate issues of some principle.
  - The possible existence of exotic states (eg. states not in the dumbest version of the quark model) is such an issue.
    - The existence of pentaquarks, alas, is not
  - The determination of a consistent way to treat certain types of models is another such issue.

- Large Nc gives an organizing principle for the treatment of models.
  - Eg. What is the correct way to quantize exotic states in chiral solitons. (A subject of bloody controversy in the past couple of years---which I thought had been resolved)
  - As a matter of principle large Nc requires the treatment of exotic states as “vibrations” in the fashion of the Princeton mafia (Itzhaki, Klebanov, Ouyang,Rastelli) and not as rigid rotations. TDC(2003,20004), Pobylitsa (2004) A. Cherman, TDC, A. Nellore (2004)
  - Whether this is relevant at Nc=3 is open (Private communication with D. Diakonov)
  - But if not, then do we know how to treat the models?.

Sisyphus

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Or TDC showing yet again that rigid rotor quantization fails to capture the correct large Nc dynamics for exotics.

On the Existence of Pentaquarks

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- Large Nc requires the existence of certain types of exotic hadrons
  - Hybrid mesons
  - Heavy pentaquarks (combined heavy quark and large Nc limits)

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- Large Nc requires the existence of certain types of exotic hadrons
  - Hybrid mesons
  - Heavy pentaquarks (combined heavy quark and large Nc limits)
- General existence of long-lived "pentaquark" resonant states is neither required nor forbidden at large Nc by standard counting rules
  - The existence of such states is a matter of dynamical detail

Why is there so much excitement about pentaquarks?

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- Pentaquarks have been *very* hot since their reported discovery in 2003:
  - 21,800 sites listed in Google search for “Pentaquark” in February of 2005.
  - Multiple workshops, sessions at conferences…
  - Numerous press reports: *New York Times, BBC …*
  - Latest Particle Data Book (July 2004)
    - Devotes entire section to pentaquarks
    - Lists as a three star resonance

- The significance of these states is partly cultutal/linguistic and depends on the word “quark”--- which has three meanings
  - A nonsense word invented by James Joyce
  - A “fundamental degree of freedom in QCD (the field theory which describes the strong interactions.

“Three quarks for Muster Mark”
from *Finnegan’s Wake*

  - An effective degree of freedom in the quark model (aliases: the constituent quark model, the naïve quark model …)

$$L = -\tfrac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \bar{q}_i \left(-i\not{D} + m_i \right) q_i$$

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + V$$

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- A state is called “exotic” if by quantum numbers it cannot be described in the naïve quark model.
- The search for manifestly exotic states has been a holy grail for many hadron physicists.
  - **Why the intense desire to find exotics? Finding one proves the limitation of the simplest constituent quark model--- but we already know that it is not fundamental.**
  - We *know* QCD has states with exotic quantum numbers. The issue is whether they resonate to form narrow states.
  - ***Large Nc can provide insight here***

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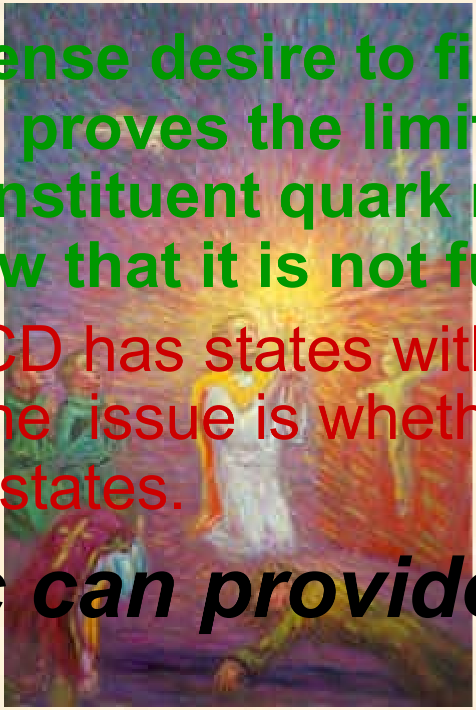

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- Example: hybrids (states with quantum numbers of quark-antiquark-gluon which cannot be pure quark-antiquark, *eg.* $J^{pc}=0^{-+}$)

Large Nc and Hybrids

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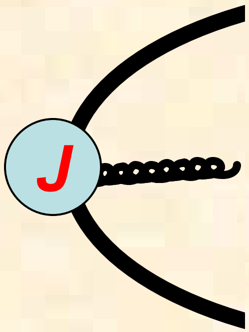
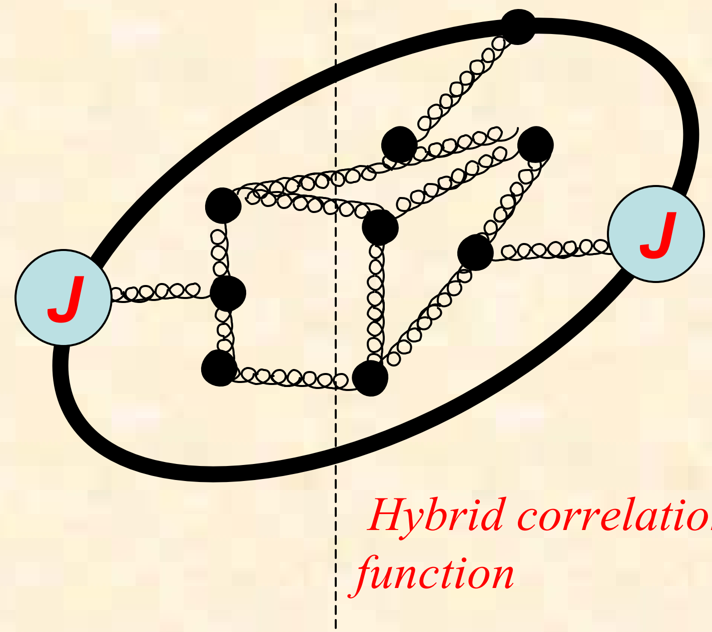
- Generalize Witten/`tHooft approach from mesons.
  - Construct current *J* with hybrid quantum numbers. (*i.e.* quark-antiquark-gluon)
  - Formally consider infinite class of diagrams contributing to correlation function: <*JJ*>
    - Only planar graphs with graph bounded by quark loop contribute at leading order.
  - Cutting the diagram to reveal intermediate states yields only one color singlet combination.



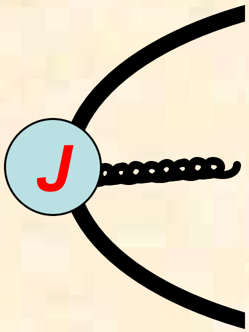
Hybrid current



Hybrid correlation function

Σ_j

j^{th} Hybrid propagator



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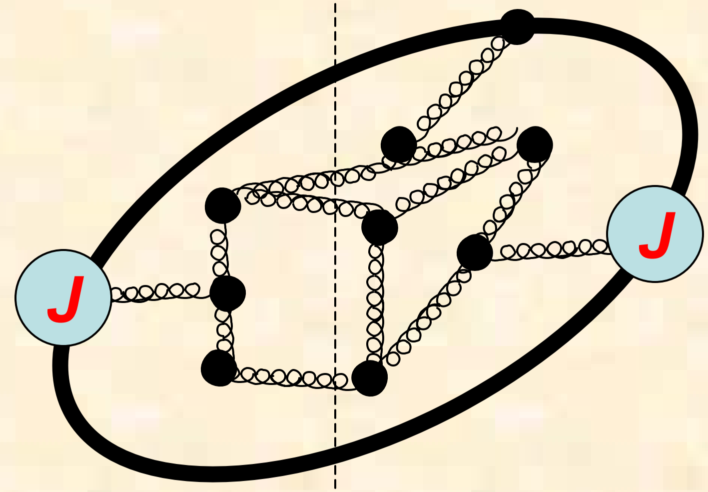

Hybrid correlation function

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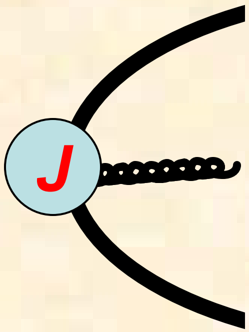

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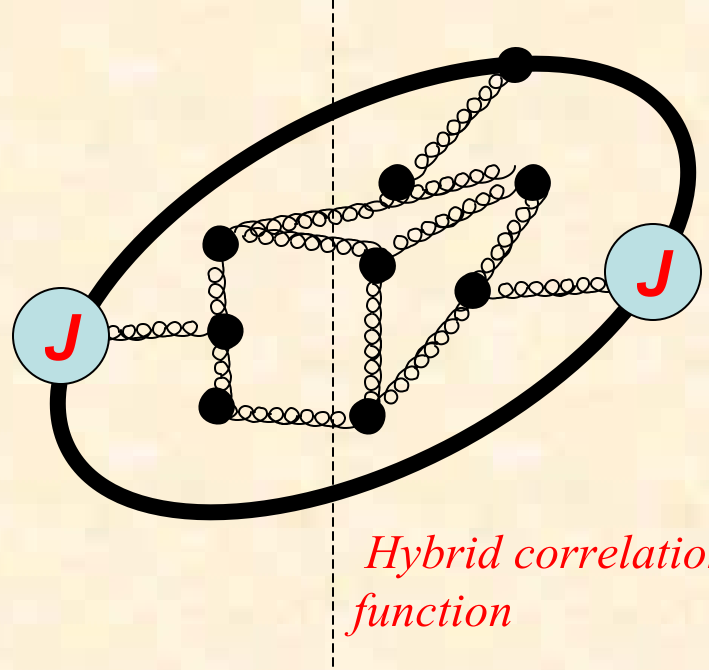

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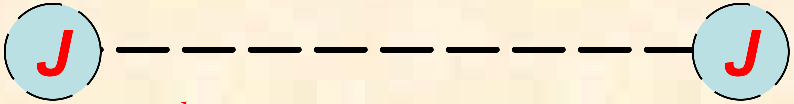
Σ_j 

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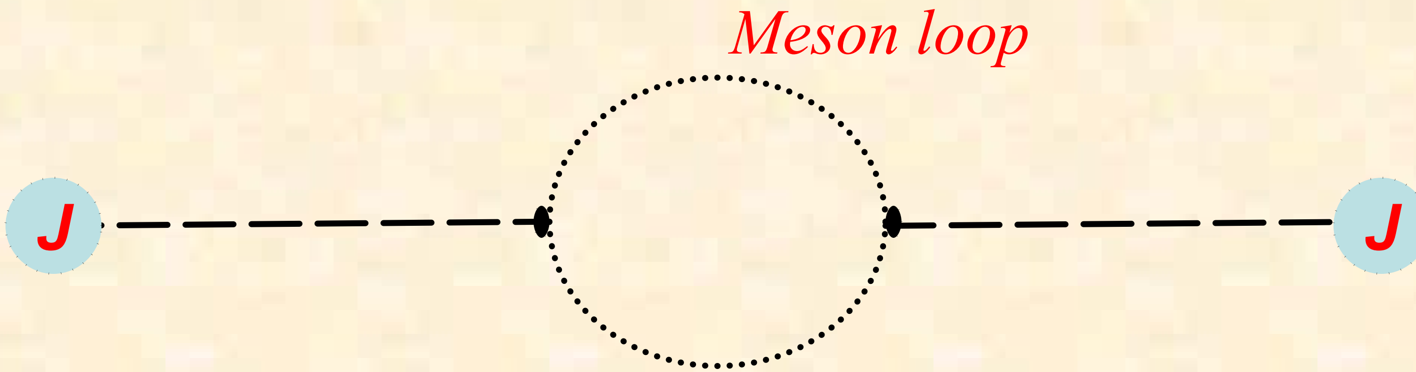
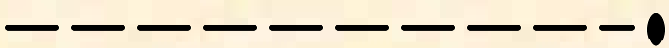
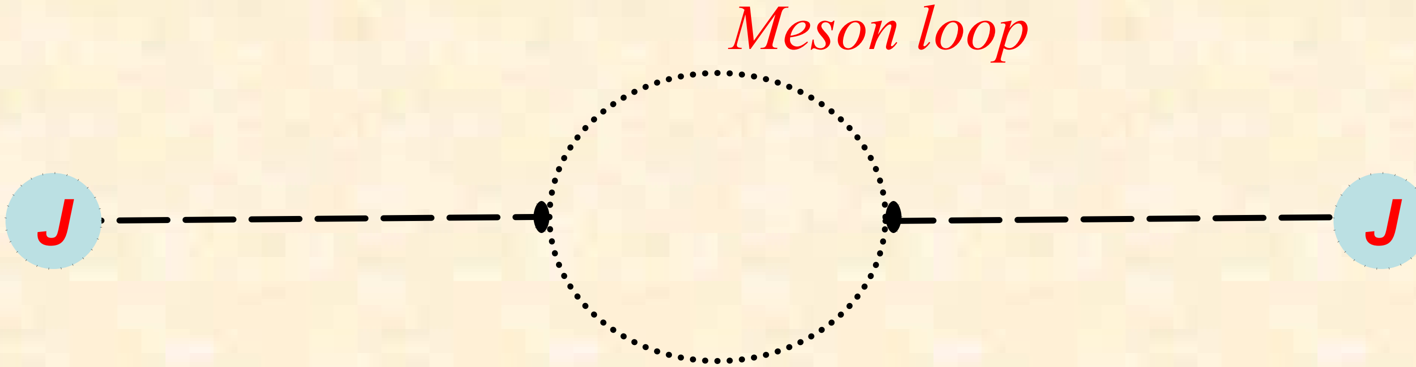


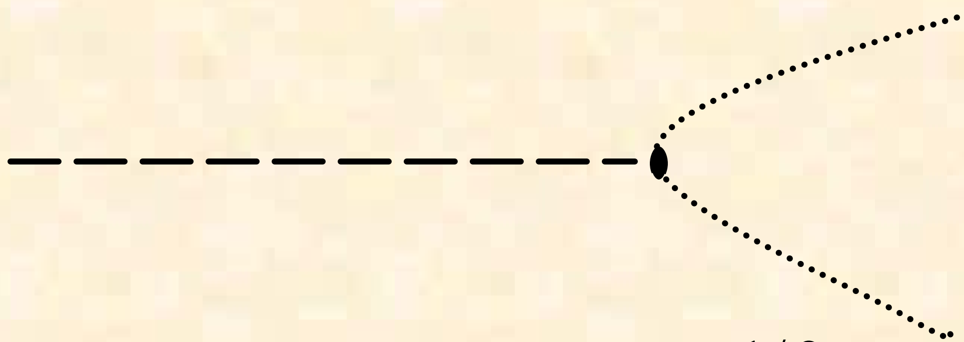
⚐*(1/Nc)*



$$g \sim N_c^{-1/2}$$



$\mathcal{O}(1/Nc)$



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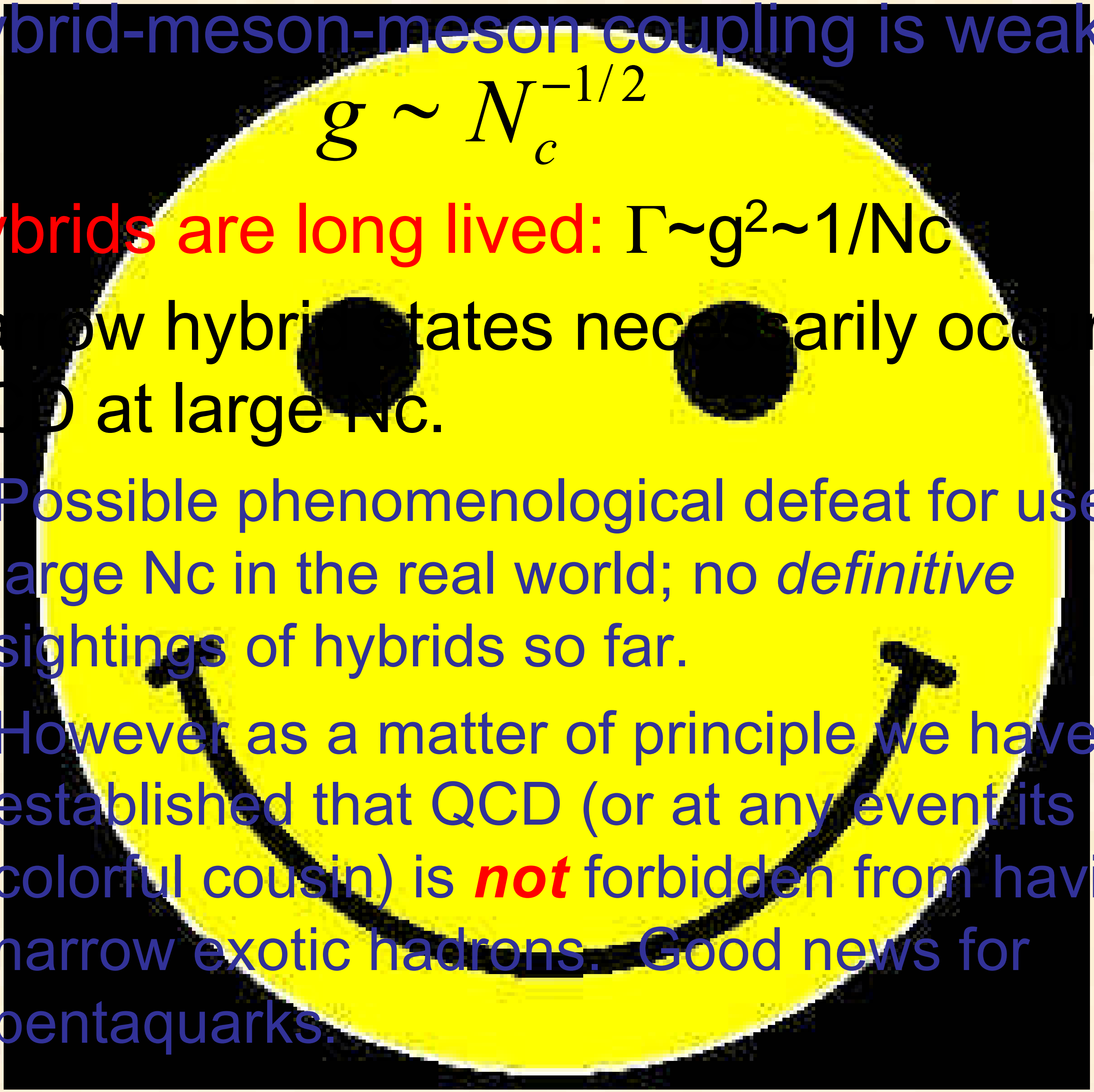
- Hybrids are long lived: $\Gamma \sim g^2 \sim 1/Nc$
- Narrow hybrid states necessarily occur in QCD at large Nc.
  - Possible phenomenological defeat for use large Nc in the real world; no *definitive* sightings of hybrids so far.
  - However as a matter of principle we have established that QCD (or at any event its colorful cousin) is ***not*** forbidden from having narrow exotic hadrons. Good news for pentaquarks.



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On the existence of heavy pentaquarks

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- QCD does not forbid exotic baryons

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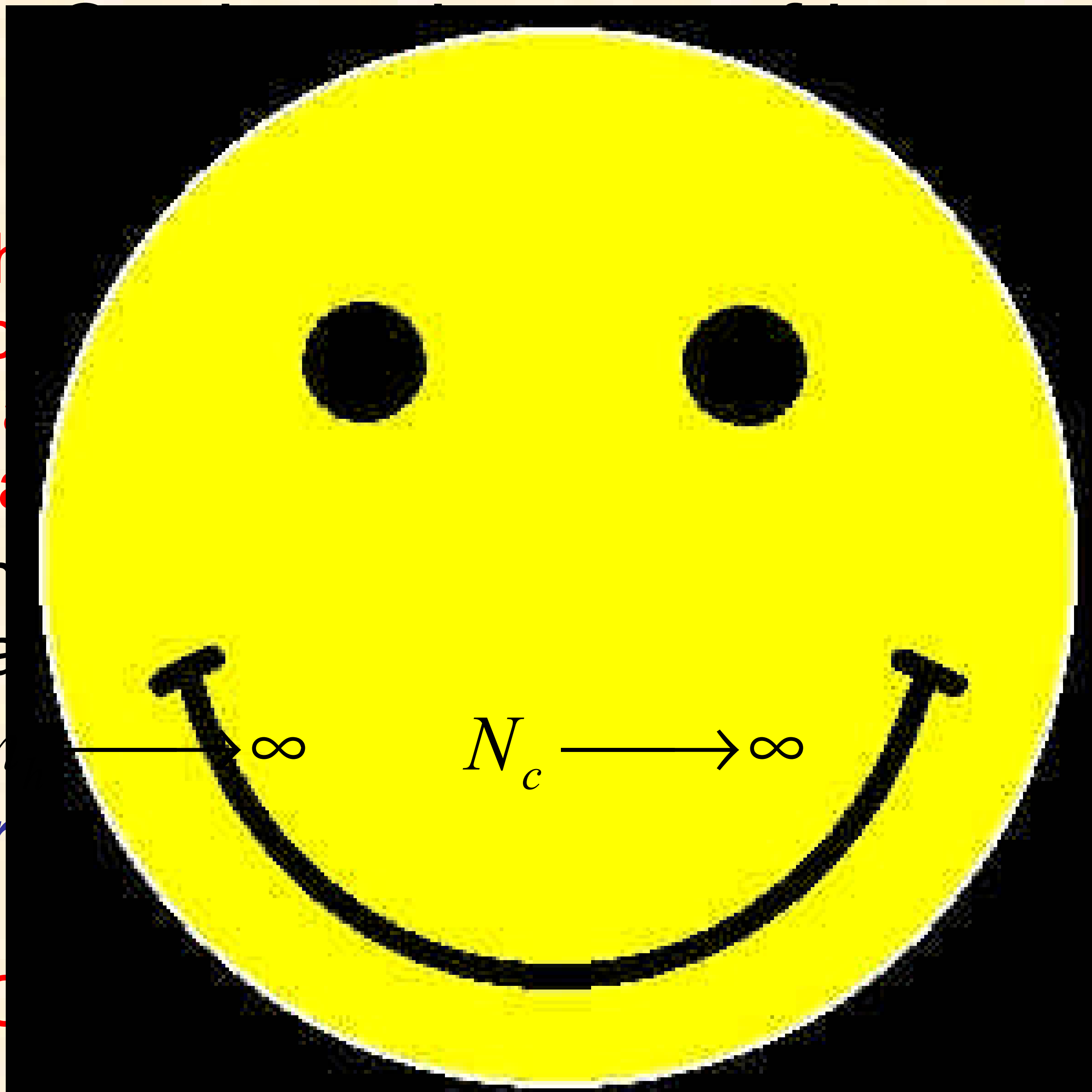

- General argument is based on effective field theory (TDC&R.F.Lebed in preparation)
- Hilbert space stace in QCD with
  - Baryon number equal to unity
  - Heavy quark number of -1 (One net anti-heavy quark)
  - Energies less than the $M_N + M_H + m_\pi$ (H is heavy meson.
  - Work below threshold for three-body final states. “Integrate out” (“project out”) all three hadron final states.
  - QCD is this regime is necessarily completely equivalent to a (nonrelativistic) two-body quantum theory.
  - Theory is non-local

- Power Counting (λ as a common prameter; modeled on counting for nonexotic heavy baryons at large Nc C.K. Chow &TDC PRL 84 (2000) 5474; NPA 688 (2001) 842)
  - $\lambda \sim 1/\mathrm{Nc}$; $\lambda \sim \Lambda/m_h$ (Λ is typical hadronic scale)
  - Expansion in $\lambda^{1/2}$ (marginal as a quantitive description of Nc=3 world)
  - At threshold relative p: $p=(2\mu m_\pi)^{1/2}$;

    $\mu = M_N M_H (M_N + M_H) \sim 1/\lambda$; $p \sim \lambda^{-1/2}$
  - Nonlocality at length scale $1/p \sim \lambda^{1/2}$.
  - Typical velocity $p/\mu \sim \lambda^{1/2}$.

- Effective theory assumes the form of a non-relativistic Schrödinger equation with a local potential at leading order.
  - nonlocalities & relativistic corrections $\sim\lambda^{1/2}$
  - Potential has strength and range of $\mathcal{O}(\lambda^0)$
    - Check via Witten approach of quark line counting
    - Easily seen in meson-exchange picture:

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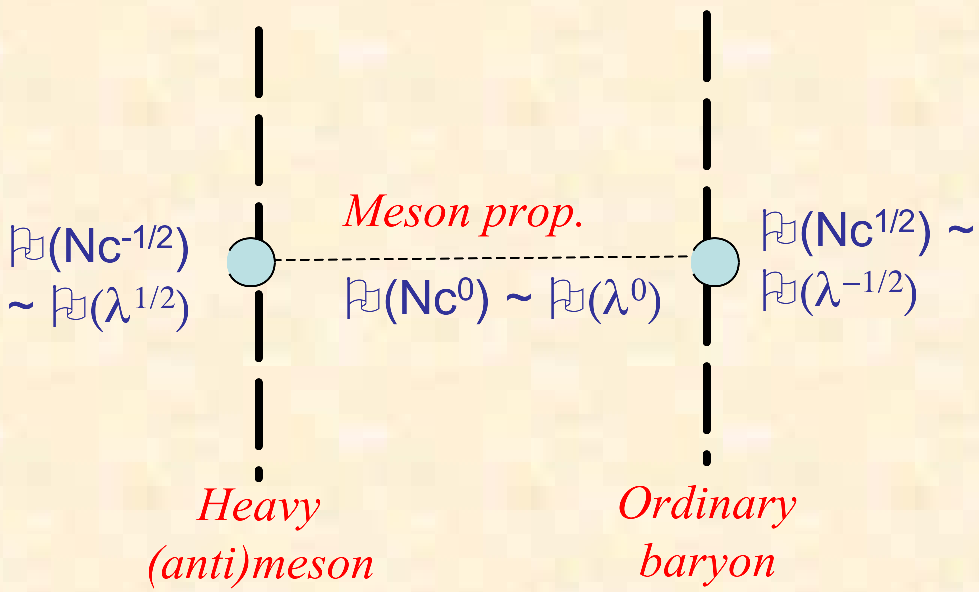

- In general a Schrödinger equation of the form $$\left(-\lambda\frac{\nabla^2}{2\bar{\mu}}+V(r)\right)\psi(\vec{r})=E\psi(\vec{r}) \quad \text{where } \bar{\mu}=\mu\lambda$$ has (many) bound states of both parities with many spins as λ gets large provded there exists at least some region where *V(r)<0.*
- We know that for large *r*, *V(r)* , is well described by a OPEP potential.
  - OPEP is necessarily attractive in some channels (and repulsive in others) depending on the relative sign of the heavy quark coupling constant to pions and the nucleons.

- Large Nc QCD **must** have (strong interaction) **stable** pentaquarks in the combined large Nc and heavy quark limits.
  - Pentaquarks of both parities and many angular momenta must exist.
- Nearly degenerate multiplets of heavy pentaquarks exist as the limit is approached.
  - Fall into representations of contracted

$$\underset{\textit{Collective vibrations}}{\mathrm{SO}(8)}\mathrm{x}\underbrace{\mathrm{SU}(4)}_{\textit{Spin-flavor for light quarks}}\mathrm{x}\underbrace{\mathrm{SU}(2)}_{\textit{Heavy quark spin}}$$

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$$\underbrace{SO(8)}_{\textit{Collective vibrations}}x\underbrace{SU(4)}_{\textit{Spin-flavor for light quarks}}x\underbrace{SU(2)}_{\textit{Heavy quark spin}}$$

- Contracted SO(8) associated with vibrations of heavy antiquark against other degrees of freedom. (Otained from QCD counting rules assuming a ground state exists.) Identical to nonexotic sector C.K. Chow &TDC PRL 84 (2000) 5474; NPA 688 (2001) 842)

Motion is collective since excitations of the muck are ~N_c^0 while relative excitation are $N_c^{1/2}$

- Contracted SU(4) is the standard light quark spin-flavor symmetry. (*i.e*. which relates N to Δ) Gervais & Sakita (1984), Dashen & Manohar (1993)
- Applied to heavy pentaquarks (M.E. Wessling PLB 603 (2004) 152; D. Parjol&C. Schact **hep-ph/0408293**)
  - Done in quark model language
  - Generally this is dangerous for exicted states as shown in series of papers with R. Lebed as connection of group theory only matches onto quark model for a set of stable states but not for resonances.
  - Kosher here since the heavy pentaquarks are stable in the combined limit.

- Proof of principle QCD can have exotic baryons.
- What about the real world? Does this argument strongly suggest that stable heavy pentaquarks exist for Nc=3?
- General argument of this type also suggests that many deeply bound 2-nucleon states should exist but in practive we have one barely bound state---the deuteron.

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Caveat Emptor

- Optimistic View: large Nc argument gives a hint that stable heavy pentaquarks exist.
- Genenric argument that a potential model is the correct effective theory is legit.
- My student, Paul Hohler has been modeling such potentials using an OPEP plus a variety of short distance (1-2 fermi) forms (attractive and repulsive).
  - The existence of several bound heavy pentaquarks seems to be a robust feature regardless of the sign of the pion-heavy meson coupling unless coupling is anomalously small. Results are preliminary.
- H1?

What about ordinary pentaquarks

What about ordinary pentaquarks

- Large Nc Agnosticism
  - Counting rules neither require the existence of narrow pentaquarks (at large Nc) nor do they forbid them.
    - Narrow here means narrow enough to discern in spectrum
  - It is a matter of dynamical detail.
  - Essential reason is simply scales of the problem. *If* pentaquarks exist

$M_\theta - M_N \sim \mathcal{O}(Nc^0)$ $g_{\theta NK} \sim \mathcal{O}(Nc^0)$ $\Gamma_\theta \sim \mathcal{O}(Nc^0)$

- Scaling rules are not controversial
  - Derivable via Witten quark line counting
  - Seen directly chiral soliton models (regardless of quanitzation procedure) D.Kaplan&I Klebanov (1990);TDC(2004);D. Diakonov&V.Petrov(2004); M. Praszalowicz (2004).
- Distinct from behavior of mesons or "ground band" baryons such as the Δ.

$$\Gamma_{meson} \sim \mathcal{O}(1/Nc)$$

$$\Gamma_{\Delta} \sim \mathcal{O}(1/Nc) \text{ (chiral limit)} \qquad \Gamma_{\Delta} = 0 \text{ } (m_{\pi} \text{ fixed})$$

- Not surprising: same scaling seen in non-exotic excited baryons.

The bottom line

- As a matter of principle, large Nc QCD shows that nothing in the strucure of QCD forbids exotics.

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- As a matter of principle, large Nc QCD shows that nothing in the strucure of QCD forbids exotics.
- Whether there exist any “ordinary” pentaquark resonances narrow enough to observe depends on the details of QCD dyanamicseven at large Nc.
  - Heavy pentaquarks must exist in the combined limit

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- This is a consequence of the general spin-flavor contracted SU(2 N_f) symmetry which emerges for ground band baryons at large Nc.

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- Identical results seen in Skyrme model and other chiral solitons..
- Large Nc quark model also yields this emergent symmetry.

Some Consequences

- ***States in ground band have I=J***
- ***Lowest two identified as N & Δ***
- ***Higher ones artifacts***
- ***N – Δ mass splitting goes as 1/Nc***
- ***MEs of operators between states in representation proportional to each other; proportionality constants are Clebsch's for this group.***

- One key result: I=J rule all leading order matrix elements have quantum numbers with I=J. Operators violating this suppressed by factor of $\left(\frac{1}{N_c}\right)^{|I-J|}$

(Kaplan& Manohar 1998; I=J rule seen in Skyrme model by Mattis & colloborators in the late 1980's)

- Leading order nucleon operators are either *scalar-isoscalar* or *vector-isovector ;* *scalar-isoscalar* and *vector-isovector* operators are *1/Nc* suppressed.

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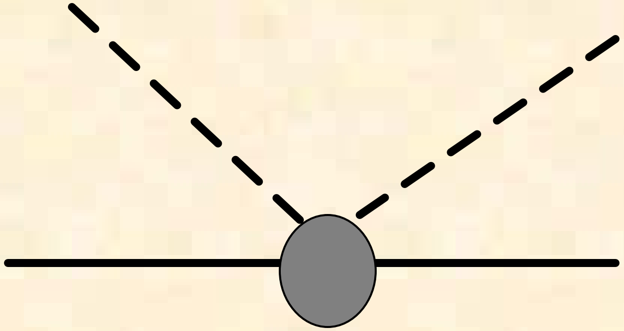
- Scattering observables are just operators acting on nucleons. TDC&RF.Lebed (2003)
- Logic is directly applicable to pentaquarks.Conservative scheme only rely isospin symmetry but not SU(3). (Suggestion out there by Jaffe&Wilczek that SU(3) may be badly broken in cases of ideal mixing)

Focus on K-N scattering

- Label scattering amplitudes by $S_{LL'I_sJ_s}$

  L (L') initial (final) L for K;

  I_s (J_s) total isospin (angular momentum of state



I=J for nucleon operator; t channel for scattering

- Most general amplitude does not have $I_t=J_t$ but large Nc QCD does. Fewer amplitudes at large Nc than in general: large Nc QCD requires relations among amplitudes. (Modulo 1/Nc corrections)
- Express large Nc amplitude in terms of most general amplitude with $I_t=J_t$ (requires recoupling) and then use 6-J coefficient identities:

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- Express large Nc amplitude in terms of most general amplitude with $I_t=J_t$ (requires recoupling) and then use 6-J coefficient identities:

$$S_{LL'IJ} = \sum_k 2(2k+1) \times \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L' & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L & \frac{1}{2} \end{Bmatrix} S_{kLL'}$$

- Note the same “reduced” amplitude contributes to many physical channels.
- A resonance is a pole in the scattering amplitude (at a complex energy).
- If there is a pole in a physical amplitude there must be a pole in some reduced amplitude---which implies a pole another physical channel with the same mass and width.

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- *Eg.* Suppose θ^+ has *I=0 and* $J^p=(1/2)^+$ (we don't this for sure---indeed we don't the state exists) and is created in p-wave scattering (initial and final state).
- Six-J coefficient: k=1/2 is only possibility: $S_{1/2\ 1\ 1}$ has pole at resonance position.
- $S_{1/2\ 1\ 1}$ also contributes to channel with L=L'=1; I=0, J=3/2

- *Ergo: at large Nc there is resonant state with* $I=0$, $J^p=(3/2)^+$ *at the same mass and width as orginal* θ^+ *.*

- 1/Nc correction shifts mass slightly (*eg.* M_{Δ}-M_{N}).
- Width could have larger shift due to phase space effects but coupling constant will be the same as the θ^+ (+ 1/Nc corrections)
- **Model Independent Prediction of large Nc QCD.**
  - ***Similar analysis yields same qualitative conclusion regardless of quantum #s.***