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**DYNAMICAL STUDY OF THE PENTAQUARK  
ANTIDECUPLET**

Trento, February 10-12, 2004

- Historical note, Light and Heavy pentaquarks
- How to accommodate  $\Theta^+$   
in constituent quark models
- The mass spectrum of the pentaquark antidecuplet
- Outlook

References:

- Fl. Stancu, PRC58 (1998) 111501
- Fl. Stancu and D. O. Riska, PLB575 (2003) 242
- Fl. Stancu, arXiv:hep-ph/0402044

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## HISTORICAL NOTE

### • LIGHT PENTAQUARKS

$$Z^+ = |uudd\bar{s}\rangle$$

*R. L. Jaffe, Talk at the Topical Conference on Baryon resonances, Oxford, 1976*

*H. Hogaasen and P. Sorba, NPB145(1978)119,  
missed the lowest state*

*D. Strottman, PRD20(1979)748,  
deliberately studied  $P = -1$  only*

→ SEARCHES IN THE MASS RANGE 1.74 GeV to 2.16 GeV (Y. Ohashi, hep-ph/0402005)

### NEW WAVE

*M. Praszalowicz Workshop on Skyrmions and Anomalies, World Scientific, 1987 and hep-ph/0308114*

*D. Diakonov, V. Petrov, M. Polyakov, Z. Phys. A359(1997)305*

*H. Weigel, Eur. Phys. J. A2(1998)391*

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- HEAVY PENTAQUARKS

NEGATIVE parity pentaquarks with  $\bar{c}$  or  $\bar{b}$

$$P^0 = |uuds\bar{c}\rangle, \quad P^- = |udds\bar{c}\rangle$$

*C.Gignoux, B.Silvestre-Brac, J.-M. Richard, PLB193(1987)323*

*H.J.Lipkin, PLB195(1987)484*

*M. Genovese, J.-M. Richard, F. S, S. Pepin, PLB425(1998)171*

Experiment is inconclusive

POSITIVE parity pentaquarks with  $\bar{c}$  or  $\bar{b}$

$$\Theta_c = |uudd\bar{c}\rangle, \quad \Theta_b = |uudd\bar{b}\rangle$$

*D.O.Riska, N.Scoccola, PLB299(1993)338*

*Fl. Stancu, PRD58(1998)111501*

NO experimental search so far

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## CONSTITUENT QUARK MODELS: POSITIVE PARITY SCENARIOS

$$8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8)_F + 1_F$$
$$8_F \times 10_F = 35_F + 27_F + 10_F + 8_F$$

**B. K. Jennings, K. Maltman, arXiv:hep-ph/0308286**  
*Z<sup>\*</sup> resonances: Phenomenology and models*

**C. E. Carlson, C. D. Carone, H. J. Kwee, V. Nazaryan,  
PLB 579 (2004) 52**  
*Positive parity pentaquarks pragmatically predicted*

**F. Huang, Z. Y. Zhang, Y. W. Yu, B. S. Zou, arXiv:hep-ph/0310040**  
*A study of pentaquark Θ in the chiral SU(3) quark model*

## MASS FORMULAE

**M. Karliner and H. J. Lipkin, PLB575 (2003) 249**  
**R. Jaffe and F. Wilczek, PRL 91 (2003) 232003**  
**D. Diakonov, V. Petrov, arXiv:hep-ph/0310212**  
**E. Shuryak and I. Zahed, arXiv:hep-ph/0310270**  
**R. Bijker, M. M. Giannini, E. Santopinto, arXiv:hep-ph/0310281**

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**Stable  $uudd\bar{s}$  pentaquarks in the constituent quark model**

*Fl. Stancu, D. O. Riska* PLB575,242(2003),  
hep-ph/0307010

**1)  $qq$  pairs, schematic GBE interaction**

*Glozman and Riska*, PR268(1996)263

$$V_\chi = - C_\chi \sum_{i < j}^4 \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$$

$$C_\chi \sim 30 \text{ MeV} \quad (\Delta - N \text{ splitting})$$

enough for infinitely heavy antiquarks

**2)  $q\bar{s}$  pairs, schematic  $\eta$ -meson exchange**

$$V_\eta = V_0 \sum_i^4 \vec{\sigma}_i \cdot \vec{\sigma}_j$$

*Laehde and Riska* NPA710(2002)99

Pion decay  $D_s^* \rightarrow D_s\pi^0$  requires  $\pi^0 - \eta$  mixing  
 $\rightarrow \eta$  meson couples to  $s$  or  $\bar{s}$

$$f_{\eta ss} \sim -1.66$$

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## THE BASIS

**The light quark  $q^4$  subsystem, POSITIVE PARITY**

$$|1\rangle = ([31]_O[211]_C[1^4]_{OC} ; [22]_F[22]_S[4]_{FS})$$

$$|2\rangle = ([31]_O[211]_C[1^4]_{OC} ; [31]_F[31]_S[4]_{FS})$$

**Expectation value of  $V_\chi$**

$$\langle 1 | V_\chi | 1 \rangle = -28 C_\chi$$

$$\langle 2 | V_\chi | 2 \rangle = -64/3 C_\chi$$

**In the stability problem the relevant quantity is**

$$\Delta E = E(q^4\bar{q}) - E(q^3) - E(q\bar{q})$$

**GBE schematic interaction for the lowest state  $|1\rangle$**

$$\Delta E = \frac{5}{4}\hbar\omega - 14C_\chi = -107.5\text{MeV}$$

**with**

$$\langle N | V_\chi | N \rangle = -14C_\chi, \quad \hbar\omega = 250\text{MeV}$$

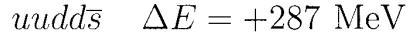
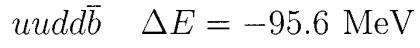
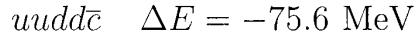
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**Realistic calculations,** *F.S.* *PRD58(1998)111501*

**Graz parametrization,** *Glozman, Papp & Plessas,* *PLB381(1996)311*

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{\vec{P}^2}{2M} + \sum_{i < j} V_{conf}(r_{ij}) + \sum_{i < j} V_\chi(r_{ij})$$

$$\begin{aligned} V_\chi(r_{ij}) &= \left\{ \sum_{F=1}^3 V_\pi(r_{ij}) \lambda_i^F \lambda_j^F \right. \\ &\quad \left. + \sum_{F=4}^7 V_K(r_{ij}) \lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j \end{aligned} \quad (1)$$



## THE PENTAQUARK $\Theta^+(uudd\bar{s})$

**couple  $\bar{s}$  to  $q^4 \rightarrow 5$  particle states**

$$|1\rangle \rightarrow |\psi_1\rangle, \quad |2\rangle \rightarrow |\psi_2\rangle$$

The interaction is  $V = V_\chi + V_\eta$

	$\langle \psi_1   \psi_1 \rangle$	$\langle \psi_2   \psi_2 \rangle$
$\langle \psi_1   \psi_1 \rangle$	$-28C_\chi$	$\frac{8}{\sqrt{2}}V_0$
$\langle \psi_2   \psi_2 \rangle$	$\frac{8}{\sqrt{2}}V_0$	$-\frac{64}{3}C_\chi - 4V_0$

Take  $C_\chi = 30$  MeV

$$\langle V \rangle = -(740 + 2V_0) \pm [10,000 - 400V_0 + 36V_0^2]^{1/2}$$

$V_0 = C_\chi \rightarrow \langle V \rangle$  decreases by 130 MeV

$V_0 = 2C_\chi \rightarrow \langle V \rangle$  decreases by 350 MeV  $\rightarrow \Delta E < 0$

**Conclusion:**  $V_\chi + V_\eta$  gives stable  $uudd\bar{s}$  pentaquarks with  $J^P=1/2^+$ ,  $I=0$ ,  $S=+1$

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# THE ANTIDECUPLET. F. S. hep-ph/0402044

Basic assumption:

- the  $q\bar{q}$  interaction is spin dependent but flavour independent → global shift in the spectrum
- the lowest state

$$|1\rangle = ([31]_O[211]_C[1^4]_{OC} ; [22]_F[22]_S[4]_{FS})$$

- $qq$  radial dependent interaction: Graz parametrization

The radial part of the wave function  
Internal Jacobi coordinates

$$\begin{aligned}\vec{x} &= \vec{r}_1 - \vec{r}_2 , \\ \vec{y} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) / \sqrt{3} \\ \vec{z} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4) / \sqrt{6} , \\ \vec{t} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5) / \sqrt{10} ,\end{aligned}\tag{2}$$

$$\psi_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & & \\ \hline \end{array} = \langle \vec{x} | 000 \rangle \langle \vec{y} | 000 \rangle \langle \vec{z} | 010 \rangle , \tag{3}$$

$$\psi_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & & \\ \hline \end{array} = \langle \vec{x} | 000 \rangle \langle \vec{y} | 010 \rangle \langle \vec{z} | 000 \rangle , \tag{4}$$

$$\psi_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & & \\ \hline \end{array} = \langle \vec{x} | 010 \rangle \langle \vec{y} | 000 \rangle \langle \vec{z} | 000 \rangle , \tag{5}$$

TABLE I. Expectation values of the hyperfine interaction  $\langle V_\chi \rangle$  Eq. (3), for four quark subsystems. The upper index indicates the flavour of every interacting  $qq$  pair (see Ref. [9])

$q^4$	$I, I_3$	$\langle V_\chi \rangle$
$uudd$	0, 0	$30 V_\pi - 2 V_\eta^{uu} - 4 V_{\eta'}^{uu}$
$uuds$	$1/2, 1/2$	$15 V_\pi - V_\eta^{uu} - 2 V_{\eta'}^{uu} + 12 V_K + 2 V_\eta^{us} - 2 V_{\eta'}^{us}$
$ddss$	1, -1	$V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{2}{3}V_{\eta'}^{uu} + \frac{4}{3}V_\eta^{ss} + \frac{2}{3}V_{\eta'}^{ss} + 20V_K + \frac{16}{3}V_\eta^{us} - \frac{16}{3}V_{\eta'}^{us}$

TABLE II. Variational solutions  $E = \sum_{n=1}^5 m_i + \langle T \rangle + \langle V_c \rangle + \langle V_\chi \rangle$  of the Hamiltonian (1) and the masses M of various  $q^4\bar{q}$  systems. The mass is obtained from E by subtraction 510 MeV in order to fit the mass of  $\Theta^+$ .

$q^4\bar{q}$	$\sum_{n=1}^5 m_i$	$\langle T \rangle$	$\langle V_c \rangle$	$\langle V_\chi \rangle$	$E$	$M$
$uudd\bar{d}$	1700	1864	442	-2044	1962	1452
$uudd\bar{s}$	1800	1848	461	-2059	2050	1540
$uuds\bar{d}$	1800	1535	461	-1563	2233	1732
$uuds\bar{s}$	1900	1634	440	-1663	2310	1800
$ddss\bar{u}$	1900	1418	464	-1310	2472	1962
$uuss\bar{s}$	2000	1410	452	-1310	2552	2042

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 TABLE III. The antidecuplet mass spectrum in MeV.

Pentaquark	$Y, I, I_3$	Present result	Carlson et al. [16]	Diakonov and Petrov [19]
$\Theta^+$	2, 0, 0	1540	1540	1540
$N_{\overline{10}}$	1, 1/2, 1/2	1684	1665	1647
$\Sigma_{\overline{10}}$	0, 1, 1	1833	1786	1755
$\Xi_{3/2}^{--}$	-1, 3/2, -3/2	1962	1906	1862

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## OUTLOOK

$\Theta^+$ ,  $M \simeq 1540$  MeV,  $\Gamma < 25$  MeV,  $I = 0$   
Spin and parity to be measured

$\Xi_{3/2}^{--}$ ,  $M \simeq 1862$  MeV,  $\Gamma \simeq 18$  MeV  
resonance to be confirmed

## QUARK MODELS

Understand the nature of pentaquarks and implications  
of their existence in baryon spectroscopy

- constituent quark models accommodate  $\Theta^+$  and  $\Xi^{--}$
- absolute mass is not determined
- $\Xi^{--} - \Theta^+$  mass difference is model dependent
- more sophisticated 5 particles calculations are needed
- mixing of  $\bar{10}_F$  and  $8_F$  states
- higher Fock components
- strong decays
- spin-orbit partners