DYNAMICAL STUDY OF THE PENTAQUARK ANTIDECUPLET

Trento, February 10-12, 2004

- Historical note, Light and Heavy pentaquarks

- How to accommodate Θ^+
  in constituent quark models

- The mass spectrum of the pentaquark antidecuplet

- Outlook

References:
Fl. Stancu, PRC58 (1998) 111501
Fl. Stancu and D. O. Riska, PLB575 (2003) 242
Fl. Stancu, arXiv:hep-ph/0402044

HISTORICAL NOTE

- **LIGHT PENTAQUARKS**

$$Z^+ = |uudd\bar{s}\rangle$$

R. L. Jaffe, **Talk at the Topical Conference on Baryon resonances, Oxford, 1976**

H. Hogaasen and P. Sorba, **NPB145(1978)119,**
missed the lowest state

D. Strottman, **PRD20(1979)748,**
deliberately studied P = - 1 only

→ SEARCHES IN THE MASS RANGE 1.74 GeV to 2.16 GeV (Y. Ohashi, hep-ph/0402005)

NEW WAVE

M. Praszalowicz **Workshop on Skyrmions and Anomalies, World Scientific, 1987 and hep-ph/0308114**

D.Diakonov, V.Petrov, M.Polyakov, **Z. Phys. A359(1997)305**

H. Weigel, **Eur. Phys. J. A2(1998)391**

- **HEAVY PENTAQUARKS**

NEGATIVE parity pentaquarks with $\bar{c}$ or $\bar{b}$

$$P^0 = |uuds\bar{c}\rangle, \;\; P^- = |udds\bar{c}\rangle$$

C.Gignoux, B.Silvestre-Brac, J.-M. Richard, **PLB193(1987)323**
H.J.Lipkin, **PLB195(1987)484**
M. Genovese, J.-M. Richard, F. S, S. Pepin, **PLB425(1998)171**

Experiment is inconclusive

POSITIVE parity pentaquarks with $\bar{c}$ or $\bar{b}$

$$\Theta_c = |uudd\bar{c}\rangle, \;\; \Theta_b = |uudd\bar{b}\rangle$$

D.O.Riska, N.Scoccola, **PLB299(1993)338**
Fl. Stancu, **PRD58(1998)111501**

NO experimental search so far

CONSTITUENT QUARK MODELS: POSITIVE PARITY SCENARIOS

$$8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8)_F + 1_F$$
$$8_F \times 10_F = 35_F + 27_F + 10_F + 8_F$$

B. K. Jennings, K. Maltman, arXiv:hep-ph/0308286
Z^* resonances: Phenomenology and models

C. E. Carlson, C. D. Carone, H. J. Kwee, V. Nazaryan, PLB 579 (2004) 52
Positive parity pentaquarks pragmatically predicted

F. Huang, Z. Y. Zhang, Y. W. Yu, B. S. Zou, arXiv:hep-ph/0310040
A study of pentaquark Θ in the chiral SU(3) quark model

MASS FORMULAE

M. Karliner and H. J. Lipkin, PLB575 (2003) 249
R. Jaffe and F. Wilczek, PRL 91 (2003) 232003
D. Diakonov, V. Petrov, arXiv:hep-ph/0310212
E. Shuryak and I. Zahed, arXiv:hep-ph/0310270
R. Bijker, M. M. Giannini, E. Santopinto, arXiv:hep-ph/0310281

Stable $uudd\bar{s}$ pentaquarks in the constituent quark model

Fl. Stancu, D. O. Riska **PLB575,242(2003), hep-ph/0307010**

1) qq pairs, schematic GBE interaction

Glozman and Riska, **PR268(1996)263**

$$V_\chi \; = \; - \, C_\chi \; \sum_{i\,<\,j}^{4} \lambda_i^F \cdot \lambda_j^F \; \vec{\sigma}_i \cdot \vec{\sigma}_j$$

$$C_\chi \sim 30\text{MeV} \quad (\Delta - N \; splitting)$$

enough for infinitely heavy antiquarks

2) $q\bar{s}$ pairs, schematic η-meson exchange

$$V_\eta = V_0 \sum_{i}^{4} \; \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Laehde and Riska **NPA710(2002)99**

Pion decay $D_s^* \rightarrow D_s \pi^0$ requires $\pi^0 - \eta$ mixing $\rightarrow \eta$ meson couples to s or $\bar{s}$

$f_{\eta ss} \sim$ **- 1.66**

THE BASIS

The light quark q^4 subsystem, POSITIVE PARITY

$$|1\rangle \;=\; \left([31]_O[211]_C\left[1^4\right]_{OC}\;;\;[22]_F[22]_S[4]_{FS}\right)$$

$$|2\rangle \;=\; \left([31]_O[211]_C\left[1^4\right]_{OC}\;;\;[31]_F[31]_S[4]_{FS}\right)$$

Expectation value of V_χ

$$\langle 1|V_\chi|1\rangle = -28\ C_\chi$$
$$\langle 2|V_\chi|2\rangle = -64/3\ C_\chi$$

In the stability problem the relevant quantity is

$$\Delta E = E(q^4\overline{q}) - E(q^3) - E(q\overline{q})$$

GBE schematic interaction for the lowest state $|1\rangle$

$$\Delta E = \frac{5}{4}\hbar\omega - 14C_\chi = -107.5\mathrm{MeV}$$

with

$$\langle N|V_\chi|N\rangle = -14C_\chi, \quad \hbar\omega = 250\mathrm{MeV}$$

Realistic calculations, *F.S. PRD58(1998)111501*

Graz parametrization, *Glozman, Papp & Plessas, PLB381(1996)311*

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \frac{\vec{P}^2}{2M} + \sum_{i<j} V_{conf}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij})$$

$$\begin{aligned} V_\chi(r_{ij}) &= \left\{ \sum_{F=1}^{3} V_\pi(r_{ij}) \lambda_i^F \lambda_j^F \right. \\ &+ \left. \sum_{F=4}^{7} V_K(r_{ij}) \lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j \end{aligned} \quad (1)$$

$uudd\bar{c} \quad \Delta E = -75.6$ MeV

$uudd\bar{b} \quad \Delta E = -95.6$ MeV

$uudd\bar{s} \quad \Delta E = +287$ MeV

THE PENTAQUARK $\Theta^+(uudd\bar{s})$

couple $\bar{s}$ to q^4 $\rightarrow$ 5 particle states
$|1\rangle \rightarrow |\psi_1\rangle, \quad |2\rangle \rightarrow |\psi_2\rangle$

The interaction is $V = V_\chi + V_\eta$

	$\langle\psi_1\|\psi_1\rangle$	$\langle\psi_2\|\psi_2\rangle$
$\langle\psi_1\|\psi_1\rangle$	$-28C_\chi$	$\frac{8}{\sqrt{2}}V_0$
$\langle\psi_2\|\psi_2\rangle$	$\frac{8}{\sqrt{2}}V_0$	$-\frac{64}{3}C_\chi - 4V_0$

Take $C_\chi = 30$ MeV

$$\langle V\rangle = -(740 + 2V_0) \pm [10,000 - 400V_0 + 36V_0^2]^{1/2}$$

$$V_0 = C_\chi \quad \rightarrow \langle V\rangle \text{ decreases by 130 MeV}$$

$$V_0 = 2C_\chi \quad \rightarrow \langle V\rangle \text{ decreases by 350 MeV} \quad \rightarrow \Delta E < 0$$

Conclusion: $V_\chi + V_\eta$ gives stable $uudd\bar{s}$ pentaquarks with J^P=1/2$^+$, I=0, S=+1

THE ANTIDECUPLET. F. S. hep-ph/0402044

Basic assumption:

- **the $q\bar{q}$ interaction is spin dependent but flavour independent $\rightarrow$ global shift in the spectrum**
- **the lowest state**

$$|1\rangle = \big([31]_O[211]_C\big[1^4\big]_{OC} \, ; \, [22]_F[22]_S[4]_{FS}\big)$$

- **qq radial dependent interaction: Graz parametrization**

The radial part of the wave function
Internal Jacobi coordinates

$$\begin{aligned} \vec{x} &= \vec{r}_1 - \vec{r}_2 \, , \\ \vec{y} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)\,/\sqrt{3} \\ \vec{z} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4)\,/\sqrt{6} \, , \\ \vec{t} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5)\,/\sqrt{10} \, , \end{aligned} \tag{2}$$

$$\psi_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|000\rangle \, \langle \vec{y}\,|000\rangle \, \langle \vec{z}\,|010\rangle \; , \tag{3}$$

$$\psi_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|000\rangle \, \langle \vec{y}\,|010\rangle \, \langle \vec{z}\,|000\rangle \; , \tag{4}$$

$$\psi_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|010\rangle \, \langle \vec{y}\,|000\rangle \, \langle \vec{z}\,|000\rangle \; , \tag{5}$$

TABLE I. Expectation values of the hyperfine interaction $\langle V_\chi \rangle$ Eq. (3), for four quark subsystems. The upper index indicates the flavour of every interacting qq pair (see Ref. [9])

q^4	$I,\ I_3$	$\langle V_\chi \rangle$
$uudd$	0, 0	$30\ V_\pi - 2\ V_\eta^{uu} - 4\ V_{\eta'}^{uu}$
$uuds$	1/2, 1/2	$15\ V_\pi - V_\eta^{uu} - 2\ V_{\eta'}^{uu} + 12\ V_K + 2\ V_\eta^{us} - 2\ V_{\eta'}^{us}$
$ddss$	1, -1	$V_\pi + \frac{1}{3} V_\eta^{uu} + \frac{2}{3} V_{\eta'}^{uu} + \frac{4}{3} V_\eta^{ss} + \frac{2}{3} V_{\eta'}^{ss} + 20 V_K + \frac{16}{3} V_\eta^{us} - \frac{16}{3} V_{\eta'}^{us}$

TABLE II. Variational solutions $E = \sum_{n=1}^{5} m_i + \langle T\rangle + \langle V_c\rangle + \langle V_\chi\rangle$ of the Hamiltonian (1) and the masses M of various $q^4\bar{q}$ systems. The mass is obtained from E by subtraction 510 MeV in order to fit the mass of Θ^+.

$q^4\bar{q}$	$\sum_{n=1}^{5} m_i$	$\langle T\rangle$	$\langle V_c\rangle$	$\langle V_\chi\rangle$	E	M
$uudd\bar{d}$	1700	1864	442	-2044	1962	1452
$uudd\bar{s}$	1800	1848	461	-2059	2050	1540
$uuds\bar{d}$	1800	1535	461	-1563	2233	1732
$uuds\bar{s}$	1900	1634	440	-1663	2310	1800
$ddss\bar{u}$	1900	1418	464	-1310	2472	1962
$uuss\bar{s}$	2000	1410	452	-1310	2552	2042

TABLE III. The antidecuplet mass spectrum in MeV.

Pentaquark	$Y,\ I,\ I_3$	Present result	Carlson et al. [16]	Diakonov and Petrov [19]
Θ^+	2, 0, 0	1540	1540	1540
$N_{\overline{10}}$	1, 1/2, 1/2	1684	1665	1647
$\Sigma_{\overline{10}}$	0, 1, 1	1833	1786	1755
$\Xi_{3/2}^{--}$	-1, 3/2, -3/2	1962	1906	1862

OUTLOOK

Θ^+, M $\simeq$1540 MeV, $\Gamma < 25$ MeV, I = 0
Spin and parity to be measured

$\Xi^{--}_{3/2}$, M $\simeq$ 1862 MeV, $\Gamma \simeq$ 18 MeV
resonance to be confirmed

QUARK MODELS

Understand the nature of pentaquarks and implications of their existence in baryon spectroscopy

- **constituent quark models accommodate Θ^+ and Ξ^{--}**
- **absolute mass is not determined**
- **Ξ^{--} - Θ^+ mass difference is model dependent**
- **more sophisticated 5 particles calculations are needed**
- **mixing of $\overline{10}_F$ and 8_F states**
- **higher Fock components**
- **strong decays**
- **spin-orbit partners**