Opportunities for pentaquarks within the chiral constituent quark picture

L. Ya. Glozman

1. Physics of the low-lying 3Q baryons
2. Pentaquarks with the quark model
3. Connection to large N_c
4. Jaffe-Wilczek scenario

Where is a key problem?

N

- $\frac{1}{2}^- - \frac{3}{2}^-$ — N(1535)-N(1520)
- $\frac{1}{2}^+$ — N(1440)
- $\frac{1}{2}^+$ — N(939)

Δ

- $\frac{1}{2}^- - \frac{3}{2}^-$ — Δ(1620)-Δ(1700)
- $\frac{3}{2}^+$ — Δ(1600)
- $\frac{3}{2}^+$ — Δ(1232)

Λ

- $\frac{1}{2}^- - \frac{3}{2}^-$ — Λ(1670)-Λ(1
- $\frac{1}{2}^+$ — Λ(1600)
- $\frac{1}{2}^- - \frac{3}{2}^-$ — Λ(1405)-Λ(15
- $\frac{1}{2}^+$ — Λ(1115)

Octet-decuplet splittings (e.g. N-Δ) can be explained in many models with any suitable spin-spin force

confinement:

$\sim r^2$	$\sim r$
$\frac{1}{2}^+$ —	$\frac{1}{2}^+$ —
$\frac{1}{2}^- - \frac{3}{2}^-$ —	$\frac{1}{2}^- - \frac{3}{2}^-$ —
$\frac{1}{2}^+$ —	$\frac{1}{2}^+$ —

(i) Perturbative gluon-exchange spin-spin force (the color-magnetic interaction) is flavor-independent $\sim \frac{1}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j$

(ii) Instanton-induced spin-spin force is absent in flavor-symmetrical pairs (i.e. it does not contribute in Δ)

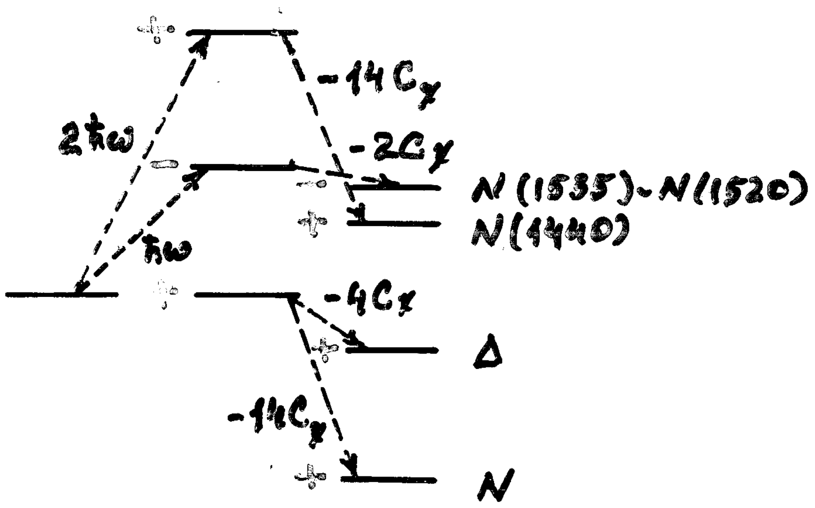
$$\sum_{i<j} -V_\chi(r_{ij})\, \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F\, \vec{\sigma}_i \cdot \vec{\sigma}_j \rightarrow \sum_{i<j} -C_\chi\, \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F\, \vec{\sigma}_i \cdot \vec{\sigma}_j$$

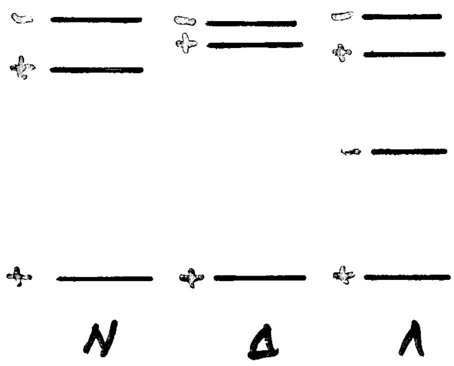
octet: N, Λ, Σ, Ξ	$[3]_{FS}$	$[21]_F$	$[21]_S$	$-14C_\chi$	$P=+$
decuplet: $\Delta, \Sigma^*, \Xi^*, \Omega$	$[3]_{FS}$	$[3]_F$	$[3]_S$	$-4C_\chi$	$P=+$
octet*: $N(1440), \Lambda(1600), \ldots$	$[3]_{FS}$	$[21]_F$	$[21]_S$	$-14C_\chi$	$P=+$
decuplet*: $\Delta(1600), \ldots$	$[3]_{FS}$	$[3]_F$	$[3]_S$	$-4C_\chi$	$P=+$
orbital excitations of negative parity:					
$N(1535)-N(1520)$, $\Lambda(1670)-\Lambda(1690), \ldots$	$[21]_{FS}$	$[21]_F$	$[21]_S$	$-2C_\chi$	$P=-$
$\Delta(1620)-\Delta(1700)$	$[21]_{FS}$	$[3]_F$	$[21]_S$	$+4C_\chi$	$P=-$
$\Lambda(1405)-\Lambda(1520)$	$[21]_{FS}$	$[111]_F$	$[21]_S$	$-8C_\chi$	$P=-$

$\Delta - N$ splitting $\Rightarrow$ $C_\chi = 29.3$ MeV

$N(1440) - N$ splitting $\Rightarrow$ $\hbar\omega = 250$ MeV

Predictions

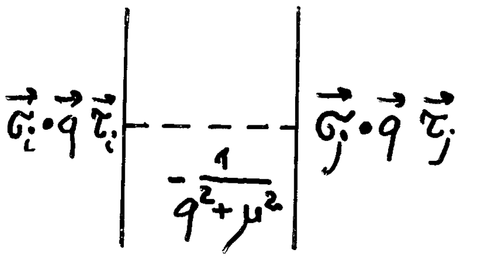




L. Ya. G., D. O. Riska, 96

Simplest example

$$\mathcal{L} = \bar\Psi(i\gamma_\mu\partial^\mu - m)\Psi + \partial_\mu\vec\varphi\,\partial^\mu\vec\varphi^* - \mu^2|\vec\varphi|^2 + \frac{g}{2m}\bar\Psi\gamma_\mu\gamma_5\vec\tau\Psi\partial^\mu$$



$$V(\vec q) \sim -\frac{\vec\sigma_i\cdot\vec q\,\vec\sigma_j\cdot\vec q}{q^2+\mu^2}\,\vec\tau_i\cdot\vec\tau_j$$

$$V(\vec r) \sim \left(\mu^2\frac{e^{-\mu r}}{r} - 4\pi\delta(\vec r)\right)\vec\sigma_i\cdot\vec\sigma_j\,\vec\tau_i\cdot\vec\tau_j + t.$$

1. due to both the constituent quark and pion finite sizes the $\delta(\vec r)$ is smeared out
2. the Goldstone boson field $\vec\varphi$ should in fact satisfy some nonlinear equation:

$$\partial_\mu\varphi\,\partial^\mu\varphi^* \to \frac{f_\pi^2}{4}Tr(\partial_\mu U\,\partial^\mu U^+) + \ldots$$

$$U = e^{i\frac{\vec\varphi\cdot\vec\tau}{f_\pi}}$$

Callan, Coleman, Wess, Zumino

$$\mathcal{D} \neq -\frac{1}{q^2+\mu^2} \quad \text{where } |\varphi| \text{ is large}$$

However $\mathcal{D} \to \mathcal{D}_0 = -\frac{1}{q^2+\mu^2}$ at $q^2 \to 0$

Most general theorem: **at "short" range**   L.Ya.G. PLB,199[illegible]

$$V(\vec r) \sim -V(r)\,\vec\sigma_i\cdot\vec\sigma_j\,\vec\tau_i\cdot\vec\tau_j$$

Proof: $V(\vec q) \sim \vec\sigma_i\cdot\vec q\,\vec\sigma_j\cdot\vec q\;\vec\tau_i\cdot\vec\tau_j\;\mathcal{D}(q^2)F^2(q^2)$

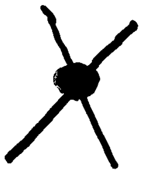
$$V(\vec q=0) = 0 \Rightarrow \int V(\vec r)\,d\vec r = 0$$

at large $\vec r$: $V(\vec r) \sim \mu^2\frac{e^{-\mu r}}{r} \Longrightarrow$ at small $\vec r$ $V(r)$ must be negative and strong

Chiral limit: ($\mu = 0$) $V(\vec q=0) \neq 0$;

The long-range Yukawa tail vanishes, but the short-range interaction **does not!**

Why Goldstone boson exchange is so important?



← interaction between color quark currents in the local approximati

← instanton-induced interaction

. . .

$$H_{eff} = -G[(\bar{\Psi}\Psi)^2 + (\bar{\Psi} i\gamma_5 \vec{\tau}\Psi)^2] + \dots$$

Nambu and Jona-Lasin

if $G \geq G_0$, then

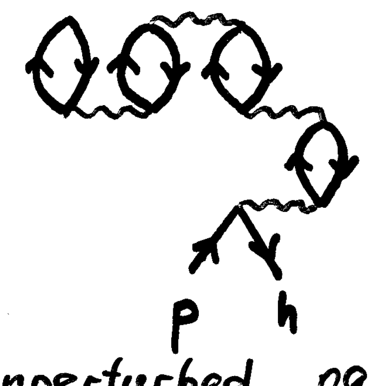
(i) $-G(\bar{\Psi}\Psi)^2$: $\langle \bar{q}q \rangle \neq 0$, $m^* \sim -G\langle \bar{q}q \rangle$

Schwinger-Dyson — gap equation

(ii) $-G(\bar{\Psi} i\gamma_5 \vec{\tau}\Psi)^2$:

Bethe-Salpeter eq. in $\bar{q}q$ $T=1$ channel

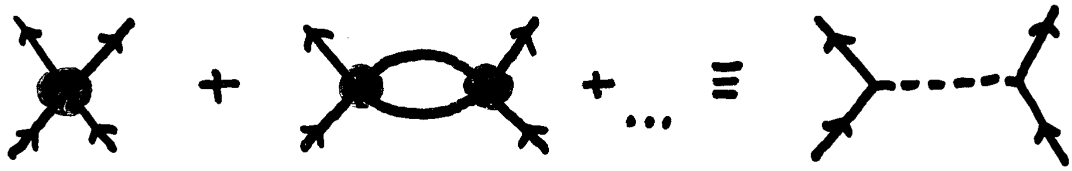
Pion is a pole at $s=0$, $m_\pi = 0$



Unperturbed particle-hole state going into the collective Anderson mode, which comes down to zero energy as a result of the quasiparticle intera

What happens in qq systems?

L. Ya. G, 6
K. Varga
PRD, 2000

$T =$ 

$$T = 2G + 2G\,J_p(q^2)\,2G + \ldots = \frac{2G}{1-2G\,J_p(q^2)}$$

$J_p(q^2) \equiv$  = vacuum polarization

$$J_p(q^2=\varepsilon) = \frac{1}{2G} \Rightarrow \frac{2G}{1-2G\,J_p(q^2)}$$ has a pole!

antiscreening

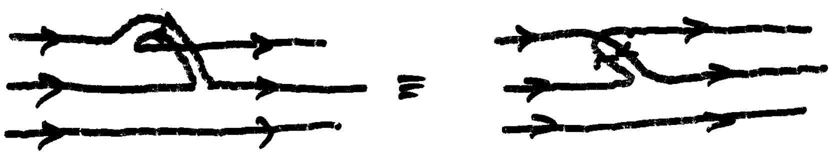
$$-\frac{2G}{1-2G\,J_p(q^2)} \equiv \frac{g_{\pi q}^2\, F_{\pi q}^2(q^2)}{q^2} \to V_\chi(r)$$

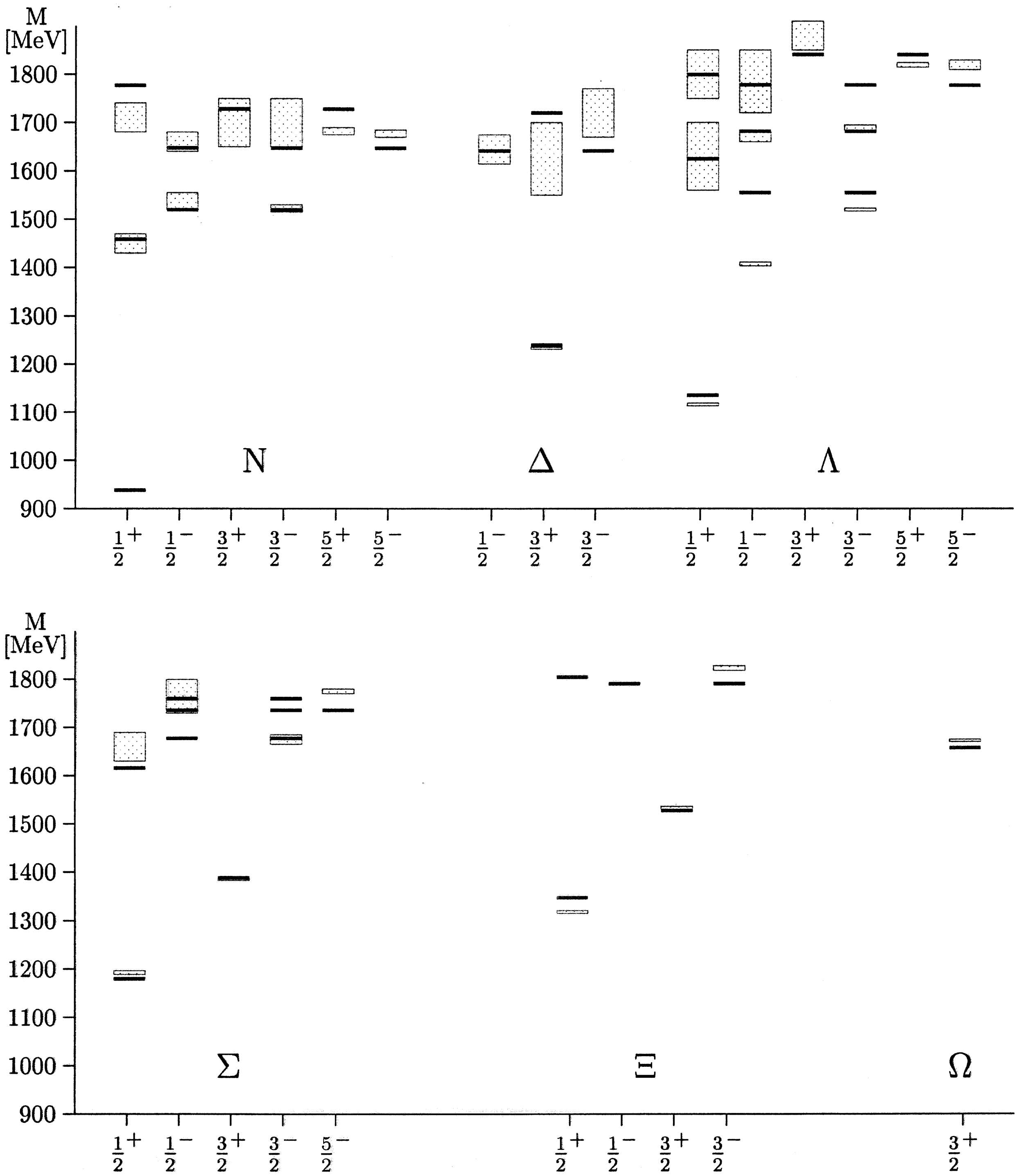
$$V_\chi(r) = -\frac{g_{\pi q}^2}{4\pi}\,\frac{1}{12m^2}\,\vec{\sigma}_i\cdot\vec{\sigma}_j\,\vec{\tau}_i\cdot\vec{\tau}_j\,4\pi\delta(\vec{r})$$

if $F_{\pi q}(q^2) \to 1$

in reality $4\pi\delta(\vec{r}) \to$ finite function, with the range Λ_χ^{-1}

At the quenched QCD level:

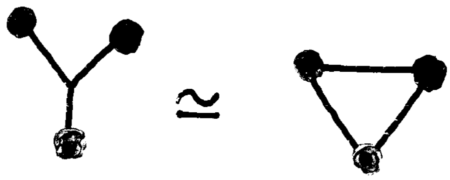




L. Ya. G., W. Plessas, K. Varga, R.F. Wagenbrunn
PRD 58 (1998) 094030

How to model effective confining interaction — severe problem!

<u>3Q baryons</u>



(exact for r^2)

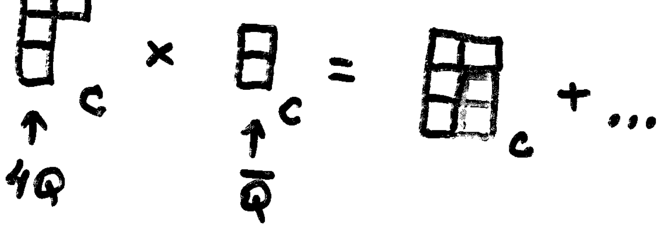
Then

$$V_{conf} \sim \sum_{i<j} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \, V_c(r_{ij})$$

 $\Rightarrow$ in each quark pair $\langle \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \rangle = -\frac{8}{3}$

So in the 3Q baryon one has an attraction in each quark pair

<u>Pentaquarks</u>



If confinement $\sim \sum \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \, V_c(r_{ij})$, then there is an attraction in some pairs and repulsion in others! Your result will critically depend on the model for confinement.

How to model interaction between the valence quarks? The most reasonable model $-\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \, \vec{\sigma}_i \cdot \vec{\sigma}_j$

How to model interaction between the valence antiquark and valence quarks? – open problem

Simplest model: 1) assume $-\vec{\lambda}_i^F \vec{\lambda}_j^F \, \vec{\sigma}_i \cdot \vec{\sigma}_j$ for $Q-Q$

2) neglect spin-dependent interaction in $\bar{Q}-Q$

L. Ya. G., D. O. Riska, 1998
Fl. Stancu, 1998
Fl. Stancu, D. O. Riska, 2003
L. Ya. Glozman, 2003
Carlson, Carone, Kwee, Nazaryan, 2003
Jenkins, Manohar, 2004

What will be the ground state system? How to find it? $\Longrightarrow$
Analyze symmetry properties of the interaction

$$\left\langle [f_{ij}]_F \times [f_{ij}]_S : [f_{ij}]_{FS} \right| -\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \, \vec{\sigma}_i \cdot \vec{\sigma}_j \left| [f_{ij}]_F \times [f_{ij}]_S : [f_{ij}]_{FS} \right\rangle =$$

$$= \begin{cases} -\frac{4}{3}, & (\square\square)_F, \ (\square\square)_S, \ (\square\square)_{FS} \\ -8, & \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_F, \ \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_S, \ (\square\square)_{FS} \\ +4, & (\square\square)_F, \ \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_S, \ \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_{FS} \\ +\frac{8}{3}, & \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_F, \ (\square\square)_S, \ \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_{FS} \end{cases}$$

1) attraction in the FS symmetric pairs and repulsion in the FS antisymmetric pairs

2) given the FS (SU(6)) representation, the most attractive contribution comes from the "most antisymmetric" $SU(3)_F$ representation

Consider first naive model – all quarks and the antiquark are in the 1S state:

$$\Theta^+: \quad \underset{\substack{\uparrow \\ QQQQ}}{(1S)^4} \ \underset{\substack{\uparrow \\ \bar{s}}}{1S} \rightarrow \text{Negative parity!}$$

What is the wave function?

$$(1S)^4 \Rightarrow [4]_O$$

$$[211]_C \Rightarrow [211]_C \odot [4]_O = [211]_{CO}$$

Pauli principle: $[1111]_{COFS}$

Then $[31]_{FS}$, since

$$[1111]_{COFS} = [211]_{CO} \odot [31]_{FS}$$

Conclusion: if all quarks are in the ground orbital state, then $[31]_{FS}$

With such a wave function the pentaquarks were considered in
C. Gignoux, B. Silvestre-Brac, J.M. Richard, PLB 193 (1987) 323;
H.J. Lipkin, PLB 195 (1987) 484;
M. Genovese, J.M. Richard, Fl. Stancu, S. Pepin, PLB 425 (1998) 171

Is it ground state? No!

what is the most favorite configuration from the point of view $-\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$?

$[4]_{FS}$; $[31]_F \odot [31]_S \supset [4]_{FS}$, $[22]_F \odot [22]_S \supset [4]_{FS}$, ...

the most favourable

What will be an orbital wave function?

Pauli principle: $[1111]_{COFS} = [4]_{FS} \odot [1111]_{CO}$

$[211]_C$: $[1111]_{CO} \subset [211]_C \odot [31]_O \Rightarrow (1s)^3 1p$

$|(1s)^3 1p;\ [4]_{FS};\ [22]_F;\ [22]_S\rangle$ Positive parity!

$1\hbar\omega$ excitation

Let's compare

$$\langle (1s)^4 [31]_{FS} [211]_F [22]_S | -C_\chi \sum_{i<j} \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j | (1s)^4 [31]_{FS} [211]_F [22]_S\rangle = -17C$$

$$\langle (1s)^3 1p\, [4]_{FS} [22]_F [22]_S | -C_\chi \sum_{i<j} \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j | (1s)^4 [4]_{FS} [22]_F [22]_S\rangle = -28$$

$C_\chi \simeq 30\ \mathrm{MeV}$; $\Delta E_{kin} \sim \frac{\hbar\omega}{2} \sim 125\ \mathrm{MeV}$

$|P=+, (1s)^3 1p\, [4]_{FS} [22]_F [22]_S;\ L=1;\ I=0;\ S=0\rangle$ is a ground state by a wide margin

$4Q \rightarrow [22]_F \times [11]_F\ (\bar{Q}) = [33]_F\ (\overline{10}) + [321]_F\ (8)$

Given uncertainties, should we try to model this? If yes, there is no predictive power — too many parameters to be constrained.

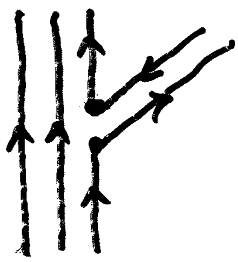
More pragmatic approach — absorb parameters into input Θ^+, N, $N(1440)$, $N(1535)$ and try to predict using symmetry of the model other antidecuplet members — Carlson, Carone, ...

$$M_{\Xi_5} = 1906\,; \quad M_{N_5} = 1665 \text{ MeV}$$

before "observation"!

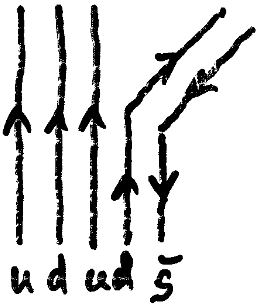
Why Θ^+ is so narrow?

Usual baryons — creation $\bar{q}q$ from the vacuum



In Θ^+ this mechanism is impossible!

Fall apart mode for the leading Fock component



$$S \sim C \, \langle \psi_{5Q} \,|\, \underset{\substack{\uparrow \\ N}}{\psi_{3Q}} \;\; \underset{\substack{\uparrow \\ \pi,\, K,\, \dots}}{\psi_{Q\bar{Q}}} \rangle$$

$$\left| \langle P=+,\, (1s)^3 1p \;\Box\Box\Box\Box_{FS} \cdots \,|\, \psi_{3Q}\; \psi_{Q\bar{Q}} \rangle \right|^2 = \frac{5}{96}$$

If one takes naive model instead

$$\left| \langle P=-,\, (1s)^4 \;\boxplus\!\Box_{FS} \cdots \,|\, \psi_{3Q}\; \psi_{Q\bar{Q}} \rangle \right|^2 \sim \frac{1}{4}$$

Very important: Chiral quark picture predicts $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states within $\overline{10}$ (contrary to soliton picture).

$$(1s)^3\, 1p;\; S=0 \;+\; \bar{s} \;\Rightarrow\; \frac{1}{2}^+;\; \frac{3}{2}^+$$

$\frac{1}{N_c}$ study.

First crucial step – Cohen

$N, \Delta - \Theta^+$ splitting $\sim N_c^0$ (finite at any N_c

contrary to $N-\Delta$ splitting $\sim \frac{1}{N_c}$

$\Rightarrow$ rotational and vibrational modes must be mixed

More important: $1s \to 1p$ transition (particle-hole excitation) $\sim N_c^0$!

Hence, there must be in Θ^+

$(1s)^3 1p \; 1s$ mode! (predicted by chiral quark model))

$\uparrow$ QQQQ $\quad \uparrow$ $\bar{Q}$

What is more important, rot – vibr excitation or particle-hole excitation?

If both $\Theta = \frac{1}{2}^+$ and $\Theta^* = \frac{3}{2}^+$ discovered, then it is a particle-hole state

Second crucial step – Jenkins, Manohar

They show that $\frac{1}{N_c}$ expansion of the collective $\frac{1}{2}^+$ excitation and of $\frac{1}{2}^+$ chiral quark $(1s)^3 1p \; 1s$ is the same!

Jaffe - Wilczek scenario

Θ^+: $\quad ud \quad ud \quad \bar{s} \quad \leftarrow \overline{10}$

$N(1440)$: $\quad ud \quad ud \quad \bar{d} \quad \leftarrow \overline{10}+8$

$N(1710)$: $\quad ud \quad us \quad \bar{s} \quad \leftarrow \overline{10}+8$

......

Ξ $\quad us \quad us \quad \bar{d} \quad \leftarrow \overline{10}$

It combines Θ^+ and some of the "perplexing" (anomalously low-lying) Roper states within the same picture

Phenomenologically unsatisfactory

Roper states: $N(1440), \Delta(1600), \Lambda(1600), \Sigma(1660)$.

All — $P=+$

All — $\Gamma \sim 200-300$ MeV

All - anomalously low-lying

$\Delta(1600)$ — $J=\frac{3}{2}, I=\frac{3}{2}$ cannot be constructed as diquark - diquark - antiquark;

$N(1440)\to\Delta\pi$; $N(1710)\to\Delta\pi$ is large! (Impossible for penta)