Phenomenological & Theoretical Constraints on Models of Exotic Baryons

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Talk by Tom Cohen in Trento

Outline

- Experimental Situation
 - Fact that these states were not observered in past experiments puts very stringent bounds on θ^+ width: Γ < 2 Mev (?)
- Quark Models
 - Focus on Jaffe-Wilczek Model & Ideal Mixing
 - Demonstration that widths imply N*(1440) and N*(1710) cannot both be ideally mixed states in the model. There must be a new state.
 - JW model is not special. All quark models without substantial OZI violations are nearly ideally mixed
 - Previous conclusion hold for all pure pentaquark models
 - Theoretically OZI violations generally are 1/Nc suppressed. But not in present context
 - JW models predict a light negative parity Octet which has not been seen.
- Chiral Soliton Models
 - Virtually all treatments to date rely on collective quantization for exotic modes. This is not justified from large Nc QCD.
 - Proper treatment is via meson-baryon scattering but results are highly model dependent

State seen in many experiments Clearly exotic in quark model sense



- The state cannot be a pure three quark state since it has S=1 (namely one more anti-strange quark then strange quark)
- Minimum configuration 4quarks +1 antiquark ("pentaquark")
- -First unambiguous evidence of an exotic state
- Death of the naïve quark model?

Widths

- All reported pentaquarks are narrow---the experimental bounds are given by machine resolution ~15MeV.
- If states are real the must be *much* narrower or they would have been seen in prior experiments. (K⁺ D scattering)



- Analysis with very limited data bounds the θ^+ width as less the 6 MeV (Nussinov).
- GWU mafia redid this analysis (nucl-th/0311030) with more complete data set and got a much more stringent limit Γ_{θ} <1.5 MeV.
- Full reanalysis of K⁺ scattering by GWU group, bounds scattering gives upper bound for width of approximately 1 MeV (Phys.Rev. C68 (2003) 042201)
- "Conservative" Estimate Γ_{θ} <2 MeV

- The extremely narrow widths are remarkable. Indeed this is more remarkable than the exotic quantum numbers.
- There is no good explanation for the extreme narrowness of the state. (Personal Opinion)
- One can use the narrowness of the state to constrain models even if we do not understand its source.

- Basic argument by Nussinov hep-ph/0307357:
 - K⁺N resonance is elastic and its height is fixed by unitarity. Area under curve proportional to width.
 - K⁺D scattering is essentially independent scattering on P and N since D is loosely bound.
 - D wave function is broad compared to width of θ^+ so smears over full K⁺N resonance which is proportional to width.
 - Absence of structure in data bounds the width in region near the θ^{+} , bounds the width

Quark Models

- Simplest quark models are pentaquark models
- Here I explore one well known variant---the Jaffe-Wilczek (JW) model---and show that the extremely narrow width puts severe constraints on the viability of the model
- Basic idea quarks like to form diquarks

- Most favored channel has diquark in an antitriplet in both color and flavor (see high density QCD)
- Effectively get three body problem---2 diquarks and one anti-quark
- θ⁺ must have the two diquarks in a relative pwave.
 - Diquarks are bosons so must be symmetrric
 - Total state is color singlet so anti-symmetric in color
 - Anti-decuplet in flavor so symmetric in flavor
 - Thus anti-symmetric in space---p-wave

- Analogous state can be formed in the octet representation
- States will be nearly degenerate in exact SU(3) flavor limit
- SU(3) Symmetry will break this degeneracy leading to nearly ideal mixing.



 Effective interaction counts number of s quarks and anti-quarks. Each costs the mass of the quark. Also s-quarks have a cost inside of diquarks

$$H_{I} = (a(n_{s} + n_{\bar{s}}) + b n_{s})(2m_{s} - m_{u} - m_{d})$$

Note this vanishes in SU(3) limit. Away from SU(3) limit mixes the octet and the anti-decuplet. This interaction induces nearly ideal mixing

 Physical states are (almost) eigenstates of the number of s quarks and s anti-quarks seperately

Spectrum



- $-\Sigma$ (mixed: 2 s ,1 anti s)
- Ξ (I=3/2) Ξ (I=1/2) (nearly degenerate and unmixed)
- -N (mixed: 1 s ,1 anti s)
- Λ (unmixed octet) Σ (mixed: 1 s ,0 anti s)
- θ (unmixed anti-decuplet)
- N (mixed: 0 s ,10anti s)

Tentative Identification of States



- New exotics θ⁺ (1540) and Ξ⁺⁺ (1860) fit in in an obvious way. These are largely unmixed anti-decuplets
- States with nucleon quantum numbers identified as the N^{*} (1440) (the Roper) and the N^{*} (1710). These states are in the right mass ranges and it stretches credulity for there to be two states at nearly the same mass for this quantum number.

Troubling Issue



- The Roper and the N^{*} (1710) do not look like brothers.
 - The Roper is very wide (~350 MeV) with big partial width to N π (~225MeV)
 - N^{*} (1710) is reasonably narrow ((~100 MeV) with very small partial width to N π (~15MeV)
- Does this rule out model?

• By itself, it does not.

- Combined with the very narrow bound for the θ^+ width, however either

– The model is wrong

 The identification of at least one of the states with nucleon quantum numbers is wrong

• Tool to see this is an inequality relating widths.

Analysis of the widths

- Focus on the two mixed states with nucleon quantum numbers.
- Take seriously J-W model
 - Assume pure diquark model (three body problem)
 - Assume exact degeneracy in SU(3) limit
 - Assume ideal mixing due to SU(3) violations.

Label diquarks by there SU(3) flavor

$$\overline{U} = (ds) \quad \overline{D} = (us) \quad \overline{S} = (ud)$$

Possible SU(3) N* flavor states ($I_3=1/2$)

$$\begin{split} \left| \overline{10} \right\rangle_{flavor} &= \frac{1}{\sqrt{3}} \left(\left| \overline{S} \, \overline{D} \, \overline{S} \right\rangle + \left| \overline{D} \, \overline{S} \, \overline{S} \right\rangle + \left| \overline{S} \, \overline{S} \, \overline{d} \right\rangle \right) \\ \left| 8SA \right\rangle_{flavor} &= \frac{1}{\sqrt{6}} \left(\left| \overline{S} \, \overline{D} \, \overline{S} \right\rangle + \left| \overline{D} \, \overline{S} \, \overline{S} \right\rangle - 2 \left| \overline{S} \, \overline{S} \, \overline{d} \right\rangle \right) \\ \left| 8AS \right\rangle_{flavor} &= \frac{1}{\sqrt{2}} \left(\left| \overline{S} \, \overline{D} \, \overline{S} \right\rangle - \left| \overline{D} \, \overline{S} \, \overline{S} \right\rangle \right) \end{split}$$

Possible full wave functions
$$\left|\overline{10}\right\rangle = \left|\overline{10}\right\rangle_{flavor} \otimes \left|\overline{10}\right\rangle_{space-spin} \setminus$$

$$\begin{aligned} |8\rangle = c_{SA} |8SA\rangle_{flavor} \otimes |8SA\rangle_{space-spin} \\ + c_{AS} |8AS\rangle_{flavor} \otimes |8AS\rangle_{space-spin} \end{aligned}$$

with
$$|c_{SA}|^2 + |c_{AS}|^2 = 1$$

Constraint due to ideal mixing



Linear combinations of the octet and anti
 decuplet yield eigenstates for the number of
 strange quarks and strang anti-quarks
 separately. Look at state with no s quarks:

$$0 = n_{s} \left[\alpha \left| \bar{10} \right\rangle + \beta \left| 8 \right\rangle \right] =$$

$$\left(\left| \bar{S} \, \bar{D} \, \bar{s} \right\rangle + \left| \bar{D} \, \bar{S} \, \bar{s} \right\rangle \right) \otimes \left(\alpha \frac{\left| \bar{10} \right\rangle_{space-spin}}{\sqrt{3}} + \beta \, C_{SA} \frac{\left| 8SA \right\rangle_{space-spin}}{\sqrt{6}} \right)$$

$$+ \beta \, C_{SA} \left(\left| \bar{S} \, \bar{D} \, \bar{s} \right\rangle - \left| \bar{D} \, \bar{S} \, \bar{s} \right\rangle \right) \otimes \left| 8AS \right\rangle_{space-spin}$$

Can be satisfied only if

$$\beta = -\sqrt{2}\alpha, \quad c_{AS} = 0, \quad c_{SA} = 1$$
$$|8SA\rangle_{space-spin} = |\overline{10}\rangle_{space-spin}$$

- The non-flavor wave function for the octet and the anti-decuplet must be identical to get ideal mixing
- The ratio of the octet to the anti-decuplet is fixed

Wave functions

$$|N_1\rangle = \sqrt{\frac{2}{3}} \left|\overline{10}\rangle - \sqrt{\frac{1}{3}} \left|8\rangle \quad (Roper?) \\ |N_0\rangle = \sqrt{\frac{1}{3}} \left|\overline{10}\rangle + \sqrt{\frac{2}{3}} \left|8\rangle \quad (N^*(1710)?) \right|$$

 Ideal mixing fixes ratio of octet and antidecuplet for the N^{*} states SU(3) Flavor is typically reliable at the ~30% level for generic observables.

- Large SU(3) violations can occur
 - Ideal mixing (when SU(3) unbroken states are nearly degenerate).
 - Masses of psuedo-Goldstone bosons.
 - Large violations in widths near thresholds but not in coupling constants.
- These are the *only* known sources of large SU(3) violations.
- An inequality for widths can be derived assuming *no* other SU(3) violation+ideal mixing.

Widths are given by coupling constants and a kinematical factor incorporating phase space and the p-wave nature of coupling.

$$\Gamma_{B'BP} = |g_{PB'B}|^2 \kappa_{PB'B}$$

$$\kappa_{PB'B} = \frac{q}{4\pi M_{B'}} \left(\sqrt{q^2 + M_B^2} - M_B \right)$$

$$M_{B'} = \sqrt{q^2 + M_B^2} + \sqrt{q^2 + m_P^2}$$

- B,B' are baryons; P is a pseudo-scalar
- q is the momentum of decay fragments.

Given assumptions of SU(3) symmetry and ideal mixing

$$g_{\pi N_1 N} = -\sqrt{\frac{1}{3}}g_{\pi N_8 N} + \sqrt{\frac{2}{3}}g_{\pi N_{\bar{1}\bar{0}}N}$$
$$g_{\pi N_0 N} = \sqrt{\frac{2}{3}}g_{\pi N_8 N} + \sqrt{\frac{1}{3}}g_{\pi N_{\bar{1}\bar{0}}N}$$

Simple algebra yields an inequality

$$\left\| g_{\pi N_0 N} \right\|^2 - 2 \left| g_{\pi N_1 N} \right|^2 + \left| g_{\pi N_{\overline{10}} N} \right|^2 \right\| \leq 2\sqrt{2} \left| g_{\pi N_{\overline{10}} N} \right| \sqrt{\left| g_{\pi N_0 N} \right|^2 + \left| g_{\pi N_1 N} \right|^2 - \left| g_{\pi N_{\overline{10}} N} \right|^2}$$

- Inequality rather than equality due to unknown phases.
- SU(3) Clebsch-Gordons imply:

$$g_{\pi N_{\bar{1}\bar{0}}N} = \frac{1}{2}g_{K\theta^{+}N}$$

Combining the inequality, the SU(3) relation for the coupling and the expression for the width yields

$$\left| \frac{\Gamma_{\pi N_0 N}}{\kappa_{\pi N_0 N}} - 2 \frac{\Gamma_{\pi N_1 N}}{\kappa_{\pi N_1 N}} + \frac{\Gamma_{\pi \theta^+ N}}{4\kappa_{\pi \theta^+ N}} \right| \leq$$

$$\sqrt{\left(\frac{2\Gamma_{\pi\theta^{+}N}}{\kappa_{\pi\theta^{+}N}}\right)\left(\frac{\Gamma_{\pi N_{0}N}}{\kappa_{\pi N_{0}N}}+\frac{\Gamma_{\pi N_{1}N}}{\kappa_{\pi N_{1}N}}-\frac{\Gamma_{\pi\theta^{+}N}}{4\kappa_{\pi\theta^{+}N}}\right)}$$

- Limitations to this inequality
 - Assumes ideal mixing is exact
 - Assume only SU(3) violations are due to ideal mixing, threshold effects and pseudo Goldstone masses
- Minor violation of inequality consistent with J-W model.
- Gross violations are not.
- Inequality is grossly violated if $N_0 = N^*(1440)$ and $N_1 = N^*(1710)$

N* Properties from the PDG

• N*(1440)



- Mass: ~1440 Mev (1430-1470 Mev)
- Width: ~350 (MeV) (250-450 Mev)
- Branching Fraction to $N\pi$: 60%–70%
- N*(1710)
 - Mass: ~1710 Mev (1680-1740 Mev)
 - Width: ~100 (MeV) (50-250 Mev)
 - Branching Fraction to $N\pi$: 10%–20%
- Qualitatively different: $\Gamma_{N^*(1440)\Re N\pi} \sim 225 \text{ MeV}$ $\Gamma_{N^*(1440)\Re N\pi} \sim 15 \text{ MeV}.$

- Plugging in best estimate values for the N* states and using the direct experimental bound for the θ⁺ width (20 Mev)
 - LHS of inequality: 72
 - RHS of inequality: 62
- Small violation of the inequality; does not exclude model for reasons discussed above.

- Plugging in best estimate values for the N* states and using the bound for the θ⁺ width deduced from not seeing it in prior experiments(2 Mev)
 - LHS of inequality: 77
 - RHS of inequality: 20
- Plugging in best esitmate values for the N* states and using the lowest bound for the θ⁺ width measured (9 Mev)
 - LHS of inequality: 75
 - RHS of inequality: 42
- Gross violation of the inequality; excludes model with states identified as in J-W

 Uncertainties in masses, widths, and branching fractions for the N* states do not alter this conclusion. Taking the most optimistic of these still yields a factor of two violation of the inequality assuming θ⁺ width of 2 MeV.

• Extremely important to pin down θ^+ width experimentally.

Reason for the violation of the inequality:

- The extracted θ⁺ width is so small that the coupling of the anti-decuplet can almost be neglected compared to the coupling of the N* (1440) which then fixes the octet coupling to be large.
- A large octet coupling with a small antidecuplet coupling implies strong coupling of N* (1710) but the N* (1710) partial width is very small.

What does this mean?

- The bounds on the θ^+ width are wrong
- The J-W model is wrong

Or

- The identification N₀=N*(1440) and N₁=N*(1710) is wrong
 - Scenario i: N*(1710) is a pentaquark; N₀ is a narrow state presently unobserved.
 - Scenario ii: N*(1440) is a pentaquark; N₁ is a wide state presently unobserved.

- Both scenarios are implausible in some ways
 - Requires two states with same quantum numbers right on top of each other. (*Eg.* Scenario i has Roper and new narrow state both at ~1450 MeV). Unknown in hadronic physics.
 - Unknown states are strongly constrained by old searches.
 - If there is a narrow state at ~1450 MeV it will have to be very narrow not have been seen. Analysis along line of GWU group would be useful.
 - If there is a wide state at ~1700 MeV it would have to be very wide to avoid detection heretofore. This cannot be ruled out if very wide but is unattractive on theoretical grounds.

- General problem of disparate widths noted by JW. They argue that internal and flavor wave functions for the Roper is different than that of the θ^+ . More acute issue with narrower bound on θ^+ .
 - Argument is wrong. Ideal mixing requires same internal wave function as shown earlier.
 - Glozman has argued that Roper and θ^+ cannot be pentaquark due to its width.
 - No theorem states that the same internal wave function cannot lead to both narrow and wide states but does seems quite perverse.
- This makes Scenario ii quite unattractive.

Constraints More General Than for JW Model

- Constraints due to widths apply for *any* model with ideal mixing.
- Ideal mixing occurs when all OZI violating amplitudes are negligible. Thus any pure pentaquark model will mix ideally.
 - In pure SU(3) limit, the octet and anti-decuplet are only split be diagrams which change number of quarks (*eg.* octet mixing with three quark component).

- Nothing special about diquark model in this.
- How good is OZI rule?
 - Generally pretty good except for pseudoscalar channels.
- Does this mean OZI rule holds here?
 - Not necessarily.
 - In most circumstances OZI rule follows from large Nc QCD.
 - In present case both OZI preserving and OZI violating graphs have same Nc counting
- How well motivated is ideal mixing?

Other States in JW model

- JW model gives positive parity due to pwave between diquarks.
- This is consequence of boson nature of diquarks, anti-symmetry in color and symmetry in flavor for the anti-decuplet θ^{+}
- However this argument does not hold for octets. One can have an AS configuration with a symmetric space state. Implies negative parity state.

- One would expect that this symmetric state would be lower in energy than the anti-symmetric combination
 - Possible interaction between spin of antiquark and orbital of the diquarks could alter this.
 - In any event a negative parity octet should lower or comprable to the mass of the antidecuplet-octet positive parity states.
- Possible candidates: N*(1535) and N*(1650)

Problem

- Both the N*(1535) and N*(16500) are well accounted for in the constituent quark model.
- If either is a pentaquark one does damage to explanation of ordinary baryon spectroscopy.
- If not where is the light negative parity pentaquark?

Summary of JW model

 Inequality for widths imply that either model is wrong or the identification N₀=N*(1440) and N₁=N*(1710) is wrong. No plausible candidate states.

 Model implies light negative parity octet pentaquarks and there are no plausible candidates.

Chiral Solitons Models

- Analysis done in context of collective quantization for almost all calculations
- Result for the mass is almost completely insensitive to details of model.
 - Details of profile completely irrelevant to prediction. Only structure of model plus parameters of SU(3) breaking and the identification of the nucleon state in the multiplet



Good news if collective quantization is legitimate

- Is it?
 - Yes for non-exotic states
 - No for exotic states (TDC PLB531 175 (2004); TDC hepph/031219, Princeton Mafia hep-ph/0309305, P.V. Poblylitsa, hepph/0310221)



• Collective quantization amounts to quantizing the motion of a slowly rotating hedgehog.



 Only legitimate if the motion described is slow at large Nc. (Model only justified at large Nc. Large Nc justifies the classical treatment of the soliton profile as well as collective quantization) Semiclassical Quanatization of SU(3) Skyrmions

- Assume exact SU(3) Symmetry (ms perturbatively)
- Hedgehog solution (assume in u-d subspace)

$$U = \begin{pmatrix} 2x2 \text{ hh} \\ \widetilde{U_h} & 0 \\ 0 & 1 \end{pmatrix}$$

• Follow ANW approach (Guadagnini 1984...)

$$U(\vec{r},t) = A^{+}(t)U_{o}(\vec{r})A^{+}(t)$$

- Constraint due to Wess-Zumino term: $J'_8 = -\frac{N_c B}{2\sqrt{3}}$
 - Analog of intrinsic angular momentum for monopole problem.
 - Derivable at quark level

• Hamiltonian:

$$H = M_0 + \sum_{A=1,2,3} \frac{J_A'^2}{2I_1} + \sum_{A=4,5,6,7} \frac{J_A'^2}{2I_2}$$

- Two moments of inertia (in SU(2) space and out.)
- No kinetic energy in 8 direction (leave hedgehog unchanged. Note analogy to monopole.

• Energies:

$$M = M_0 + \frac{C_2}{2I_2} + \frac{(I_2 - I_1)J(J+1)}{2I_1I_2} - \frac{N_c^2}{2I_2}$$

$$C_2 = \frac{p^2 + q^2 + pq + 3(p+q)}{3}$$

• Constraint:

- Representation must have Y=Nc/3
 (2J+1) = # of states with S=0
- For Nc=3 lowest (p,q) rep J representations (1,1) 8 1/2 (3,0) 10 3/2 (0,3) 10 1/2

- The anti-decuplet is manifestly exotic
- Masses:

$$M_8 = M_0 + \frac{3}{8I_1}$$
$$M_{10} = M_0 + \frac{15}{8I_1}$$
$$M_{\overline{10}} = M_0 + \frac{3}{8I_1} + \frac{3}{2I_2}$$

• SU(3) symmetry breaking added perturbatively.

Problems with rigid rotor quantization for exotic excitations?

Is semi-classical rigid-rotor quantization Kosher for exotic states?

- Superficially yes. It depends on an adiabatic scale separation between collective motion and intrinsic motion, *i.e.* $\tau_{collective} >> \tau_{intrinsic}$
 - Standard semi-classical relation $\tau \sim 1/(\Delta E)$
 - Intrinsic (vibrational motion) $\Delta E \sim N_c^0$
 - For exotic (nonexotic) motion $\Delta E \sim 1/I_1$ (1/I₂) so in both cases $\tau_{\text{collective}} \sim Nc$
 - For both cases $\tau_{collective} >> \tau_{intrinsic}$

- Actually this argument is a complete swindle. To test whether the exotic motion is slow at large Nc we must go to large Nc limit. But Nc =3 was built in to constraint condition!!!
- Redo analysis for arbitrary Nc and take large Nc limit.
 - Issue in identifying states as all representations are larger than for Nc=3. (Issue does not arise in SU(2) models)

(Standard approach identify representations whose lowest members match on to Nc=3 and dismiss other states as large Nc artifacts)

- Lowest representation $(p,q) = (1, \frac{N_c 1}{2})$ (analog of octet) J = 1/2 denoted "8"
- Next representation $(p,q) = \left(3, \frac{N_c 3}{2}\right)$ (analog of decuplet) J = 3/2 denoted "10"

• Lowest representation containing s=+1 state $(p,q) = \left(0, \frac{N_c+3}{2}\right)$ (analog of antidecuplet) J = 1/2 denoted "10"



• Use mass formula from before: - Nonexotic excitations $M_{"10"} - M_{"8"} = \frac{3}{2I_1} \sim \frac{1}{N_c}$ $\tau_{\text{collective}} \sim N_c >> \tau_{\text{intrinsic}}$

Adiabatic : collective quantization justified.

Exotic excitations

$$M_{"\overline{10}"} - M_{"8"} = \frac{3 + N_c}{4I_2} \sim N_c^0 \quad \tau_{\text{collective}} \sim N_c^0 \sim \tau_{\text{intrinsic}}$$

Nonadiabatic: collective quantization not justified!!!

 Diakonov and Petrov agree with Nc counting but disagree with conclusion in hep-ph/0309203.

 As shown in hep-ph/0312191 these DP counter arguments are not valid and collective quantization is not legitimate for exotic states.

• Other ways to see this:

Widths

- Diakonov, Petrov, Polyakov stress narrow numerical width to justify approach selfconsistently. Largely this is due to phase space
- there is still a fundamental formal issue.
 - If approach is legitimate it should give exact mass at large Nc. Otherwise ad hoc corrections need to be added.
 - This implies width must be zero at large Nc.
 If not, the state doesn't really exist and concept of an exact mass is silly. Alternatively, view width as an imaginary contribution to mass

 Width computed from coupling constant which in turn depends on asymptotic profile function and collective wave function.
 Explicit computation in the context of rigid rotor quantization was done by Praszalowicz:

$$\frac{\left|\left\langle \theta^{+}, s = \downarrow \right| \hat{O}_{K3} \left| N, s = \downarrow \right\rangle\right|^{2}}{9(N_{c} + 1)}$$

$$\frac{9(N_{c} + 1)}{(M_{N} + M_{\theta^{+}})^{2} (N_{c} + 3)(N_{c} + 7)}$$

$$\left[G_0 - \frac{N_c + 1}{4}G_1\right]p^2$$

$$G_0 \sim N_c^{3/2} - G_0 \sim N_c^{1/2}$$

Where the operator gives the coupling to a Kaon in direction J.

 Including phase space and scaling one deduces that

 $\Gamma \sim N_c^0$

• This indicates an inconsistency

Large Nc Consistency

- Reason to chiral soliton prediction seriously in first place was model insensitivity; this typically means that relation derivable directly by large Nc consistency rules.
- These rules known for three flavor QCD.
 - Give exactly the same states as in a large Nc Quark model. (Dashen, Jenkins Manohar 94).
 - Exotic collective states not predicited by this model indpendent approach. But all nonexotic ones are.
- Does not mean exotic states from semiclassical quantization is wrong. But it does mean we have no reason to believe them a priori.

Quark Models

- Derivation of collective quantization does not depend on chiral symmetry. It only needs large Nc scaling, and a mean-field hedgehog.
- Large Nc SU(3) quarks model with Nc quarks in Hilbert space satisfy this so have identical collective quantization.
- By construction quark model has no exotics; collective quantization predicting exotics is wrong. (Pobylitsa)

- Previous arguments show rigid rotor quantization fails for exotic states but works for nonexotic states. Why?
- Fundamental reason---mixing of collective and intrinsic (vibrational) modes at leading order in Nc due to Wess-Zumino term. Collective and vibration modes not orthogonal.
- This can be illustrated in toy models--analog of collective quantization works only when vibrations and rotations decouple for reasons other than Nc.

Conclusions For Chiral Solitons

- Rigid rotor quantization is not justified for exotic motion. We don't understand θ^+ from first principles
 - Room for many models
 - theorist will remain employed for the foreseeable future
 - Successful prediction of θ^+ properties in chiral soliton models with rigid rotor qunatization is fortutious.

- Large Nc does not provide automatic method for understanding existence of θ^+
 - Chiral soliton models (vibrational approach) does not predict θ⁺ but can accommodate it. (Princeton group---Itzhaki, Klebanov,Ouyang&Rastelli 03)
 - Large N_c QCD does not predict θ⁺ but given the existence of θ⁺ predicts the existence of S=1 large N_c "partners" in much the same way that Δ is partner of nucleon. The mass splitting to these new states is O(1/N_c). Derivation use QCD consistency applied to scattering amplitudes & a neat crossing from t to s channels(Cohen & Lebed 2003)