Parametrization of the quark-quark correlator of a spin-1/2 hadron

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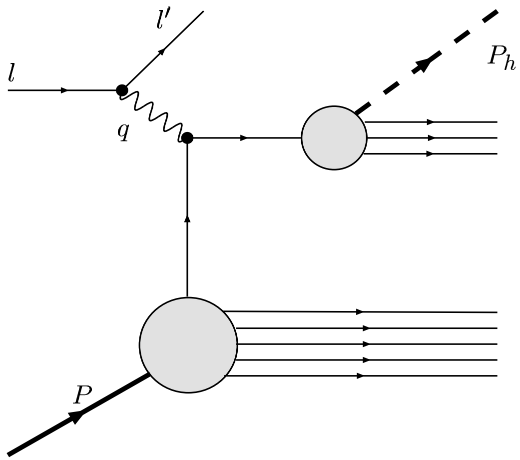
- Semi-Inclusive Deep Inelastic Scattering (SIDIS)
- Parametrization of Correlators (I)
- Gauge Links
- Parametrization of Correlators (II)

Semi-inclusive DIS

Collision of leptons and hadrons

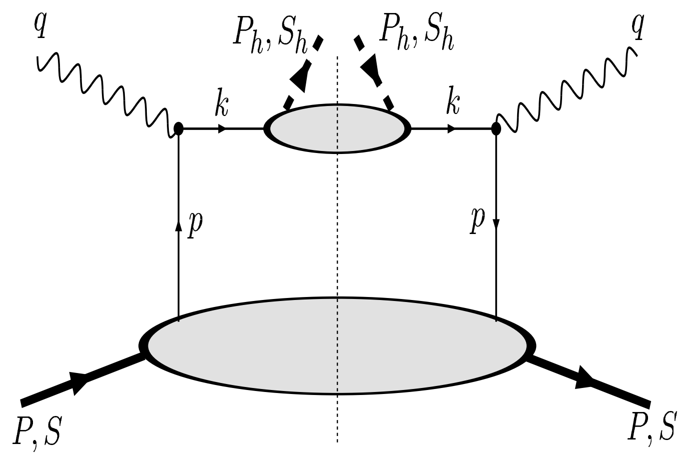


Kinematical invariants:

$$
\begin{aligned}
q^2 &= -Q^2 \\
x_B &= \frac{Q^2}{2P\cdot q} \\
y &= \frac{P\cdot q}{l\cdot P} \\
z_h &= \frac{P_h\cdot P}{q\cdot P}
\end{aligned}
$$

Infinite momentum frame: P^+ large!

Hadronic tensor



Correlators

$$\Phi_{ij}(x, \vec{p}_T) = \frac{1}{(2\pi)^3} \int d\xi^- d^2\xi_T \; e^{ip\cdot\tilde{\xi}} \, \langle P, S|\, \bar{\Psi}_j(0) \; \mathcal{L}^{[+]}[0, \tilde{\xi}] \; \Psi_i(\tilde{\xi})\, |P, S\rangle$$

$$\Delta_{ij}(z, \vec{k}_T) = \sum_X \int \frac{d\xi^+}{2\pi} \frac{d^2\xi_T}{(2\pi)^2} \; e^{ik\cdot\xi'} \, \langle 0|\, \mathcal{L}^{[-]}[0, \xi']\Psi_i(\xi')\, |P_h, S_h; X\rangle \, \langle X; P_h, S_h|\, \bar{\Psi}_j(0)\, |0\rangle$$

Parametrization of Correlators (I)

- The correlators $\Phi_{ij}(x,\vec{p}_T)$ and $\Delta_{ij}(z,\vec{k}_T)$ are matrices in Dirac-space.

  $\Rightarrow$ Decomposition into the 16 covariant basis matrices $\mathbb{1}$, γ_5, γ^μ, $\gamma^\mu\gamma_5$, $\sigma^{\mu\nu}/i\sigma^{\mu\nu}\gamma_5$.
  $(\Phi^{[\Gamma]}(x,\vec{p}_T) \equiv \frac{1}{2}Tr[\Phi(x,\vec{p}_T)\Gamma])$

$$\Phi_{ij}(x,\vec{p}_T) = \frac{1}{2}\Phi^{[\gamma_\mu]}\gamma^\mu - \frac{1}{2}\Phi^{[\gamma_\mu\gamma_5]}\gamma^\mu\gamma_5 - \frac{1}{4}\Phi^{[i\sigma_{\mu\nu}\gamma_5]}i\sigma^{\mu\nu}\gamma_5 + \frac{1}{2}\Phi^{[\mathbb{1}]}\mathbb{1} - \frac{1}{2}\Phi^{[i\gamma_5]}i\gamma_5$$

$$\Downarrow$$

  Coefficients are the transverse-momentum dependent (TMD) parton distributions / fragmentation functions.

- Historically, TMD correlators were parametrized using an "unintegrated" correlator

$$\boxed{\Phi_{ij}(p;P,S) \equiv \int \frac{d^4\xi}{(2\pi)^4}\, \mathrm{e}^{ip\cdot\xi}\, \langle P,S|\, \bar{\Psi}_j(0)\, \mathcal{L}[0,\xi|\mathrm{path}]\, \Psi_i(\xi)\, |P,S\rangle}$$

  Connection to TMD correlators: $\Phi_{ij}(x,\vec{p}_T) = \int dp^- \;\; \Phi_{ij}(p;P,S)\big|_{p^+=xP^+}$.

$$\Downarrow$$

  Parametrization of the "unintegrated" correlator

1. Ralston, Soper [Nucl. Phys. B 152 (1979) 109]:

$$\begin{aligned}\Phi(p;P,S) &= A_1\mathbb{1} + A_2\not{P} + A_3\not{p} + A_4\gamma_5\not{S} + A_5[\not{P},\not{S}]\gamma_5 \\ &+A_6[\not{p},\not{S}]\gamma_5 + A_7(p\cdot S)\not{P}\gamma_5 + A_8(p\cdot S)\not{p}\gamma_5\end{aligned}$$

Eight amplitudes $A_i = A_i(p^2, p\cdot P)$, structures are constraint by $(\bar{A} = (A_0, -\vec{A}))$

$$\begin{aligned}\Phi^\dagger(p;P,S) &= \gamma_0\Phi(p;P,S)\gamma_0 && (\text{Hermiticity}) \\ \Phi(p;P,S) &= \gamma_0\Phi(\bar{p};\bar{P},-\bar{S})\gamma_0 && (\text{Parity}) \\ \Phi^*(p;P,S) &= (-i\gamma_5 C)\Phi(\bar{p};\bar{P},-\bar{S})(-i\gamma_5 C) && (\text{Time reversal}).\end{aligned}$$

$\Longrightarrow \Phi^{[\gamma^+]}(x,\vec{p}_T) = 2P^+\int dp^-(A_2 + xA_3) \equiv f_1(x,\vec{p}_T)$

2. Mulders, Tangerman [Nucl. Phys. B 461 (1996) 197]:

Addition of three more structures $\sigma^{\mu\nu}P_\mu p_\nu$, $(p\cdot S)i\gamma_5$ and $\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma$ which do not obey time reversal constraint + one T-even structure $A_9(p\cdot S)[\not{P},\not{p}]\gamma_5$

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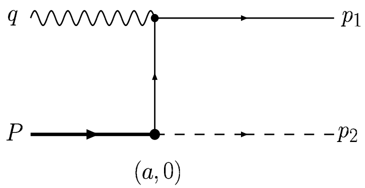
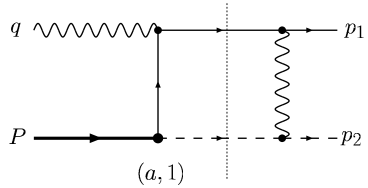
Parton Distributions which are extracted from these structures: T-odd PDFs!

Gauge links

- Model calculation of Single Spin Asymmetry A_{UT}: [Brodsky, Hwang, Schmidt, Phys. Lett. B 530, 99 (2002)]

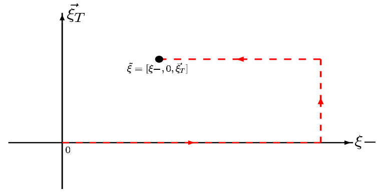




$\Longrightarrow$ non-vanishing asymmetry due to *rescattering* ("final state interactions")

- *Explanation* [Collins, Phys. Lett. B 536, 43 (2002)]:T-odd Sivers-function $f_{1T}^{\perp}$ includes a gauge link, makes $f_{1T}^{\perp}$ non-vanishing.

$$\boxed{\mathscr{L}[0,\xi|\text{path}] \equiv \mathscr{P}\exp\left\{-ig\int_0^{\xi} ds^{\mu}\, A_{\mu}(s)\right\}}$$



- Consequence for the parametrization of $\Phi(p;P,S)$:
  1. Time reversal constraint $\Phi^*(p;P,S) = (-i\gamma_5 C)\Phi(\bar{p};\bar{P},-\bar{S})(-i\gamma_5 C)$ doesn't hold.
  2. Goeke, Metz, Pobylitsa, Polyakov [Phys. Lett. B 567 (2003) 27]:

     TMD correlator $\Phi(x,\vec{p}_T)$ depends implicitly on a light cone vector n due to the gauge link.
     $\Rightarrow$ Unintegrated $\Phi(p;P,S)$ also depends on n!
     $\Rightarrow$ Additional light cone dependent, spin independent structures $\frac{\not{n}}{(P\cdot n)}$, $\frac{[\not{P},\not{n}]}{(P\cdot n)}$ and $\frac{[\not{p},\not{n}]}{(P\cdot n)}$.

- Gauge link affects also twist-3 observables, longitudinal single-spin asymmetries A_{UL} and A_{LU}.

  $\Rightarrow$ Generalization of the BHS-model calculation [Metz, Schlegel, Eur. Phys. J. A 22, 489 (2004)], [Afanasev, Carlson, hep-ph/0308163]

  $\Rightarrow$ non-vanishing result due to rescattering $\boxed{A_{UL} \neq 0}$, $\boxed{A_{LU} \neq 0}$

- Model-independent analysis: [Bacchetta, Mulders, Pijlman, Phys. Lett. B 595 (2004) 309]:

Additional term in the parametrization of $\Phi(p; P, S|n)$

$$\Phi(p; P, S|n) = M A_1 \mathbb{1} + \ldots + B_1 \frac{M^2 \not{n}}{P \cdot n} + B_2 \frac{iM[\not{P}, \not{n}]}{2(P \cdot n)} + B_3 \frac{iM[\not{p}, \not{n}]}{2(P \cdot n)} + B_4 \frac{1}{(P \cdot n)} \varepsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma_5 P^\nu n^\rho p^\sigma$$

$\Rightarrow$ generates a new T-odd, twist 3 PDF $\boxed{g^\perp(x, \vec{p}_T) = 2P^+ \int dp^- B_4}$!

Results:

$$\boxed{A_{LU,jet}^{\sin\phi_h} = \frac{M^2}{Q} \frac{2y\sqrt{1-y}}{1-y+\frac{y^2}{2}} \frac{x_B g^{\perp(1)}}{f_1}}$$

$$\boxed{A_{UL,jet}^{\sin\phi_h} = -\frac{M^2}{Q} \frac{2(2-y)\sqrt{1-y}}{1-y+\frac{y^2}{2}} \frac{x_B f_L^{\perp(1)}}{f_1}}$$

Parametrization of Correlators (II)

- Up to this point: only spin-independent light-cone structures of the parametrization of $\Phi(p;P,S|n)$ have been presented.

What happens if spin is included? [Goeke, Metz, Schlegel, Phys. Lett. B 618 (2005) 90]:

$$\Downarrow$$

16 additional structures including the light-cone vector n and spin vector S.

Altogether, there are 32(!) structures of $\Phi(p;P,S|n)$ restricted by hermiticity and parity, 12 T-odd structures.

- In order to avoid redundant terms, make use of the identity

$$g^{\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = g^{\mu\beta}\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\varepsilon^{\mu\nu\rho\alpha}$$

- Twist classification: convenient to use Sudakov decomposition

$$\begin{aligned}
P^\mu &= P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu, \\
p^\mu &= xP^+ n_+^\mu + p^- n_-^\mu + p_T^\mu \\
S^\mu &= \lambda \frac{P^+}{M} n_+^\mu - \lambda \frac{M}{2P^+} n_-^\mu + S_T^\mu
\end{aligned}$$

- Result: two new twist-3 parton distributions $e_T^\perp$ and $f_T^\perp / f_T^{\perp\prime}$, all T-odd!

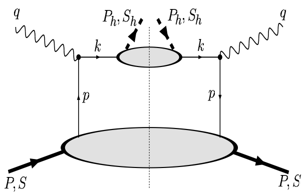
Twist-2 parametrization:

$$\begin{aligned}
\Phi^{[\gamma^+]}(x,\vec{p}_T) &= f_1 - \frac{(\vec{p}_T \times \vec{S}_T)}{M} f_{1T}^\perp \\
\Phi^{[\gamma^+\gamma_5]}(x,\vec{p}_T) &= \lambda g_{1L} + \frac{\vec{p}_T \cdot \vec{S}_T}{M} g_{1T} \\
\Phi^{[i\sigma^{i+}\gamma_5]}(x,\vec{p}_T) &= S_T^i h_{1T} + \frac{p_T^i}{M}\left(\lambda h_{1L}^\perp + \frac{\vec{p}_T \cdot \vec{S}_T}{M} h_{1T}^\perp\right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M} h_1^\perp
\end{aligned}$$

Twist-3 parametrization:

$$\begin{aligned}
\Phi^{[\mathbb{1}]}(x,\vec{p}_T) &= \frac{M}{P^+}\left[e - \frac{(\vec{p}_T \times \vec{S}_T)}{M} e_T^\perp\right] \\
\Phi^{[\gamma^i]}(x,\vec{p}_T) &= \frac{M}{P^+}\left[\frac{p_T^i}{M}\left(f^\perp - \frac{(\vec{p}_T \times \vec{S}_T)}{M} f_T^{\perp\prime}\right) + \frac{\varepsilon_T^{ij} p_{Tj}}{M}\left(\lambda f_L^\perp + \frac{\vec{p}_T \cdot \vec{S}_T}{M} f_T^\perp\right)\right] \\
\Phi^{[\gamma^i\gamma_5]}(x,\vec{p}_T) &= \frac{M}{P^+}\left[S_T^i g_T^\prime + \frac{p_T^i}{M}\left(\lambda g_L^\perp + \frac{\vec{p}_T \cdot \vec{S}_T}{M} g_T^\perp\right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M} g^\perp\right] \\
\Phi^{[i\sigma^{+-}\gamma_5]}(x,\vec{p}_T) &= \frac{M}{P^+}\left[\lambda h_L + \frac{\vec{p}_T \cdot \vec{S}_T}{M} h_T\right] \\
\Phi^{[i\sigma^{ij}\gamma_5]}(x,\vec{p}_T) &= \frac{M}{P^+}\left[\frac{S_T^i p_T^j - p_T^i S_T^j}{M} h_T^\perp - \varepsilon_T^{ij} h\right]
\end{aligned}$$

- In [Mulders, Tangerman], the trace $\Phi^{[\gamma^i]}$ contains a term $\varepsilon_T^{ij} S_{Tj} f_T$.
  $\Rightarrow$ Can be eliminated by means of the identity $\vec{p}_T^2 \varepsilon_T^{ij} S_{Tj} = -p_T^i(\vec{p}_T \times \vec{S}_T) + \varepsilon_T^{ij} p_{Tj}(\vec{p}_T \cdot \vec{S}_T)$
- Number of PDFs $=$ number of amplitudes $\Longrightarrow$ no linear dependence between PDFs in terms of amplitudes, no Lorentz-invariance relations.
- Where do new PDFs show up?



$$2MW^{\mu\nu}_{tree} = \int d^2p_T d^2k_T \delta^{(2)}(\vec{p}_T + \vec{q}_T - \vec{k}_T) Tr[\Phi(x_B, \vec{p}_T)\gamma^\mu \Delta(z_h, \vec{k}_T)\gamma^\nu]$$

$e_T^\perp$ enters the double polarized cross section σ_{LT} multiplied with $H_1^\perp$.

$f_T^{\perp\prime}$ and $f_T^\perp$ enter the transversely polarized cross section σ_{UT} multiplied with D_1.

- For the "integrated" correlator $\Phi_{ij}(x) = \int d^2p_T \; \Phi_{ij}(x, \vec{p}_T)$, time-reversal constraint holds. $\Longrightarrow$ *no* T-odd "integrated" PDFs.
  $\Longrightarrow$ Constraint on some of the T-odd PDFs:

$$\begin{aligned} \int d^2p_T \; e_L(x, \vec{p}_T^2) &= 0 \\ \int d^2p_T \; \vec{p}_T^2 \left(f_T^{\perp\prime}(x, \vec{p}_T^2) + f_T^\perp(x, \vec{p}_T^2) \right) &= 0 \\ \int d^2p_T \; h(x, \vec{p}_T^2) &= 0. \end{aligned}$$

Summary

- The general structure of the fully unintegrated correlator $\Phi(p; P, S|n)$ was derived. This made it possible to write down the most general form of the transverse momentum dependent correlator $\Phi(x, \vec{p}_T, S)$ appearing in the description of various hard scattering processes. Two new twist-3, T-odd parton distributions were found.
- The gauge link which is contained in the definition of the correlators influences their parametrization.
  1. It invalidates the time-reversal constraint and enables T-odd structures.
  2. It generates an additional dependence of the correlators on a lightcone vector n. This adds new structures to the parametrization.