Restoration of chiral symmetry in the $U(2)_L \times U(2)_R$ linear sigma model at finite temperature

(work still in progress. . .)

Stefan Michalski

Theoretische Physik IIIb
Universität Dortmund

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Outline

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   - Patterns of symmetry breaking
2. $U(N_f)_L \times U(N_f)_R$ linear sigma model
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Chiral symmetry

Massless QCD

$$\mathscr{L} = \sum_{i=1}^{N_f} \bar{\psi}_i^{(L)} \, i\not{D} \, \psi_i^{(L)} + \bar{\psi}_i^{(R)} \, i\not{D} \, \psi_i^{(R)} - \sum_{a=1}^{N_c^2-1} \frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu}$$

What we observe. . .

- Langrangian of massless QCD is chirally symmetric

$$U(N_f)_L \times U(N_f)_R = SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_V$$

- $U(1)_V$ symmetry corresponds to baryon number conservation
- $SU(N_f)_L \times SU(N_f)_R \times U(1)_A = SU(N_f)_V \times U(N_f)_A$

Patterns of symmetry breaking

spontaneous

- non-vanishing expectation value of quark condensate $\langle \bar{\psi}^{(L)} \psi^{(R)} \rangle \neq 0$ breaks axial symmetry spontaneously:

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_A \rightarrow SU(N_f)_V$$

- QCD instantons break the $U(1)_A$ symmetry (axial anomaly)
- $N_f^2 - 1$ Goldstone bosons

explicit

Patterns of symmetry breaking

spontaneous

explicit

- Mass terms mix right- and left-handed quarks, hence, break chiral symmetry explicitly

$$\mathscr{L}_{\text{mass}} = -\sum_{i=1}^{N_f} m_i \left[\bar{\psi}_i^{(L)} \psi_i^{(R)} + \bar{\psi}_i^{(R)} \psi_i^{(L)}\right]$$

- $N_f^2 - 1$ pseudo-Goldstone bosons
- for $M \leq N_f$ degenerate quark flavors an $SU(M)_V$ symmetry remains

How we investigate this symmetry breaking

Linear sigma models

- linear sigma models are low-energy effective theories of QCD
- same symmetry and patterns of symmetry breaking as QCD
- contain all scalar mesons of $SU(N_f)_V \times SU(N_f)_A$ multiplet, for example $N_f = 2$:
  - $J^P = 0^+$: $f_0(600) \equiv \sigma$, $a_0(980)$
  - $J^P = 0^-$: $\eta(548)$, $\pi(140)$
- expect (only partial) restoration of chiral symmetry at high temperature (value from lattice QCD ~ 175 MeV)
- still mesonic degrees of freedom at high temperature as in QCD for only partial symmetry restoration

$U(N_f)_L \times U(N_f)_R$ linear sigma model

Lagrangian

Φ is a complex $N_f \times N_f$ matrix $\Phi = T_a \left(\phi_a^{(\mathrm{s})} + i\phi_a^{(\mathrm{p})}\right)$ in the Lagrangian

$$\begin{aligned}
\mathscr{L}[\Phi] \quad = \quad & \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi\right) \\
& - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left[(\Phi^\dagger \Phi)^2\right] \\
& + c\left[\det \Phi + \det \Phi^\dagger\right] + \mathrm{Tr}\left[H(\Phi + \Phi^\dagger)\right]
\end{aligned}$$

Scalars for $N_f = 2$

$$\sum_{a=0}^{N_f-1} T_a\, \phi_a^{(\mathrm{s})} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\sigma + a_0^0\right) & a_0^+ \\ a_0^- & \frac{1}{\sqrt{2}}\left(\sigma - a_0^0\right) \end{pmatrix}$$

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Pseudoscalars for $N_f = 2$

$$i \sum_{a=0}^{N_f-1} T_a\, \phi_a^{(\mathrm{p})} = \frac{i}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\eta + \pi^0\right) & \pi^+ \\ \pi^- & \frac{1}{\sqrt{2}}\left(\eta - \pi^0\right) \end{pmatrix}$$

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Symmetry breaking...

- always respect the quantum numbers of the vacuum
- spontaneous symmetry breaking by $m^2 < 0 \Rightarrow \langle \Phi \rangle \neq 0$
- $c \neq 0$ corresponds to $U(1)_A$ anomaly
- limit $c \to \infty$ removes a_0 and η from theory $\Rightarrow O(4)$ model
- matrix H to break symmetry explicitly

Effective action

two-loop approximation

Effective action

$$\Gamma\left[\phi, \mathcal{M}^2; T\right] = \Gamma_1[\phi, \mathcal{M}^2; T] + \Gamma_q\left[\phi, \mathcal{M}^2; T\right]$$

- From a stationarity condition $\delta\Gamma = 0$ it follows that
  - $\frac{\partial \Gamma}{\partial \phi} = 0$ yields expectation value $\phi = \langle \Phi \rangle_T$
  - $\frac{\partial \Gamma}{\partial \mathcal{M}^2} = 0$ defines self-consistency equations for the mass matrix
- $\phi(T) = \langle \Phi \rangle = f_\pi(T)$ by PCAC
- $\mathcal{M}^2(T) = M^2_{\text{tree}}(\Phi) + \Delta(T; \Phi, \mathcal{M}^2)\big|_{\Phi=\phi(T)}$

Graphical representation

Lines are propagators with $iG^{-1} = k^2 - \mathcal{M}^2$

Sunsets graphs in the $U(2)_R \times U(2)_L$ model

There's more than just one sunset...

$$\Gamma_{\text{sunset}} = i\hbar\phi_0^2 \Bigg[\left(\lambda_1 + \frac{3}{2}\lambda_2\right)^2 (3\,S_{\sigma\sigma\sigma} + 3\,S_{\sigma a_0 a_0}) + \left(\lambda_1 + \frac{\lambda_2}{2}\right)^2 (S_{\sigma\eta\eta} + 3\,S_{\sigma\pi\pi}) + \left(\lambda_1 + \frac{\lambda_2}{2}\right) \lambda_2 (2\,S_{\sigma\eta\eta} + 3\,S_{\sigma\eta\pi} + 6\,S_{a_0\eta\eta} + 9\,S_{a_0\eta\pi}) + \frac{\lambda_2^2}{2}(2\,S_{\sigma\eta\eta} + S_{a_0\eta\pi}) \Bigg]$$

Numerical Results

What we do...

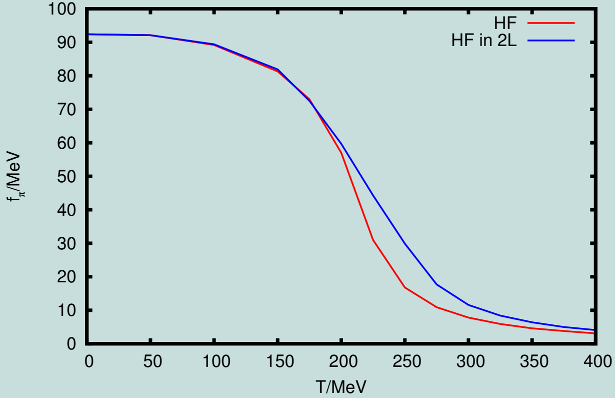
- let $N_f = 2$ and $m_u = m_d$
- fix parameters at $T = 0$
    - masses of the mesons σ, a_0, η and π
    - pion decay constant $\phi_0 = f_\pi = 92.4$ MeV
- solve equations for $\delta\Gamma = 0$ numerically for different temperatures (4 masses + 1 decay constant)

What makes this work preliminary ...

- equations for the masses are only solved up to one-loop (HARTREE–FOCK approximation)
- two-loop potential is calculated with HF masses

Numerical Results
order parameter

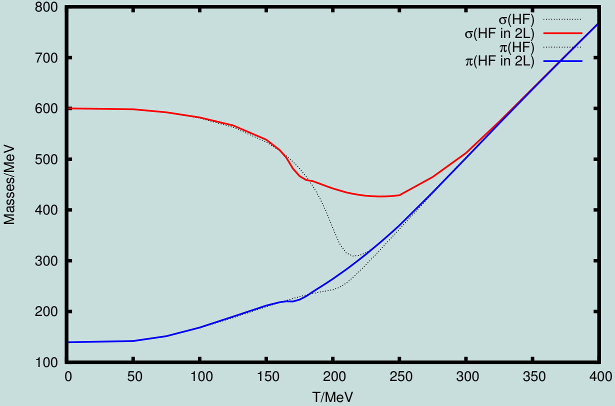
$f_\pi(T)$ in the $U(2)_L \times U(2)_R$ model



Numerical Results

masses

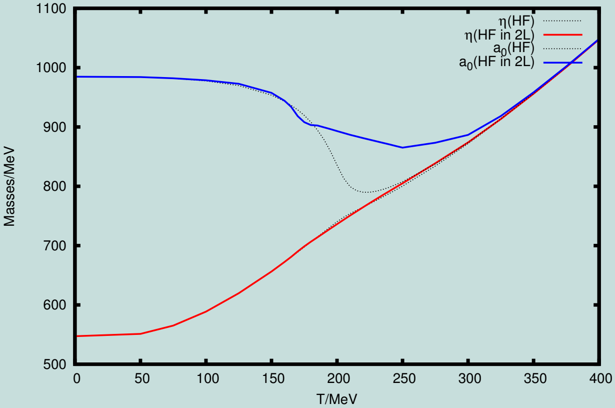
masses of σ and pion in the $U(2)_L \times U(2)_R$ model



Numerical Results

masses

masses of a_0 and η in the $U(2)_L \times U(2)_R$ model



Conclusions and Outlook

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- two-loop approximation with only HF masses yields a higher critical temperature than HF approximation (unwanted)
- sunset graph broadens crossover region
- need lower masses in this region $\longrightarrow$ need fully self-consistent calculation to two-loop order

Outlook

- solution of mass gap equation $\longrightarrow$ expect slightly lower T_c than in HF [works fine in $O(4)$]
- thermodynamic quantities like pressure or entropy
- fermions (baryons or quarks)?
- strange mesons $(N_f = 3)$?

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