Tunneling in Quantum Field Theory on a Compact Space

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based on: J.Baacke and N. K., hep-th/0505118

Dortmund, 11.10.2005

Outline

1. Introduction
2. Quantum field theory on compact space in 1+1 D
3. Real-time approach: time dependent Hartree-Fock approximation
4. Numerical results
5. Conclusions

Introduction

The decay of the false vacuum

- various models in quantum field theory (e.g. GUTs) have local and global minima, which represent "false" and "true" vacua

- in cosmology the universe may be trapped in a false vacuum and subsequently decay to the true vacuum, either thermally or by tunneling

- for infinite space these transitions are local, formation of bubbles of true vacuum $>>$ bubble nucleation, "bounces", with gravity: Coleman-deLuccia bounce

- for finite space extension, like a closed spherical universe, the transition may also occur globally, e.g. Hawking-Moss instanton

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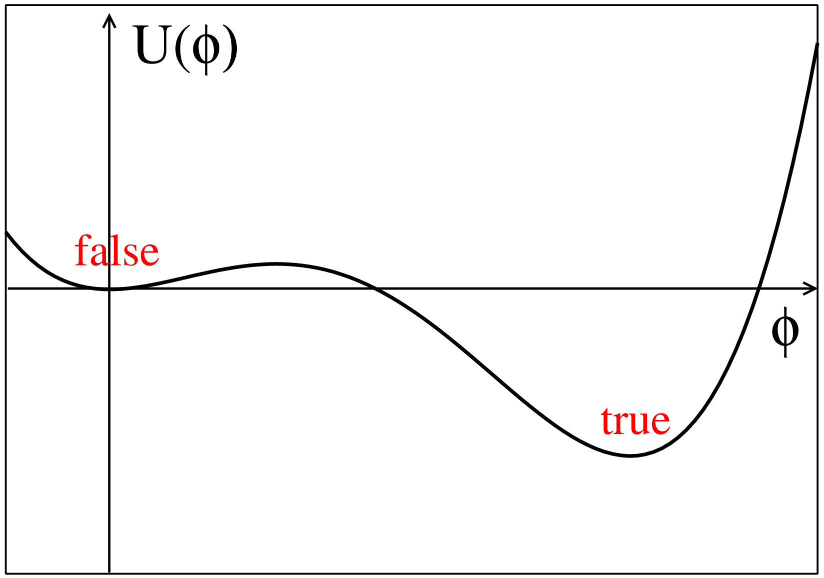
- purpose of our present research: to study such transitions, including the fluctuations of the quantum fields living in these spaces

The most simple renormalizable toy "Higgs potential"



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Tunnelling in quantum mechanics

- seems (but is not always) well understood
- is usually considered in the WKB approximation
- continuation to imaginary time
- prefactors “of order 1” computed in the eikonal expansion
- however: study of evolution of a wave packet in a real time formalism by F. Cooper, S.-Y. Pi and P. N. Stancioff, Phys. Rev. D34, 3831 (1986), shows that “false vacuum decay is not so simple”, computation of the “barrier transition probability” is by far not sufficient or even irrelevant.

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Tunneling in quantum field theory

- in addition to a "classical field" that describes the local or global tunneling we have quantum fluctuations, their interactions and their back-reaction
- recent work done in Dortmund on such quantum corrections
  * 4D case in the 1-loop approximation

    J. Baacke and G. Lavrelashvili, Phys.Rev. D69:025009, 2004 [hep-th/0307202]

  * 2D bounce treated in Hartree approximation

    J. Baacke and N. K, Phys.Rev. D71:025008, 2005 [hep-th/0411162]

- most recently: quantum corrections to global tunneling in real time

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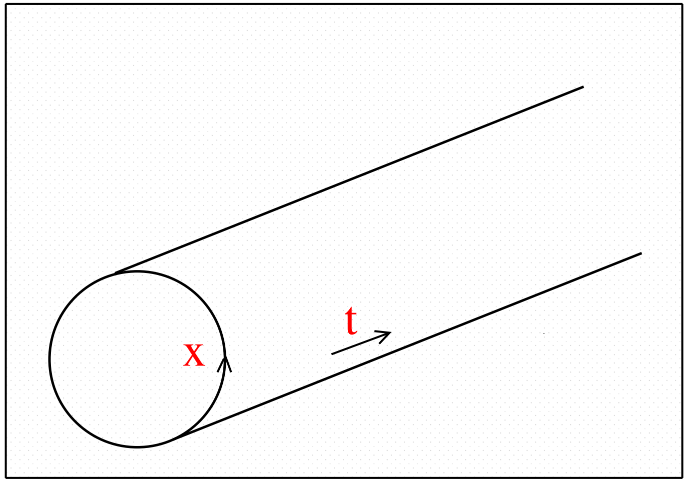
Quantum field theory

on compact space (1+1 dimensions)

1+1 dimensional QFTh on a compact space $R \times S_1$

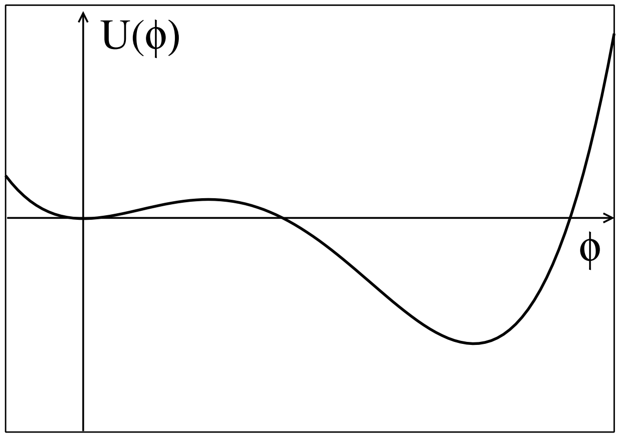


Action:

$$S = \int_0^{2\pi a} dx \int dt \left[\frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2 - U(\Phi) \right]$$

Potential

$$U(\Phi) = \frac{1}{2}m^2\Phi^2 - \eta\Phi^3 + \frac{\lambda}{8}\Phi^4$$



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Expansion into normal modes

$$\Phi(x,t) = \varphi_0(t) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \varphi_n(t) e^{ik_n x}$$

with $k_n = n/a$.
The homogeneous part $\varphi_0(t)$ describes the tunneling, the φ_n are the quantum fluctuations. The action

$$\begin{aligned} S &= 2\pi a \int dt \left[\frac{1}{2}\dot{\varphi}_0^2 + \frac{1}{2}\sum_{n\neq 0} \dot{\varphi}_n \dot{\varphi}_{-n} - \frac{1}{2}\sum_n \omega_n^2 \varphi_n \varphi_{-n} \right. \\ & \left. + \eta \sum_{nn'} \varphi_n \varphi_{n'} \varphi_{-n-n'} - \frac{\lambda}{8} \sum_{nn'n''} \varphi_n \varphi_{n'} \varphi_{n''} \varphi_{-n-n'-n''} \right] \end{aligned}$$

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Hamiltonian takes the form (interaction terms include the zero mode)

$$
\begin{aligned}
H \quad = \quad & \Bigg\{ -\frac{1}{2}\left[\frac{\partial^2}{\partial\chi_0^2} + \sum_{n>0,j}\frac{\partial^2}{\partial\chi_{nj}^2}\right] \\
& +\frac{1}{2}\left[m^2\chi_0^2 + \sum_{n>0,j}\omega_n^2\chi_{nj}^2\right] \\
& -\eta'\sum_{nn'}\chi_n\chi_{n'}\chi_{-n-n'} + \frac{\lambda'}{8}\sum_{nn'n''}\chi_n\chi_{n'}\chi_{n''}\chi_{-n-n'-n''}\Bigg\}
\end{aligned}
$$

where

$$
\omega_n^2 = m^2 + \frac{n^2}{a^2} \qquad \varphi_{\pm n} = \frac{1}{\sqrt{2}}(\varphi_{n1} \pm i\varphi_{n2}) \qquad n > 0
$$

$$
\phi_k = \chi_k/\sqrt{2\pi a} \qquad \eta' = \eta/\sqrt{2\pi a} \qquad \lambda' = \lambda/2\pi a
$$

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Time dependent Hartree-Fock approximation

- Time dependent Hartree-Fock variational principle

$$\delta \int d\chi_0 \prod_{n>0,j} d\chi_{nj} \Psi^\dagger(\chi_0, \chi_{nj}, t) \left(i\partial_t - H\right) \Psi(\chi_0, \chi_{nj}, t) = 0$$

- variational *Ansatz* for the wave function: product

$$\Psi(\chi_0, \chi_n, t) = \psi_0(\chi_0, t) \prod_{n>0} \psi_n(\chi_{nj}, t)$$

- for the modes with $n \neq 0$ a Gaussian wave function
- **but** zero mode, ψ_0 remains a **general** Schrödinger wave function, i.e. its Schrödinger equation will be **integrated exactly**

$$i\partial_t \psi_0(\chi_0, t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial \chi_0^2} + \tilde{U}(\chi_0) + (-6\eta'\chi_0 + \frac{3\lambda'}{2}\chi_0^2)\mathcal{F} \right] \psi_0(\chi_0, t)$$

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- For the modes with $n \neq 0$ we find

$$i\partial_t \psi_n(\chi_n, t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial \chi_{nj}^2} + \frac{1}{2}(\omega_n^2 + W(t))\chi_{nj}^2 \right] \psi_n(\chi_n, t)$$

- The Gaussian wave packet then takes the form

$$\psi_n(\chi_n, t) = e^{-i\alpha_n(t)} \left[\frac{2\omega_{n0}}{2\pi |f_n(t)|^2} \right]^{1/4} \exp\left[\frac{i}{2} \frac{\dot{f}_n^*(t)}{f_n^*(t)} \chi_n^2 \right]$$

where $f_n(t)$ satisfy the equation

$$\ddot{f}_n(t) + \Omega_n^2(t) f_n(t) = 0$$

- The fluctuation sum becomes

$$\mathcal{F}(t) = \sum_{n>0} \frac{|f_n(t)|^2}{2\omega_{n0}}$$

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- The system of equations we have presented here is consistent with an conserved energy which serves as a numerical cross check

- The fluctuation sum describes the backreaction

- The regularization can be done using the Plana formula, which replaces the sum by integrals, dimensional or PV regularization may be applied to the resulting integrals over traces of Minkowski-space Green functions

- Renormalization for Hartree - 2PPI formalism of Verschelde

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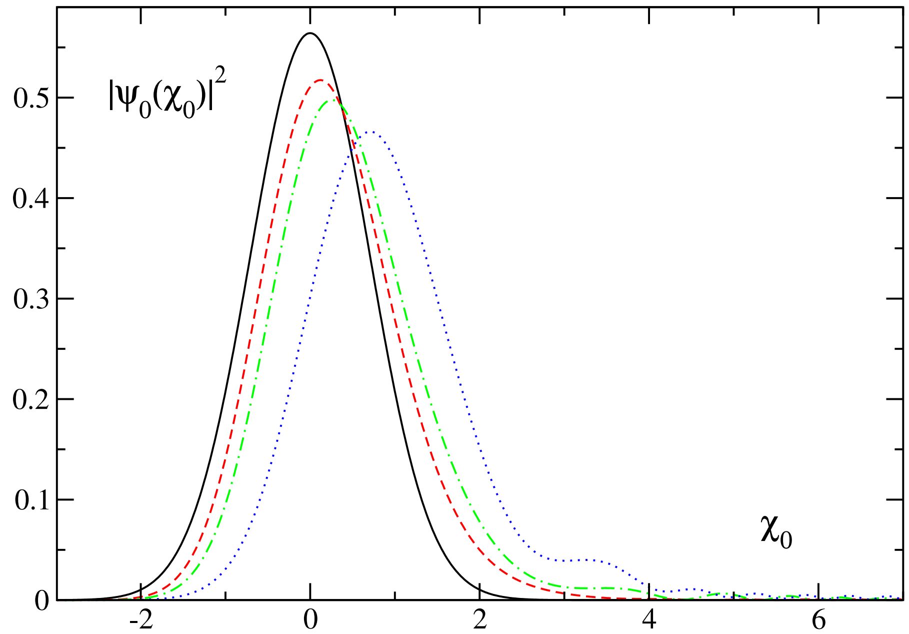
Numerical results



Initial evolution of the wave function. $m = \eta = \lambda = 1$,$a = 6$

(solid line: $t = 0$; dashed line: $t = 5$; dash-dotted line: $t = 6$; dotted line: $t = 7$.)

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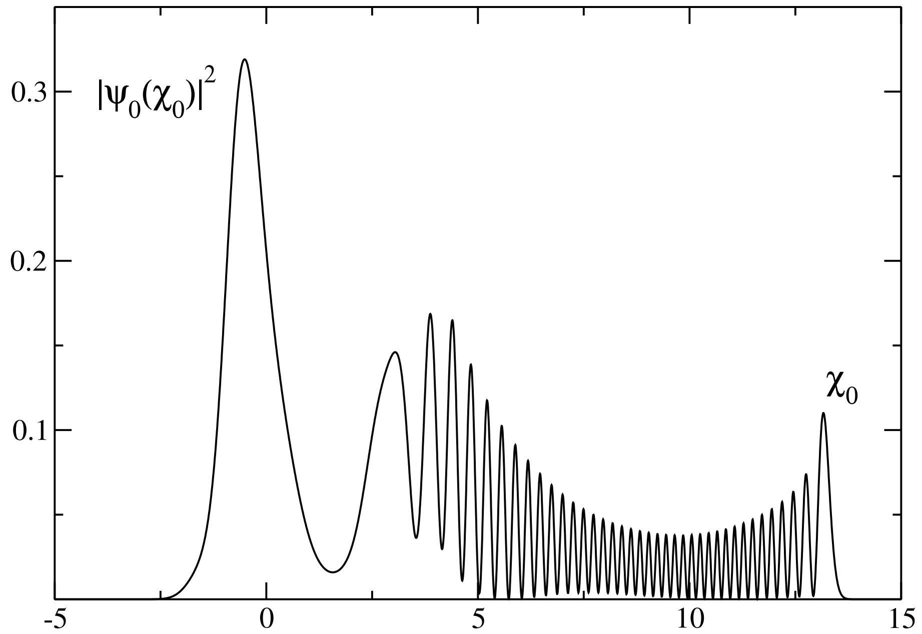

A typical wave function in the ordinary tunneling regime.

$$m = \eta = \lambda = 1,\ a = 0.5.$$

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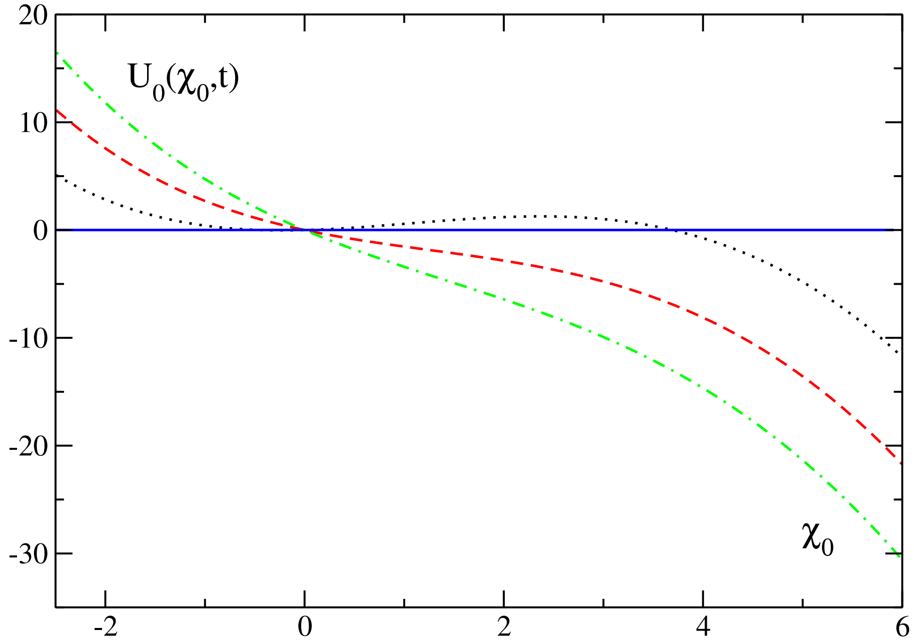

Evolution of the potential with time for large a.
$m = \eta = \lambda = 1$; $a = 6$.
(dotted line: $t = 0$;dashed line: $t = 6.9$; dash-dotted line: 7.54.)

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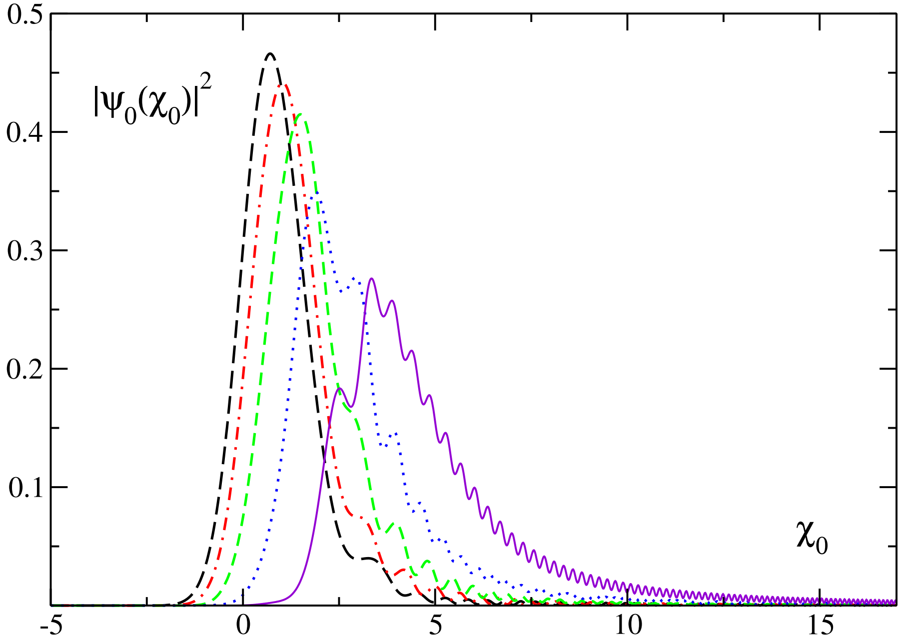

Evolution of the wave function. $m = \eta = \lambda = 1$; $a = 6$.

(long-dashed line: $t = 7$; dash-dotted line: $t = 7.4$; short-dashed line: $t = 7.8$; dotted line: $t = 8.2$; solid line: $t = 8.6$.)

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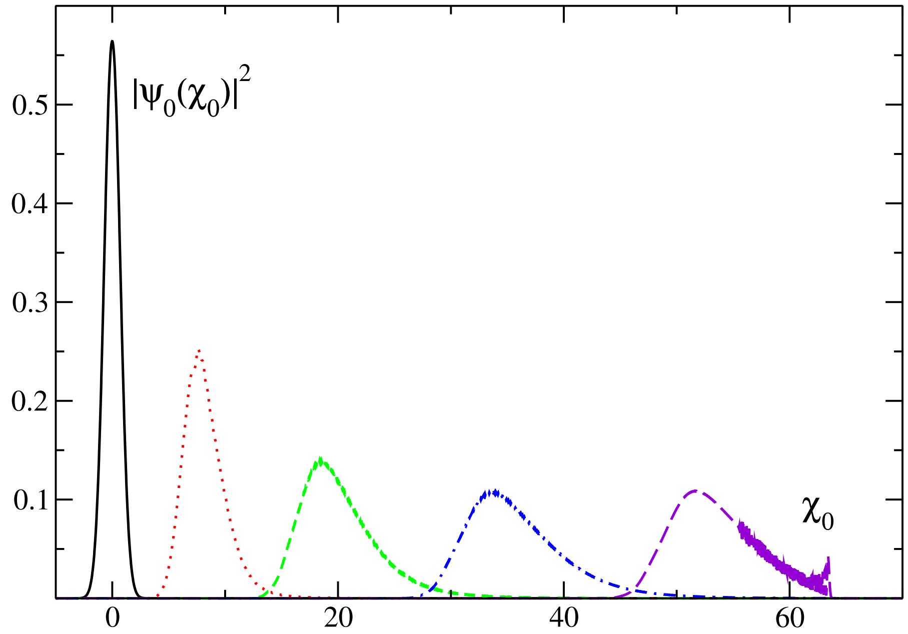

Evolution of the wave function. $m = \eta = \lambda = 1$; $a = 20$.

(solid line: $t = 0$; dotted line: $t = 12.6$; short-dashed line: $t = 13.2$; dash-dotted line: $t = 13.6$; long-dashed line: $t = 14$.)

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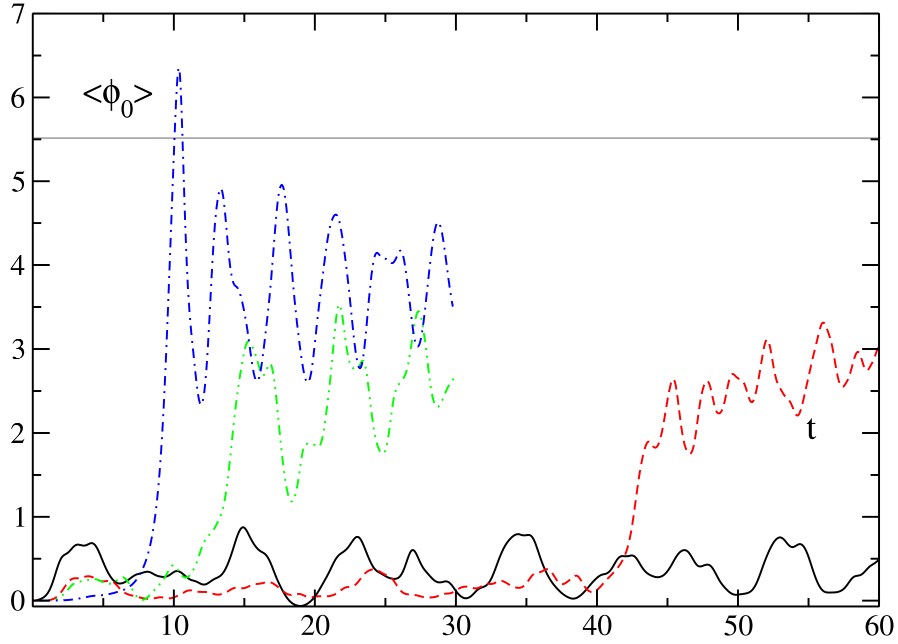

Evolution of the mean value of the zero mode. $m = \eta = \lambda = 1$.

(straigth line: $a = 0.3$; dashed line: $a = 1$; dash-double-dotted line: $a = 1.5$; dash-doted line: $a = 6$; horizontal line: minimum of the tree level potential $U(\phi_0)$.)

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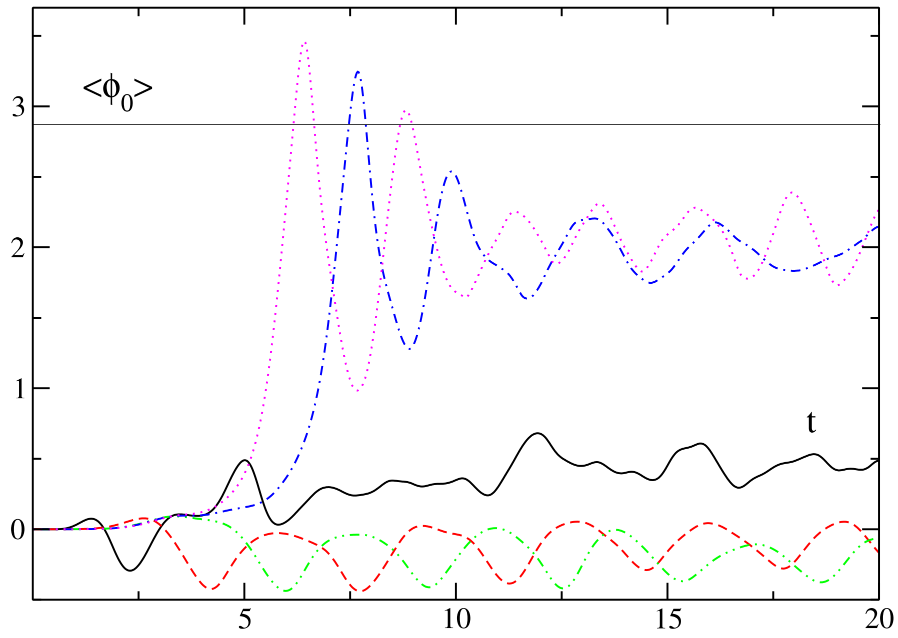

Evolution of the mean value of the zero mode. $m = 1, \eta = 3, \lambda = 6$.

(straigth line: $a = 1$; dashed line: $a = 10$; dash-double-dotted line: $a = 18$; dash-dotted line: $a = 19$; dotted line: $a = 20$; horizontal line: minimum of the potential $U(\phi_0)$.)

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Tunneling occurs in two different regimes:

at small values of a

- normal tunneling and suppression, as for the quantum mechanical case;
- the wave function gradually spreads into the region of the barrier and then into the deeper well.

at large values of a

- the initial Gaussian wave function becomes skew;
- the shift of $< \chi_0 >$ changes the fluctuations and the backreaction of the quantum fluctuations tilts the potential in such a way that the barrier disappears;
- an essential Gaussian wave packet "surfs" on the changed potential;
- the transition becomes a "sliding" transition.

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Conclusions

- we have applied a new (not standard) approach to our simple model;

- we have found a new regime of tunneling.

- issues to be explored:

    * mechanism and reason of growth of fluctuations;

    * the same processes in 3D and with gravity.

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Thanks!