Recent Developments in QCD I

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IPPP, University of Durham

Workshop on Hard Processes,

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General aims of perturbative QCD

1. Measuring fundamental quantities α_s
2. Determining ‘pseudo-fundamentals’ PDF’s
3. Predicting signals and backgrounds Tevatron and LHC
4. Understanding new structures in field theory
   twistor inspired developments for computing
   (Supersymmetric)-Yang Mills amplitudes

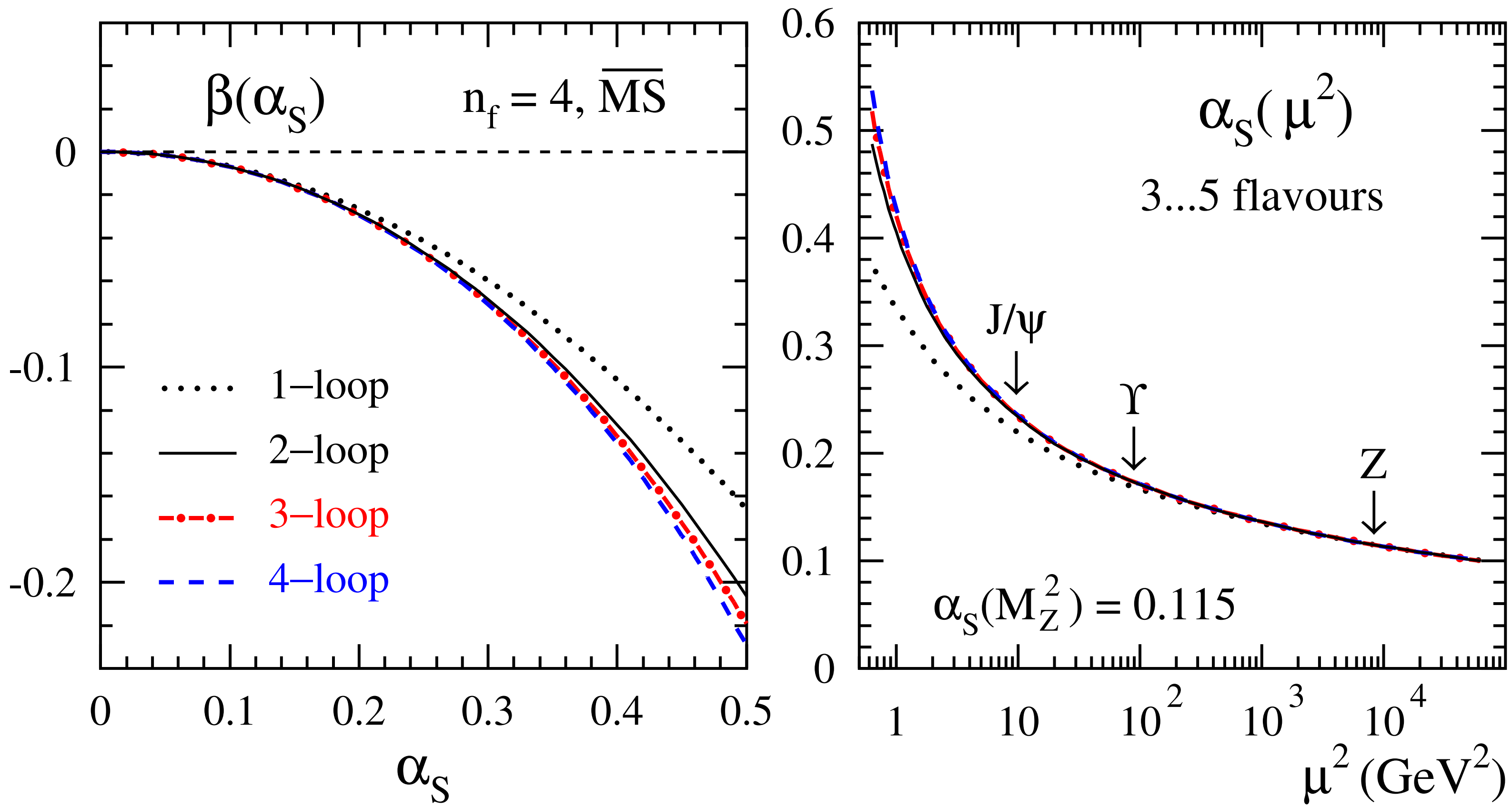
Will discuss 1-3 today, and leave 4 for tomorrow.

The running coupling in perturbative QCD

$$d\alpha_s/d\ln\mu^2 = -\beta_0\,\alpha_s^2 - \beta_1\,\alpha_s^3 - \beta_2\,\alpha_s^4 - \beta_3\,\alpha_s^5 - \ldots$$

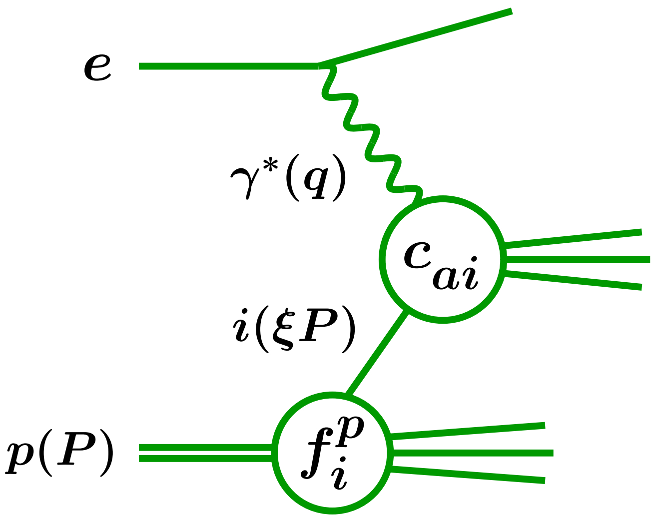
Four-loop coeff.:

van Ritbergen, Vermaseren, Larin; Czakon



Hard processes in perturbative QCD

Example: inclusive deep-inelastic scattering (DIS)



Kinematic variables

$$\begin{aligned} Q^2 &= -q^2 \\ x &= Q^2/(2P\cdot q) \end{aligned}$$

Lowest order : $x = \xi$

Structure functions F_a [up to $\mathcal{O}(1/Q^2)$]

$$F_a^p(x, Q^2) = \sum_i \left[c_{a,i}(\alpha_s(\mu^2), \mu^2/Q^2) \otimes f_i^p(\mu^2)\right](x)$$

Coefficient functions $c_{a,i}$, renormalization/factorization scale μ

Hard processes in perturbative QCD

Parton distributions f_i: evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

Initial conditions incalculable in pert. QCD.

Splitting functions P, Coefficient functions c_a

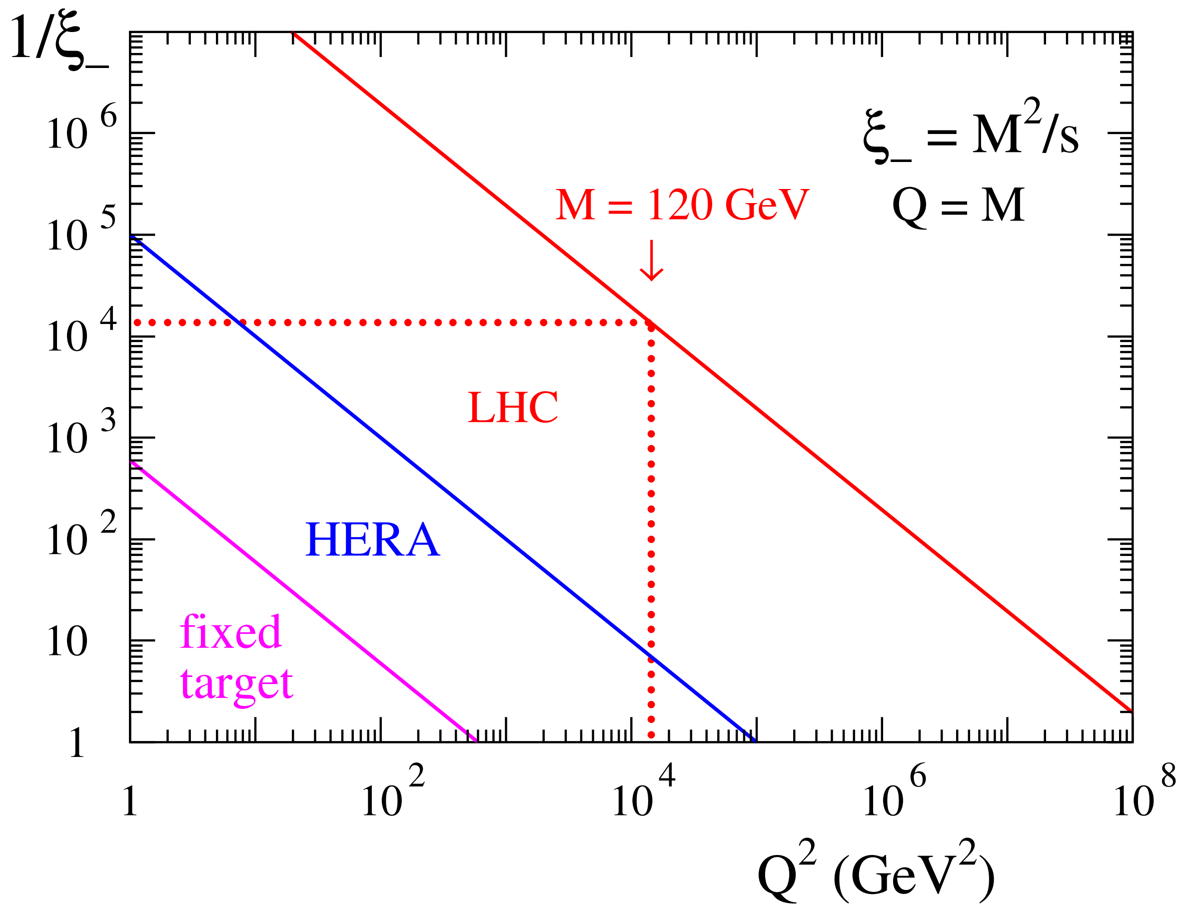
$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \alpha_s^{n_a}\left[c_a^{(0)} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots\right] \end{aligned}$$

NLO: standard approximation
NNLO: new emerging standard

Moch, Vermaseren, Vogt

Parton evolution from HERA to LHC

Kinematics: parton momenta $\xi_- < \xi < 1$ probed

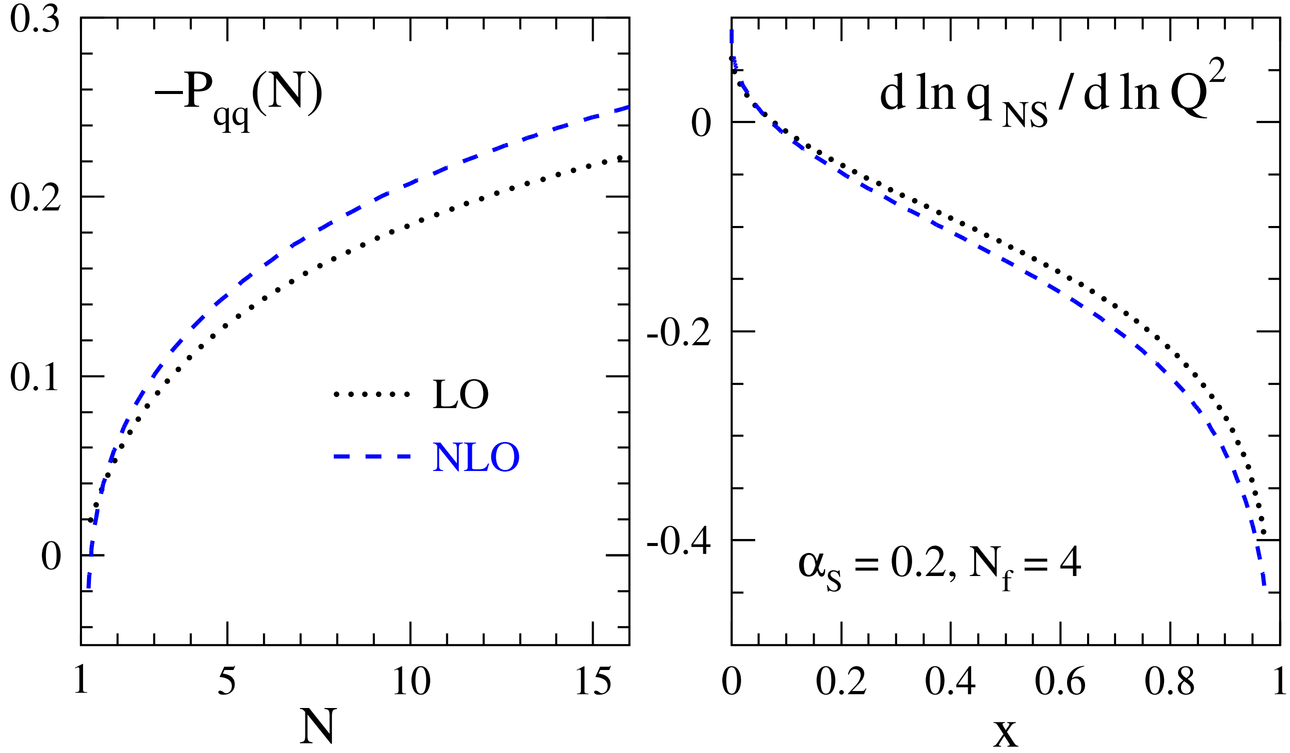


HERA $\longrightarrow$ LHC:

Q^2 evolution across up to three orders of magnitude

Parton evolution at large x

$A(N) = \int_0^1 dx\, x^{N-1} A(x)$.   Non-singlet : $u+\bar{u} - (d+\bar{d})$ etc



Moch, Vermaseren, Vogt

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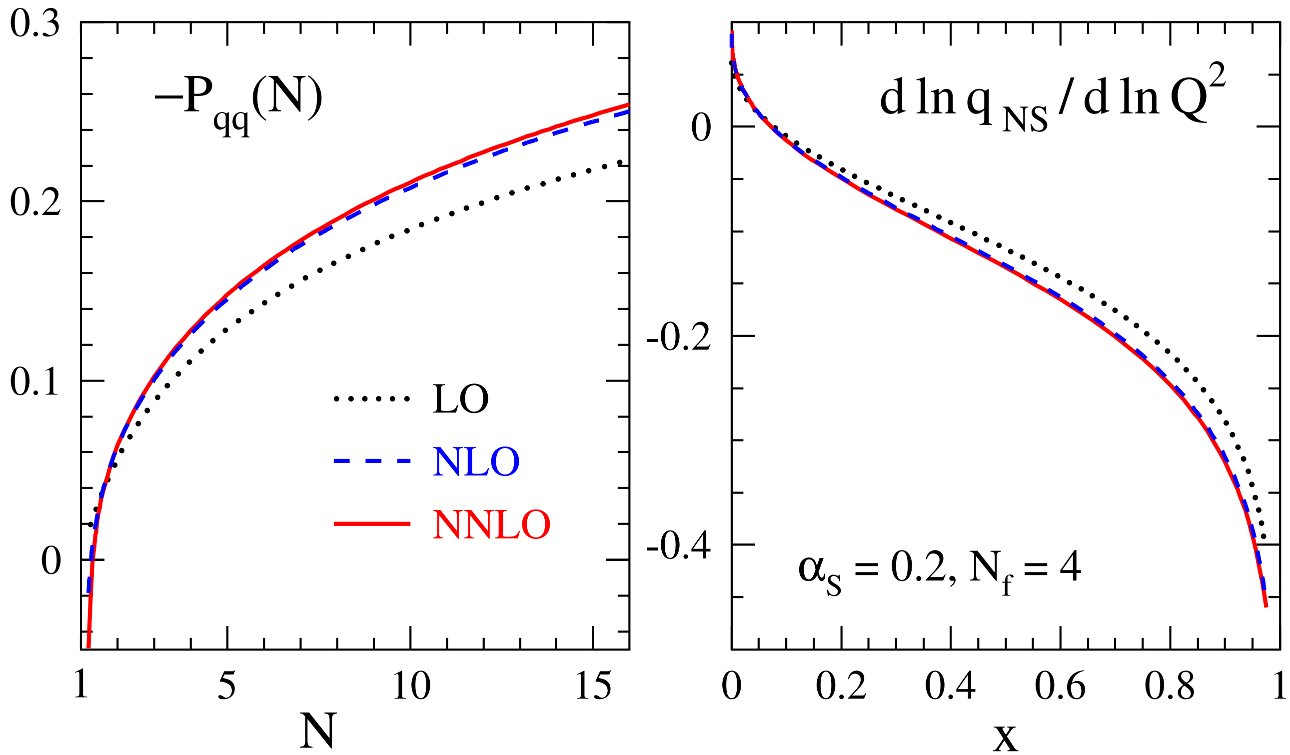
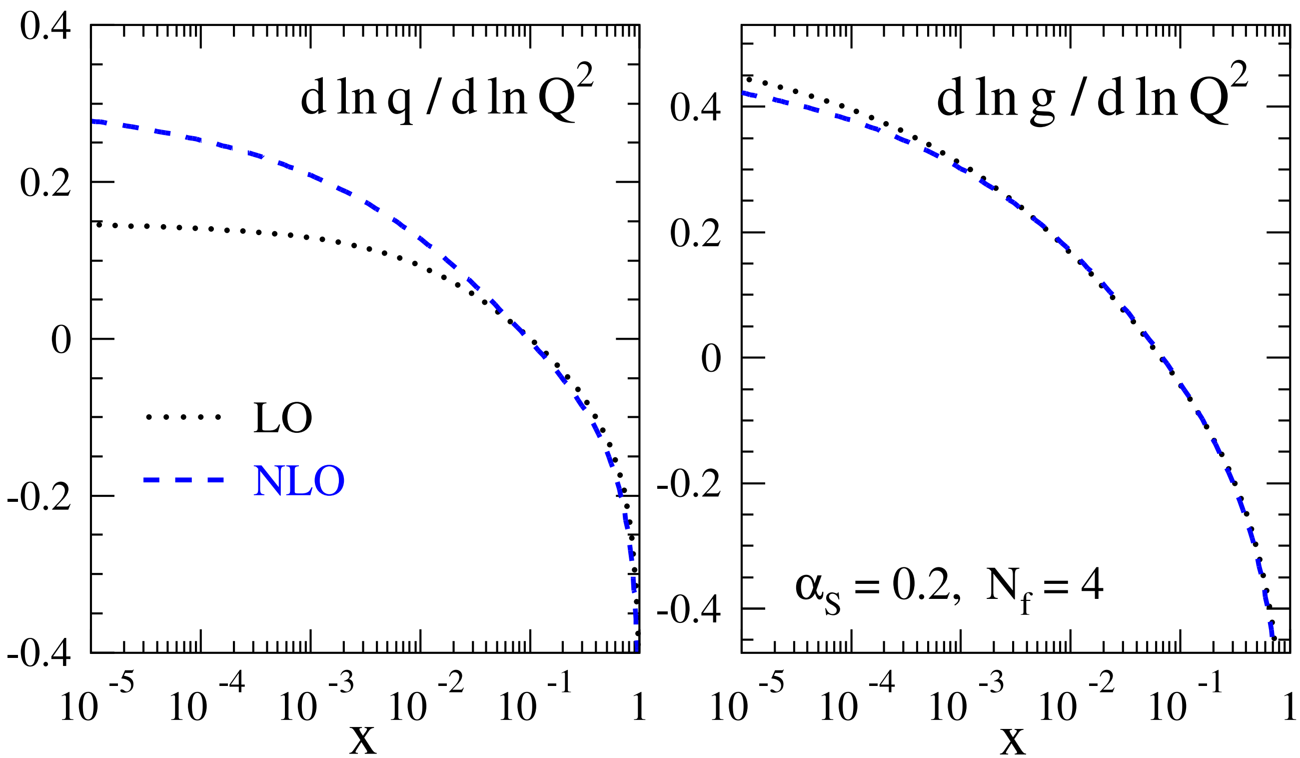


Moch, Vermaseren, Vogt

Perturbative expansion very benign: expect $< 1\%$ beyond NNLO

Parton evolution at small x

Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



Moch, Vermaseren, Vogt

Parton evolution at small x

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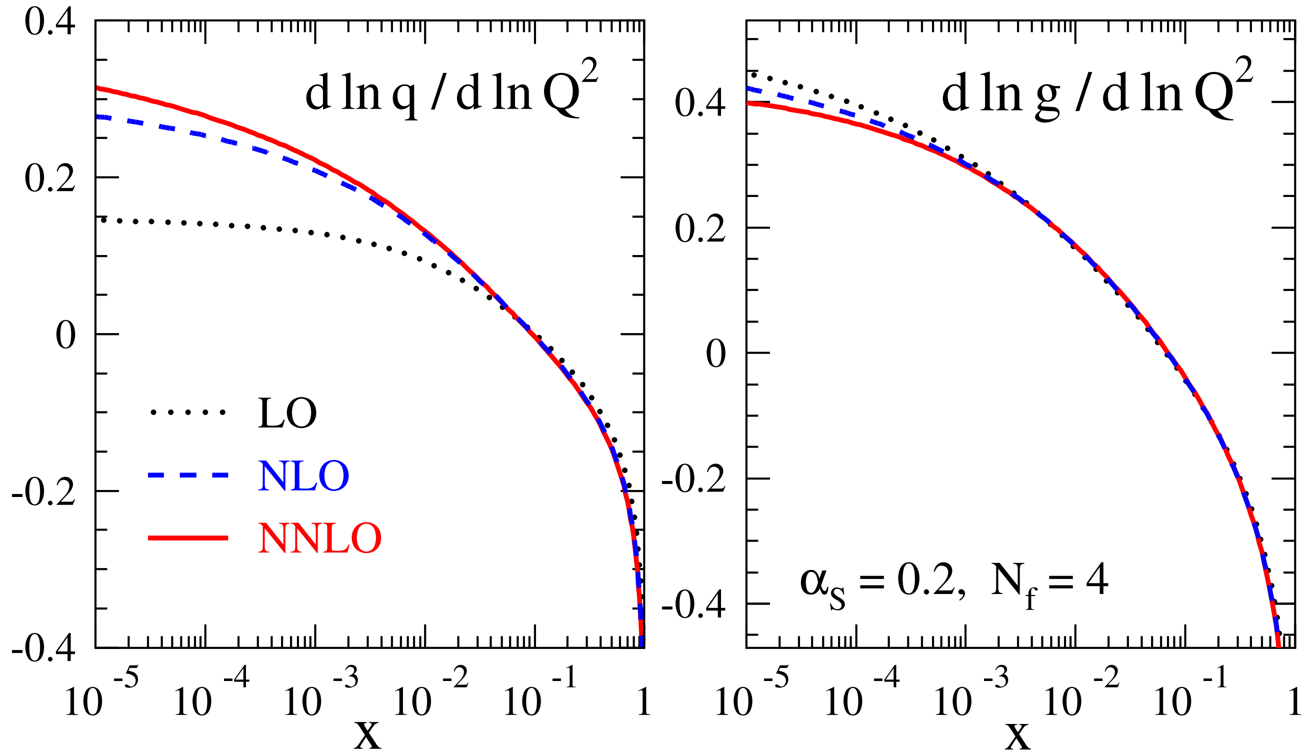
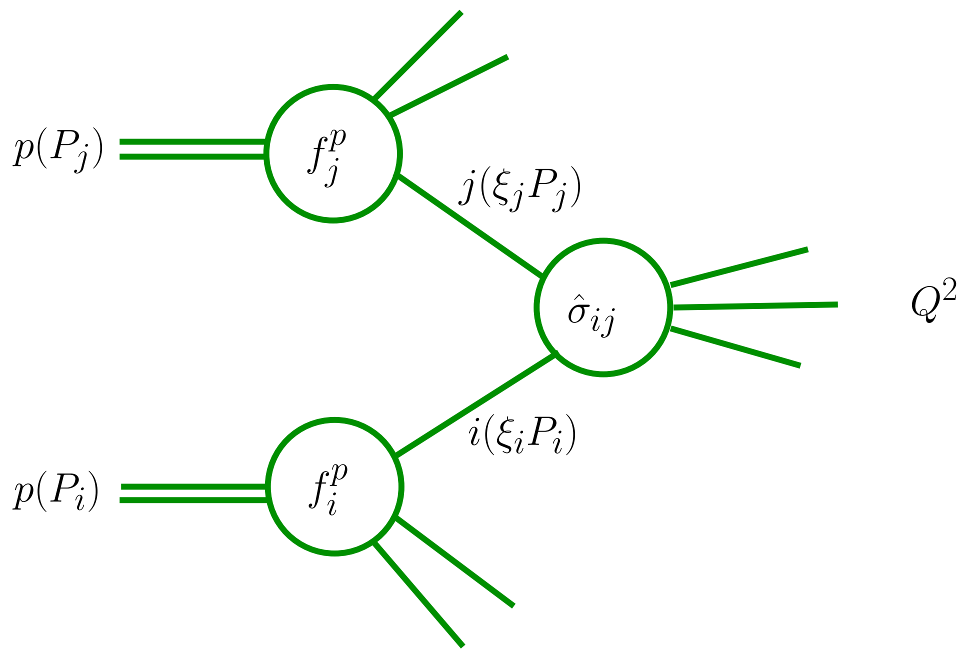


Moch, Vermaseren, Vogt

Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

Hard processes in perturbative QCD

Example: Hadron-hadron collisions



$$\sigma(Q^2) = \sum_{i,j} \left[\hat{\sigma}_{ij}(\alpha_s(\mu^2), \mu^2/Q^2) \otimes f_i^p(\mu^2) \otimes f_j^p(\mu^2)\right]$$

partonic cross sections σ_{ij},
parton distributions f_i,
renormalization/factorization scale μ

Collider Physics

1. Predictions for multiparticle final states that occur at high rate and form background to New Physics

   High multiplicity, but low order - typically LO or NLO

   For example, $pp \to V + 3$ jets is background to $pp \to t\bar{t}$ and other new physics.

2. Precise predictions for hard pp processes involving “standard particles" like W, Z, jets, top, Higgs, ..

   Low multiplicity, but high order - NNLO is emerging standard

   For example, Drell Yan cross section.

State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

LO

- matrix elements automatically generated up to $2 \to 6$ or even $2 \to 8$ or more
- plus automatic integration over phase space HELAC/PHEGAS, MADGRAPH/MADEVENT, SHERPA/AMEGIC++, COMPHEP, GRACE, ...
- able to interface with parton showers - CKKW
- very good for estimating importance of various processes in different models

Multiparticle production at LO

Example: Multi-jet production at the LHC using `HELAC/PHEGAS`

Draggiotis, Kleiss, Papadopoloulos

# of jets	2	3	4	5	6	7	8
# of dist.processes	10	14	28	36	64	78	130
total # of processes	126	206	621	861	1862	2326	4342
$\sigma(nb)$	-	91.41	6.54	0.458	0.030	0.0022	0.00021
% Gluonic	-	45.7	39.2	35.7	35.1	33.8	26.6

Sizeable cross sections for multi-jet events

Large uncertainty since $\sigma(n \text{ jets}) \sim \alpha_s^n$

State of the Art

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1	LO					
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α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NLO

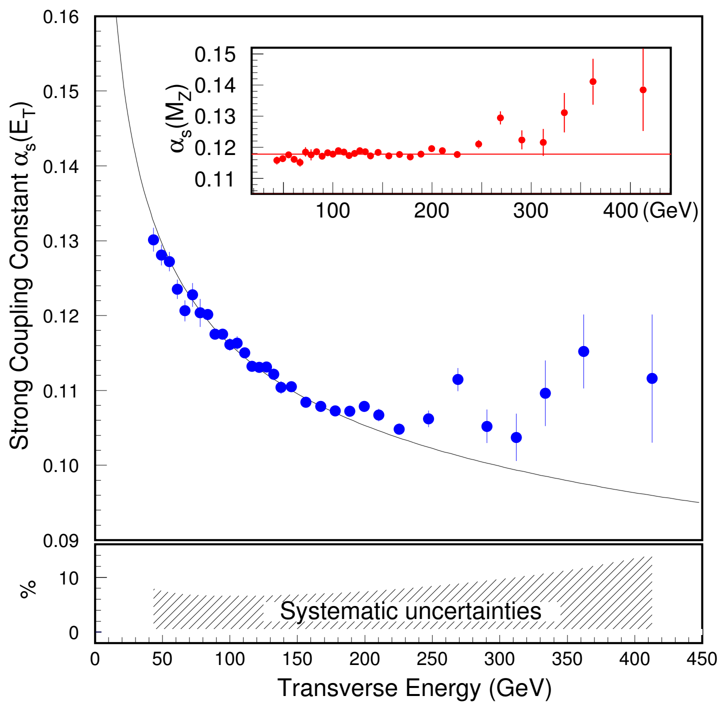
- parton level integrators available for most Standard Model and MSSM processes for some time
- extensively used at LEP, TEVATRON and HERA
  EVENT, JETRAD, MCFM, DISENT, etc
- starting to be matched with parton shower MC@NLO
- reduced renormalisation scale uncertainty

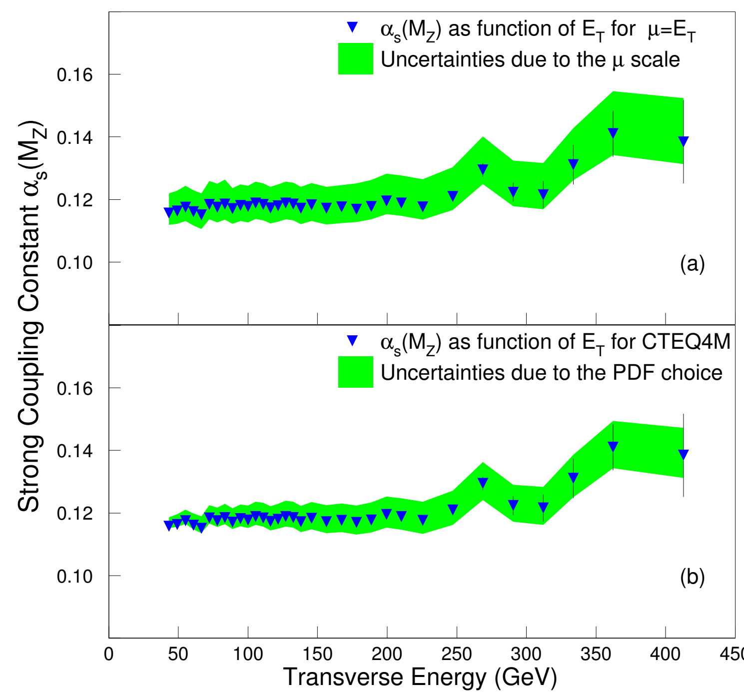
Example from the TEVATRON

Single jet inclusive transverse energy distribution (CDF)

$$\frac{d\sigma}{dE_T} = A\alpha_s^2(E_T) + B\alpha_s^3(E_T)$$

and solve for α_s





$$\alpha_s(M_Z) = 0.1178 \quad \pm 0.0001(stat)^{+7\%}_{-8\%}(syst)$$

Example from LEP

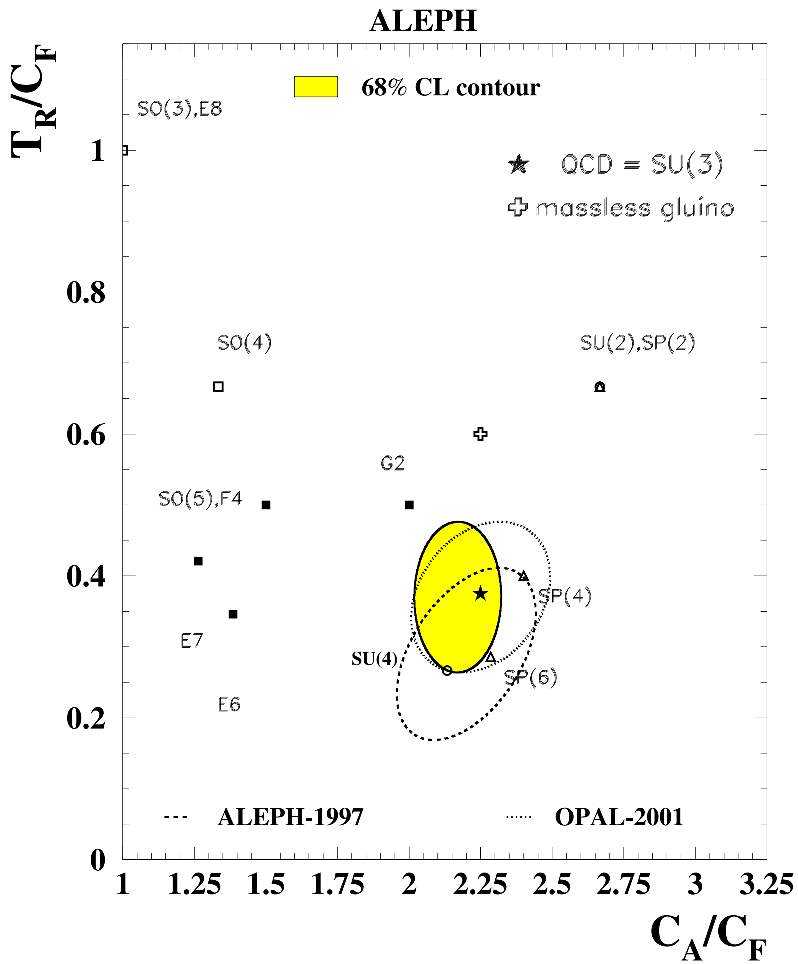
Fixing the quadratic casimirs of QCD in 4 jet events

OPAL

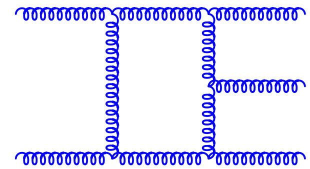
C_A	$3.02 \pm 0.25 \pm 0.49$
C_F	$1.34 \pm 0.13 \pm 0.22$
$\alpha_s(M_Z)$	$0.120 \pm 0.011 \pm 0.020$

ALEPH

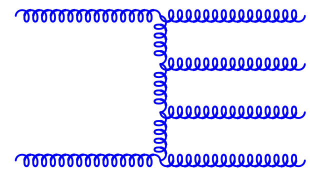
C_A	$2.93 \pm 0.14 \pm 0.49$
C_F	$1.35 \pm 0.07 \pm 0.22$
$\alpha_s(M_Z)$	$0.119 \pm 0.006 \pm 0.022$



Anatomy of a NLO calculation



- one-loop $2 \rightarrow 3$ process
  looks like 3 jets in final state



- tree-level $2 \rightarrow 4$ process
  looks like 3 or 4 jets in final state
- plus method for combining the infrared divergent parts - dipole subtraction

Catani, Seymour

One-loop matrix elements - $2 \rightarrow 3$ difficult, $2 \rightarrow 4$ almost impossible - see later for twistor inspired developments
and recently completed calculation of the full one-loop electroweak corrections to $e^+e^- \rightarrow 4$ fermions

Denner and Dittmaier

Automated NLO calculations

Diagram generation easy QGRAF, FeynArts.

Almost all one-loop scalar integrals are known analytically.
The problem is evaluating the tensor integrals - or reducing them to known scalar integrals.

Several attempts, combination of analytic and numerical methods.

- Identify and subtract divergences before doing the loop integral, do rest numerically.

  Nagy, Soper

- Reduce all integrals to a basis set of known integrals

  Binoth, Guillet, Heinrich, Pilon, Schubert; Ellis, Giele, EWNG, Zanderighi; Denner, Dittmaier; ...

  Many different combinations of analytic/numeric

- Recent numeric calculation of $H \to 4$ partons

  Ellis, Giele, Zanderighi

NLO Wish List - Salam

Experiments priorities

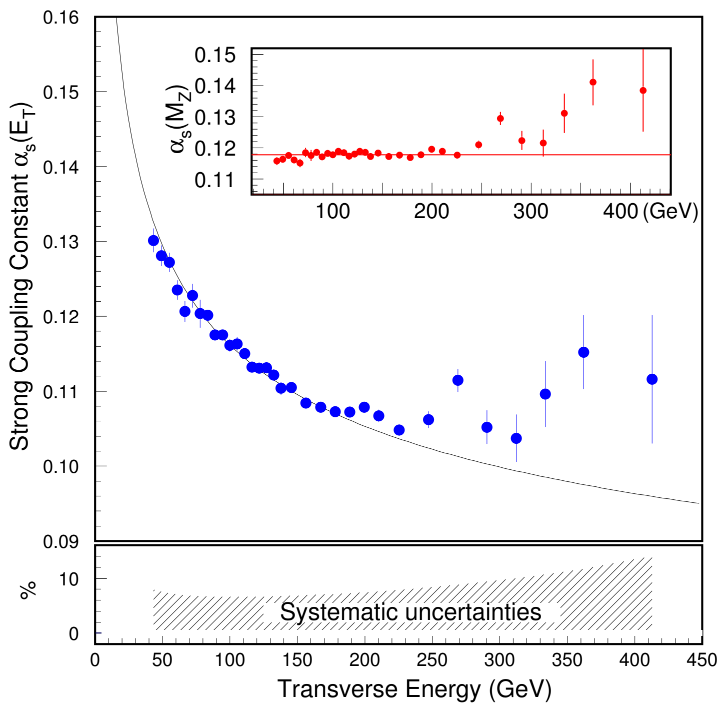
1. $pp \to WW$+ jet
2. $pp \to H + 2$ jets
   $\to$ VBF Higgs background
3. $pp \to t\bar{t}b\bar{b}$
4. $pp \to t\bar{t}$ + 2 jets
   $\to t\bar{t}H$ backgrounds
5. $pp \to WWb\bar{b}$
6. $pp \to VV + 2$ jets
   $\to$ WW scattering background
7. $pp \to V$+3 jets
8. $pp \to VVV$+jet
   $\to$ SUSY background

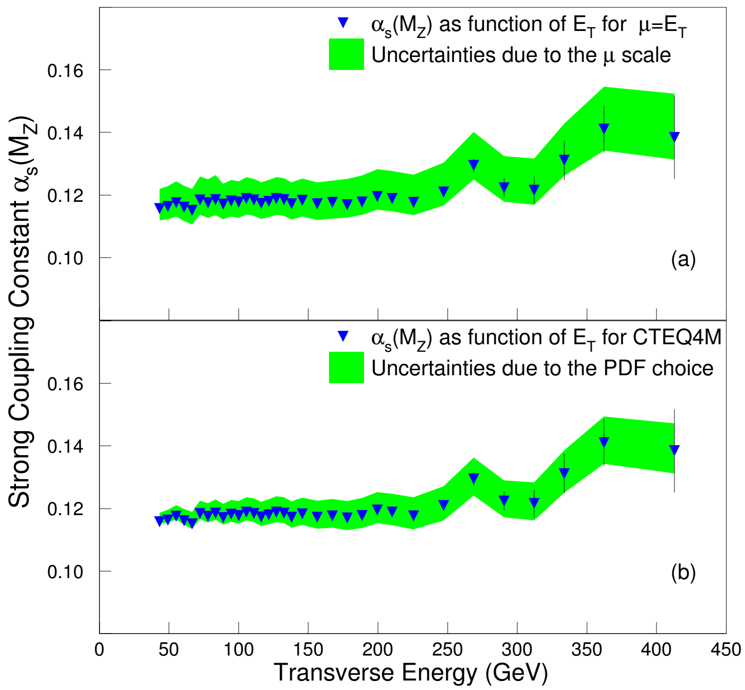
Already available
NLOJET++, MCFM, PHOX, ...
`http://www.cedar.ac.uk/hepcode`
Still to come

- $pp \to WW$+jet
- $pp \to VVV$
- $pp \to H$+2 jet
- $pp \to 4$ jets
- $pp \to t\bar{t}$+2 jets
- $pp \to t\bar{t}b\bar{b}$
- $pp \to VV$+2 jets
- $pp \to VVV$+jet
- $pp \to WWb\bar{b}$

Why go beyond NLO?

In many cases, the uncertainty from the pdf's and from the choice of renormalisation scale give uncertainties that are as big or bigger than the experimental errors.
e.g. theoretical uncertainties in α_s extraction from $p\bar{p} \rightarrow$ jet are due to renormalisation scale and pdf's





$$\alpha_s(M_Z) = 0.1178 \; {}^{+6\%}_{-4\%}(scale) \; {}^{+5\%}_{-5\%}(pdf)$$

Why do we vary renormalisation scale?

- The theoretical prediction should be independent of μ_R
- The change due to varying the scale is formally higher order. If an observable $\mathcal{O}bs$ is known to order α_s^N then,

$$\frac{\partial}{\partial \ln(\mu_R^2)} \sum_0^N A_n(\mu_R)\alpha_s^n(\mu_R) = \mathcal{O}\left(\alpha_s^{N+1}\right).$$

- So the uncertainty due to varying the renormalisation scale is way of guessing the uncalculated higher order contribution.

Why do we vary renormalisation scale?

- ... but the variation only produces copies of the lower order terms

$$\mathcal{O}bs = A_0\alpha_s(\mu_R) + \left(A_1 + b_0 A_0 \ln\left(\frac{\mu_R^2}{\mu_0^2}\right)\right)\alpha_s(\mu_R)^2$$

A_1 will contain logarithms and constants that are not present in A_0 and therefore cannot be predicted by varying μ_R.
For example, A_0 may contain infrared logarithms L up to L^2, while A_1 would contain these logarithms up to L^4.

- μ_R variation is only an estimate of higher order terms
- A large variation probably means that predictable higher order terms are large - but doesnt say anything about A_1.

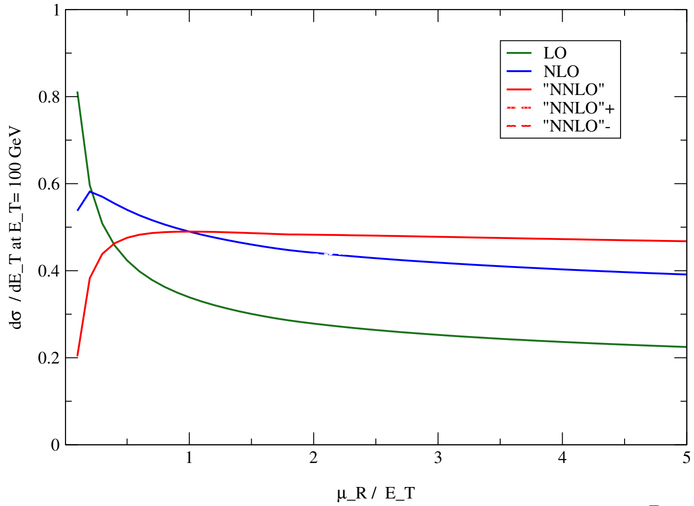
Renormalisation scale dependence

For example, $p\bar{p} \to$ jet, scale dependence

$$\begin{aligned}\frac{d\sigma}{dE_T} &= \alpha_s^2(\mu_R)A \\ &+ \alpha_s^3(\mu_R)\left(B + 2b_0LA\right) \\ &+ \alpha_s^4(\mu_R)\left(C + 3b_0LB + (3b_0^2L^2 + 2b_1L)A\right)\end{aligned}$$

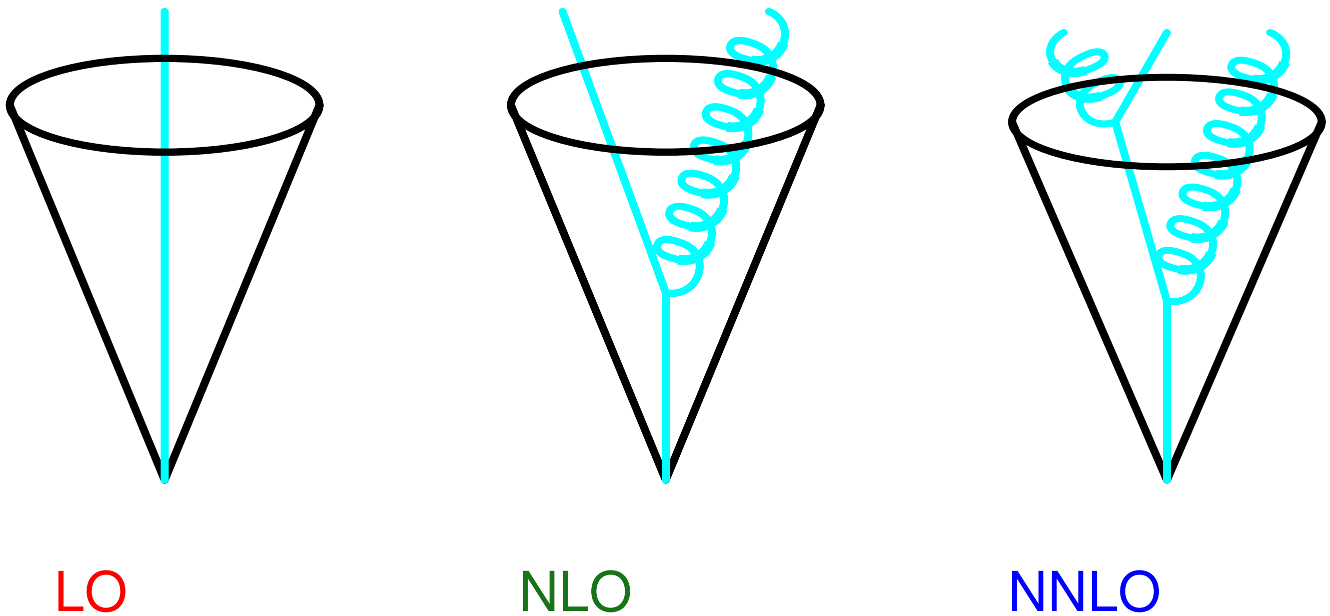
with $L = \log(\mu_R/E_T)$. The NNLO coefficient C is unknown.

The curves show guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed).
Scale dependence is significantly reduced.



Jet algorithms

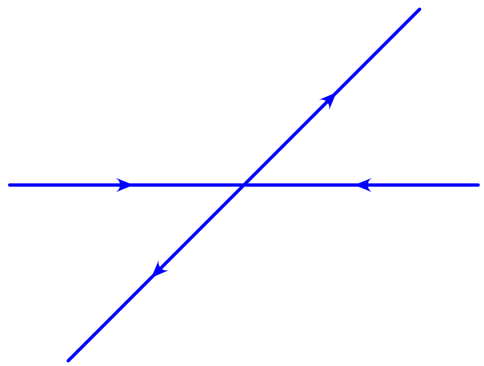
Also there is a mismatch between the number of hadrons and the number of partons in the event. At NLO at most two partons make a jet - while at NNLO three partons can combine to form the jet



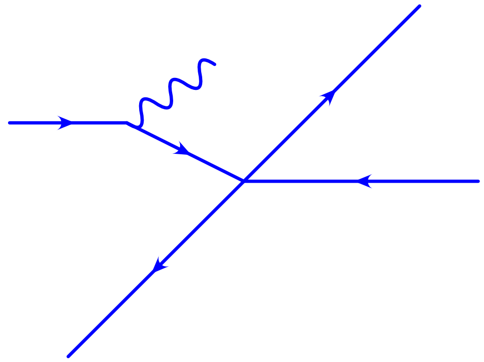
Perturbation theory starts to reconstruct the shower
$\Rightarrow$ better matching of jet algorithm between theory and experiment
$\Rightarrow$ need for better jet algorithms

Description of the initial state

LO At lowest order final state has no transverse momentum

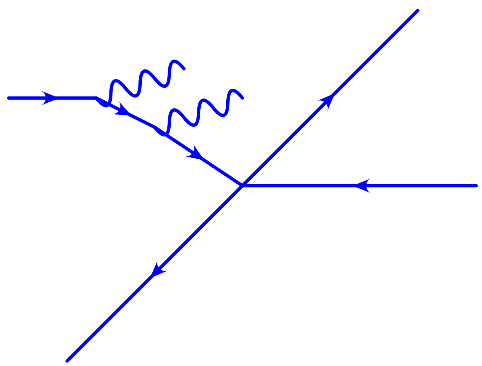


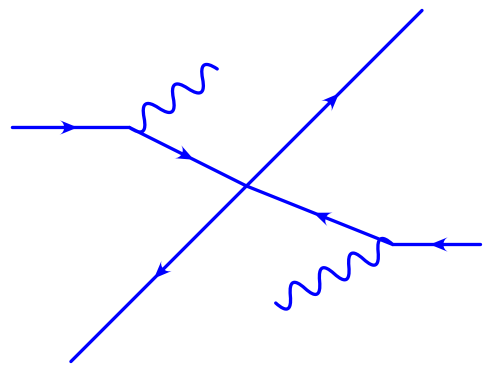
NLO Single hard radiation gives final state transverse momentum, even if no additional jet observed



Description of the initial state

NNLO Double radiation on one side or single radiation off each incoming particle gives more complicated transverse momentum to final state





Higher orders and power corrections

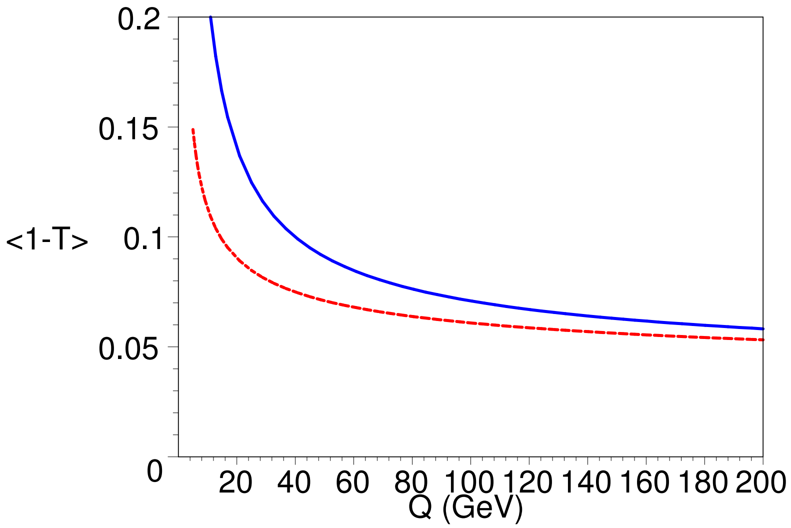
NLO Phenomenological power corrections match data with coefficient of $1/Q$ extracted from data.

$$\langle 1 - T \rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + \frac{\lambda}{Q}$$

At NLO, $\lambda \sim 1$ GeV gives a good description of the data.

$\langle 1 - T \rangle$ with NLO and no power correction and NLO with power correction $\lambda = 1$ GeV.

The power correction parameterises the unknown higher orders as well as the genuine non-perturbative correction



Higher orders and power corrections

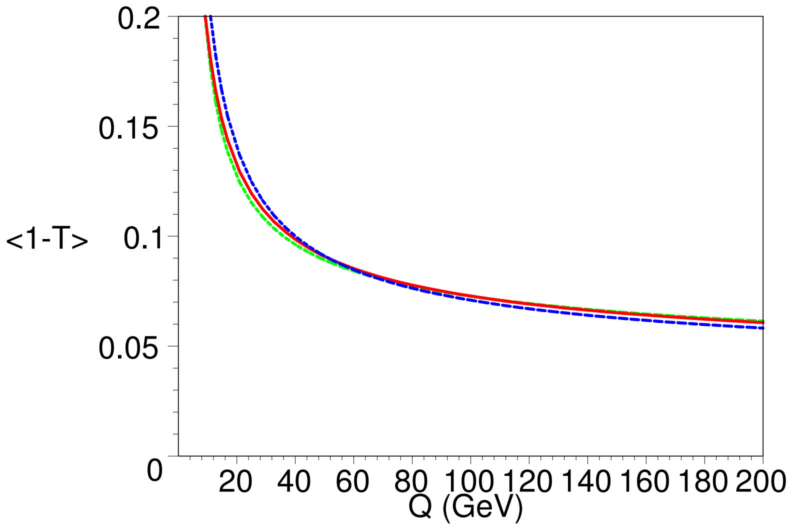
NNLO Higher orders partially remove need for power correction

$$\langle 1-T\rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + A\alpha_s^3 + \frac{\lambda\ \mathrm{GeV}}{Q}$$

If we guess $A = 3$, then $\lambda = 0.5$ GeV is good fit.

$\langle 1-T\rangle$ with NLO and $\lambda = 1$ GeV, "NNLO" with $\lambda = 0.5$ GeV and "All orders" with no power correction.

At present data not good enough to tell difference between $1/Q$ and $1/\log(Q/\Lambda)^3$.



Why go beyond NLO - Summary

- Reduced renormalisation scale dependence
- Event has more partons in the final state hence closer to real world
- Better description of transverse momentum of final state due to double radiation off initial state
- Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$
- Full NNLO global fit of PDF's should also reduce the factorisation scale uncertainty
- NNLO is the first serious estimate of the error
- Obvious application: reduction of uncertainty in α_s in e^+e^- annihilation

  Currently: $\alpha_s = 0.121 \pm 0.001(\mathrm{expt}) \pm 0.006(\mathrm{theory})$ resummed NLO

State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NNLO

- Drell-Yan and Higgs total cross sections

  Harlander, Kilgore; Anastasiou, Melnikov; ven Neerven, Ravindram, Smith

- Drell-Yan rapidity distribution, can do for W, Higgs etc

  Anastasiou, Melnikov, Petriello

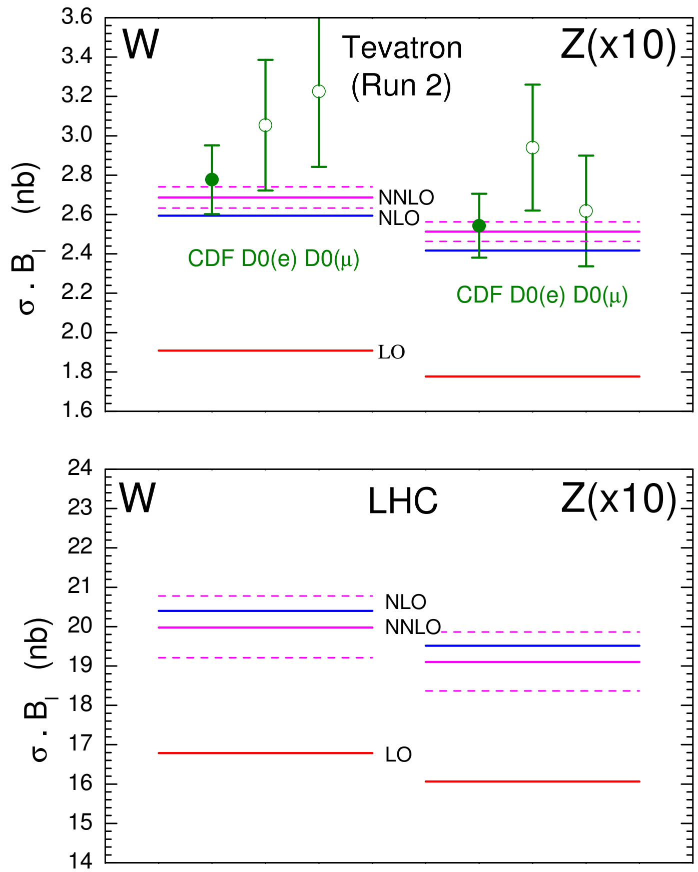
- NNLO splitting functions

  Moch, Vermaseren, Vogt

NNLO

- want to calculate $2 \to 2$ to few percent accuracy and use as standard candle to determine pdfs and α_s more accurately
- with global pdf fit, gives impact on all observables

Drell Yan production



Most accurate prediction yet

- √ NNLO splitting functions
- √ NNLO PDF fits
- √ NNLO Drell-Yan cross section

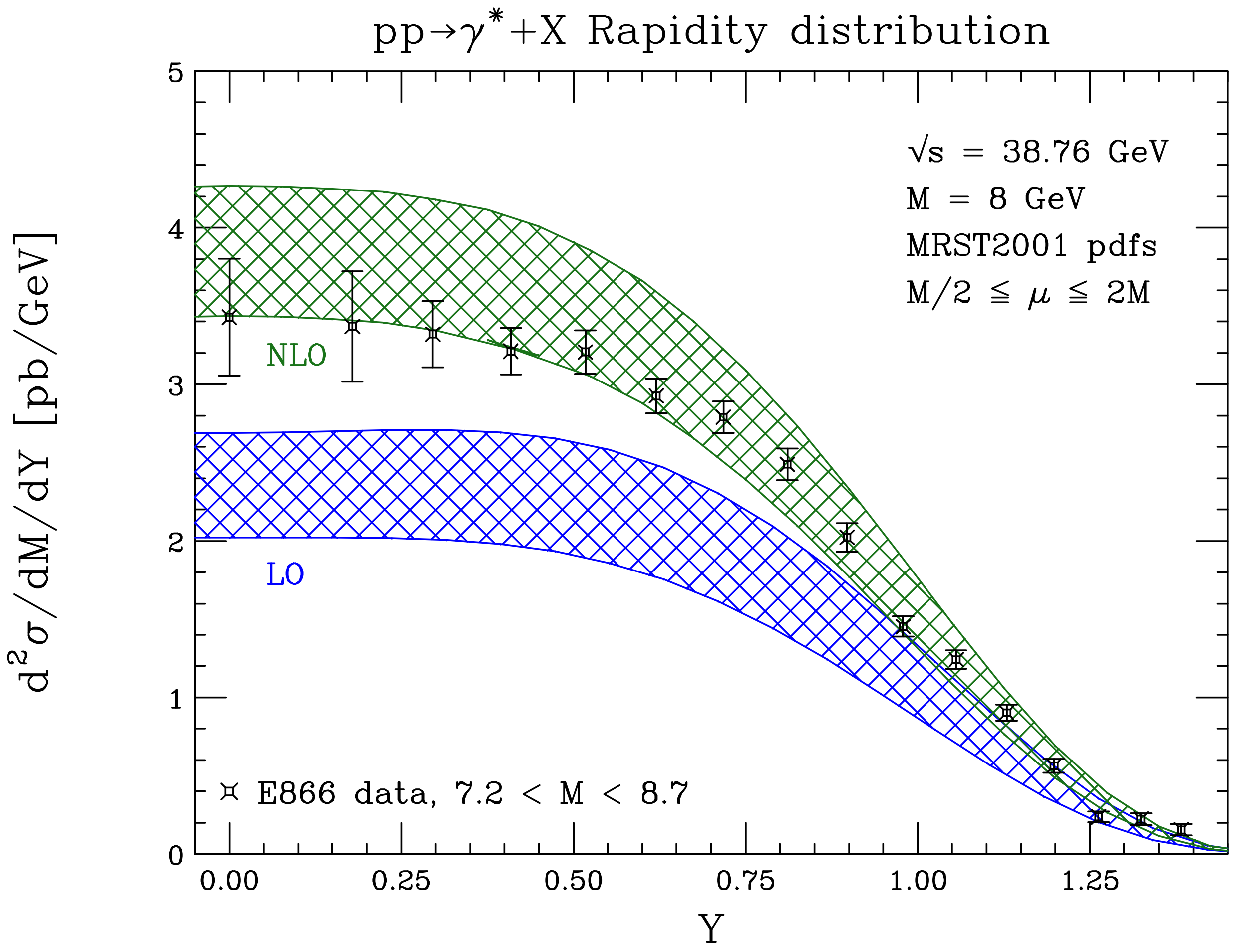
→ High precision

Total error of $4\% - -5.5\%$

Martin et al

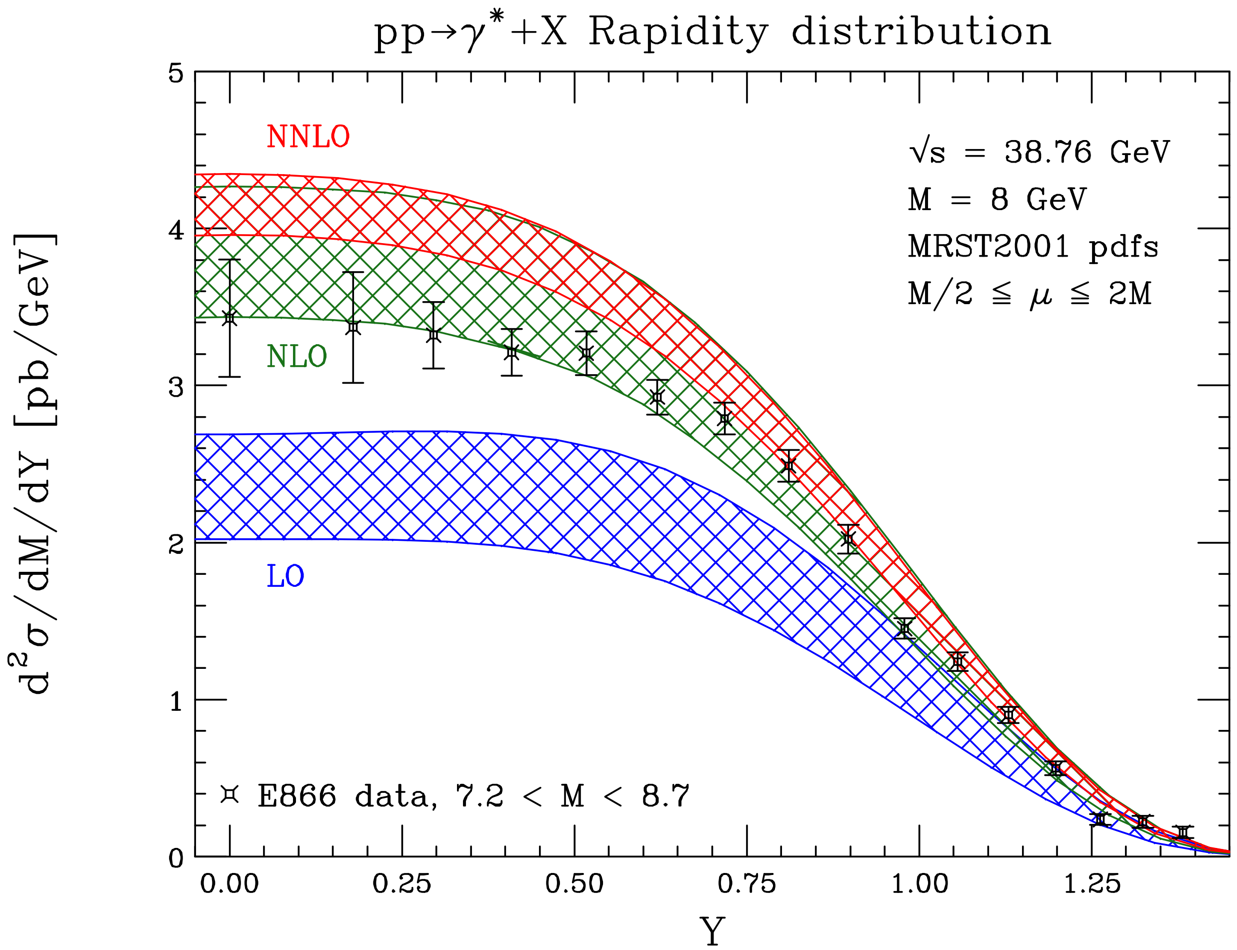
Aim to able to use as Standard Candle for luminosity measurements.

Gauge boson production at E866



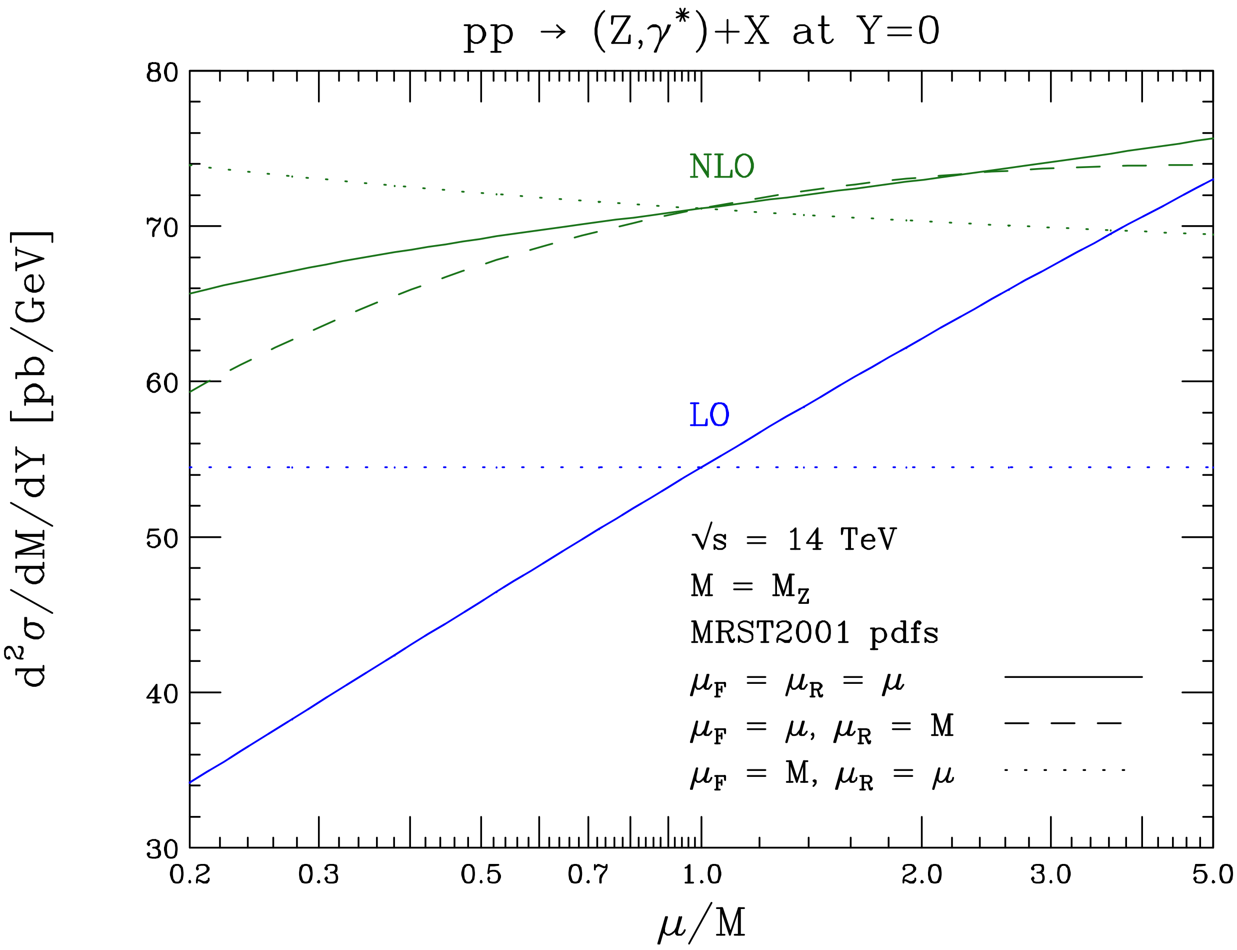
Anastasiou, Dixon, Melnikov, Petriello

Gauge boson production at E866



Anastasiou, Dixon, Melnikov, Petriello

Gauge boson production at the LHC



Anastasiou, Dixon, Melnikov, Petriello

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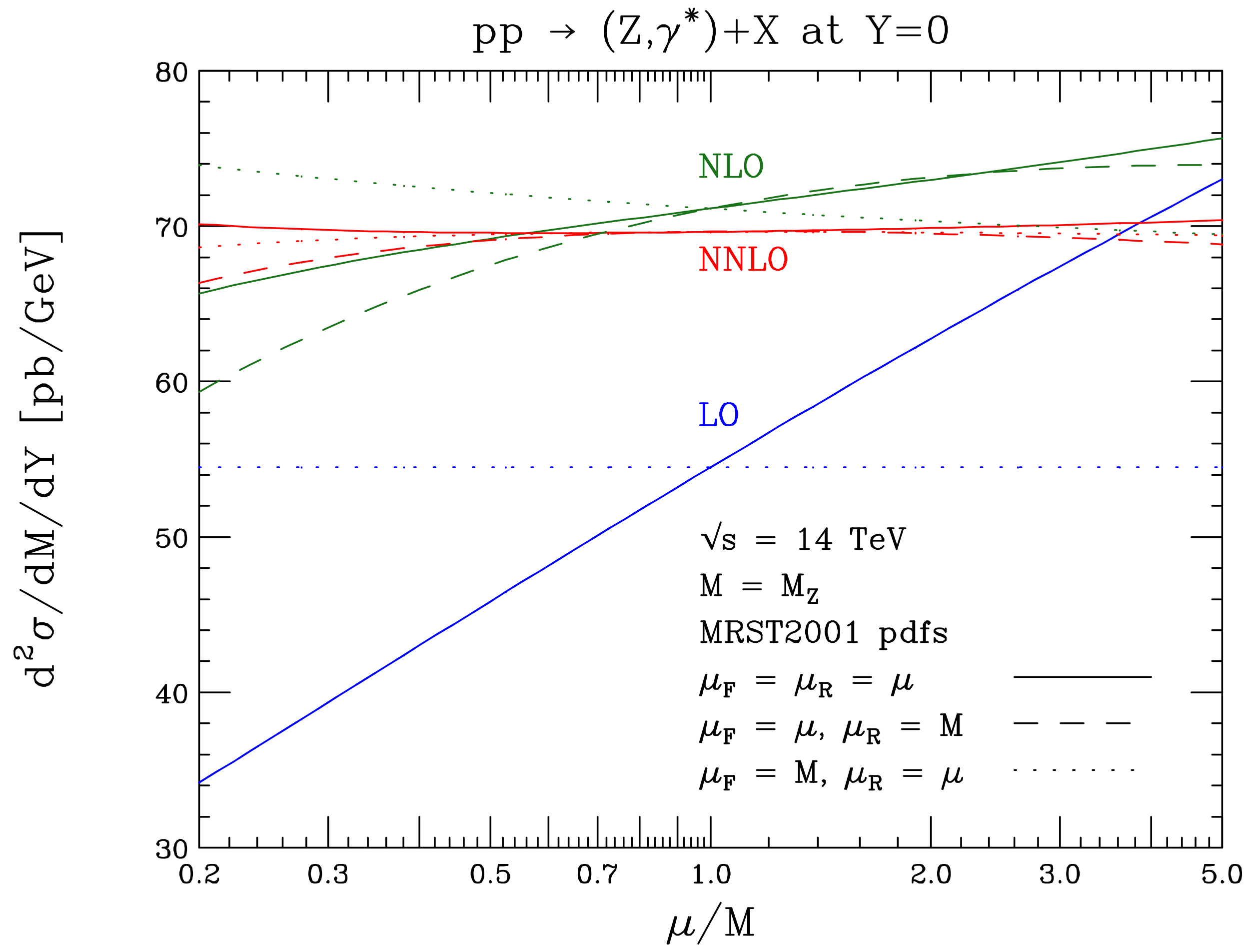
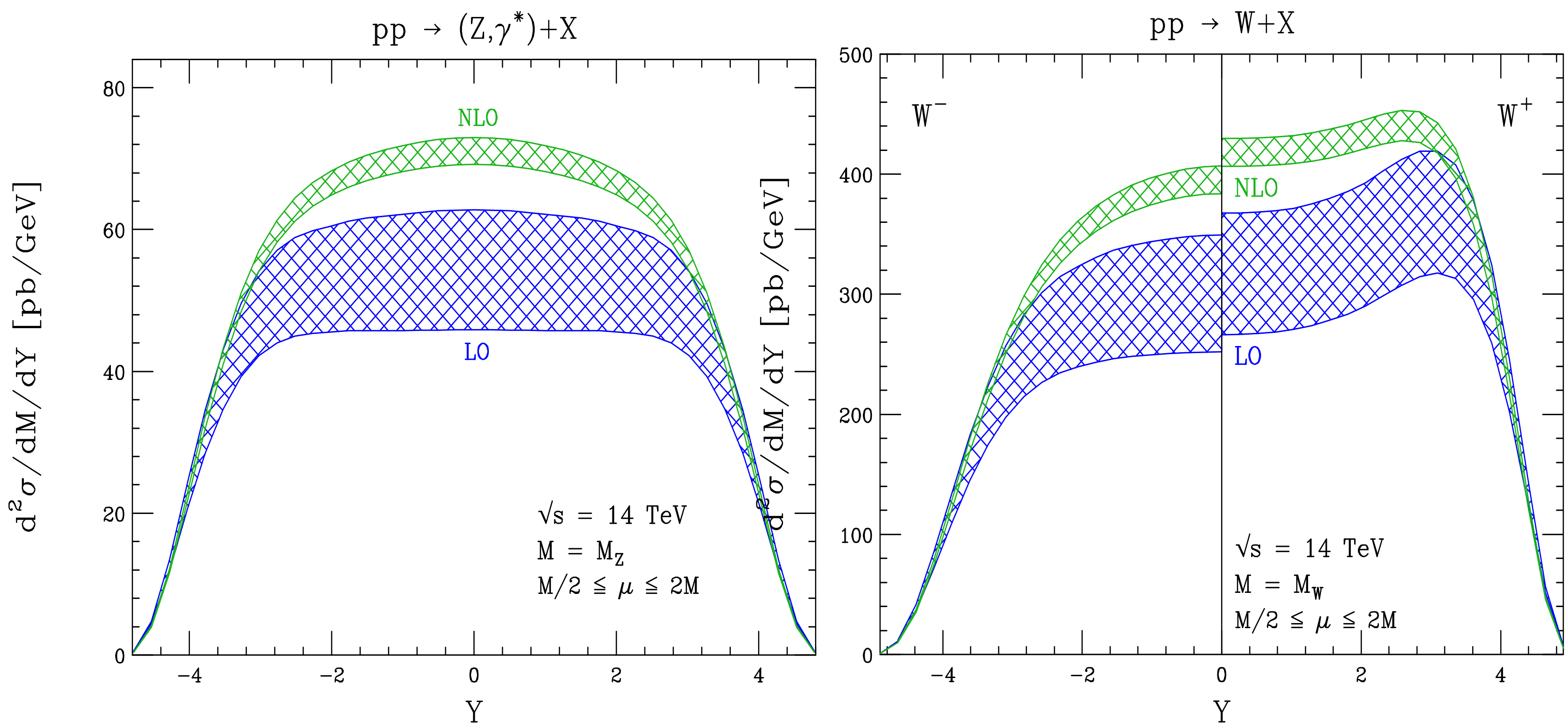


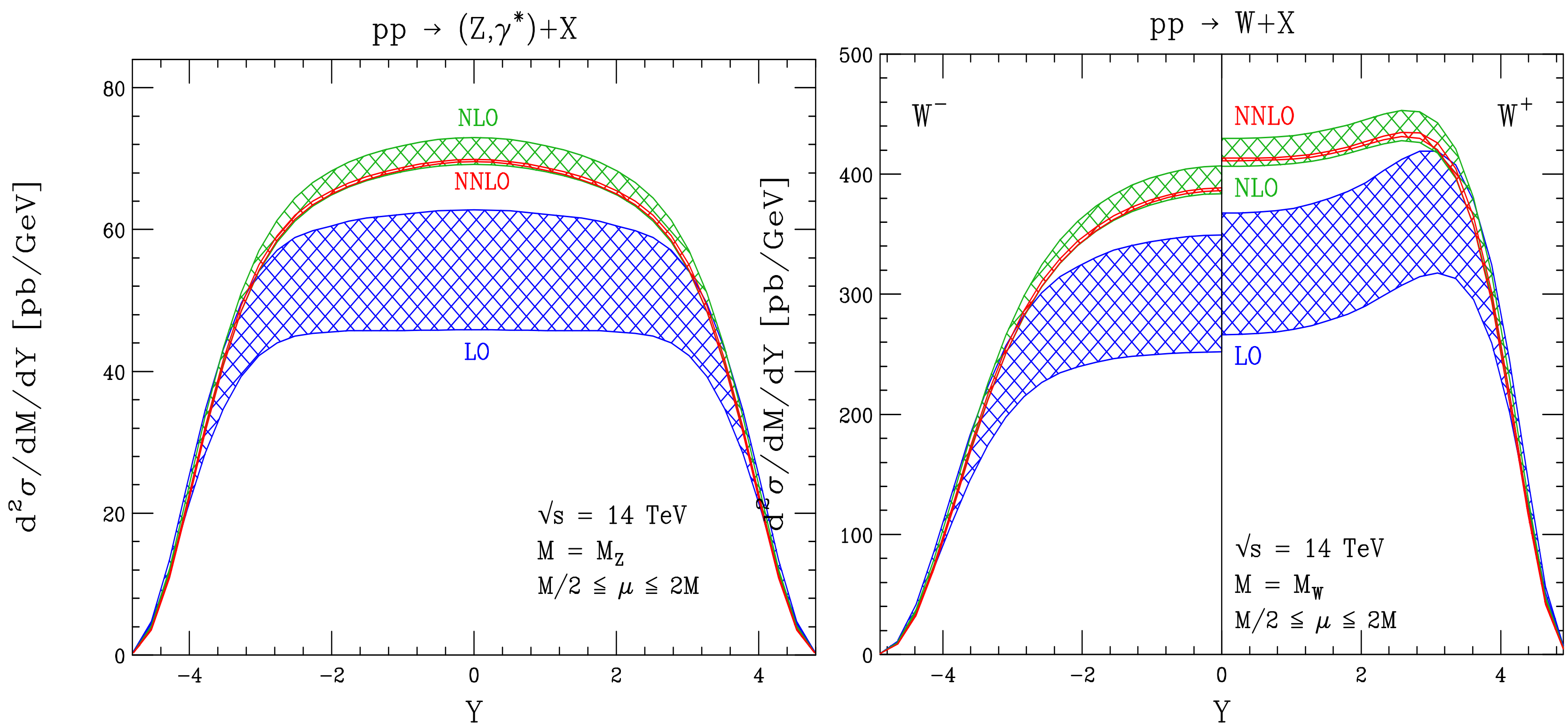
Anastasiou, Melnikov, Petriello

Significantly reduced scale uncertainty.

Gauge boson production at the LHC



Gauge boson production at the LHC



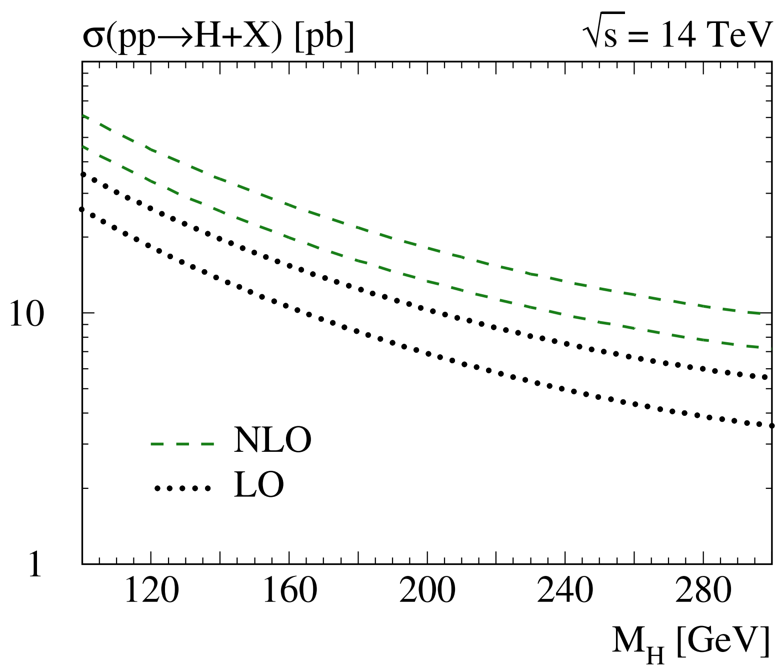
Gold-plated process

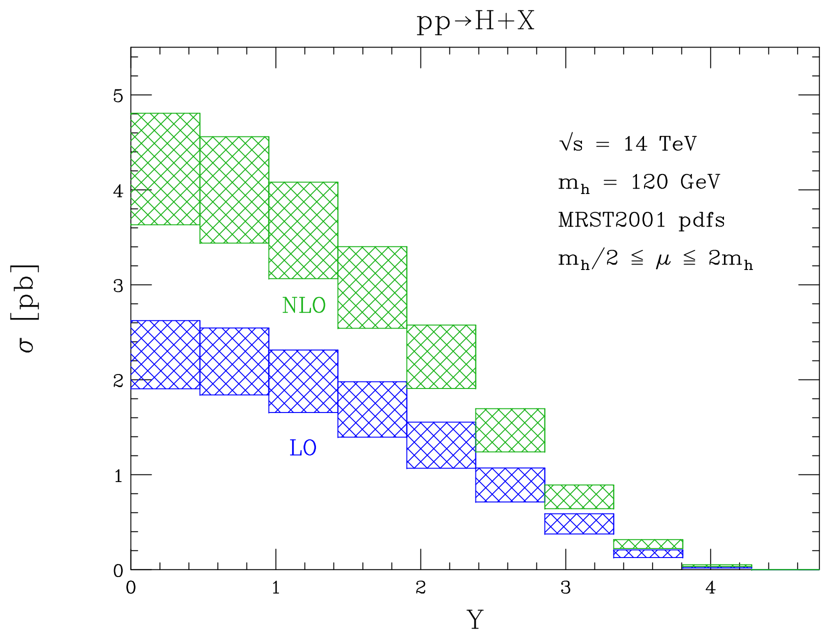
Anastasiou, Dixon, Melnikov, Petriello

At LHC NNLO perturbative accuracy better than 1%

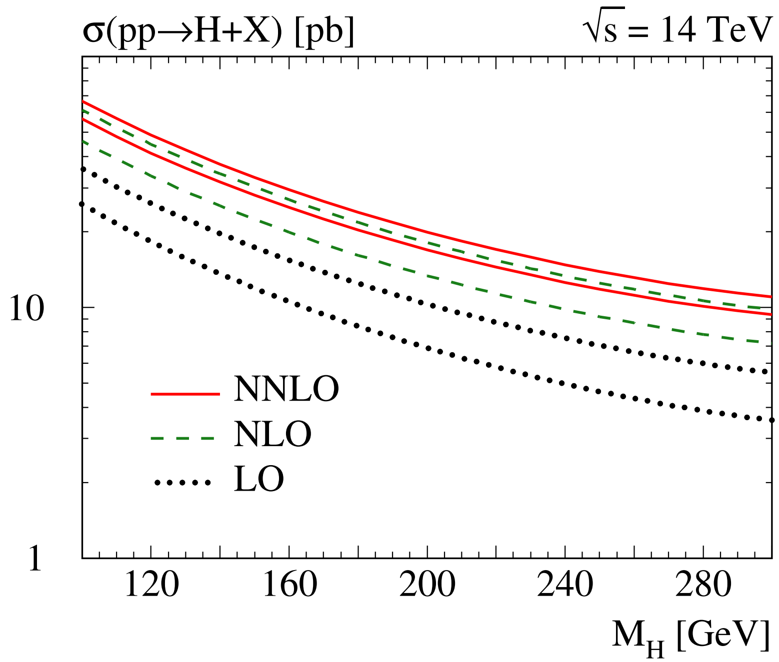
⇒ use to determine parton-parton luminosities at the LHC

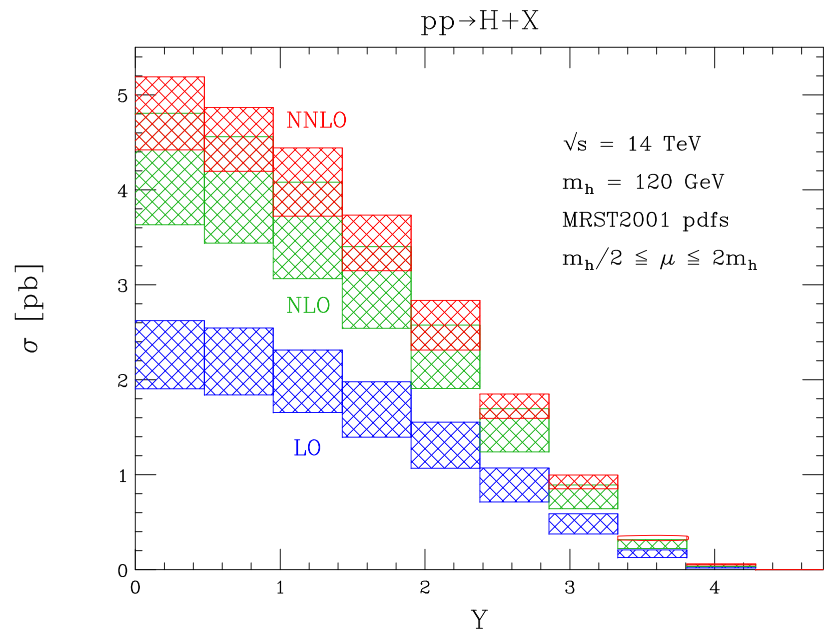
Higgs boson production at the LHC





Higgs boson production at the LHC





Total cross section

Harlander, Kilgore; Anastasiou, Melnikov, Petriello; . . .

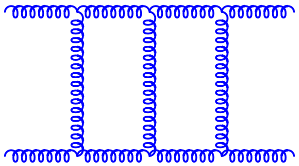
Fully differential

Anastasiou, Melnikov, Petriello

NNLO needed for reliable predictions

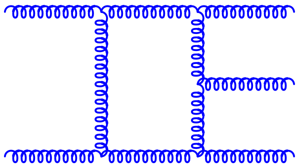
Anatomy of a NNLO calculation

- two-loop $2 \to 2$ matrix element



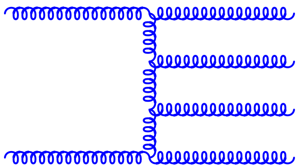
  looks like 2 jets in final state

- one-loop $2 \to 3$



  looks like 2 or 3 jets in final state

- tree-level $2 \to 4$

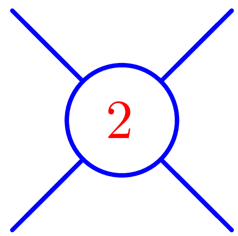


  looks like 2, 3 or 4 jets in final state

- plus method for combining the infrared divergent parts

Structure of two-loop contribution

- The many (thousands) of tensor integrals appearing in two-loop graphs can be written in terms of a few Master Integrals MI_j

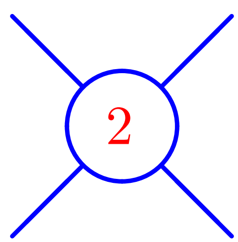


$$= \sum_j a_j \; \mathrm{MI}_j$$

where the a_j are polynomials in kinematical variables and the space-time dimension D.

- The MI_j can be expanded in $\epsilon = (4 - D)/2$ so that

$$\sum_{\mathrm{diagrams}}$$



$$= \sum_{i=1}^{4} \frac{X_i}{\epsilon^i} + X_0$$

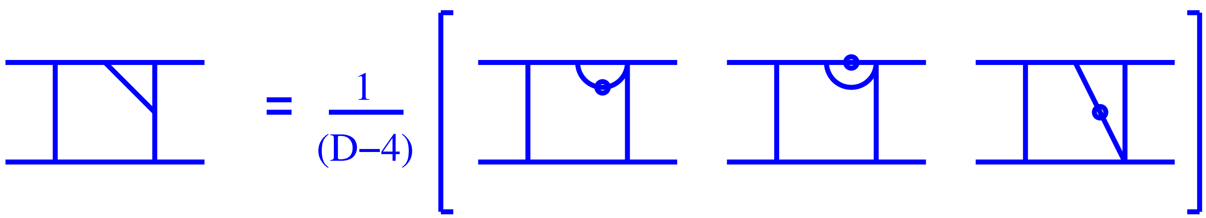
Reduction of Tensor Integrals

- Integration by Parts

$$\int d^D k_1 \int d^D k_2 \frac{\partial}{\partial k_i^\mu} \left[\frac{v^\mu}{A_1^{\nu_1} \cdots A_n^{\nu_n}} \right] \equiv 0$$

where v is any momentum in the problem, k_i, p_i.

Chetrykin, Kataev, Tkachov

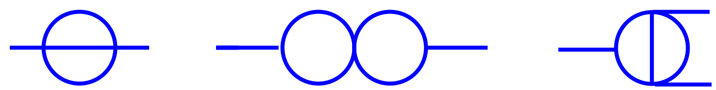


- Lorentz Invariance
  Invariance of integral under infinitessimal rotation yields extra identities

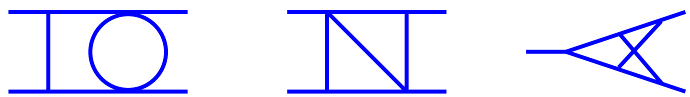
Gehrmann, Remiddi

Master integrals - on-shell

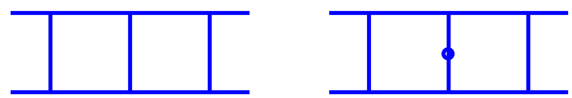
- √ The trivial topologies



- √ The less trivial topologies

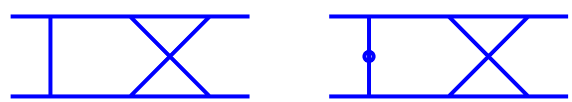


- √ The planar boxes



Smirnov; Smirnov and Veretin

- √ The non-planar boxes

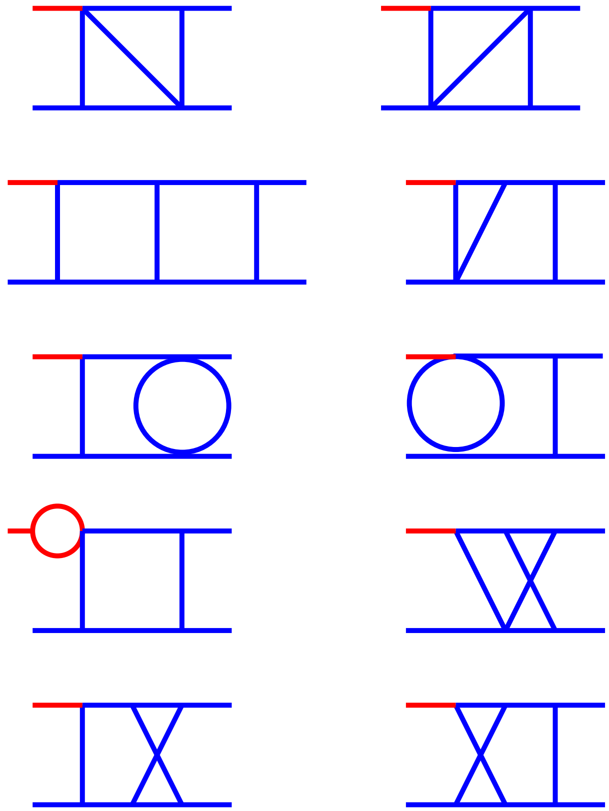


Tausk; Anastasiou, Gehrmann, Oleari,Remiddi and Tausk

Master integrals - off-shell

- Each extra mass scale introduces many more master integrals
- One off-shell leg gives 10 (+5 tensor) master four-point intregrals
- New functions (2-dimensional harmonic polylogarithm) functions needed to describe the MI

Gehrmann and Remiddi



Methods for calculating master integrals

- Mellin-Barnes contour integrals

  Davydychev; Smirnov; Tausk

- Differential equations in external scales and match to boundary conditions with fewer scales

  Remiddi, Gehrmann

- Nested sums from Schwinger parameterisation together with Hopf algebra techniques to relate to standard sums

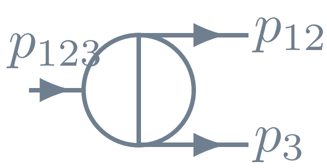
  Moch, Uwer, Weinzierl

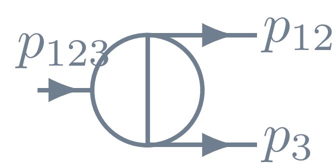
- Numerical method based on iterated sector decomposition - used to check many of above results

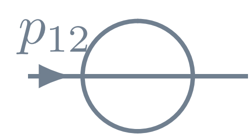
  Binoth, Heinrich

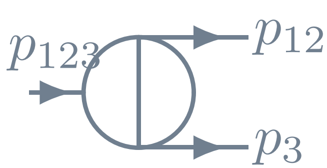
Differential equations

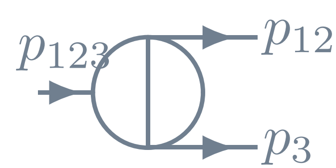
- Multiscale master integrals satisfy inhomogeneous differential equations in terms of invariants.

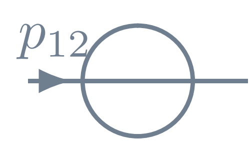
$$s_{123} \frac{\partial}{\partial s_{123}}$$


$$= \frac{d-4}{2} \frac{2s_{123} - s_{12}}{s_{123} - s_{12}}$$


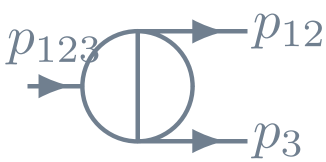
$$- \frac{3d-8}{2} \frac{1}{s_{123} - s_{12}}$$


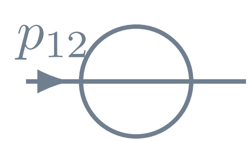
$$s_{12} \frac{\partial}{\partial s_{12}}$$


$$= - \frac{d-4}{2} \frac{s_{12}}{s_{123} - s_{12}}$$


$$+ \frac{3d-8}{2} \frac{1}{s_{123} - s_{12}}$$


- The boundary equations are in terms of simpler integrals.



$$(s_{123} = 0) = \frac{3d-8}{d-4} \frac{1}{s_{12}}$$


Recent progress on two-loop master integrals

Vertex integrals with one internal mass scale

- equal external masses: $\gamma^* \to Q\bar{Q}$ QCD and QED form factors

  Bonciani, Mastrolia, Remiddi

- QCD heavy quark form factor

  Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia and Remiddi

- $\gamma^* \to Q\bar{Q}$ electroweak form factors

  Aglietti,Bonciani

- $H \to gg, \gamma\gamma$

  Aglietti,Bonciani,Degrassi,Vicini

Four point integrals with one internal mass scale

- massive Bhabha scattering and heavy quark production

  Smirnov; Heinrich, Smirnov; Bonciani, Ferroglia, Mastrolia, Remiddi, van der Bij; Czakon, Gluza, ...

Recent progress on two-loop amplitudes

On-shell Process	Tree $\times$ Two-loop	Helicity amplitudes
$e^+e^- \to \mu^+\mu^-(e^+e^-)$	$\color{blue}{\surd}$ (00)	
$q\bar{q} \to q\bar{q}(\bar{q}'q')$	$\color{red}{\surd}$ (00)	$\color{red}{\surd}$ (04)
$q\bar{q} \to gg$	$\color{red}{\surd}$ (01)	$\color{blue}{\overline{\surd}}$ (03), $\color{red}{\surd}$ (03)
$gg \to gg$	$\color{red}{\surd}$ (01)	$\color{blue}{\surd}$ (00), $\color{blue}{\overline{\surd}}$ (02)
$gg \to \gamma\gamma$	—	$\color{blue}{\surd}$ (01)
$\gamma\gamma \to \gamma\gamma$	—	$\color{blue}{\surd}$ (01), $\color{red}{\overline{\surd}}$ (02)
$q\bar{q} \to g\gamma(\gamma\gamma)$	$\color{red}{\surd}$ (02)	$\color{red}{\surd}$ (03)
Off-shell Process		
$e^+e^- \to q\bar{q}g$	$\color{red}{\surd\!\!\surd}$ (01)	$\color{red}{\surd\!\!\surd}$ (02), $\color{green}{\surd}$ (02)

$\color{blue}{\surd}$ Bern, De Freitas, Dixon, Ghinculov, Kosower, Wong

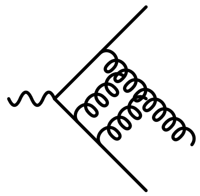
$\color{red}{\surd}$ Anastasiou, Binoth, EWNG, Marquard, Oleari, Tejeda-Yeomans, van der Bij

$\color{red}{\surd\!\!\surd}$ Garland, Gehrmann, EWNG, Koukoutsakis, Remiddi

$\color{green}{\surd}$ Moch, Uwer, Weinzierl

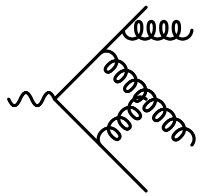
Jet physics at NNLO

- Two-loop matrix elements



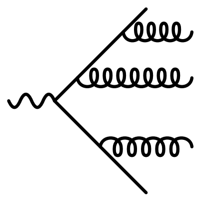
explicit poles from loop integration

- One-loop matrix elements



explicit poles from loop integration
implicit poles from single unresolved radiation

- Tree level matrix elements



implicit poles from double unresolved radiation

How can we make infrared poles cancel in the sum?

Different approaches to infrared cancellation

- Subtraction
  Provides completely exclusive description of final state
  Non trivial extension of dipole subtraction at NLO

  Weinzierl; Kosower; Gehrmann, Gehrmann-De Ridder, EWNG; Del Duca, Trocsanyi; Grazzini, De Florian

  Antenna subtraction scheme of Gehrmann et al complete and in numerical implementation stage

- Unitarity based approach
  Good for suitably inclusive quantities such as σ_H

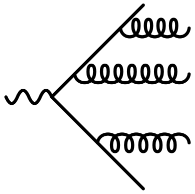
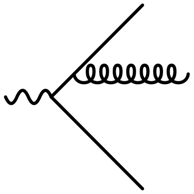
  Anastasiou, Melnikov

- Sector decomposition
  Also good for more exclusive quantities - e.g. $e^+e^- \to 2$ jets

  Binoth, Heinrich; Anastasiou, Melnikov, Petriello

Infrared subtraction terms

$n+2$ parton final state forming n jets:

 $\mathrm{d}\Phi_2 \quad \longrightarrow \quad C$ 

- Singular configurations:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft

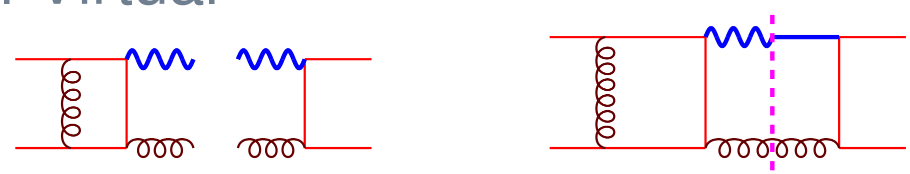
Campbell, EWNG; Grazzini, Catani

- Issue: find subtraction functions which
  - √ approximate full $n+2$ matrix element in all singular limits
  - √ are sufficiently simple to be integrated analytically

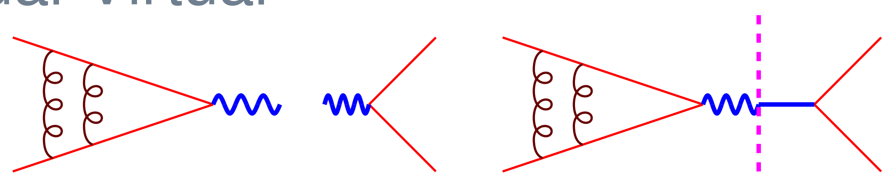
Unitarity approach

- Real-Real

- Real-Virtual



- Virtual-Virtual



For suitably inclusive quantities, can relate real and virtual contributions using unitarity

$$\delta(p^2 - M^2) \rightarrow \frac{1}{p^2 - M^2 - i\delta} - c.c.$$

Sector decomposition

Key result:

$$x^{-1-\epsilon} = -\delta(x)/\epsilon + [1/x]_+ - \epsilon[\ln(x)/x]_+ + \ldots$$

At NLO: $$I_1 = \int_0^1 \mathrm{d}x\mathrm{d}y\, x^{-1-\epsilon} y^{-1-\epsilon}\, J(x,y).$$

At NNLO: $$I_2 = \int_0^1 \mathrm{d}x\mathrm{d}y\;\; x^{-1-\epsilon} y^{-1-\epsilon} (x+y)^{-\epsilon}\, J(x,y).$$

Overlapping divergences so cannot directly apply prescription
Split integral into two parts $x < y$ and $y < x$

$$\begin{aligned} I_{2a} &= \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\;\; x^{-1-\epsilon} y^{-1-\epsilon} (x+y)^{-\epsilon}\, J(x,y) \\ &= \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, x^{-1-3\epsilon} y^{-1-\epsilon} (1+y)^{\epsilon}\, J(x,xy) \end{aligned}$$

Now apply key result

Summary - Precise predictions

- Last few years has seen substantial progress in pQCD
- Many NLO predictions still needed
  Significant steps forward in automation for NLO pQCD
- NNLO pQCD for basic observables is becoming new standard
  - Inclusive DIS coefficient functions completed
  - Unpolarised three-loop splitting functions completed
  - Differential distributions for Higgs and gauge bosons completed
  - NNLO Jet cross sections on horizon for e^+e^- - and then pp/ep
  - NNLO heavy quarks still a long way away