Generalized parton distributions

M. Diehl

Deutsches Elektronen-Synchroton DESY

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More on impact parameter dependence

in following specialize to $\xi = 0$

- impact parameter distribution

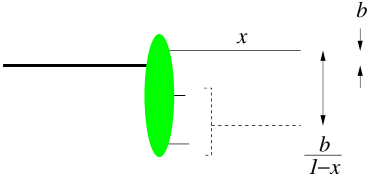
$$q(x, b^2) = (2\pi)^{-2} \int d^2\boldsymbol{\Delta}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\, H^q(x, \xi = 0, t = -\boldsymbol{\Delta}^2)$$

  gives distribution of quarks with
  - longitudinal momentum fraction x
  - transverse distance b from proton center

- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2b\; b^2\, q(x, b^2)}{\int d^2b\; q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log H^q(x, \xi, t) \Big|_{t=0}$$

Constraint for large x



- for $x \to 1$ get $b \to 0$
  nonrel. analog:
  center of mass of atom
- $\Leftrightarrow$ t dependence becomes flat

- $d = b/(1-x)$
  $=$ distance of selected parton from spectator system
  gives lower bound on overall size of proton
- finite size of configurations with $x \to 1$ implies

$$\langle b^2 \rangle_x \sim (1-x)^2$$

M. Burkardt, '02, '04

Ansatz for small x

$$H(x,t) \sim x^{-\alpha-\alpha' t}\, e^{tB} \quad \rightsquigarrow \quad \langle b^2 \rangle_x \sim B + \alpha' \log(1/x)$$

- suggested by phenomenology of high-energy hadron-hadron scattering
- generalizes ansatz $q(x) \sim x^{-\alpha}$ for usual parton densities
- for valence quarks typically find $\alpha \sim 0.4 \ldots 0.5$ at low scale μ
- situation more complicated for sea quarks (mixing with gluons under evolution)

Evolution

- $q(x, b^2)$ fulfils usual DGLAP evolution equation for non-singlet (e.g. $q_{\mathrm{NS}} = q - \bar{q}$ or $q_{\mathrm{NS}} = u - d$):

$$\mu^2 \frac{d}{d\mu^2} q_{\mathrm{NS}}(x, b^2, \mu^2) = \int_x^1 \frac{dz}{z} \left[P\Big(\frac{x}{z}\Big) \right]_+ q_{\mathrm{NS}}(z, b^2, \mu^2)$$

evolution local in b (let $1/\mu \ll b$ to be safe)

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- average

$$\langle b^2 \rangle_x = \frac{\int d^2b\; b^2\, q_{\rm NS}(x, b^2)}{\int d^2b\; q_{\rm NS}(x, b^2)}$$

  evolves as

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\, \frac{1}{q_{\rm NS}(x)} \int_x^1 \frac{dz}{z}\, P\Big(\frac{x}{z}\Big)\, q_{\rm NS}(z) \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

M.D. et al. '04

Information from electromagnetic form factors

- ff's constrain interplay of x and b dependence

M.D. et al. '04, see also M. Guidal et al. '04

- e.m. current $\rightsquigarrow$ only valence combination $q - \bar{q}$

$$H_v^q(x,t) = H^q(x,t) - H^{\bar{q}}(x,t)$$

$$\begin{aligned} F_1^p(t) &= \int_0^1 dx \left[\tfrac{2}{3} H_v^u(x,t) - \tfrac{1}{3} H_v^d(x,t) \right] \\ F_1^n(t) &= \int_0^1 dx \left[\tfrac{2}{3} H_v^d(x,t) - \tfrac{1}{3} H_v^u(x,t) \right] \end{aligned}$$

- ansatz: $H_v^q(x,t) = q_v(x) \exp[\, t f_q(x)\,]$ $\qquad \langle b^2 \rangle_x^q = 4 f_q(x)$
- ansatz for $f_q(x)$ interpolates between

$f_q(x) \to \alpha' \log(1/x)$ for $x \to 0$

$f_q(x) \sim (1-x)^2$ for $x \to 1$

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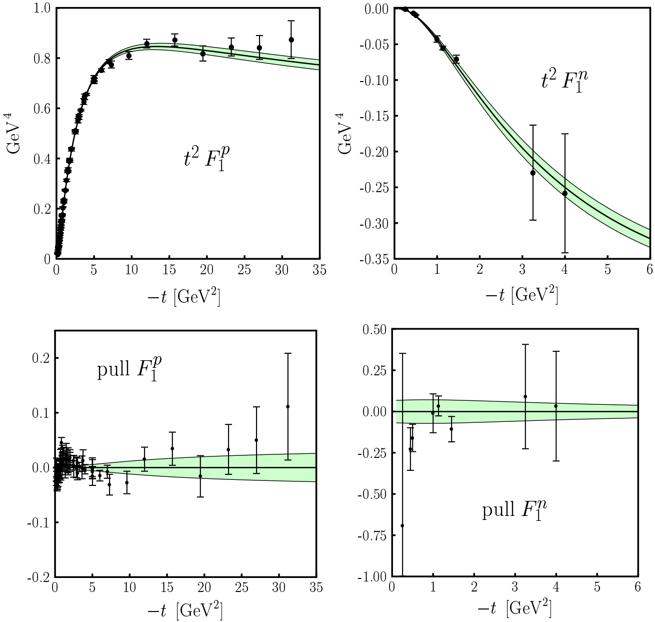
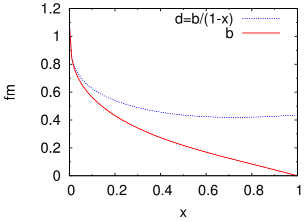
- good description of data with $\alpha' = 0.9$ to 1 GeV^{-2}

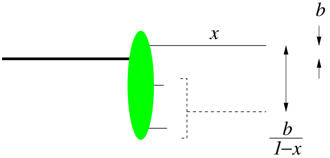


χ^2/d.o.f. $= 1.93$

pull = data/fit -1

Lesson from the fit





- clear drop with x of average distance $d = b/(1-x)$
  $\leftrightarrow$ strong correlation of x and t dependence

Compare with lattice results

matrix elements of local operators $\leftrightarrow$ form factors
calculate in lattice QCD

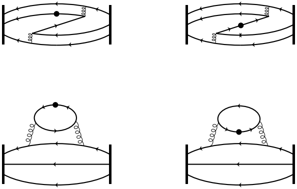


figure:
J. Negele, hep-lat/0211022

- main systematic uncertainties from
  - omission of "disconnected" diagrams
    but: cancel in difference of u and d
  - extrapolation to physical pion mass

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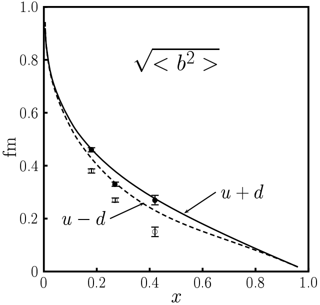
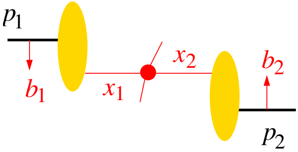


J. Negele et al., hep-lat/0404005

- Wilson fermions
- $m_\pi = 870$ MeV
- typical x in $\int dx\, x^n q(x,b)$ estimated as

$$\langle x \rangle = \frac{\int dx\, x^{n+1} q(x)}{\int dx\, x^n q(x)}$$

Consequences for hadron-hadron collisions



- hard inclusive process, e.g. $pp \to \text{jet jet} + X$
  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently

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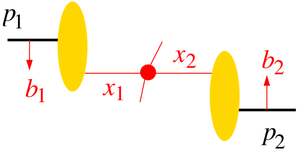
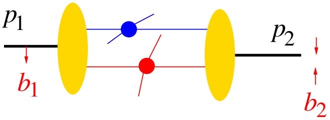




- hard inclusive process, e.g. $pp \to \text{jet jet} + X$
  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently
- secondary soft or hard interactions
  do not affect inclusive cross section
  but change event structure
- larger mom. fractions x_1, x_2 in hard subprocess
  $\rightsquigarrow$ more central collision
  $\rightsquigarrow$ more secondary interactions Frankfurt, Strikman, Weiss '03

Spin and the Pauli form factors

- $E \leftrightarrow$ nucleon helicity flip $\langle\downarrow|\mathcal{O}|\uparrow\rangle$

  $\leftrightarrow$ transverse pol. difference $|X\pm\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow\rangle \pm |\downarrow\rangle\,)$

  $$\langle X{+}|\mathcal{O}|X{+}\rangle - \langle X{-}|\mathcal{O}|X{-}\rangle = \langle\uparrow|\mathcal{O}|\downarrow\rangle + \langle\downarrow|\mathcal{O}|\uparrow\rangle$$

- quark density in proton state $|X{+}\rangle$

$$q_v^X(x,\mathbf{b}) = q_v(x,b) - \frac{b^y}{m}\frac{\partial}{\partial b^2}\, e_v^q(x,b)$$

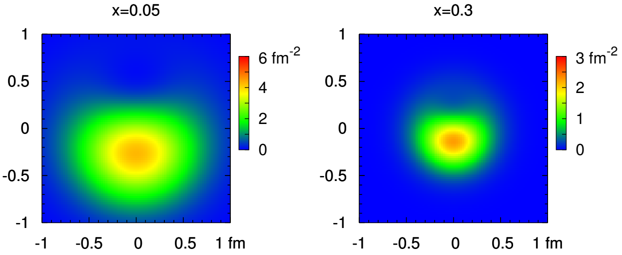
shifted in y direction M. Burkardt '02

$e_v^q(x,b)$ is Fourier transform of $E_v^q(x,t)$

quark density in proton state $|X+\rangle$

$$q_v^X(x,\mathbf{b}) = q_v(x,b) - \frac{b^y}{m}\frac{\partial}{\partial b^2}\, e_v^q(x,b)$$

is shifted



$(d-\bar{d})$ density in transverse plane M.D. et al. '04

- from anomalous magnetic moments of p and n
  $\int dx\, E^u(x,0) = \kappa^u \approx 1.67$ and $\int dx\, E^d(x,0) = \kappa^d \approx -2.03$
  $\rightarrow$ large spin-orbit correlations

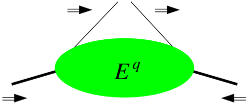
- density representation

$$q_v^X(x, \mathbf{b}) = q_v(x, b) - \frac{b^y}{m} \frac{\partial}{\partial b^2}\, e_v^q(x, b)$$

gives positivity bound M. Burkardt '03

$$\Big[E^q(x, t=0)\Big]^2 \quad \le \quad m^2 \Big[q(x) + \Delta q(x)\Big] \Big[q(x) - \Delta q(x)\Big] \times 4\, \frac{\partial}{\partial t} \ln \Big[H^q(x,t) \pm \tilde{H}^q(x,t)\Big]_{t=0}$$

$\Rightarrow$ E^q must fall faster than H^q at large x



- $E \leftrightarrow$ orbital angular momentum
  $\Rightarrow$ carried by partons with smaller x

Information from the Pauli form factors

M.D. et al. '04

- sum rules: Pauli ff's $\leftrightarrow$ $E_v^q(x,t) = E^q(x,t) - E^{\bar{q}}(x,t)$
- ansatz $E_v^q(x,t) = e_v(x) \exp[\, t\, g_q(x)\,]$

  $g_q(x)$ same form as $f_q(x)$ in ansatz for H_v^q
- shape of forward limit $e_v^q(x)$ not known $\longrightarrow$ ansatz

$$e_v^q = \mathcal{N}_q\, x^{-\alpha}(1-x)^{\beta_q}$$

$\mathcal{N}_q$ determined by p and n magnetic moments

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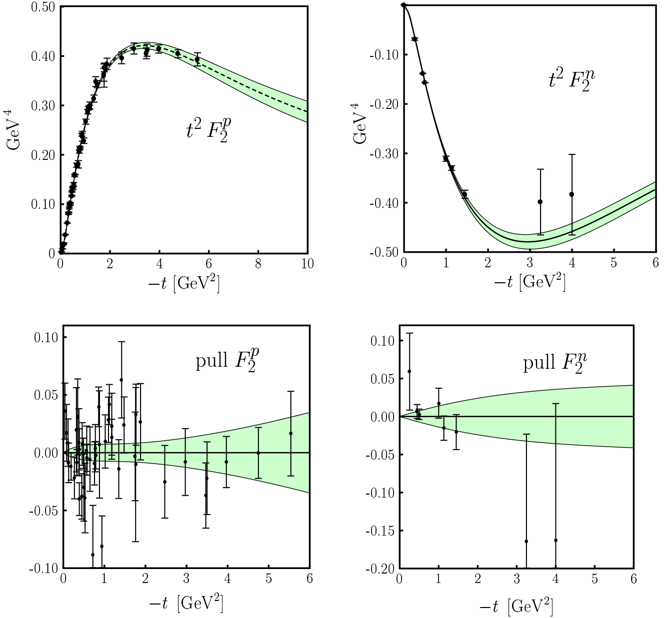
$$e_v^q = \mathcal{N}_q \, x^{-\alpha} (1-x)^{\beta_q}$$

  $\mathcal{N}_q$ determined by p and n magnetic moments
- obtain good fit of $F_2^p(t)$ and $F_2^n(t)$

  $\alpha' = 0.9$ GeV^{-2} and $\alpha = 0.55$
- large allowed regions of β_q and parameters in $g_q(x)$

  but positivity constraints seriously limit parameter space

  in particular $\beta_d \geq 5$ and $\beta_u \geq 3.5$

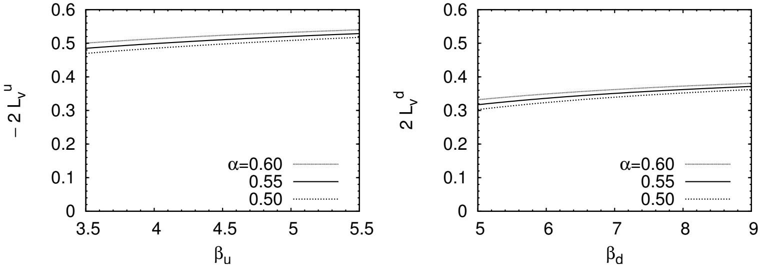


χ^2/d.o.f. $= 1.31$

pull $=$ data/fit -1

orbital angular momentum carried by valence quarks   Ji's sum rule

$$\langle L_v^q \rangle = \frac{1}{2} \int dx \left[x e_v^q(x) + x q_v(x) - \Delta q_v(x) \right]$$



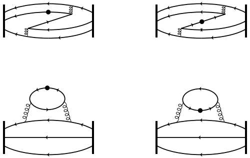
- individual u and d quite well determined
- $2\langle J_v^u \rangle = 2\langle L_v^u \rangle + 0.93$ and $2\langle J_v^d \rangle = 2\langle L_v^d \rangle - 0.34$
- $2\langle L_v^u - L_v^d \rangle = -(0.77 \div 0.92)$ at $\mu = 2$ GeV
  strong cancellations in $2\langle L_v^u + L_v^d \rangle = -(0.11 \div 0.22)$

Comparison with lattice results

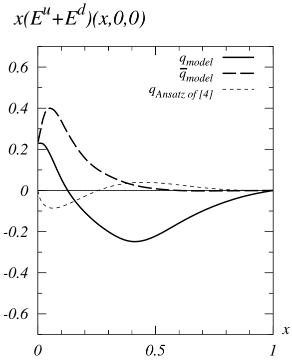
- form factor analysis — M.D. et al '04
  $2\langle L_v^u - L_v^d \rangle = -(0.77 \div 0.92)$
  $2\langle L_v^u + L_v^d \rangle = -(0.11 \div 0.22)$
- lattice results — QCDSF, G. Schierholz at LC 2005
  $2\langle L_v^u - L_v^d \rangle = -0.9 \pm 0.12$
  $2\langle L_v^u + L_v^d \rangle = \quad 0.06 \pm 0.14$
- lattice for $m_\pi = 897$ MeV — LHPC, from hep-ph/0410017
  $2\langle L_v^u - L_v^d \rangle = -0.25 \pm 0.05$
  $2\langle L_v^u + L_v^d \rangle = -0.10 \pm 0.05$

all results for $\mu = 2$ GeV

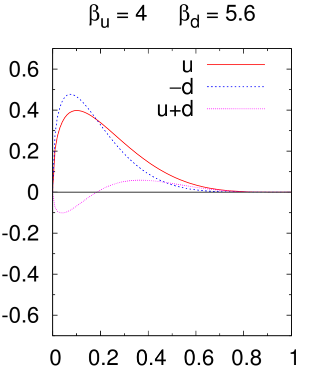
lattice "valence" contributions in sense of "connected quark diagrams" ↗



calculation of $E^u + E^d$ in chiral soliton model J. Ossmann et al. '05

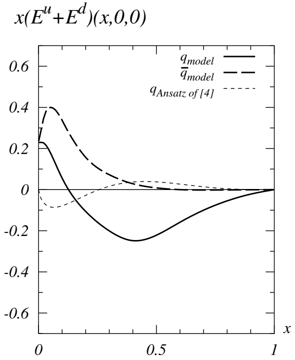


hep-ph/0411172, $\mu^2 = 5$ GeV^2



$E^q - E^{\bar{q}}$, $\mu^2 = 4$ GeV^2

- shape differs from our fit (with central values of par's) calculation of $E^u + E^d$ in chiral soliton model J. Ossmann et al. '05



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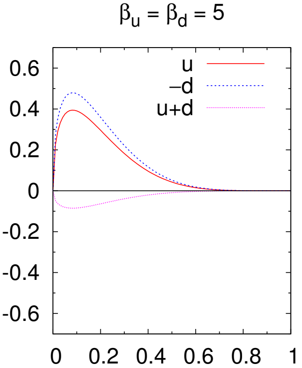


$E^q - E^{\bar{q}}$, $\mu^2 = 4$ GeV^2

- shape differs from our fit (with central values of par's)
- but fit has large parameter uncertainties for $E^u + E^d$
- normalization: chiral soliton model: $\kappa_u + \kappa_d = 0.35$

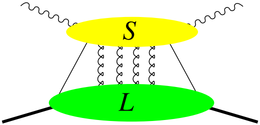
  experiment: $\kappa_u + \kappa_d = 0.12$

Summary

- Dirac form factor data and lattice
  $\rightsquigarrow$ strong decrease of impact parameter $\langle \boldsymbol{b}^2 \rangle$ with x
- attempts at quantitative understanding of helicity flip distribution E $\rightsquigarrow$ orbital angular momentum

  "valence" contributions: L^{u-d} big and L^{u+d} small
  need direct measurements to learn more

Wilson lines in short-distance factorization



- resum gluon exchange between hard-scattering subprocess and spectator partons in hadron

- $S^\mu(l)$ all components big
  $L^\mu(l) \propto p^\mu$ only plus-component big (for right-moving hadron)
  $\rightsquigarrow \quad S_\mu L^\mu \approx S^- L^+$
- Wilson line $W[a,b] = P \exp\left[ig \int_a^b dz^- \, A^+(z) \right]_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$

$$q(x) \propto \int dz^- \, e^{ixp^+z^-} \langle p| \, \bar{q}(-\tfrac{1}{2}z) \, W[-\tfrac{1}{2}z, \tfrac{1}{2}z] \, \gamma^+ q(\tfrac{1}{2}z) \, |p\rangle \Big|_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$$

- Ward identities $\rightsquigarrow$ appear explicitly in parton distribution but not in hard-scattering kernel

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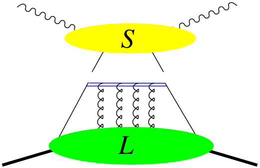

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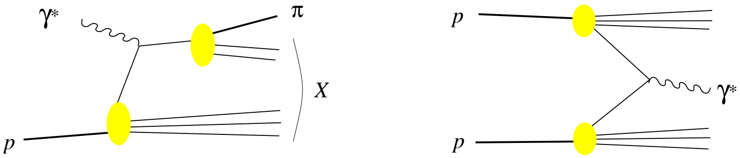
Unintegrated parton distributions

- naturally appear when "observe" parton k_t in final state
- e.g. semi-inclusive DIS $\gamma^* p \longrightarrow \pi(p_t)\, X$
  and Drell-Yan $pp \longrightarrow \gamma^*(p_t)\, X$ with $p_t \sim \Lambda$ non-perturbative



$$q(x,\boldsymbol{k}) \propto \int dz^- \, d^2\boldsymbol{z}\, e^{ixp^+z^-} e^{-i\boldsymbol{k}\boldsymbol{z}} \langle p|\, \bar{q}(-\tfrac{1}{2}z)\, W_P[z]\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0}$$

- space-time structure of process $\rightsquigarrow$ path P in Wilson line
- SIDIS: interactions after quark struck by photon
  DY: interactions before quark annihilates

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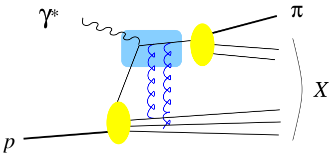

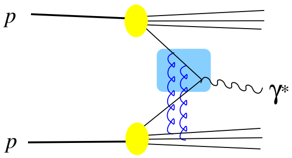

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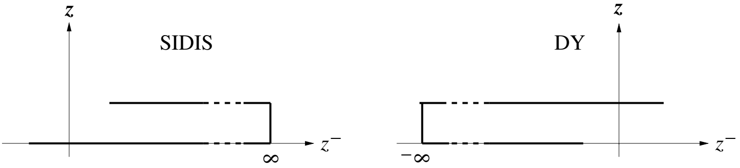

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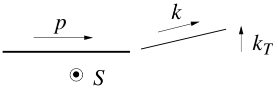
Wilson line has physical consequences

- transverse proton polarization $\rightsquigarrow$ anisotropic $\boldsymbol{k}$ distribution

$$f_{q/p\uparrow}(x, \boldsymbol{k}) = f_{\text{unpol}}(x, \boldsymbol{k}^2) + \frac{(\vec{S} \times \vec{p}) \cdot \vec{k}}{m_p \, |\vec{p}|} \, f_{\text{Sivers}}(x, \boldsymbol{k}^2)$$

- induces anisotropic p_t distribution in SIDIS (Sivers effect) observed at HERMES
- time reversal changes sign of $(\vec{k} \times \vec{S}) \cdot \vec{p}$
  $\rightsquigarrow$ Sivers function $= 0$?!



Wilson line has physical consequences

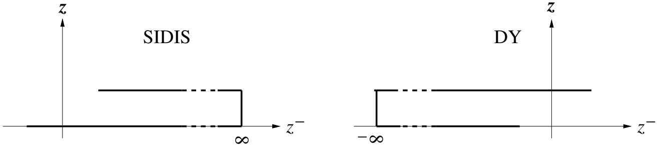
- transverse proton polarization $\rightsquigarrow$ anisotropic $\boldsymbol{k}$ distribution

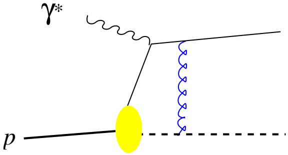
$$f_{q/p\uparrow}(x, \boldsymbol{k}) = f_{\text{unpol}}(x, \boldsymbol{k}^2) + \frac{(\vec{S} \times \vec{p}) \cdot \vec{k}}{m_p \, |\vec{p}|} \, f_{\text{Sivers}}(x, \boldsymbol{k}^2)$$

- induces anisotropic p_t distribution in SIDIS (Sivers effect)
  observed at HERMES
- time reversal changes sign of $(\vec{k} \times \vec{S}) \cdot \vec{p}$
  $\rightsquigarrow$ Sivers function $= 0$?!
- no: time reversal interchanges Wilson lines for
  SIDIS (future pointing) and DY (past pointing)

$$\rightsquigarrow \quad f_{\text{Sivers}}^{\text{SIDIS}}(x, \boldsymbol{k}^2) = -f_{\text{Sivers}}^{\text{DY}}(x, \boldsymbol{k}^2)$$

J. Collins '02





- Sivers effect found in explicit model calculation

  J. Brodsky et al. '02

- same model relates anisotropies in impact parameter distribution $q^X(x, \boldsymbol{b})$ and transv. momentum distribution $f_{q/p\uparrow}(x, \boldsymbol{k})$

  M. Burkardt, D.-S. Hwang '03

- sign of anomalous magnetic moments κ_u, κ_d
  ⇝ signs of Sivers functions for u and d quarks consistent with HERMES measurement

Section summary

- A^+ gluon exchange between spectators and hard scattering
  $\rightsquigarrow$ Wilson line in parton distributions
- has observable consequences: Sivers effect
- promising development: connection
  impact parameter dep't $\leftrightarrow$ transverse momentum dep't parton distributions