GENERALIZED PARTON DISTRIBUTIONS

M. DIEHL (DESY)

SOME LITERATURE

BRIEF AND BASIC :

X. JI	hep-ph /9604317	SPIN SUM RULE
M. BURKARDT	0005108	SPATIAL INTERPRETATION
M.D. et al.	9706344	DVCS
J. COLLINS	9907513	FACTORIZATION

REVIEWS :

M. BURKARDT	0207047
K. GOEKE et al.	0106012
M. D.	0307382
A. BELITSKY, A. RADYUSHKIN	0504030

HOW ARE QUARKS AND GLUONS DISTRIBUTED IN A HADRON?

IMPORTANT INFORMATION FROM PARTON DESITIES

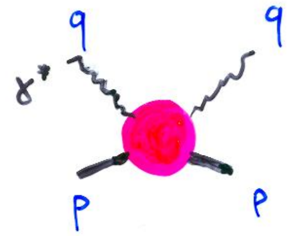
- LONGITUDINAL MOMENTUM $\sim 50\%$ FROM GLUONS
- PROTON SPIN $\lesssim 30\%$ FROM q AND $\bar{q}$ HELICITY

FOR MORE DETAILED INFORMATION NEED MORE DETAILED QUANTITIES

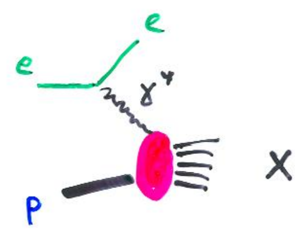
FROM INCLUSIVE TO EXCLUSIVE SCATTERING

1. COMPTON SCATTERING AND GENERALIZED PARTON DISTRIBUTIONS

INCLUSIVE DIS :



MEASURE IN $ep \to eX$



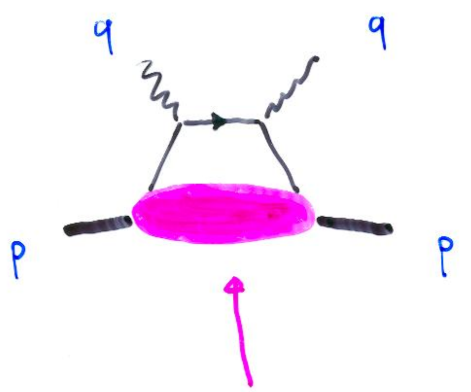
$$\sigma_{tot}(\gamma^* p) \propto \mathrm{Im}\, \mathcal{A}(\gamma^* p \to \gamma^* p)$$

BJ LIMIT

$$Q^2 = -q^2 \to \infty$$

$$W^2 = (p+q)^2 \to \infty$$

$$x_B = \frac{Q^2}{2pq} \quad \text{FIXED}$$



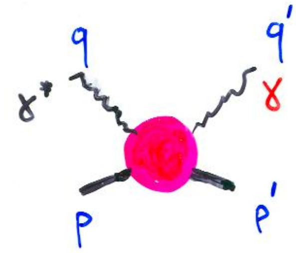
$+ \; O(\alpha_s)$

PARTON DISTRIBUTION

1. COMPTON SCATTERING AND GENERALIZED PARTON DISTRIBUTIONS

~~INCLUSIVE DIS~~ :

COMPTON AMPLITUDE



MEASURE IN $ep \to ep\gamma$

DEEPLY
VIRTUAL
COMPTON
SCATTERING

$$\cancel{\sigma_{tot}(\gamma^* p) \propto \mathrm{Im}\, A(\gamma^* p \to \gamma^* p)}$$

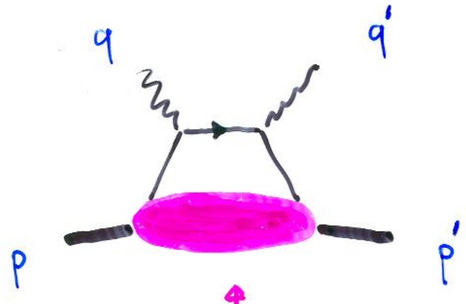
BJ LIMIT

$$Q^2 = -q^2 \to \infty$$

$$W^2 = (p+q)^2 \to \infty$$

$$x_B = \frac{Q^2}{2pq} \quad \text{FIXED}$$

$$t = (p-p')^2 \quad \text{FIXED}$$



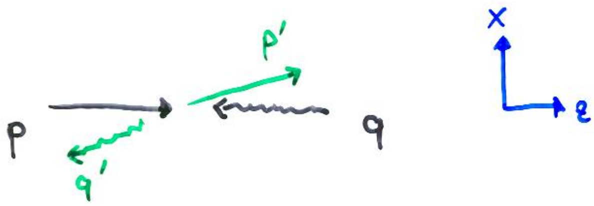
$+ \; O(\alpha_s)$

GENERALIZED PARTON DISTRIBUTION

GPD

IN MORE DETAIL :

$\gamma^*(q) + p(p) \rightarrow \gamma(q') + p(p')$ IN c.m.



LARGE W^2 $\rightarrow$ p AND p' MOVE FAST TO THE RIGHT

LIGHT - CONE COORDINATES (*)

$p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3)$; p'^+ LARGE $\sim W \sim Q$

$\vec{p}_T'$ SMALL $\sim \sqrt{-t}$

p^- ; $p'^- = \dfrac{m^2 + \vec{p}_T'^{\,2}}{2p'^+}$ VERY SMALL

LARGE Q^2 $\rightarrow$

q^+ ; q^- LARGE $\sim Q$

(*) $v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$ $\qquad v^2 = 2v^+v^- - \vec{v}_T^{\,2}$

EVALUATING THE HANDBAG DIAGRAMS

(VERY SCHEMATICALLY)

$$\mathcal{A} \sim \int d^4k \;\; \text{DIRAC MATRICES} \times \frac{1}{(k+q)^2 + i\epsilon}$$

$$\times \int d^4z \; e^{izk} \; \langle p' | \, \bar{q}_\alpha(0) \;\; q_\beta(z) \, | p \rangle$$

$$+ \text{ 2nd GRAPH}$$

- QUARK MOMENTUM k ~ ALONG p

  $\Rightarrow$ $k^+ \sim p^+$ BIG , $\vec{k}_T$ SMALL , $k^- \sim p^-$ YET SMALLER

- IN QUARK PROPAGATOR + DIRAC MATRICES

  NEGLECT k^- $(\ll |q^-| \sim Q)$ AND $\vec{k}_T$

  COLLINEAR APPROXIMATION

$$\Rightarrow \mathcal{A} \sim \int dk^+ \;\; \text{DIRAC MATRICES} \times \frac{1}{2(k^+ + q^+) q^- + i\epsilon}$$

$$\times \int dk^- \, d^2k_T \int dz^+ \, d^2z_T \, dz^- \; e^{iz^+k^- + iz^-k^+ - i\vec{z}_T\vec{k}_T}$$

$$\times \langle p' | \, \bar{q}_\alpha(0) \;\; q_\beta(z) \, | p \rangle \quad + \text{ 2nd GRAPH}$$

- INTEGRATION OVER k^-, k_T $\Rightarrow$

  z BECOMES LIGHT-LIKE

AFTER SIMPLIFICATION OF DIRAC MATRICES AND CHOOSING γ^* RIGHT-HANDED

$$\mathcal{A} \sim \int dk^+ \frac{1}{k^+ + q^+ - i\epsilon}$$

$$\times \int dz^- \, e^{i z^- k^+} \langle p' | \bar{q}(0) \left(\gamma^+ + \gamma^+ \gamma_5 \right) q(z) | p \rangle_{\substack{z^+ = 0 \\ \vec{z}_T = 0}}$$

+ 2nd GRAPH

$q^+ = - x_B \, p^+$ (EXERCISE)

$k^+ = x \, p^+$ (DEFINITION)

$$\mathcal{A} = \sum_q e_q^2 e^2 \int dx \frac{1}{x - x_B - i\epsilon}$$

$$\times \boxed{\int \frac{dz^-}{4\pi} e^{i x z^- p^+} \langle p' | \bar{q}(0) \left(\gamma^+ + \gamma^+ \gamma_5 \right) q(z) | p \rangle_{\substack{z^+ = 0 \\ z_T = 0}}}$$

+ 2nd GRAPH

$$= \sum_q e_q^2 e^2 \left[\text{P.V.} \int dx \frac{1}{x - x_B} \times \text{2nd LINE } (x) + i\pi \times \text{2nd LINE } (x_B) \right]$$

PARTON DISTRIB'S

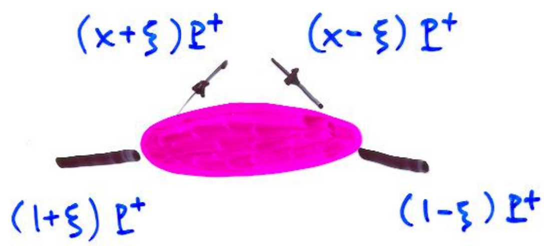
+ 2nd GRAPH

GPDs:

BASIC PROPERTIES AND PHYSICAL INTERPRETATION

DEFINITION OF NUCLEON GPDs

TAKE DIFFERENT DEF. OF x



$$P = \frac{1}{2}(p+p')$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$F^q = \int \frac{dz^-}{4\pi} e^{ix z^- P^+} \langle p', s' | \bar{q}(-\tfrac{z}{2}) \gamma^+ q(\tfrac{z}{2}) | p, s \rangle \Big|_{\substack{z^+=0 \\ z_T=0}}$$

$$= \frac{1}{2P^+} \bar{u}(p',s') \gamma^+ u(p,s) \; H^q(x,\xi,t)$$

$$+ \frac{1}{2P^+} \bar{u}(p',s') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m} u(p,s) \; E^q(x,\xi,t)$$

$$\tilde{F}^q = (\;\ldots \quad \gamma^+ \to \gamma^+\gamma_5 \;)$$

$$= (\;\ldots\; \gamma^+ \to \gamma^+\gamma_5\;) \; \tilde{H}^q(x,\xi,t) + \frac{1}{2P^+} \bar{u}(p',s') \frac{(p'-p)^+}{2m} \gamma_5 u(p,s)$$

$$\times \; \tilde{E}(x,\xi,t)$$

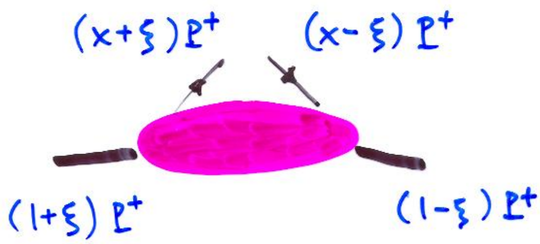
DVCS $\qquad \xi = \dfrac{x_B}{2-x_B}$

$$A(\gamma^*(+)p \to \gamma(+)p) = \sum_q (e e_q)^2 \Big[\int dx \; F^q \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

$$+ \int dx \; \tilde{F}^q \left(\frac{1}{\xi - x - i\epsilon} + \frac{1}{\xi + x - i\epsilon} \right) \Big]$$

DEFINITION OF NUCLEON GPDs

TAKE DIFFERENT DEF. OF x



$$P = \frac{1}{2}(p+p')$$

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$$= (\dots \gamma^+ \to \gamma^+\gamma_5) \, \tilde{H}^q(x,\xi,t) + \frac{1}{2P^+} \bar{u}(p',s') \frac{(p'-p)^+}{2m} \gamma_5 u(p,s)$$

$$\times \tilde{E}(x,\xi,t)$$

AT $p = p'$ $\quad (\xi = 0, \; t = 0)$ "FORWARD LIMIT"

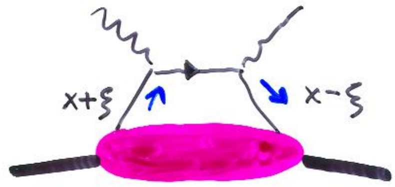
$$H^q(x,0,0) = \begin{cases} q(x) & x>0 \\ -\bar{q}(-x) & x<0 \end{cases}$$

$$\tilde{H}^q(x,0,0) = \begin{cases} \Delta q(x) & x>0 \\ \Delta\bar{q}(-x) & x<0 \end{cases}$$

E^q AND $\tilde{E}^q$ DECOUPLE

2. PARTON INTERPRETATION

ONE COVARIANT DIAGRAM : $\int_{-\infty}^{\infty} dx$



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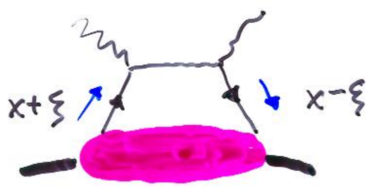
THREE REGIONS

PARTON PLUS-MOMENTUM ≥ 0

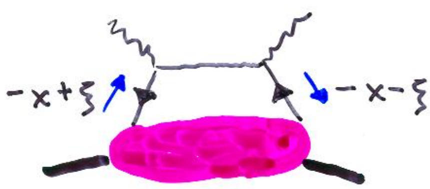
$\xi < x < 1$



q out

q back in

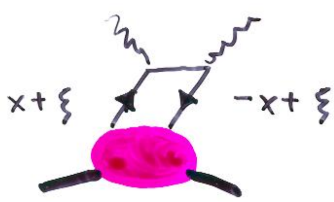
$-1 < x < -\xi$



$\bar{q}$ out

$\bar{q}$ back in

$-\xi < x < \xi$



$q\bar{q}$ out

FOR $|x| > 1$

$\int dk^- d^2k_T$

$= 0$

EVOLUTION

- GPDs DEPEND ON FACTORIZATION SCALE μ

  SCHEMATICALLY, IN THEIR DEFINITION $\int dk^- \int^{\mu^2} d^2k_T$

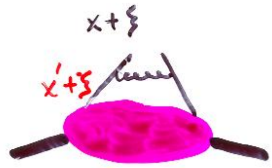
$$\mu^2 \frac{d}{d\mu^2} \, GPD(x,\xi,t,\mu^2) = \int \frac{dx'}{\xi} \, GPD(x',\xi,t\ \mu^2)$$

$$\times \, K\left(\frac{x'}{\xi}, \frac{x}{\xi}\right)$$

- 2 REGIONS :

DGLAP

$\xi < x < 1$



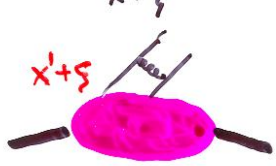
SAME FOR

$-1 < x < -\xi$

- SIMILAR TO USUAL DGLAP EVOLUTION

ERBL

$-\xi < x < \xi$



- LIKE MESON DISTRIBUTION AMPLITUDE

WAVE FUNCTION REPRESENTATION

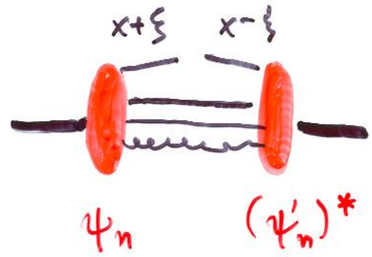
DECOMPOSE HADRON ON "PARTON STATES"

$$|p\rangle = \sum_{\text{QUANTUM \#S}} \int_{\text{MOMENTA}} \Big[\psi_3 \, |qqq\rangle + \psi_4 \, |qqq\,g\rangle + \psi_5 \, |qqqq\bar{q}\rangle + \ldots \Big]$$

LIGHT-CONE WAVE FUNCTIONS

$\Rightarrow$ GPD IN DGLAP REGION :

$= \sum_n \sum_{Q.\#s} \int_{MOM.}$



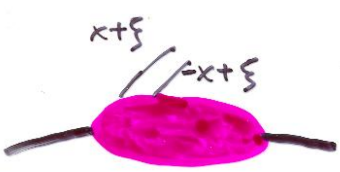
IN FORWARD LIMIT : $= \sum \sum \int \Big|$  $\Big|^2$ [ANALOGOUS FOR $-1 < x < -\xi$]

$\longrightarrow$ SQUARED AMPLITUDE

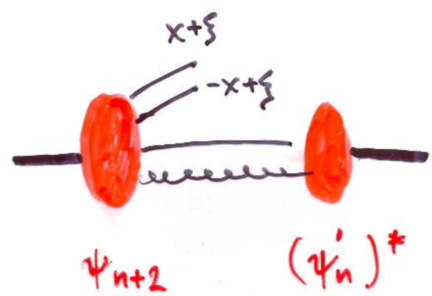
= PROBABILITY

ELSE : INTERFERENCE BETWEEN DIFFERENT CONFIGURATIONS

IN ERBL REGION :



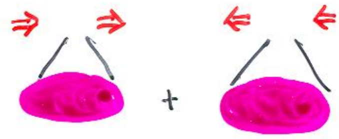
$= \sum_n \sum_{Q.\#s} \int_{MOM}$



3. HELICITY STRUCTURE

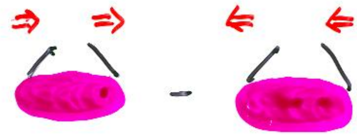
- PARTON SIDE

H^q, E^q OPERATOR $\bar{q}\gamma^+ q$



"UNPOLARIZED"

$\tilde{H}^q$, $\tilde{E}^q$ $\bar{q}\gamma^+\gamma_5 q$



"POLARIZED"

ALSO : QUARK HELICITY FLIP

GENERALIZES TRANSVERSITY DISTRIBUTIONS

VERY DIFFICULT TO MEASURE

ALL THIS ALSO FOR GLUONS

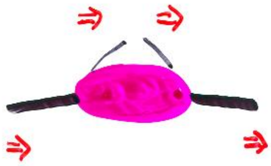
H^g, E^g UNPOL.

$\tilde{H}^g$, $\tilde{E}^g$ POL.

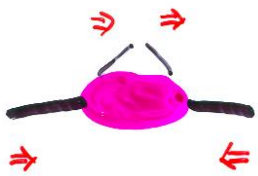
AND g HELICITY FLIP (QUITE DIFFICULT TO MEASURE)

- HADRON SIDE

EVALUATE PROTON SPINOR PRODUCTS IN DEF'S p. 5



$$\propto \quad H - \frac{\xi^2}{1-\xi^2} E \quad + \left(\tilde{H} - \frac{\xi^2}{1-\xi^2} \tilde{E} \right)$$

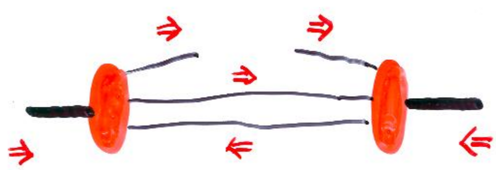


$$\propto \quad \left(D^x + i D^y \right) \frac{1}{2m} \left(E + \xi \tilde{E} \right)$$

$$D = \frac{p'}{1-\xi} - \frac{p}{1+\xi}$$

HELICITY NOT CONSERVED $\Rightarrow$ ORBITAL ANGULAR MOMENTUM

e.g.



$L_z = 0$ $\qquad$ $L_z = +1$

5. SUM RULES

INTEGRATE DEF. OF H, E OVER x :

$$\int dx \;\; \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{i x z^- P^+} \langle p' | \bar{q}(-\tfrac{z}{2}) \, \gamma^+ \, q(\tfrac{z}{2}) \, | p \rangle \Big|_{\substack{z^+=0 \\ \vec{z}=0}}$$

$$= \frac{1}{2P^+} \langle p' | \bar{q}(0) \, \gamma^+ \, q(0) \, | p \rangle$$

LOCAL OPERATOR

$$= \frac{1}{2P^+} \bar{u}(p') \gamma^+ u(p) \underbrace{\int_{-1}^{1} dx \, H^q(x, \xi, t)}_{F_1^q(t)} + \frac{1}{2P^+} \bar{u}(p') \frac{i \sigma^{+\alpha} (p'-p)_\alpha}{2m} u(p) \times \underbrace{\int_{-1}^{1} dx \, E^q(x, \xi, t)}_{F_2^q(t)}$$

DIRAC F.F. PAULI F.F.

- ξ DEPENDENCE GONE AFTER $\int_{-1}^{1} dx$

LORENTZ INVARIANCE !

- HIGHER MOMENTS IN x

$$\frac{1}{2P^+}\,\bar u \gamma^+ u \boxed{\int_{-1}^{1} dx\; x\, H^q} + \frac{1}{2P^+}\,\bar u \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u \boxed{\int_{-1}^{1} dx\; x\, E^q}$$

$$= \int dx\; x\; \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ix z^- P^+} \langle p' |\, \bar q(-\tfrac{z}{2})\, \gamma^+ q(-\tfrac{z}{2})\, | p\rangle_{\substack{z^+=0\\ \vec z=0}}$$

$$= \left(\frac{1}{2P^+}\right)^2 \langle p' | \underbrace{\frac{i}{2}\,\bar q(0)\left(\overleftarrow{\partial}^+ - \overrightarrow{\partial}^+\right)\gamma^+ q(0)}_{T_q^{++}} | p \rangle$$

QUARK CONTRIB'N TO ENERGY – MOMENTUM TENSOR

$$= \left(\frac{1}{2P^+}\right)^2 \Big[\; A^q(t)\; \bar u \gamma^+ u\; P^+ + B^q(t)\; \bar u \frac{\sigma^{+\alpha}\Delta_\alpha}{2m} u\; P^+$$

$$+\; C^q(t)\; \bar u u\; \frac{\Delta^+ \Delta^+}{m} \;\Big]$$

GORDON IDENTITY: $\quad P^\mu \dfrac{\bar u u}{m} = \bar u \gamma^\mu u - \bar u \dfrac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m} u$

$$\Rightarrow \quad \int_{-1}^{1} dx\; x\; H^q(x,\xi,t) = A^q(t) + 4\xi^2\, C^q(t)$$

$$\int_{-1}^{1} dx\; x\; E^q(x,\xi,t) = B^q(t) - 4\xi^2\, C^q(t)$$

POLYNOMIAL IN ξ

- JI'S SUM RULE: $\quad J^q = \frac{1}{2}\left[A^q(0) + B^q(0)\right]$

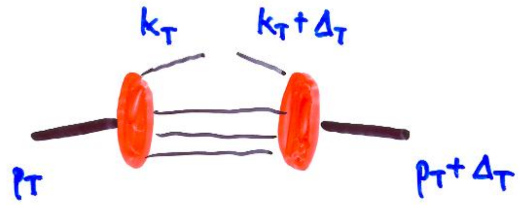
$$= \frac{1}{2} \lim_{t\to 0} \int_{-1}^{1} dx\; x\, \left(H^q + E^q\right)$$

TOTAL ANGULAR MOM. ALONG z- AXIS

4. SPATIAL INTERPRETATION

LOOK AGAIN AT WAVE FCT. REPRESENTATION:

$$H(x, \xi, t) = \sum_{\dots} \int d^2 k_T$$



= CONVOLUTION IN PARTON k_T

$\downarrow$

BECOMES SIMPLER AFTER FOURIER TRANSFORM

- PROTON STATE

$$|p^+, \vec{b}_T\rangle = \frac{1}{(2\pi)^2} \int d^2 p_T \; e^{-i \vec{p}_T \vec{b}_T} \; |p^+, \vec{p}_T\rangle$$

IS STATE OF DEFINITE TRANSVERSE POSITION

(= IMPACT PARAMETER) $\vec{b}_T$

- "MIXED REPRESENTATION" : KEEP DEFINTE p^+

$\rightarrow$ CAN STILL GO TO FRAME WHERE HADRON MOVES FAST

- LOCALIZATION IN 2D IS POSSIBLE

IN 3D IS RESTRICTED WITHIN COMPTON WAVE LENGTH $1/m$

PROTON IS EXTENDED OBJECT.

WHAT MEANS " LOCALIZED AT $\vec{b}_T$ " ?

$\rightarrow$ LOOK AT SYMMETRIES

TRANSVERSE BOOSTS : PARTICULAR LORENTZ TRF'S

$k^+ \rightarrow k^+$

$\vec{k}_T \rightarrow \vec{k}_T - k^+ \vec{v}_T$ $\qquad \vec{v}_T$ FIXED

$k^- \rightarrow \ldots$ (SO THAT $k^2 \rightarrow k^2$)

ANALOG IN NONRELATIVISTIC MECHANICS :

GALILEI TRANSFORMATIONS

$m \rightarrow m$

$\vec{k} \rightarrow \vec{k} - m\vec{v}$

$\xrightarrow[\text{NOETHER}]{}$ CENTER OF MASS $\vec{R} = \dfrac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$

$\Rightarrow$ IN RELATIVISTIC CASE :

" CENTER OF MOMENTUM " $\vec{b} = \dfrac{\sum_i \vec{b}_i k_i^+}{\sum_i k_i^+}$ (*)

$\vec{b}$  $\vec{b}_i , \; x_i = \dfrac{k_i^+}{\sum_j k_j^+}$

- SEE THIS EXPLICITLY WHEN FOURIER TRF. WAVE FUNCTIONS FROM TRANSVERSE MOMENTUM TO IMPACT PARAMETER

(*) OMIT "T" IN $\vec{b}_T$ FROM NOW

NOW TAKE $\xi = 0$ FOR SIMPLICTY

$$\Rightarrow \quad t = -(\vec{p} - \vec{p}\,')^2$$

$$\langle p^+ \vec{b}\,' | \underbrace{\int \frac{dz^-}{4\pi} e^{i x z^- p^+} \bar{q}(-\tfrac{z}{2}) \gamma^+ q(\tfrac{z}{2}) \Big|_{\substack{z^+=0 \\ \vec{z}=0}}}_{\mathcal{O}(\vec{z}=\vec{0})} | p^+ \vec{b} \rangle$$

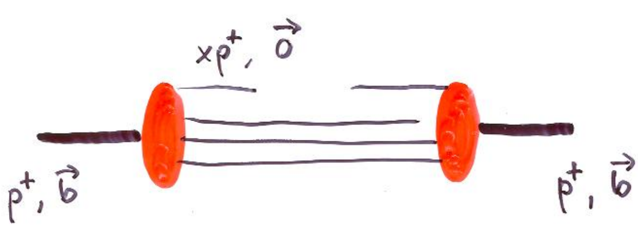
$$= \frac{1}{(2\pi)^4} \int d^2\vec{p}\,' \, d^2\vec{p} \; e^{i\vec{p}\,'\vec{b}\,'} e^{-i\vec{p}\vec{b}} \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$$= \frac{1}{(2\pi)^4} \frac{1}{4} \int d^2(\vec{p}+\vec{p}\,') \, d^2(\vec{p}\,'-\vec{p}) \; e^{i(\vec{p}\,'-\vec{p})\frac{\vec{b}\,'+\vec{b}}{2} - i(\vec{p}\,'+\vec{p})\frac{\vec{b}\,'-\vec{b}}{2}}$$

$$\times \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$$= \frac{1}{(2\pi)^2} \delta^{(2)}(\vec{b}\,'-\vec{b}) \int d^2(\vec{p}\,'-\vec{p}) \; e^{i\vec{b}(\vec{p}\,'-\vec{p})} \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$\propto$ FOURIER TRF. OF $\quad$ GPD $(x, \xi=0, t=-(\vec{p}\,'-\vec{p})^2)$



DIAGONAL IN PLUS-MOMENTUM AND IMPACT PARAMETER

$\rightarrow$ DENSITY OF PARTONS IN $(x, \vec{b})$ SPACE

$\int d^2\vec{b} \; \ldots \; = \;$ GPD$(x, \xi=0, t=0)$ $=$ USUAL PARTON DENSITY

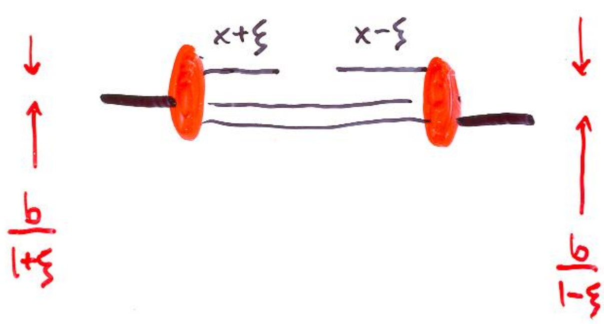
- AS $x \to 1$

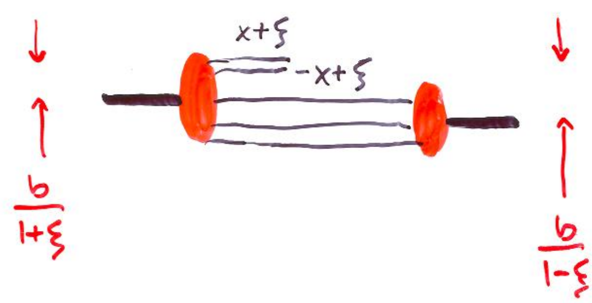
PARTON DOMINATES
CENTER OF MOMENTUM

$\Rightarrow$ DISTRIBUTION IN b
BECOMES PEAKED AT 0

$\Rightarrow$ t - DEPENCE OF
$GPD(x \to 1, \xi, t)$
BECOMES FLAT

- FOR $\xi \neq 0$





- OVERALL SHIFT OF
PROTON POSITION

- NO DENSITY
ANY LONGER

SUMMARY

- GPDs = MATRIX ELEMENTS LIKE USUAL PARTON DENSITIES, BUT

  $\langle p', s' | \ldots | p, s \rangle$

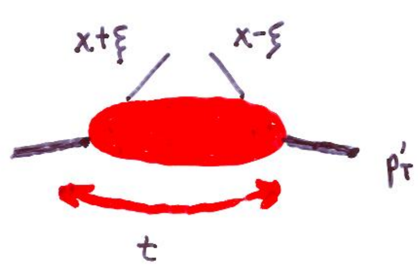
- DEPENDENCE ON PARTON AND HADRON HELICITIES

  ORBITAL ANGULAR MOMENTUM

- $\int dx \ldots$ GIVES FORM FACTORS

- $p'_T \to b_T$ IMPACT PARAMETER

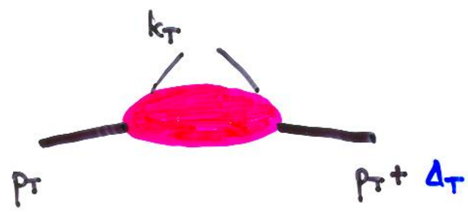
  "3 d PICTURE"



DISTINGUISH

IMPACT PARAMETER DISTRIBUTIONS

$\int d^2 k_T$



$H(x, \Delta_T) \xrightarrow{\text{FOURIER}} q(x, b_T)$

TRANSVERSE MOMENTUM DEPEND'T DISTRIB'S

$f(x, k_T)$

APPEAR e.g. IN