universitätbonn

RHEINISCHE FRIEDRICH-WILHELMS-UNIVERSITÄT

Nucleon spin structure & Δ-physics

(Project C.3)

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Collaborators: V. Bernard (Strasbourg), T.R. Hemmert (Munich), K. Kumar (Mangalore)

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Research directions

- Effective field theories with spin-$\frac{3}{2}$ and spin-1 fields

  Covariant small scale expansion, Compton scattering, Gerasimov-Drell-Hearn and Burkhardt-Cottingham sum rules, ...

- Chiral extrapolations and lattice QCD

  Nucleon and Δ properties, N^* mass, detailed analysis of the theoretical uncertainty,...

- Resonances on the lattice

  Chiral perturbation theory in a finite volume, parameters of the Δ-resonance,...

Examples:

- *Example 1:*

  Quark mass expansion of the axial coupling constant g_A

- *Example 2:*

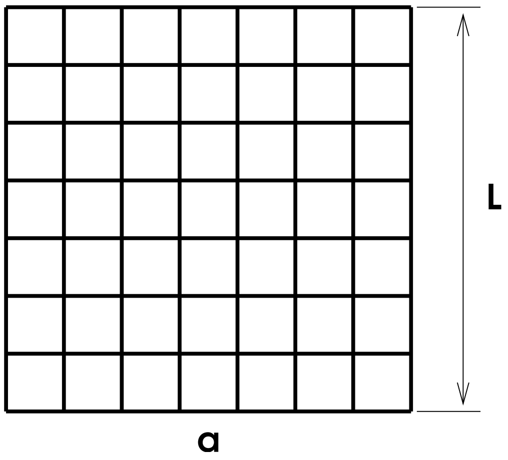
  Quark mass expansion of the Roper mass

- *Example 3:*

  Extraction of the Δ-resonance parameters from calculations in a finite volume

Lattice extrapolations

- Lattice calculations are always made at a finite lattice spacing, at a finite volume and (at present) not at physical values of the quark masses $\Rightarrow$ extrapolations necessary
- Effective field theory framework can be used in all three cases to perform the extrapolations:
  - *Finite lattice spacing* $a \to 0$
  - *Quark masses* $m_q \to m_q^{\text{phys}}$*:*
    Using ChPT to predict the quark mass dependence of the axial-vector coupling g_A *and the Roper mass*
  - *Finite volume* $V = L^4 \to \infty$
    Extracting the parameters of the resonance states from the spectrum of the Hamiltonian, calculated in a finite volume



ChPT in the one-nucleon sector: the Lagrangian

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \cdots$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i \not{D} - \overset{0}{m} + \frac{1}{2}\overset{0}{g}_A \not{u}\gamma_5\right)\Psi$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}\left(c_1\langle\chi_+\rangle + c_2\left(-\frac{1}{8\overset{0}{m}{}^2}\langle u_\mu u_\nu\rangle D^\mu D^\nu + \text{h.c.}\right) + c_3\frac{1}{2}\langle u\cdot u\rangle\right.$$
$$\left. + \; c_4\frac{i}{4}\sigma^{\mu\nu}[u_\mu, u_\nu] + c_5\tilde{\chi}_+ + c_6\frac{1}{8\overset{0}{m}}\sigma^{\mu\nu}F^+_{\mu\nu} + c_7\frac{1}{8\overset{0}{m}}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\right)\Psi$$

$$\mathcal{L}_{\pi N}^{(3)} = \sum_{i=1}^{23} d_i\bar{\Psi}O_i^{(3)}\Psi\,, \qquad \mathcal{L}_{\pi N}^{(4)} = \sum_{i=1}^{118} e_i\bar{\Psi}O_i^{(4)}\Psi\,,$$

- c_i, d_i, e_i: The LECs at $O(p^2)$, $O(p^3)$ and $O(p^4)$, respectively
- For details, see N. Fettes et al, Ann. Phys. 283 (2000) 273

Chiral extrapolations: a general strategy

ChPT $\Rightarrow$ dependence of the observables on the quark (pion) mass:

masses, magnetic moments, electromagnetic radii, axial-vector coupling,...

- Needed: global fit to many observables + phenomenological input (constraining the LECs)
- Needed: sufficiently small (dynamical) quark masses – already in the chiral regime
- Regularization independence should be provided (higher-order terms small)

Example 1: Nucleon axial-vector coupling

▲ LPC/MILC

R.G. Edwards *et al* [LHPC Coll.], PRL 96 (2006) 052001

■ QCDSF/UKQCD

A. Khan *et al*, hep-lat/0409161, hep-lat/0603028

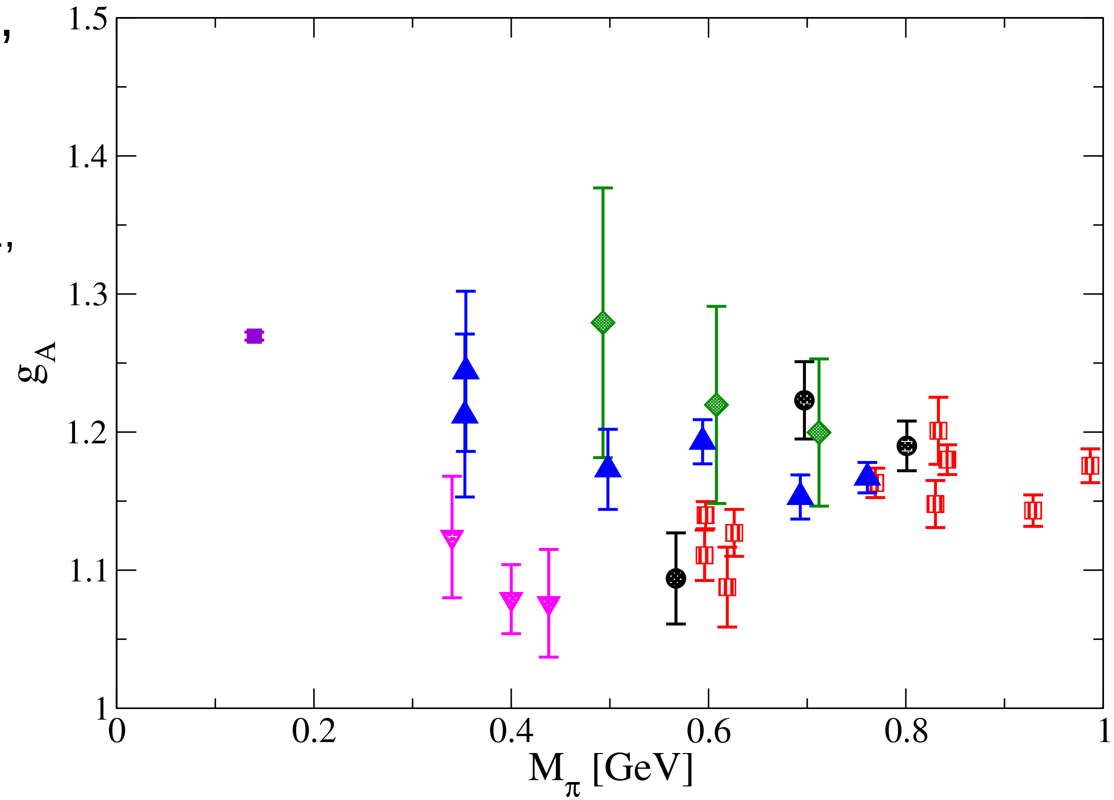
▼ QCDSF (preliminary)

◆ RBC

S. Sasaki *et al* [RBC Coll.], hep-lat/0110053

● LHPC/SESAM

D. Dolgov *et al* [LHPC Coll.], PRD 66 (2002) 034506



Dependence of g_A on the pion mass is rather flat.
Can one understand such a dependence in the framework of ChPT?
How far can one go in the pion mass?

Axial-vector coupling: one loop

J. Kambor and M. Možiš, JHEP 9904 (1999) 031

$$g_A = \overset{0}{g}_A \left\{ 1 + \frac{4M_\pi^2}{\overset{0}{g}_A} \left(d_{16}(\lambda) - \frac{\frac{1}{2}\,\overset{0}{g}_A + \overset{0}{g}_A^3}{16\pi^2 F^2} \ln \frac{M_\pi}{\lambda} \right) - \frac{\overset{0}{g}_A^2 M_\pi^2}{16\pi^2 F^2} \right.$$

$$\left. + \frac{M_\pi^3}{24\pi F^2 \overset{0}{m}} \left(3 + 3\,\overset{0}{g}_A^2 - 4\,\overset{0}{m}\,c_3 + 8\,\overset{0}{m}\,c_4 \right) \right\} + O(M_\pi^4)$$

- Natural correction at $O(M_\pi^2)$: $\Delta g_A \simeq 0.16$
- Unnaturally large correction at $O(M_\pi^3)$: $\Delta g_A \simeq 0.3$.
  The reason understood: large values of c_3 and c_4.
  The curve is not flat!
- Explicit Δ? [T.R. Hemmert, M. Procura and W. Weise, PRD 68 (2003) 075009]

Need to understand quark mass dependence at two loops!

Axial-vector coupling at two loops

V. Bernard and U.-G. Meißner, PLB 639 (2006) 278 [hep-lat/0605010]

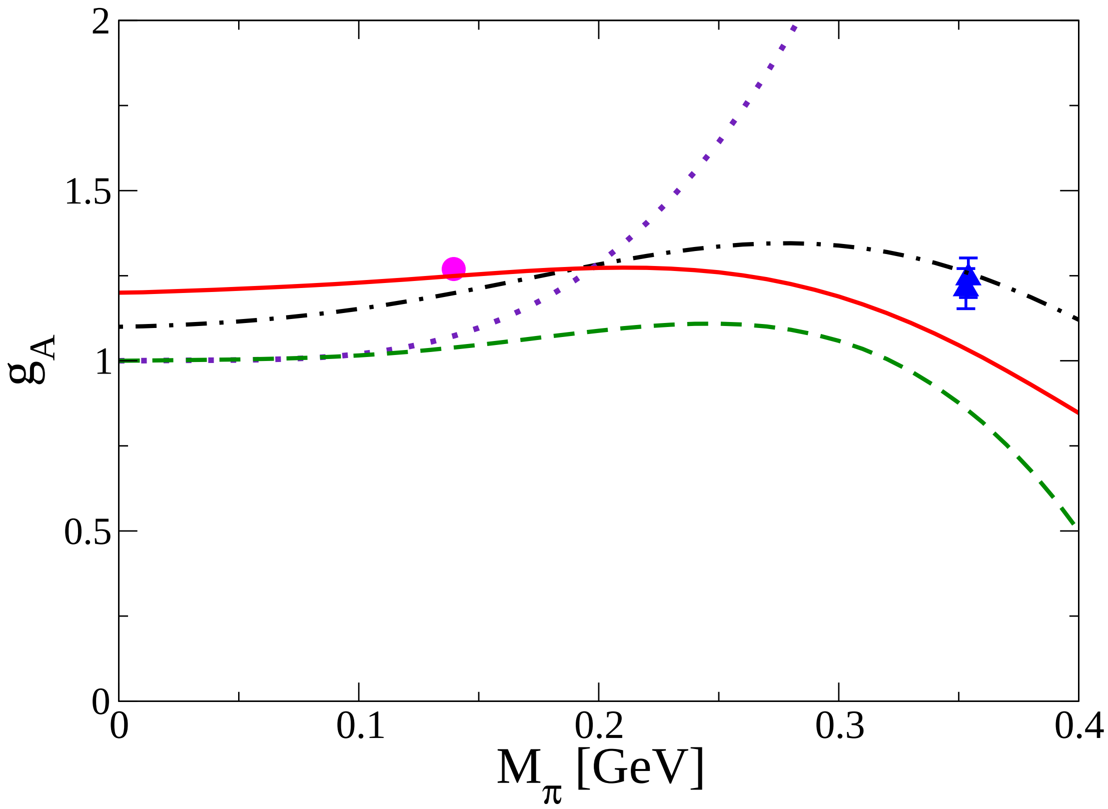
$$\begin{aligned} g_A &= \overset{0}{g}_A \left\{ 1 + \left(\frac{\alpha_2}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_2 \right) M_\pi^2 + \alpha_3 M_\pi^3 \right. \\ &+ \left. \left(\frac{\alpha_4}{(4\pi F)^4} \ln^2 \frac{M_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_4 \right) M_\pi^4 + \alpha_5 M_\pi^5 \right\} + \cdots \\ &= \overset{0}{g}_A \left\{ 1 + \Delta^{(2)} + \Delta^{(3)} \underbrace{+\Delta^{(4)} + \Delta^{(5)}}_{\text{two loops}} \right\} + O(M_\pi^6) \end{aligned}$$

- Calculate α_4 exactly, using RGE technique
- Calculate dominant contributions to $\beta_4, \gamma_4, \alpha_5$ + naturalness

$$\boxed{\Delta^{(2)} = -15.3\%,\ \Delta^{(3)} = 25.6\%,\ \Delta^{(4)} = -5.6\%,\ \Delta^{(5)} = -0.1\%}$$

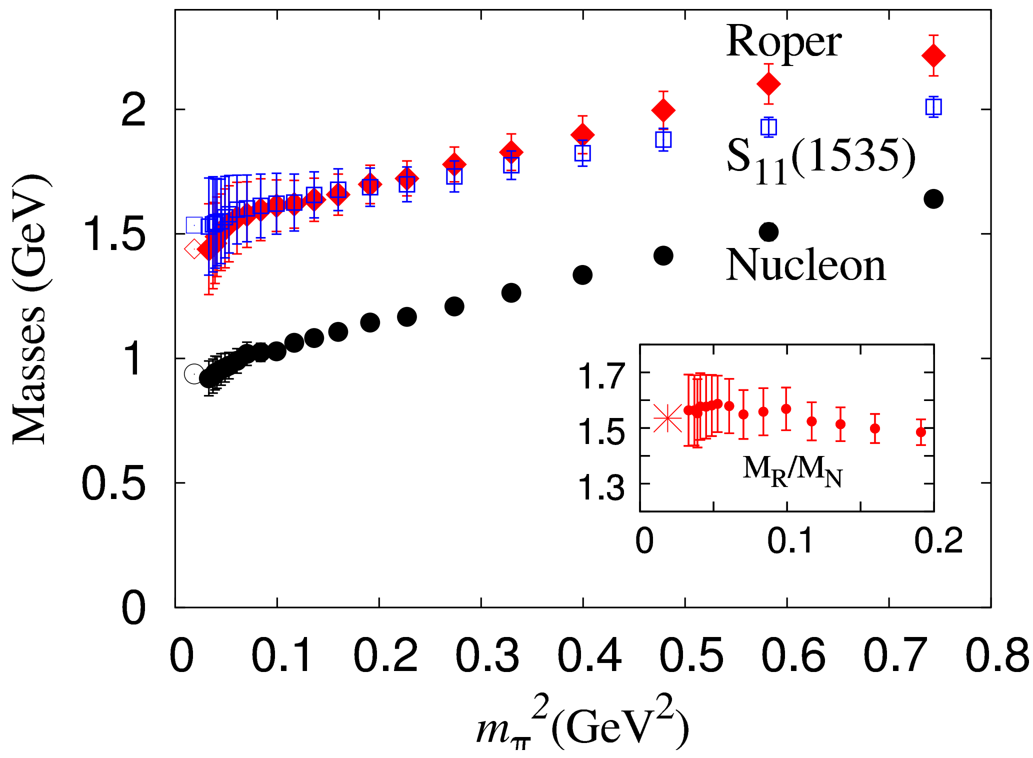
$\Rightarrow$ Convergence!

Axial-vector coupling: numerical results



- Strong pion mass dependence at one loop
- Theoretical uncertainty reasonable only for $M_\pi \le 300$ MeV

Example 2: The Roper resonance



- The Roper $N^*(1440)$ is an "irregular" nucleon excitation:

  Unexpected: lies below $S_{11}\ (1535)$

  Strongly decays into two pions + nucleon

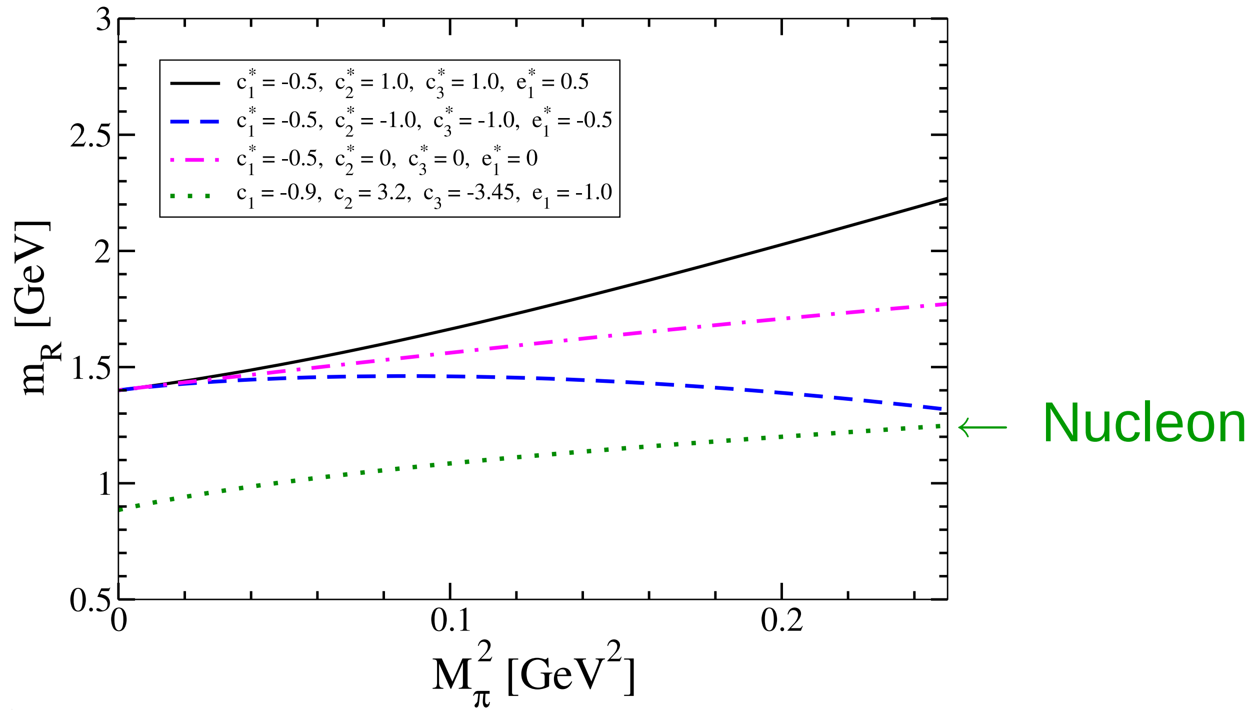
- Status of the Roper in lattice QCD under debate:

  Level crossing with the $S_{11}\ (1535)$ *at small quark masses?*

The Roper resonance in baryon ChPT

B. Borasoy, P. Bruns, U.-G. Meißner, R. Lewis, PLB 641 (2006) 294:

- Infrared regularization extended to include Roper resonance
- Provides chiral extrapolation at one loop



Similar to nucleon case; large variation at a small m_q?

Example 3: the Δ-resonance in a finite volume

stable particle: $c(t)=\int d^3\mathbf{x}\langle\phi(t,\mathbf{x})\phi(0)\rangle \quad \propto \exp(-mt)$

. . . does not apply to the case of unstable particles

- *Can be used when the resonance becomes stable that happens for* $M_\pi > M_\pi^{\text{phys}}$
- *The mass* $m_{\text{res}}(M_\pi)$ *at* $M_\pi = M_\pi^{\text{phys}}$ *is then obtained through a chiral extrapolation in the infinite volume*

- Is it possible (in principle) to perform the calculation of $m_{\text{res}}(M_\pi)$ all way down to $M_\pi \to M_\pi^{\text{phys}}$?
- How does one extract the width $\Gamma_{\text{res}}(M_\pi^{\text{phys}})$?

$\Rightarrow$ *Explore the case of the* Δ*-resonance!*

. . . maybe, feasible at some time in the future

Unstable particles

M. Lüscher, lectures given at Les Houches (1988); NPB 364 (1991) 237

U. Wiese, NPB (Proc. Suppl.) 9 (1989) 609

C. Michael, NPB 327 (1989) 515

T. DeGrand, PRD 43 (1991) 2296

. . .

Investigating scattering in the framework of the non-relativistic EFT:

S.R. Beane, P.F. Bedaque, A. Parreño and M.J. Savage, PLB 585 (2004) 106; NPA 747 (2005) 55

see also: C.h. Kim, C.T. Sachrajda and S.R. Sharpe, NPB 727 (2005) 218
N.H. Christ, C.h. Kim and T. Yamazaki, PRD 72 (2005) 114506

- *Lattice calculations are always done in a finite volume. The energy spectrum is real*
- *Studying the behavior of the energy levels (in Euclidian space) for a different box size L, one may determine the scattering phase in Minkowski space, infinite volume*

Energy levels in a finite box (NR EFT)

Find poles of the resolvent: $\langle \mathbf{q}' | \frac{1}{z - H_0 - H_I} | \mathbf{q} \rangle$

⇒ Scattering phase shift is determined from:

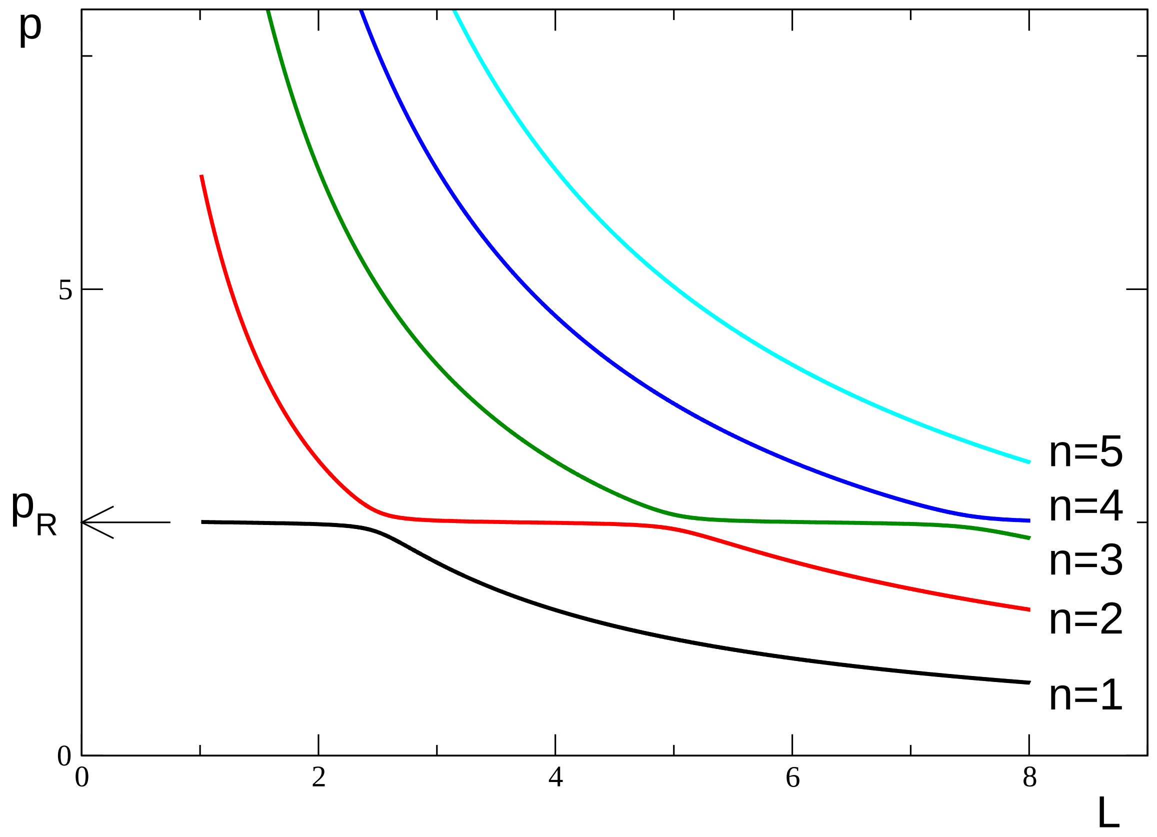
$$-p \cot \delta(p) - \frac{4\pi}{L^3} \sum_{\mathbf{k}} \frac{1}{\mathbf{k}^2 - p^2} = 0$$

$$z = \frac{p^2}{M}\,, \qquad \mathbf{k} = \frac{2\pi}{L}\mathbf{n}\,, \quad \mathbf{n} \in Z^3$$

- Reproduces Lüscher's perturbative formulae in case of weak coupling
- In the presence of resonances, an "avoided level crossing" occurs

Avoided level crossing

Example: Breit-Wigner resonance parameterization for $\delta(p)$



Finite-volume effects in ChPT

V. Bernard, U.-G. Meißner and A. Rusetsky, in preparation

Obtaining avoided level crossing order by order in ChPT:

- Invoke chiral perturbation theory in a finite volume, study the pole structure of the Δ-propagator in the Euclidian space
- If propagator has (real) poles at $p^2 = -E_n^2$,

$$c_\Delta(t) = \int d^3\mathbf{x} \langle \phi_\Delta(t, \mathbf{x}) \phi_\Delta(0) \rangle \to \sum_n Z_n \exp(-E_n t) \quad \text{(large } t\text{)}$$

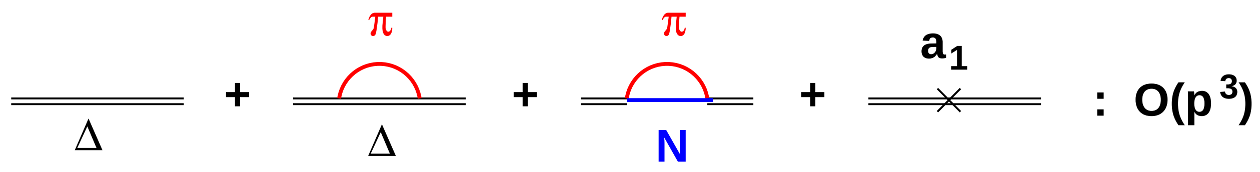
- Study the dependence $E_n(L)$ on the size of the box L. Fit the parameters of the chiral Lagrangian
- Calculate m_Δ and Γ_Δ using these parameters
  - *Advantageous: studying the dependence of the energy levels on L and M_π simultaneously*
  - *Results may depend on the convergence of chiral expansion*

Poles of the Δ-propagator in a finite volume

Lagrangian with an explicit Δ: Small Scale Expansion

(T.R. Hemmert, B.R. Holstein and J. Kambor, J. Phys. G 24 (1998) 1831)

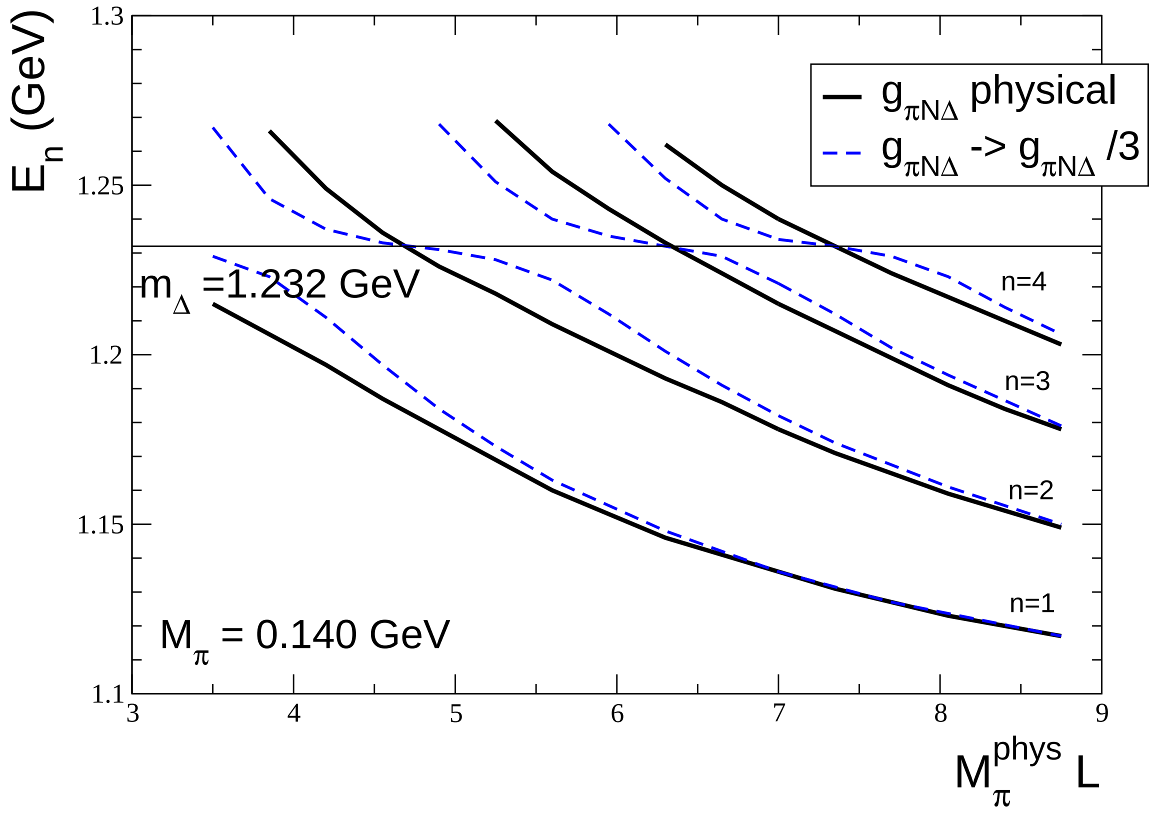
+ infrared regularization in a finite volume



$$S_\Delta^L(p) = -\frac{1}{\mathring{m}_\Delta - \not p - \Sigma_\Delta^L(p)}\, P^{3/2}\, \xi^{3/2} + \text{terms with the spin-}\tfrac{1}{2}$$

- Chiral symmetry and power counting are ensured
- Poles: $\mathring{m}_\Delta - \not p - \Sigma_\Delta^L(p) = 0 \quad \Rightarrow$ Energy levels
- No off-shell effects, if $M_\pi L \gg 1$

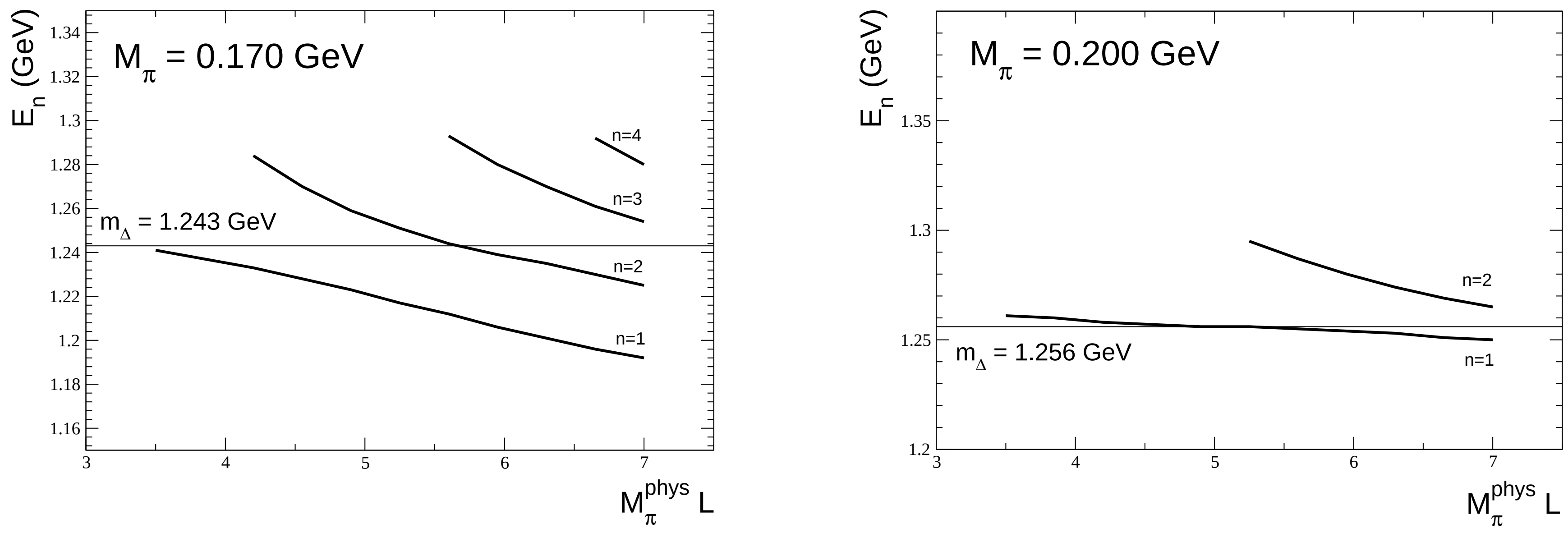
Energy levels up to and including $O(p^3)$



can one see the avoided level crossing?

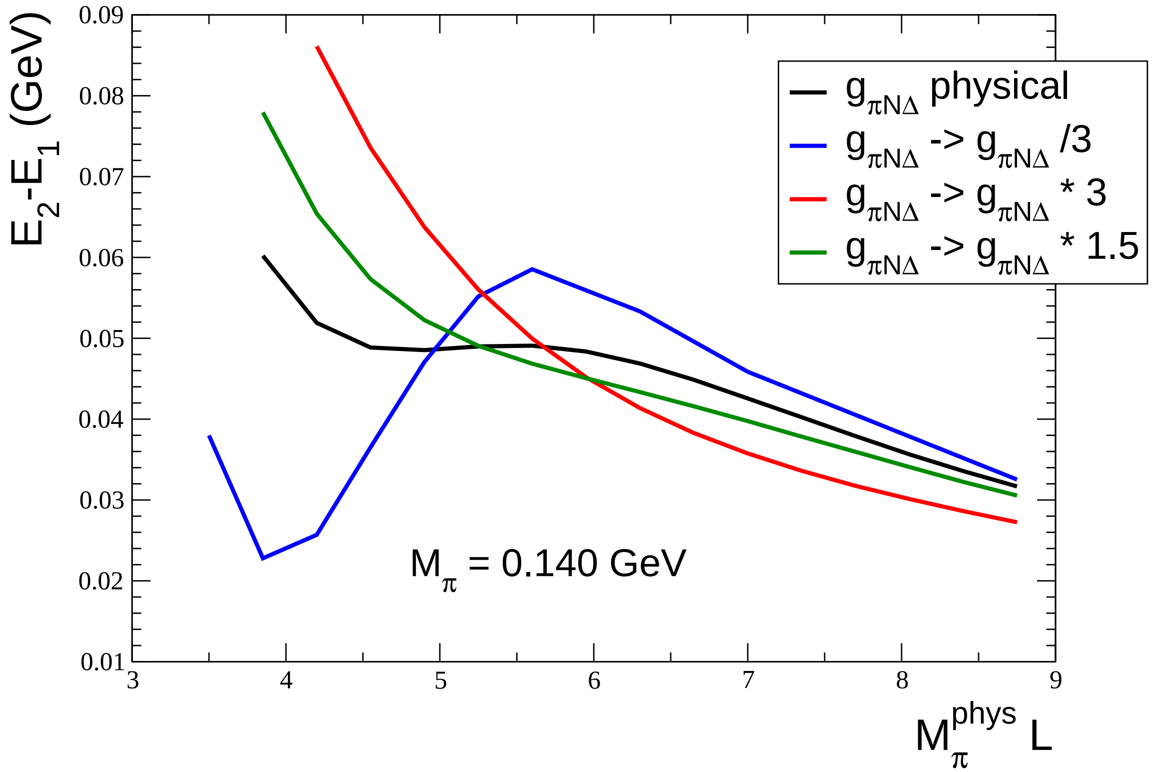
$\Rightarrow$ The width of the Δ too large, washed out!

Varying quark mass and determining m_Δ



- No irregular behavior of the energy levels near the point, where the Δ becomes stable
- The tangent of the lowest level trajectory: a monotonic function of the mass gap between the Δ and the $N+\pi$ for a fixed $g_{\pi N\Delta}$. The trajectory flattens at $m_\Delta = m_N + M_\pi$

Determination of $g_{\pi N\Delta}$ from $E_2(L) - E_1(L)$



Avoided level crossing is clearly visible for a small $g_{\pi N\Delta}$

Determining the parameters of Δ on the lattice

- ChPT in finite volume is applied to systematically calculate the energy levels of the πN system in the $I = J = \frac{3}{2}$ state, in a finite box, up to and including $O(p^3)$
- Calculations at $O(p^4)$ are underway
- It is demonstrated that fitting of the mass and width of Δ to the energy levels in the box is indeed feasible, despite the fact that the avoided level crossing is washed out due to the large width
  - Fit m_Δ to $E_1(L)$
  - Fit $g_{\pi N\Delta}$ to $E_2(L) - E_1(L)$
  - Fit $\pi\Delta$ σ-term from $m_\Delta(M_\pi^2)$ for different values of M_π^2
- One may use the known dependence of the resonance parameters on M_π, in order to improve the sensitivity of the fit

Future research

- Doubly virtual Compton scattering at low Q^2
  (project with V. Bernard and K. Kumar, to be finished)

- New research program: low-energy meson-baryon scattering from lattice QCD (all projects with V. Bernard)
  - *Δ-resonance in a finite volume at $O(p^4)$ in ChPT*
    (with Dipl. Stud. D. Hoja)
  - *Matching ChPT to the non-relativistic EFT in a finite volume*
    (with PhD Stud. M. Lage)
  - *Pion-nucleon scattering, estimation of LECs*
    (with M. Lage)
  - *Many-particle final states and the status of the Roper*
  - *Kaon-nucleon scattering, coupled channels and the nature of $\Lambda\,(1405)$*

    $\ldots$

New project: interdisciplinary links

- Coordinating efforts with lattice practitioners:

  1st workshop on “Lattice QCD, Chiral Perturbation Theory and Hadron Phenomenology,” 2-6 October (2006), Trento
  Organizers: U.-G. Meißner and G. Schierholz

- Study of the kaon-nucleon dynamics on the lattice:

  $\Rightarrow$ *SIDDHARTA, AMADEUS (DAΦNE)* $\Rightarrow$ B.3

  ECT* Workshop “Exotic Hadronic Atoms, Deeply Bound Kaonic Nuclear States and Antihydrogen,” 19-24 June (2006), Trento
  Organizers: C. Curceanu, A. Rusetsky and E. Widmann

- Decay into multiparticle final states:

  $\Rightarrow$ *Many-body physics in a finite volume (with H.-W. Hammer)*

New project: Baryon resonances on the lattice

Re-orientation of the project C.3
for phase II of TR-16

$\Downarrow$

Manpower requested:
1 postdoctoral + 1 PhD positions
related to the project

Publications (only refereed journals)

P. C. Bruns and Ulf-G. Meißner, "Infrared regularization for spin-1 fields," *Eur. Phys. J.* **C 40** (2005) 97-119 [arXiv:hep-ph/0411223].

M. Frink, Ulf-G. Meißner and I. Scheller, "Baryon masses, chiral extrapolations, and all that," *Eur. Phys. J.* **A 24** (2005) 395-409 [arXiv:hep-lat/0501024].

V. Bernard, T. R. Hemmert and Ulf-G. Meißner, "Chiral extrapolations and the covariant small scale expansion," *Phys. Lett.* **B 622** (2005) 141-150 [arXiv:hep-lat/0503022].

V. Bernard and Ulf-G. Meißner, "The nucleon axial-vector coupling beyond one loop," *Phys. Lett.* **B 639** (2006) 278-282 [arXiv:hep-lat/0605010].

B. Borasoy, P. C. Bruns, Ulf-G. Meißner and R. Lewis, "Chiral corrections to the Roper mass," *Phys. Lett.* **B 641** (2006) 294-300 [arXiv:hep-lat/0608001].