Novel Dispersion Analysis of the Nucleon Form Factors

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Status Report on the Project C.2 †

- Project leaders: HWH, Ulf-G. Meißner

- Students: M.A. Belushkin

- Collaborators: D. Drechsel (Mainz), J. Haidenbauer (Jülich), A. Sibirtsev (Bonn)

†Started as DFG project HA3203/2-1 (HWH & U.-G. Meißner)

Plan

- Introduction
- Dispersion Theoretical Framework
- Fit Results
- Conclusions & Outlook

Why Nucleon Form Factors?

- Nucleons are basic constituents of matter
- Detailed understanding of nucleon structure

$\longrightarrow$ nucleon radii, vector meson coupling constants,...

$\longrightarrow$ perturbative and nonperturbative aspects of QCD

$\longrightarrow$ input for wide variety of experiments, e.g.

  * Lamb shift measurements in atomic hydrogen
  * Sea quark dynamics in strange form factors (Sample, Happex, A4, G0)

- Theoretical tools
  - Effective Field Theory $\rightarrow$ ChPT (low momentum transfer)
  - Dispersion theory (all momentum transfers, space- & time-like)
  - Lattice QCD (future)

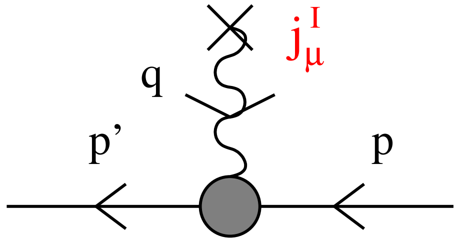
Why Dispersion Theory?

- Based on fundamental principles: unitarity, analyticity
  $\longrightarrow$ (Largely) model independent
- Connects data over full range of momentum transfers:
  $\longrightarrow$ time-like and space-like data
- Connects to data from different processes (πN scattering, ...)
- Simultaneous analysis of all four form factors
- Spectral functions encode perturbative and non-perturbative physics:
  vector meson couplings, multi-meson continua, pion cloud,...
- Constraints from ChPT, pQCD
- Extraction of nucleon radii

Electromagnetic Form Factors

- Definition:

$t \equiv q^2 = (p' - p)^2 = -Q^2$



$$\langle N(p')|j_\mu^I|N(p)\rangle = \bar{u}(p')\left[F_1^I(t)\gamma_\mu + i\frac{F_2^I(t)}{2m}\sigma_{\mu\nu}q^\nu\right]u(p)\,, \quad I = S, V$$

- Normalization: $F_1^S(0) = F_1^V(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

- Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2$, $G_M = F_1 + F_2$

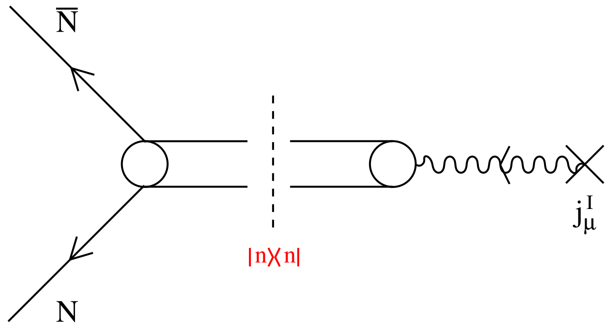
- Radii: $F(t) = F(0)\left[1 + t\langle r^2\rangle/6 + ...\right]$

Starting Point: Spectral Decomposition

- Crossing: $\langle N(p')|j_\mu^I|N(p)\rangle \longleftrightarrow \langle \bar{N}(\bar{p})N(p)|j_\mu^I|0\rangle$
- Spectral decomposition: (using microcausality, analyticity,...)
  (Federbush, Goldberger, Treiman, Chew,...)

$$\mathrm{Im}\langle \bar{N}(\bar{p})N(p)|j_\mu^I|0\rangle \sim \sum_n \langle \bar{N}(\bar{p})N(p)|n\rangle\langle n|j_\mu^I|0\rangle \quad \Longrightarrow \quad \mathrm{Im}\, F$$

- On-shell intermediate states
  $\rightarrow$ imaginary part
  $\rightarrow$ relate physical matrix elements



- Intermediate States: j_μ^S (isoscalar): $3\pi, 5\pi, ..., K\bar{K}, K\bar{K}\pi, ...$
  j_μ^V (isovector): $2\pi, 4\pi, ...$

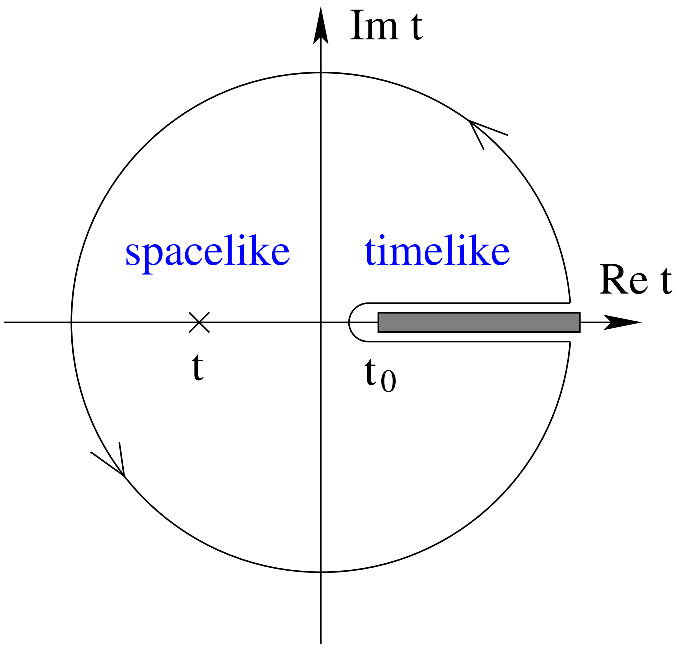
Dispersion Relations

- Form factors have multiple cuts in interval $[t_n, \infty[\quad (n = 0, 1, 2, ...)$

- Dispersion relation for F:

  –Apply Cauchy's formula in complex t-plane

  –Subtractions?



$$F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\mathrm{Im} F_i(t')}{t' - t} dt' \,, \qquad i = 1, 2$$

- Higher mass intermediate states are suppressed

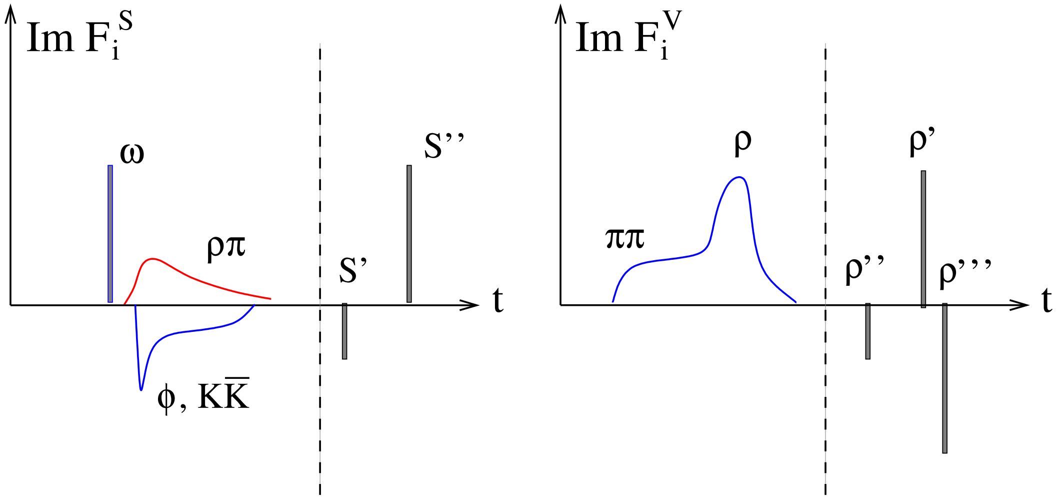
- Spectral functions are central objects $\longrightarrow$ from where to take?

EM Spectral Functions

- Include 2π and $K\bar{K}$, $\rho\pi$ $(\rightarrow 3\pi)$ continua as independent input
- Higher mass states: dominated/parametrized by vector meson poles (widths can be included)

$$\mathrm{Im} F_i(t) = \sum_v \pi a_i^v \delta(t - M_v^2), \;\; a_i^v = \frac{M_v^2}{f_v} g_{vNN} \;\Rightarrow\; F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- EM Spectral Function:



Spectral Function: 2π-, $K\bar{K}$-, $\rho\pi$-Continua

- Continuum contributions:
  - 2π: analytic continuation of πN scattering data, $F_\pi(t)$ (Höhler, Pietarinen, '75, Belushkin, HWH, Meißner, '05)
  - $K\bar{K}$: KN scattering data, $F_K(t)$ (HWH, Ramsey-Musolf, '99)
  - $\rho\pi$ ($\rightarrow 3\pi$): Jülich NN model (Meißner et al., '97)

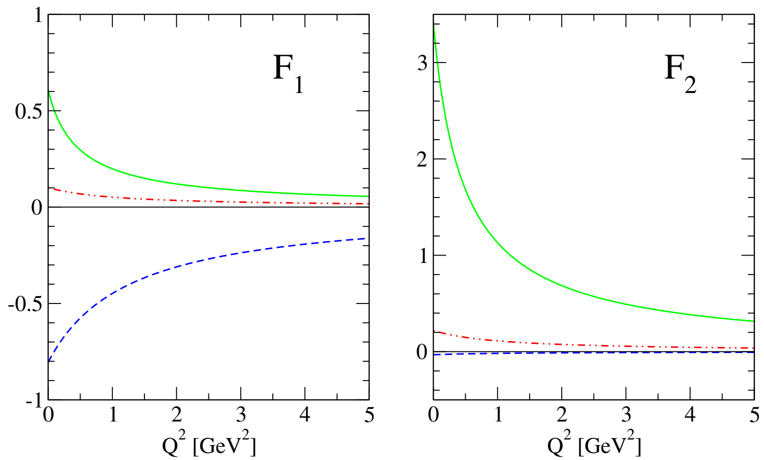
- Evaluate with DR

$\longrightarrow$ Form factor contributions

..-..-..- $K\bar{K}$

- - - - $\rho\pi$

—— 2π



Asymptotic Behavior

- pQCD prediction (modulo logs): $F_1 \sim 1/t^2$, $F_2 \sim 1/t^3$
- Various ways to implement asymptotic behavior from pQCD:
  - Superconvergence relations only
    $\rightarrow$ add broad resonance to generate imaginary part for $t \geq 4m^2$

$$F_i^{(I,broad)}(t) = \frac{a_i^I (M_I^2 - t)}{(M_I^2 - t)^2 + \Gamma_I^2}\,, \quad i = 1,2\,, \quad I = S, V\,,$$

  - Explicit pQCD term in addition to SC relations

$$F_i^{(I,pQCD)} = \frac{a_i^I}{1 - c_i^2 t + b_i^2 (-t)^{i+1}}\,, \quad i = 1,2\,, \quad I = S, V\,,$$

  - ....

Error Bands

- General Problem: What are the errors in dispersion analyses?
- Generate $1-\sigma$ error bands for best fits
- Difficult since problem is highly non-linear

$\longrightarrow$ Perform Monte Carlo sampling of fits with χ^2 in interval

$$[\chi^2_{min}, \chi^2_{min} + \delta\chi^2]\,, \qquad \delta\chi^2 \simeq 1.04$$

$\longrightarrow$ keep form factor values with maximal deviation from best fit

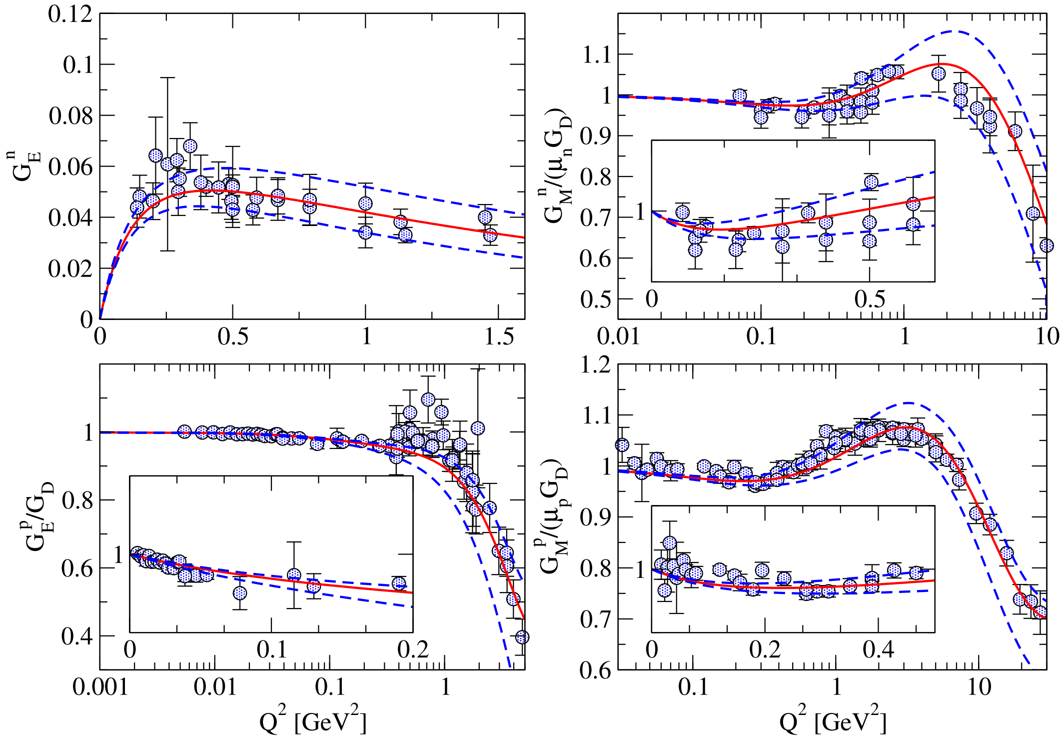
- Results, see: Belushkin, HWH, Meißner, arXiv:hep-ph/0608337

Superconvergence Approach: Space-Like

- 17 Parameters
  - ω, ϕ
  - 2 effective IS poles
  - 5 effective IV poles
  - $\chi^2/\text{dof} = 1.8$
- good description of data

—— best fit

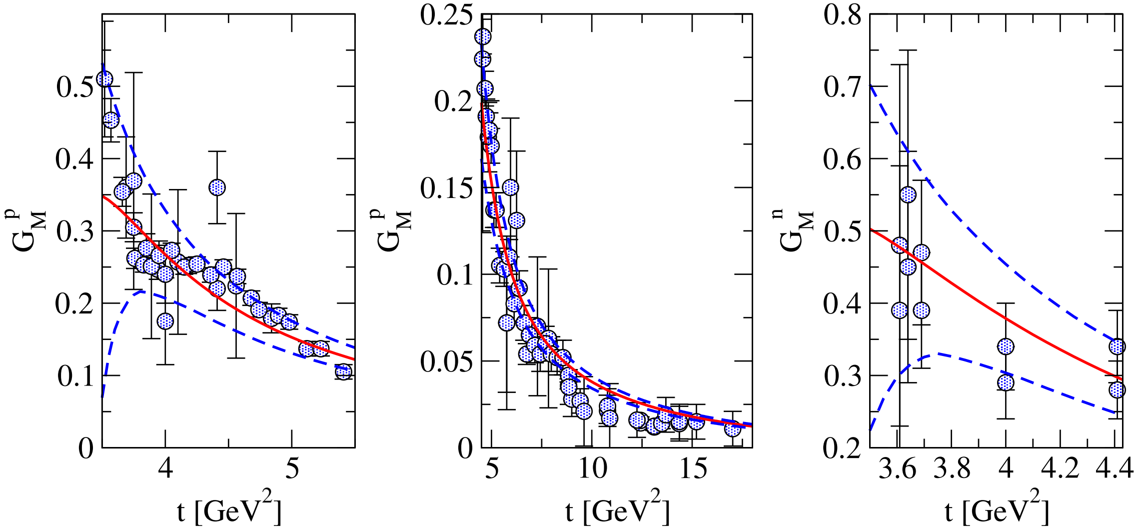
- - - - 1σ band



Superconvergence Approach: Time-Like

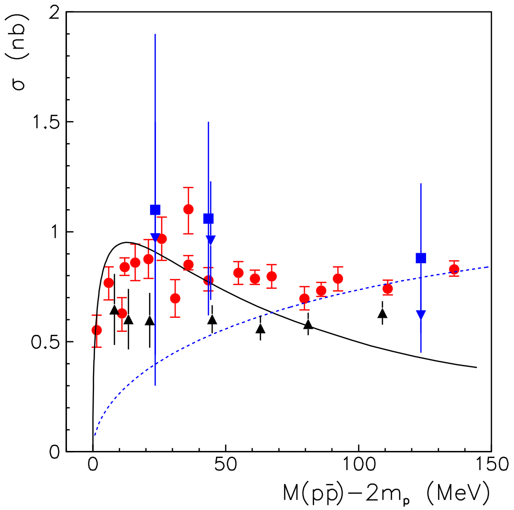
- Experimental extraction ambiguous (E/M separation)
- Subthreshold resonance? (Antonelli et al., PRD **71** ('05) 054010)

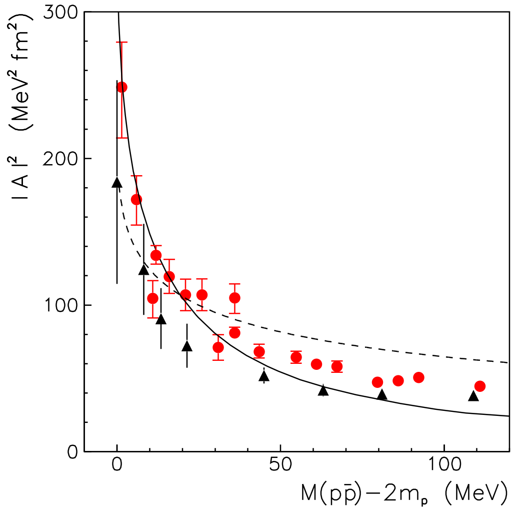
⇒ $N\bar{N}$ final-state interaction
(Haidenbauer, HWH, Meißner, Sibirtsev, PLB **643** ('06) 29)



$N\bar{N}$ Final-State Interaction

- Consider cross section/transition amplitude for $e^+e^- \rightarrow N\bar{N}$ ($\rightarrow$ source of timelike form factor data)



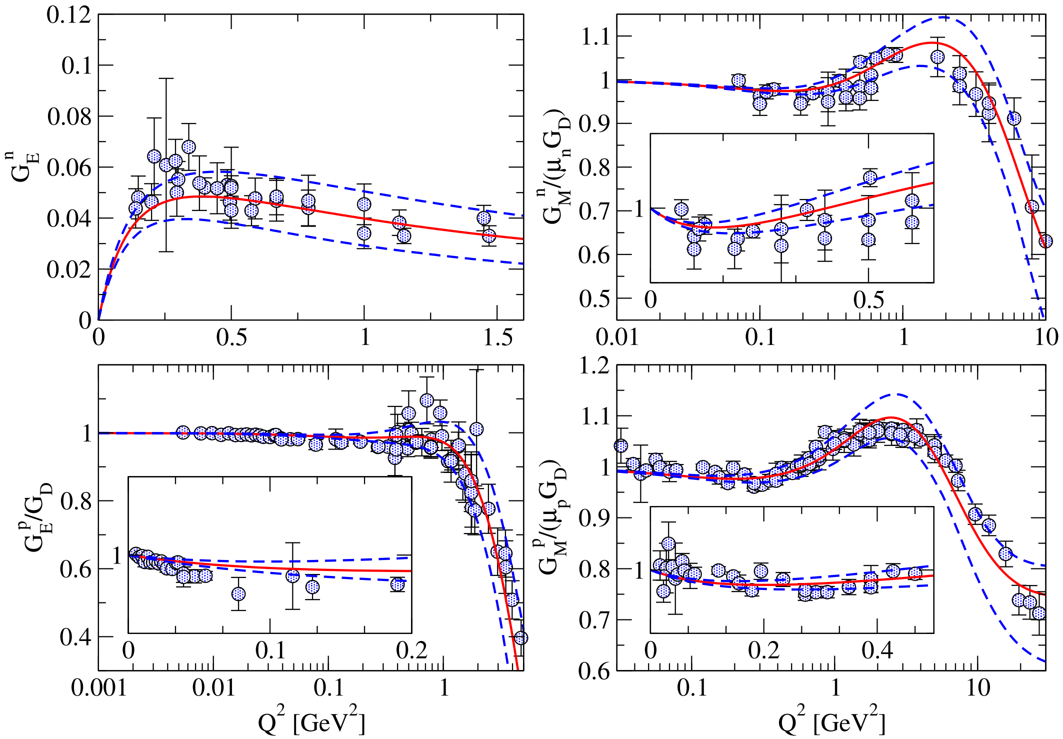


- Steep rise described by Jülich $N\bar{N}$ model/Watson-Migdal treatment (Haidenbauer, HWH, Meißner, Sibirtsev, PLB **643** ('06) 29)

- Consistent with $J/\Psi \rightarrow \gamma p\bar{p}$, $B \rightarrow p\bar{p}K$ (Haidenbauer, Meißner, Sibirtsev, '06)

Explicit pQCD Approach: Space-Like

- 14 Parameters
  - ω, ϕ
  - 1 effective IS poles
  - 3 effective IV poles
  - explicit pQCD term
  - $\chi^2/\text{dof} = 2.0$
- good description of data

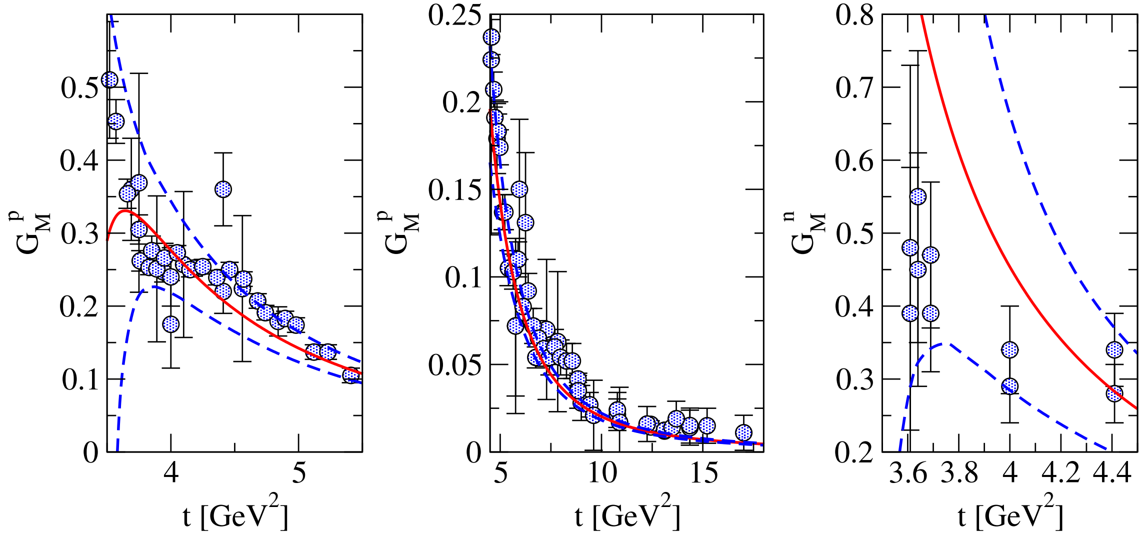


best fit

1σ band

Explicit pQCD Approach: Time-Like

- Timelike neutron data is not included in the fit (also in SC)
- No predictive power at threshold



General Comments/Radii

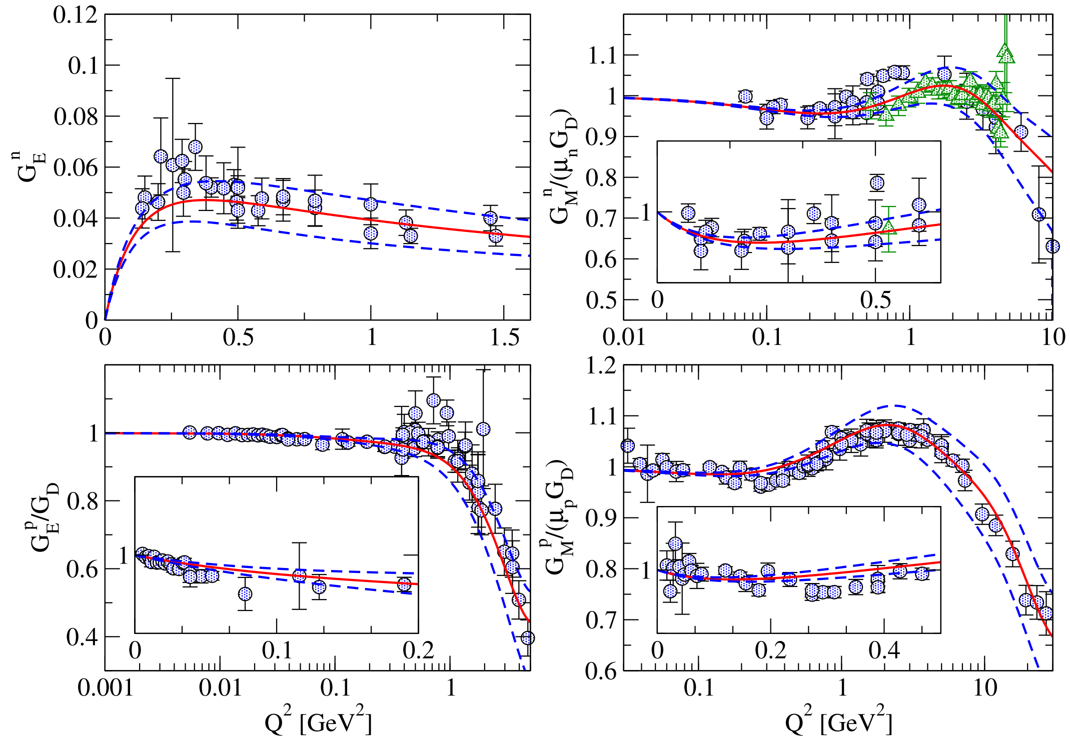
- Successive improvement by reduction of number of poles (stability constraint)
- Theoretical/systematic uncertainties? (2γ physics, consistency of data,..)
- Extraction of radii

	SC	ex. pQCD	recent determ.
r_E^p [fm]	0.84...0.85	0.82...0.84	0.886(15) [1,2,3]
r_M^p [fm]	0.85...0.86	0.84...0.85	0.855(35) [2,4]
$(r_E^n)^2$ [fm^2]	$-$0.11...$-$0.13	$-$0.11...$-$0.13	$-$0.115(4) [5]
r_M^n [fm]	0.85...0.87	0.86...0.87	0.873(11) [6]

[1] Rosenfelder, Phys. Lett. B **479** ('00) 381
[2] Sick, Phys. Lett. B **576** ('03) 62
[3] Melnikov, van Ritbergen, Phys. Rev. Lett. **84** ('00) 1673
[4] Sick, private communication
[5] Kopecky et al., Phys. Rev. C **56** ('97) 2229
[6] Kubon et al., Phys. Lett. B **524** ('02) 26

Preliminary CLAS data: Space-Like

- 14 Parameters
  - ω
  - 2 effective IS poles
  - 3 effective IV poles
  - explicit pQCD term
  - $\chi^2/\text{dof} = 2.2$
- good description of data



——— best fit

- - - - 1σ band

Summary

- Dispersion analysis of nucleon EM form factors
- Improved spectral functions: 2π, $K\bar{K}$, $\rho\pi$ continua
- Consistent description of EM FF data
- Radii: agreement with other analyses except r_E^p
  $\rightarrow$ not likely explained by 2γ physics (Blunden, Sick, '05)

Outlook

- Done since last year:
  - reduce # of poles
  - better description of time-like form factors
  - theoretical/systematic uncertainties
  - pQCD corrections at large t beyond superconvergence

- Still to be done:
  - analyse (Coulomb/2γ corrected) cross sections directly
  - consequences for strange vector form factors
  - understanding of dynamics governing timelike form factors
  - phases of timelike form factors
  - ....
  - Continue project with 1 PhD position

Publications (Refereed Journals)

- H.-W. Hammer, D. Drechsel and U.-G. Meißner,
  "On the pion cloud of the nucleon,"
  Phys. Lett. B **586**, 291 (2004) [arXiv:hep-ph/0310240].

- H.-W. Hammer and U.-G. Meißner,
  "Updated dispersion-theoretical analysis of the nucleon electromagnetic form factors,"
  Eur. Phys. J. A **20**, 469 (2004) [arXiv:hep-ph/0312081].

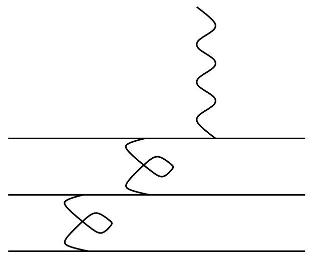
- M. A. Belushkin, H.-W. Hammer and U.-G. Meißner,
  "Novel evaluation of the two-pion contribution to the nucleon isovector form factors,"
  Phys. Lett. B **633**, 507 (2006) [arXiv:hep-ph/0510382].

- J. Haidenbauer, H.-W. Hammer, U.-G. Meißner and A. Sibirtsev, "On the strong energy dependence of the $e^+e^- \leftrightarrow p\bar{p}$ amplitude near threshold," Phys. Lett. B **643**, 29 (2006) [arXiv:hep-ph/0606064].

- M. A. Belushkin, H.-W. Hammer and U.-G. Meißner, "Dispersion analysis of the nucleon form-factors including meson continua, ' submitted to Phys. Rev. C [arXiv:hep-ph/0608337].

Constraints on Spectral Functions

- Normalization (electric charges, magnetic moments)
- Radius constraints are possible
- Superconvergence relations (pQCD)



$F_1 \sim 1/t^2$
$F_2 \sim 1/t^3$ (helicity flip)

$$\longrightarrow \quad \int_{t_0}^{\infty} \mathrm{Im}\, F_1(t) dt = 0, \qquad \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t) dt = \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t) t dt = 0$$

- Leading Logs from pQCD or other α_S corrections can be included (cf. Gari, Krümpelmann, '85)
- Inverse problem $\longrightarrow$ stabilize: minimal # of poles to describe data (Sabba-Stefanescu, '80)

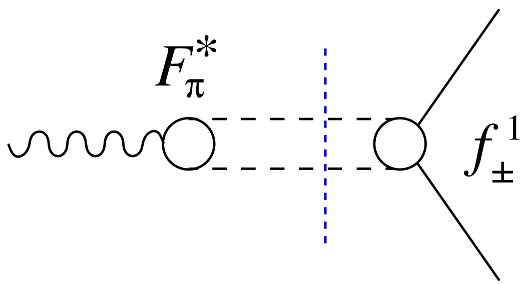
Isovector Spectral Function: 2π-Continuum

- 2π-contribution to spectral functions: (Frazer, Fulco, '59 $\longrightarrow$ predicted ρ)

$$\mathrm{Im} G_E^V(t) = \frac{q_t^3}{m\sqrt{t}} F_\pi(t)^* f_+^1(t)$$

$$\mathrm{Im} G_M^V(t) = \frac{q_t^3}{\sqrt{2t}} F_\pi(t)^* f_-^1(t)$$

where $q_t = \sqrt{t/4 - M_\pi^2}$



- $\pi\pi \to \bar{N}N$ P-waves: $f_\pm^1(t)$

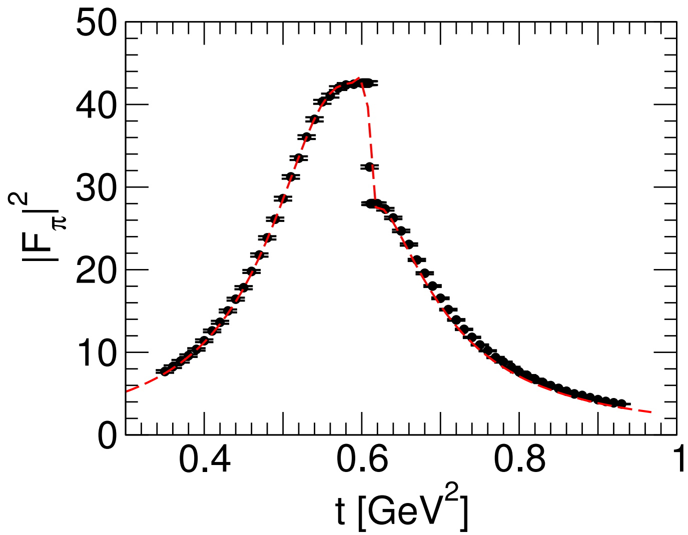
$\longrightarrow$ from analytic continuation of πN data (Höhler, Pietarinen, '75)

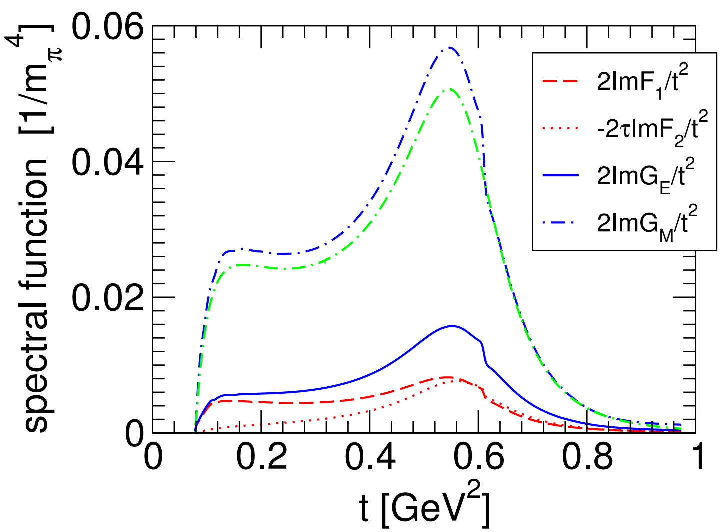
- Singularity close to threshold: $t_c \approx 3.98 M_\pi^2$ $\rightarrow$ isovector radii

- Pion EM form factor $F_\pi(t)$: from $e^+e^- \to \pi^+\pi^-$

Isovector Spectral Function: 2π-Continuum

- New data for pion form factor (CMD, KLOE, SMD)
- New determination of 2π-continuum (Belushkin, HWH, Meißner, '06)





- Pronounced ρ peak with strong $\rho - \omega$ mixing
- Contains information on long-range pion cloud (cf. Drechsel et al., '04)

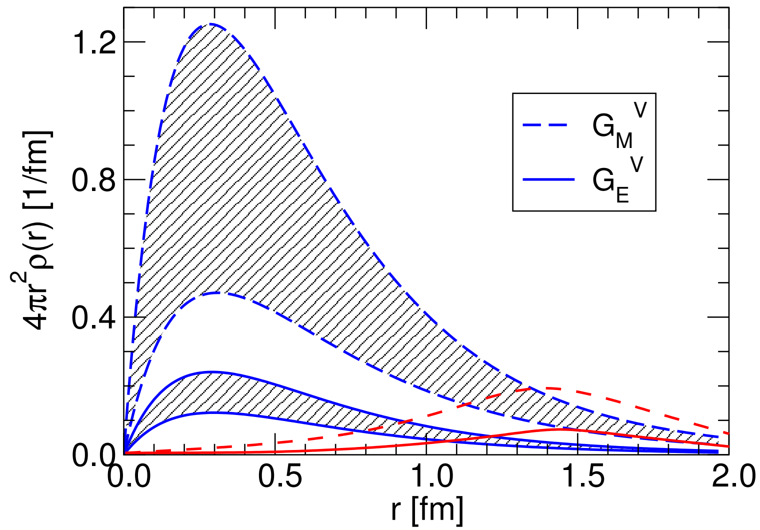
On the Pion Cloud of the Nucleon

- FW find very long-ranged pion cloud contribution: $r \simeq 2$ fm

  Friedrich, Walcher, EPJA **17** ('03) 607

- Long-range pion contribution given by 2π-continuum

HWH, Drechsel, Meißner, PLB **586** ('04) 291



$$\rho_i^V(r) = \frac{1}{4\pi^2} \int_{4M_\pi^2}^{40M_\pi^2} dt \, \mathrm{Im}\, G_i^V(t) \, \frac{e^{-r\sqrt{t}}}{r}, \qquad i = E, M$$

- Maxima around $r_{\mathrm{max}} \approx 0.3$ fm $\longleftrightarrow$ FW: $r_{\mathrm{max}} \approx 1.5$ fm
- Smaller pion cloud contribution beyond $r \sim 1$ fm compared to FW
- Independent of contribution from $t > 40M_\pi^2$

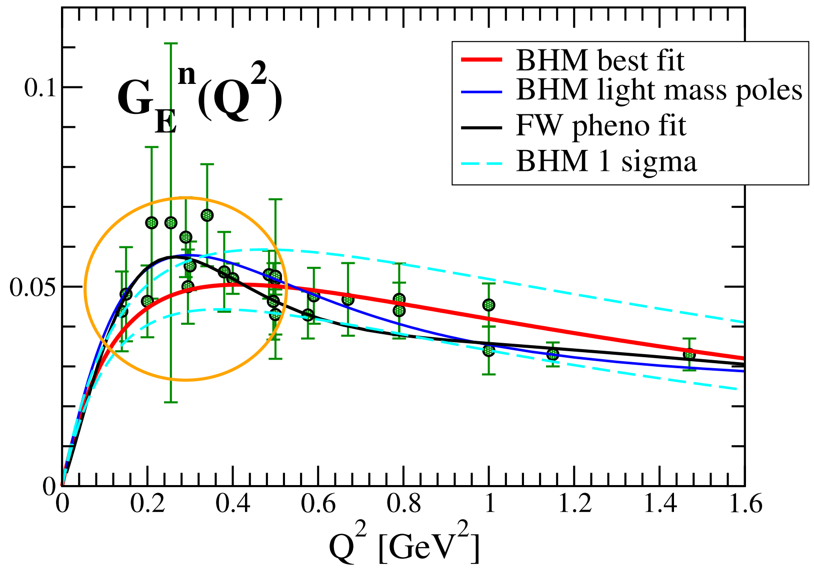
Bump-Dip Structure in G_E^n

- Can structure be generated in dispersive approach?

  $\longrightarrow$ low mass strength required!

  $\longrightarrow$ e.g. low-mass poles:

  $M_S^2 = 0.7$ GeV^2
  $M_V^2 = 0.1$ GeV^2



- No known vector mesons in this region

- Vector meson dominance applicable for $t \leq 1$ GeV^2

- Higher mass continua? ($|3\pi\rangle$: $t_{th} \approx 0.17$ GeV^2, $|4\pi\rangle$: $t_{th} \approx 0.31$ GeV^2)

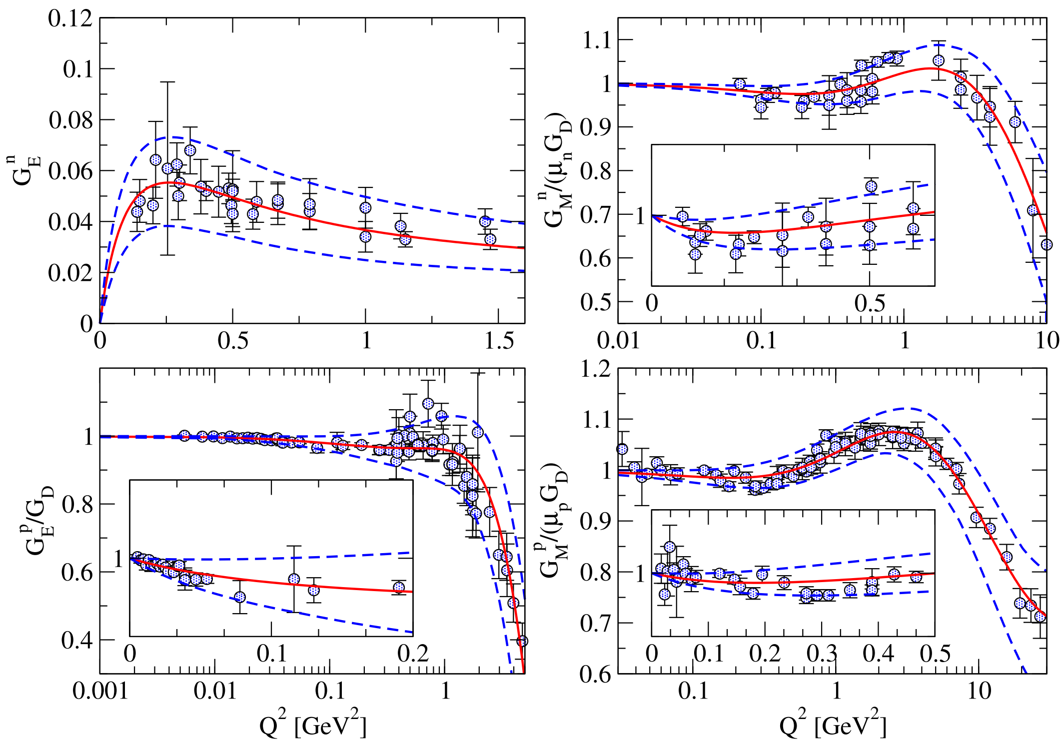
  $\Leftrightarrow$ $|3\pi\rangle$ small in ChPT (Bernard, Kaiser, Meißner, Nucl. Phys. A **611** ('96) 429)

Bump-Dip Structure: Space-Like Only

- 22 Parameters
  - ω
  - 3 effective IS poles
  - 4 effective IV poles
  - explicit pQCD term
  - $\chi^2/\text{dof} = 0.9$
    (only spacelike)
- good description of data

——— best fit

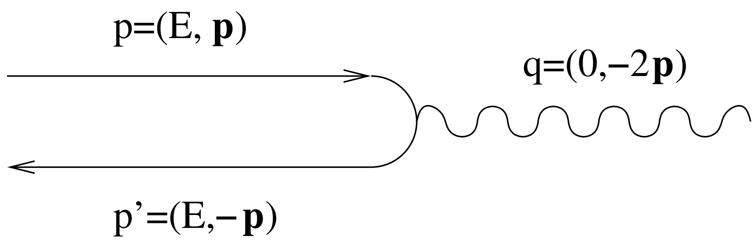
- - - - 1σ band



Interpretation of Radii

- Nucleon FF's in space-like region:

  can always find reference frame where no energy is transferred (Breit frame)



$$F(q^2) = F(-\mathbf{q}^2) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(r)$$

$$\longrightarrow \langle r^2 \rangle = \frac{4\pi}{F(0)} \int dr\, r^4\, \rho(r)\,, \qquad F(0) = 4\pi \int dr\, r^2\, \rho(r)$$

- Interpretation:
  - $G_E\,(G_M)$: FT of charge (magnetization) distribution $(\mathbf{q}^2 \ll m^2)$
  - F_1, F_2: only formal definition